Supplementary Materials: InfVC: An Inference-Enhanced Local Search Algorithm for the Minimum Vertex Cover Problem in Large-Scale Graphs

1 Applying Inference-Driven Optimization for Set Cover Problem

First, we introduce some preliminaries for the set cover problem as follows.

Definition 1 (Set Cover Problem). *Let $U = \{ele_1, ele_2, \ldots, ele_n\}$ be a universe of n elements, and let $H = \{S_1, S_2, \ldots, S_m\}$ be a collection of subsets of U, where $S_i \subseteq U$ for $i = 1, 2, \ldots, m$. The set cover problem is to find a subcollection $C \subseteq H$ such that $\bigcup_{S_i \in C} S_i = U$, and the cardinality $|C|$ is minimized.*

For each element ele_i in U, $cover(ele_i)$ represents the number of subsets comprising ele_i in H. An element ele_i is termed as uncovered if $cover(ele_i) = 0$. If $cover(ele_i) > 0$, ele_i is covered. $H(ele_i)$ denotes the set of all subsets in H that comprises ele_i. For the SCP, we denote the candidate solution as D_S, and the scoring function of a subset c is defined as follows.

$$\text{score}_{scp}(c) = \begin{cases} |H_1|, & \text{if } c \notin D_S \\ -|H_2|, & \text{if } c \in D_S \end{cases}$$

where $|H_1|$ denotes the number of elements whose $cover$ value would be changed from 0 to 1 if subset c is added to D_S, and $|H_2|$ represents the the number of elements whose $cover$ value would from 0 to 1 if subset c is removed from D_S. Note that for the subset whose $score_{scp}$ value is -1, it must comprise exactly one element ele_i such that $cover(ele_i) = 1$.

We extend the inference-driven optimization strategy to address the set cover problem (SCP). Various combinatorial optimization problems, including the MVC problem and the minimum dominating set (MDS) problem, can be transformed to the set cover problem under polynomial time complexity [Karp, 1972]. This indicates that applying our proposed inference-driven optimization to the SCP enables its application to specific versions of SCP, such as the MDS.

First, we define the potential pair for the SCP as follows:

Definition 2 (Potential Pair). *Given a candidate solution D_S and a subset $c \in D_S$. If $score_{scp}(c) = -1$, then a potential pair is defined as a tuple: (c, ele), where $e \in c$ and cover(ele) = 1. The set of all potential pairs in D_S is denoted by $P\_Set(D_S)$.*

Then, we define the critical addition subset as below.

Definition 3 (Critical Addition Subset). *Given a candidate solution D_S and its potential pair set $P\_Set(D_S)$, a subset $c' \notin D_S$ is termed a critical addition subset if there exist potential pairs (c_1, e_{c_1}) and (c_2, e_{c_2}) in $P\_Set(D_S)$ such that:*

- *$c_1 \neq c_2$,*
- *$\{e_{c_1}, e_{c_2}\} \subseteq c'$.*
- *For $\forall ele_i \in U$, if $H(ele_i) = \{c_1, c_2\}$, then $ele_i \in c'$.*

In this context, c_1 and c_2 are said to be related to c'.

Proposition 1. *Given a candidate solution D_S, a critical addition subset sub_c and the two subsets c_1 and c_2 related to sub_c, $D_S \cup \{sub_c\} \setminus \{c_1, c_2\}$ remain a feasible solution.*

