System Specification

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1 Type System

$\boxed{\Omega \vdash a :^{\ell} A}$ *(Typing)*

T-Type
$$\frac{\mathbf{axiom}\ s_1\ s_2}{\Omega \vdash s_1 :^{\ell} s_2}$$

T-Conv
$$\frac{\Omega \vdash a :^{\ell} A \qquad |\top \downarrow \Omega| \vdash A \equiv_{\ell_0} B \qquad \top \downarrow \Omega \vdash B :^{\ell_0} s}{\Omega \vdash a :^{\ell} B}$$

T-Var
$$\frac{\ell_0 \leq \ell \qquad x :^{\ell_0} A \in \Omega}{\Omega \vdash x :^{\ell} A}$$

T-Pi
$$\frac{\Omega \vdash A :^{\ell} s_1 \qquad \Omega, x :^{\ell} A \vdash B :^{\ell} s_2 \qquad \mathbf{rule\_pi}\ s_1\ s_2\ s_3}{\Omega \vdash \Pi x :^{\ell_0} A.B :^{\ell} s_3}$$

T-Abs
$$\frac{\Omega, x :^{\ell_0} A \vdash b :^{\ell} B \qquad \top \downarrow \Omega \vdash (\Pi x :^{\ell_0} A.B) :^{\ell_1} s}{\Omega \vdash \lambda^{\ell_0} x : A.b :^{\ell} \Pi x :^{\ell_0} A.B}$$

T-App
$$\frac{\Omega \vdash b :^{\ell} \Pi x :^{\ell_0} A.B \qquad \ell \downarrow \Omega \vdash a :^{\ell_0} A}{\Omega \vdash b\ a^{\ell_0} :^{\ell} B\{a/x\}}$$

T-WSigma
$$\frac{\Omega \vdash A :^{\ell} s_1 \qquad \Omega, x :^{\ell} A \vdash B :^{\ell} s_2 \qquad \mathbf{rule\_sig}\ s_1\ s_2\ s_3}{\Omega \vdash \Sigma x{:}^{\ell_0} A.B :^{\ell} s_3}$$

T-WPair
$$\frac{\top \downarrow \Omega \vdash (\Sigma x{:}^{\ell_0} A.B) :^{C} s \qquad \ell \downarrow \Omega \vdash a :^{\ell_0} A \qquad \Omega \vdash b :^{\ell} B\{a/x\}}{\Omega \vdash (a^{\ell_0}, b) :^{\ell} \Sigma x{:}^{\ell_0} A.B}$$

T-LetPair
$$\frac{\top \downarrow \Omega \vdash (\Pi y :^{\ell} B.C) :^{\ell_1} s \qquad \Omega \vdash a :^{\ell} (\Sigma x{:}^{\ell_0} A.B) \qquad \Omega, x :^{\ell_0} A \vdash c :^{\ell} (\Pi y :^{k} B.C)}{\Omega \vdash \mathbf{let}\ (x^{\ell_0},) = a\ \mathbf{in}\ c :^{\ell} C}$$

T-SSigma
$$\frac{\Omega \vdash A :^{\ell} s_1 \qquad \Omega, x :^{\ell} A \vdash B :^{\ell} s_2 \qquad \mathbf{rule\_sig}\ s_1\ s_2\ s_3}{\Omega \vdash (x{:}^{\ell_0} A) \,\&\, B :^{\ell} s_3}$$

T-SPair
$$\frac{\top \downarrow \Omega \vdash (x{:}^{\ell_0} A) \,\&\, B :^{\ell_1} s \qquad \ell \downarrow \Omega \vdash a :^{\ell_0} A \qquad \Omega \vdash b :^{\ell} B\{a/x\}}{\Omega \vdash (a^{\ell_0} \,\&\, b) :^{\ell} (x{:}^{\ell_0} A) \,\&\, B}$$

T-Proj1
$$\frac{\Omega \vdash a :^{\ell} (x{:}^{\ell_0} A) \,\&\, B \qquad \ell_0 \leq \ell}{\Omega \vdash \pi_1^{\ell_0}\ a :^{\ell} A}$$

$$\textsc{T-Proj2}\ \frac{\Omega \vdash a :^{\ell} (x{:}^{\ell_0} A) \,\&\, B}{\Omega \vdash \pi_2^{\ell_0}\, a :^{\ell} B\{\pi_1^{\ell_0}\ a/x\}}$$