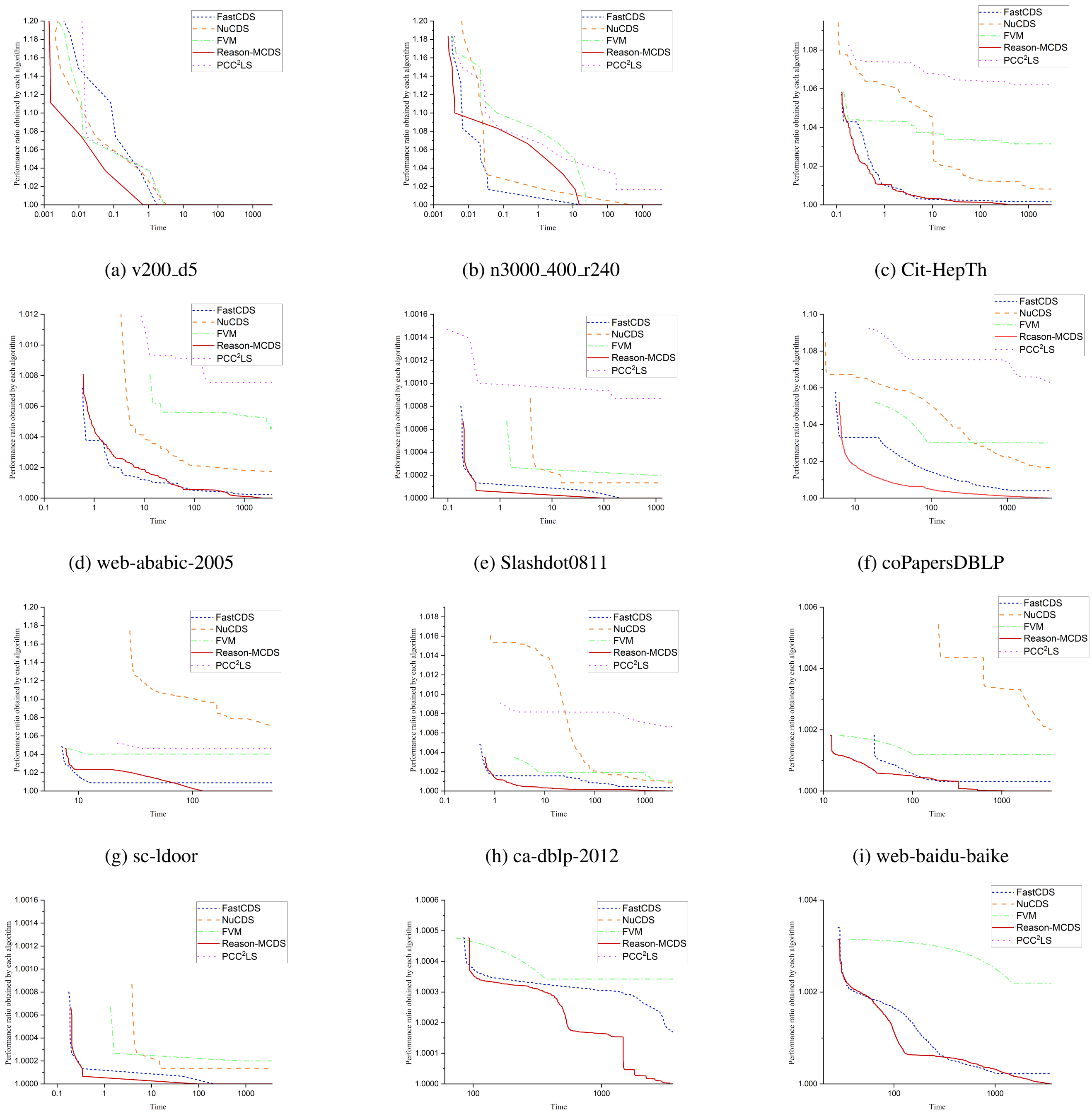
Proof. According to the definition of potential pair, removing c_1 (or c_2) alone will cause exactly one element, $\{e_{c1} \in c_1 \mid \text{cover}(e_{c1}) = 1\}$ (or $\{e_{c2} \in c_2 \mid \text{cover}(e_{c2}) = 1\}$), to become uncovered. Thus, the removal of c_1 and c_2 from D_S results in the elements set $\{ele_i \in c_1 \mid \text{cover}(ele_i) = 1\} \cup \{ele_i \in c_2 \mid \text{cover}(ele_i) = 1\} \cup \{ele_i \mid H(ele_i) = \{c_1, c_2\}\}$ becoming uncovered. However, according to the definition of critical addition subset, all these elements are included in sub_c. Consequently, $D_S \cup \{sub_c\} \setminus \{c_1, c_2\}$ remains a feasible solution. $\square$

2 Nine Examples Illustrating the Structural Characteristics of High-Quality Solutions

After obtaining high-quality solutions for 9 representative instances, we rank the vertices of the graph in descending order by the degree value, divide them into 50 equal segments, and present the proportion of vertices from each segment that belong to the achieved high-quality solution. According to Figure 1, vertices with high degrees are more likely to be included in the high-quality solution compared to vertices with low degrees.

3 Detailed Results on Exact Algorithms

We present the detailed comparison results with exact algorithms in Table 1. The bold values denote the best solution obtained among all algorithms.



(a) v200_d5 (b) n3000_400_r240 (c) Cit-HepTh

(d) web-ababic-2005 (e) Slashdot0811 (f) coPapersDBLP

(g) sc-ldoor (h) ca-dblp-2012 (i) web-baidu-baike

(j) dbpedia-link (k) soc-livejournal-user-groups (l) soc-ljournal-2008

Figure 1: Performance ratio curves on 12 instances with different sizes.

Instance	InfVC	CPLEX	B&R
Amazon0312	**261569**	261579	-
Amazon0505	**267227**	267230	-
Amazon0601	**266551**	266554	-
Cit-HepPh	**21593**	21598	-
Cit-HepTh	**17844**	17857	-
cnr-2000	95559	**95542**	-
delaunay_n22	**2871507**	2893784	-
delaunay_n23	**5743322**	6343474	-
delaunay_n24	**11487404**	12685651	-
eu-2005	410372	**410311**	-
Ga41As41H72	**233532**	268096	-
hugebubbles-00000	**9200661**	18302503	-
hugebubbles-00010	**10331837**	19439687	-
hugebubbles-00020	**11250692**	21180434	-
hugetrace-00010	**6342579**	12047187	-
hugetrace-00020	**8466522**	15990226	-
in-2004	486149	**486146**	**486146**
inf-roadNet-CA	**999497**	**999497**	-
inf-roadNet-PA	**554199**	**554199**	-
packing-500x100x*	**1614481**	2141611	-
patents	**1671949**	1939059	-
rec-dating	**89781**	136247	-
rec-libimseti-dir	93680	167628	**93676**
sc-nasasrb	**51239**	51281	-
sc-pkustk13	89227	**89215**	-
sc-pwtk	**207679**	207878	-
sc-shipsec1	**116857**	139191	-
sc-shipsec5	**146764**	176274	-
socfb-Berkeley13	**17210**	21080	-
socfb-CMU	**4986**	5003	-
socfb-Duke14	**7683**	7778	-
socfb-Indiana	**23314**	28142	-
socfb-MIT	**4657**	4669	-
socfb-OR	**36547**	36584	-
socfb-Penn94	**31159**	38319	-
socfb-Stanford3	**8518**	8538	-
socfb-Texas84	**28169**	34437	-
socfb-UCLA	**15222**	18639	-
socfb-UConn	**13230**	16199	-
socfb-UCSB37	**11261**	13815	-
socfb-UF	**27305**	33190	-
socfb-UIllinois	**24089**	29076	-
socfb-Wisconsin87	**18383**	22339	-
soc-orkut	**2170788**	2768594	-
soc-orkut-dir	**2233392**	2845450	-
soc-pokec	**843341**	**843341**	-
tech-as-skitter	525029	525024	**525022**
uk-2002	**6571007**	8017413	-
wave	**119267**	155833	-
web-Stanford	**118513**	**118513**	-
web-wikipedia2009	**648294**	**648294**	-

Table 1: Comparison between InfVC and state-of-the-art exact algorithms.

4 Detailed Results on All Heuristic Algorithms

We present the detailed results on all heuristic algorithms in Table 2. For each instance, min represents the best size found by each algorithm and avg denotes the average size obtained by each algorithm over 10 runs. When min=avg, we do not report avg. The bold values denote the best solution obtained among all algorithms.

References

[Karp, 1972] Richard M. Karp. *Reducibility among Combinatorial Problems*, pages 85–103. Boston, MA, 1972.

Instance	best solution	InfVC min(avg)	EAVC3+p min(avg)	FastVC2+p min(avg)	PEAVC min(avg)	MetaVC min(avg)	MetaVC2 min(avg)	TIVC min(avg)	QMeaMetaVC min(avg)
inf-roadNet-CA	999509	**999497**	1001093(1001160)	1001048(1001070)	999533(999547)	1000347(1000500.03)	999854(999911.64)	1001157(1001220)	999509(999510)
inf-roadNet-PA	554204	**554199**	555076(555110)	555025(555050)	554210(554213)	554530(554582.2)	554320(554346.98)	555087(555133)	554204(554210)
inf-road-usa	11519267	**11519221**	11527602(11527900)	11527593(11527700)	11520228(11520300)	11527313(11527749.68)	11525352(11525847.64)	11528663(11528800)	11519267(11519293.8)
rec-amazon	47605	**47605**	**47605**	**47605**	**47605**	**47605**	**47605**	**47605**	**47605**
sc-ldoor	856754	**856754**	**856754**	**856754**	**856754**	**856754**	**856754**	**856754**	**856754**
sc-msdoor	381558	**381558**	**381558**	**381558**	**381558**	**381558**	**381558**	**381558**	**381558**
sc-nasasrb	51237	**51236(51238.8)**	51240(51240.7)	51239(51239.2)	51240	51239(51240.9)	51238(51240.6)	51238	51237(51239.3)
sc-pkustk11	83911	**83911**	**83911**	**83911**	**83911**	**83911**	**83911**	**83911**	**83911**
sc-pkustk13	89216	89226(89230.2)	89227(89229.8)	89227(89228.5)	89226	89223(89224.66)	89232(89234.12)	89217(89218.6)	**89216(89216.2)**
sc-pwtk	207670	207675(207676.9)	**207670(207671)**	207690(207700)	207675(207678.2)	207674(207678.02)	207674(207676)	207694	207677(207680.4)
sc-shipsec1	116849	**116844(116854.3)**	117148(117167)	116865(116879)	116867(116872)	116922(116945.75)	116849(116871.75)	117219(117249)	117000(117019)
sc-shipsec5	146768	**146764(146780.6)**	146975(147002)	146833(146846)	146817(146824)	146912(146967.64)	146768(146787.57)	147013(147046)	146820(146832.7)
soc-gowalla	84222	**84222**	**84222**	**84222**	**84222**	**84222**	**84222**	**84222**	**84222**
soc-buzznet	30613	**30613**	**30613**	**30613**	**30613**	**30613**	**30613**	**30613**	**30613**
soc-delicious	85298	**85298**	85318(85328.7)	85328(85332.2)	**85298**(85298.5)	85368(85382.47)	85358(85372.75)	85329(85339.9)	**85298**
socfb-Berkeley13	17209	**17209(17209.8)**	17210(17210.8)	17219(17219.5)	17220(17220.6)	17210(17213)	**17209**(17209.9)	17211(17212.5)	**17209**(17209.9)
socfb-CMU	4986	**4986**	**4986**	**4986**(4986.6)	**4986**	**4986**(4986.4)	**4986**	**4986**(4986.8)	**4986**
socfb-Duke14	7683	**7683**	**7683**	**7683**	**7683**	**7683**	**7683**	**7683**	**7683**
socfb-Indiana	23313	**23313**(23313.9)	23314(23315)	23315(23317.1)	23315	23314(23316.9)	**23313**(23313.9)	23314(23319.8)	**23313(23313.2)**
socfb-MIT	4657	**4657**	**4657**	**4657**(4657.2)	**4657**(4657.2)	**4657**	**4657**	**4657**	**4657**
socfb-OR	36547	**36547**	**36547**	36548	36548(36548.5)	36549(36550.08)	**36547**	36548	**36547**
socfb-Penn94	31157	**31157**(31158.6)	31159(31161.1)	31161(31164)	31164	31161(31164.72)	31158(31195.82)	31162(31165.3)	**31157(31157.9)**
socfb-Stanford3	8517	**8517**(8517.3)	**8517**(8517.9)	8518	8518	8518	**8517**	**8517**(8517.9)	**8517**
socfb-Texas84	28164	**28164**(28164.8)	28166(28167.6)	28166(28170.1)	28172	28166(28170.83)	**28164**(28165.6)	28168(28173.7)	**28164(28164.3)**
socfb-UCLA	15221	**15221(15221.8)**	15222(15223.1)	15223(15224.1)	15224	15222(15223.6)	**15221**(15222.41)	15224(15225.6)	**15221(15221.8)**
socfb-UConn	13230	**13229(13229.7)**	13230(13230.8)	13237(13238.1)	13231	13230(13231.8)	13230	13230(13232.6)	13230
socfb-UCSB37	11261	**11261**	**11261**	**11261**(11262.1)	11262	**11261**(11261.9)	**11261**	**11261**(11262.3)	**11261**
socfb-UF	27303	**27303**(27304.3)	27305(27305.5)	27306(27308.4)	27305	27305(27308.52)	**27303**(27304)	27308(27310.3)	**27303(27303.2)**
socfb-UIllinois	24089	**24089**(24090.5)	24090(24091.4)	24091(24093.6)	24093	24092(24094)	24090(24090.7)	24094(24096.5)	**24089(24089.7)**
socfb-Wisconsin87	18382	18383(18383.1)	18383(18383.7)	18383(18384.6)	18384	18383(18385)	**18382**(18382.93)	18385(18388.4)	**18382(18382.5)**
soc-flickr	153271	**153271**	**153271**	**153271**	**153271**	**153271**	**153271**	**153271**	**153271**
soc-livejournal	1868903	**1868903**	1868912(1868920)	1868907(1868910)	1868904	1868907(1868910.69)	1868905(1868910.32)	1868909(1868910)	**1868903**
soc-orkut	2170773	**2170715(2170806.2)**	2170960(2171000)	2171200(2171236.4)	2171303(2171440)	2171860(2171951.4)	2170773(2170854.66)	2171212(2171291)	2170861(2170910)
soc-pokec	843345	**843341(843342.5)**	843373(843379)	843373(843378)	843345(843346)	843369(843379)	843348(843354)	843378(843384)	843345(843346)
tech-as-skitter	525025	**525024(525025.6)**	525214(525252)	525141(525149)	525027(525030)	525186(525200.99)	525183(525196.02)	525232(525266)	525025(525026)
tech-RL-caida	74593	**74593**	74617(74623.4)	74619(74620.8)	**74593**(74593.8)	74621(74629.39)	74621(74630.29)	74642(74647.7)	**74593**(74593.4)
web-arabic-2005	114420	**114420**	**114420**	**114420**	**114420**	**114420**	**114420**	**114420**	**114420**
web-it-2004	414507	**414507**	414521(414526)	414520	**414507**	414515(414516)	414515(414516.06)	414528(414534)	**414507**
web-sk-2005	58173	**58173**	**58173**	**58173**	**58173**	**58173**	**58173**	**58173**	**58173**
web-webbase-2001	2651	**2651**	2652	2652	**2651**	2652	2652	2652	**2651**
web-wikipedia2009	648294	**648294**	648296(648303)	648297(648301)	**648294**	**648294**(648296.36)	**648294**	648297(648306)	**648294**
Amazon0302	168533	**168533**	168540(168543)	168544(168546)	**168533**	168536	**168533**	168537(168541)	**168533**
Amazon0312	261569	**261569**	261585(261589)	261586(261590)	**261569(261570)**	261580(261582)	261575(261577)	261587(261592)	**261569**
Amazon0505	267227	**267226(267226.7)**	267240(267245)	267244(267246)	267227(267228)	267239(267241)	267234(267235)	267243(267247)	267227
Amazon0601	266551	**266551**	266563(266568)	266564(266567)	**266551(266553)**	266560(266562)	266558	266572.2(267565)	**266551**
caidaRouterLevel	75094	**75094**	75134(75139.6)	75165(75166)	75096(75098.5)	75128(75133.1)	75124(75132.8)	75142(75148)	**75094**(75094.2)
citationCiteseer	118115	**118115**	**118115**	118121	**118115**	**118115**	**118115**	**118115**	**118115**
Cit-HepPh	21593	**21593**	**21593**(21594.3)	21600(21602.8)	21594(21595.8)	21594(21595.4)	**21593**(21593.4)	**21593**(21594.8)	**21593**(21594.3)
Cit-HepTh	17844	**17844(17844.5)**	17845(17846.3)	17849(17850)	17845(17847)	17846	17845	17845(17846.5)	**17844**(17845.2)
cnr-2000	95581	**95556(95559.5)**	95639(95655.3)	95719(95722.2)	95614(95623.7)	95641(95652.4)	95626(95643.6)	95667(95679.1)	95581(95590.4)
delaunay_n22	2871650	**2871368(2871498.7)**	2872839(2872900)	2873973(2874015)	2871737(2871760)	2876607(2877610)	2876777(2877150)	2873207(2873239)	2871650(2871680)
delaunay_n23	5743518	**5743035(5743297.3)**	5747069(5747540)	5753390(5753730)	5743735(5743780)	5761869(5769300)	5757570(5761220)	5755880(5756100)	5743518(5743530)
delaunay_n24	11487703	**11487358(11487418.5)**	11679154(11712000)	11548840(11554500)	11488618(11488800)	11665542(11693700)	11626891(11661200)	11722909(11727200)	11487703
eu-2005	410393	**410352(410360.2)**	410491(410512)	410505(410518)	410421(410433)	410892(410984)	410852(410929)	410501(410517)	410393(410413)
friendster	1038235	**1038235**	**1038235**(1038240)	**1038235**(1038240)	**1038235**(1038240)	**1038235**(1038240)	**1038235**(1038240)	**1038235**(1038240)	**1038235**(1038240)
Ga41As41H72	233508	233516(233771)	233550(233604.9)	237101(237985)	239369(239416)	233534(233923)	233515(233582)	**233508(233572)**	239176(239215)
hugebubbles-00000	9338006	**9200321(9202030)**	10288772(10295700)	10053112(10085100)	9338006(9393190)	9753830(10183200)	10224490(10266600)	10417508(10432354)	9467569(9470780)
hugebubbles-00010	10470479	**10331620(10332200)**	11124353(11131000)	10937577(10955300)	10470479(10499100)	10897391(11087500)	11248374(11277000)	11243420(11252100)	10572433(10575500)
hugebubbles-00020	11389263	**11250526(11251533.5)**	12208601(12260500)	11631738(11719600)	11389263(11403400)	12171743(12337600)	12193210(12288500)	12290875(12294200)	11523744(11526200)
hugetrace-00010	6342139	**6342110(6342732.4)**	6622123(6643810)	6370920(6373450)	6355347(6356150)	6374256(6594470)	6342139(6423910)	6617040(6618160)	6439215(6440250)
hugetrace-00020	8518201	**8466286(8466856.7)**	9087718(9118890)	8701902(8715990)	8518201(8524540)	8807934(9088060)	8823651(8899390)	9107786(9109420)	8645429(8647160)
in-2004	486151	**486149(486152.3)**	486263(486286)	486286(486296)	486151(486161)	486358(486404)	486328(486374)	486266(486283)	486163(486176)
inf-europe_osm	25278587	**25278587**	25281962(25303400)	25284658(25284800)	25278643(25278700)	25310452(25327100)	25285042(25288900)	25344847(25345700)	**25278587**(25278600)
inf-germany_osm	5707245	**5707245**	5707605(5707630)	5707675(5707710)	**5707245**	5708699(5709000)	5707969(5708060)	5707983(5707990)	**5707245**
luxembourg_osm	56936	**56936**	**56936**(56936.8)	**56936**(56938.2)	**56936**	**56936**	**56936**	**56936**	**56936**
packing-500x100x100-b050	1614702	**1613273(1613913.2)**	1619239(1620760)	1624945(1625325)	1623374(1625660)	1615573(1616094.7)	1614702(1615460)	1623445(1625416)	1621157(1622560)
patents	1672127	**1671926(1671940)**	1672960(1672990)	1673052(1673090)	1672144(1672170)	1673290(1673350)	1672560(1672630)	1673094(1673140)	1672127(1672160)
rec-dating	89780	**89780(89780.7)**	89793(89797.8)	89806(89809.2)	89785(89787.8)	89801(89807.1)	**89780**(89780.8)	89804(89806.1)	89785(89787.8)
rec-epinions	99950	**99950**	99958(99959.2)	99963	**99950**	99998(100006)	99999(100012)	99958(99960.1)	**99950**
rec-libimseti-dir	93680	**93677(93681)**	93696(93701.8)	93704(93704.4)	93680(93684.3)	93710(96033.1)	93710(96033.1)	93696(93701.4)	93680(93684.3)
road_central	6759310	**6759227(6759230)**	6764354(6764460)	6764339(6764390)	6760067(6760110)	6763871(6763940)	6762536(6762640)	6764422(6764630)	6759310(6759340)
sc-rel9	273993	**273993**	**273993**	**273993**	**273993**	274155(274160)	274155(274160)	**273993**	**273993**
soc-flickr-und	474637	**474637**	**474637**	**474637**	**474637**	**474637**	**474637**	**474637**	**474637**
soc-livejournal-user-groups	1840718	**1840718**	1840726(1840730)	1840762	**1840718**(1840720)	1840728(1840730)	1840728(1840730)	1840760	**1840718**(1840720)
soc-ljournal-2008	2392369	**2392343(2392344.7)**	2392422(2392440)	2392450	2392383(2392390)	2392438(2392450)	2392427(2392440)	2392410(2392420)	2392369(2392380)
soc-orkut-dir	2233396	**2233363(2233416.9)**	2233660(2233690)	2233841(2233870)	2233723(2233770)	2233979(2234033.8)	2233396(2233510)	2233858(2233929)	2233670(2233692)
soc-Slashdot0902	25770	**25770**	**25770**	**25770**(25770.2)	**25770**	**25770**	**25770**	**25770**	**25770**
uk-2002	6571100	**6571007(6571044)**	6577097(6577170)	6577657(6577700)	6572795(6572820)	6586736(6587110)	6585424(6586000)	6577329(6577440)	6571100(6571141)
wave	119279	**119267**(119360.3)	119289(119411)	119306(119445.9)	119505(119714)	119279(**119309.9**)	119279(**119309.9**)	119290(119331.3)	119336(119568)
web-baidu-baike	636967	**636967**	**636967**	636973	**636967**	**636967**	**636967**	**636967**	**636967**
web-NotreDame	73878	**73878**	73927(73935)	73944(73945.9)	73890(73897.5)	73904(73907.1)	73899(73906.4)	73922(73926.8)	**73878**(73880.4)
web-Stanford	118518	**118513**	118552(118562)	118564(118569)	118532(118536)	118537(118548)	118533(118542)	118562(118568)	118518(118522)

Table 2: Experimental results across all heuristic algorithms.