

# System Specification

March 2, 2023

## 1 Type System

$\boxed{\Omega \vdash a :^\ell A}$  (*Typing*)

$\frac{\text{T-TYPE} \quad \text{axiom } s_1 \ s_2}{\Omega \vdash s_1 :^\ell s_2}$	$\frac{\text{T-CONV} \quad \begin{array}{l} \Omega \vdash a :^\ell A \\  \top \downarrow \Omega  \vdash A \equiv_{\ell_0} B \\ \top \downarrow \Omega \vdash B :^{\ell_0} s \end{array}}{\Omega \vdash a :^\ell B}$	$\frac{\text{T-VAR} \quad \ell_0 \leq \ell \quad x :^{\ell_0} A \in \Omega}{\Omega \vdash x :^\ell A}$
$\frac{\text{T-PI} \quad \begin{array}{l} \Omega \vdash A :^\ell s_1 \\ \Omega, x :^\ell A \vdash B :^\ell s_2 \\ \text{rule\_pi } s_1 \ s_2 \ s_3 \end{array}}{\Omega \vdash \Pi x :^{\ell_0} A. B :^\ell s_3}$	$\frac{\text{T-ABS} \quad \begin{array}{l} \Omega, x :^{\ell_0} A \vdash b :^\ell B \\ \top \downarrow \Omega \vdash (\Pi x :^{\ell_0} A. B) :^{\ell_1} s \end{array}}{\Omega \vdash \lambda^{\ell_0} x : A. b :^\ell \Pi x :^{\ell_0} A. B}$	$\frac{\text{T-APP} \quad \begin{array}{l} \Omega \vdash b :^\ell \Pi x :^{\ell_0} A. B \\ \ell \downarrow \Omega \vdash a :^{\ell_0} A \end{array}}{\Omega \vdash b \ a^{\ell_0} :^\ell B\{a/x\}}$
$\frac{\text{T-WSIGMA} \quad \begin{array}{l} \Omega \vdash A :^\ell s_1 \\ \Omega, x :^\ell A \vdash B :^\ell s_2 \\ \text{rule\_sig } s_1 \ s_2 \ s_3 \end{array}}{\Omega \vdash \Sigma x :^{\ell_0} A. B :^\ell s_3}$	$\frac{\text{T-WPAIR} \quad \begin{array}{l} \top \downarrow \Omega \vdash (\Sigma x :^{\ell_0} A. B) :^C s \\ \ell \downarrow \Omega \vdash a :^{\ell_0} A \\ \Omega \vdash b :^\ell B\{a/x\} \end{array}}{\Omega \vdash (a^{\ell_0}, b) :^\ell \Sigma x :^{\ell_0} A. B}$	
$\frac{\text{T-LETPAIR} \quad \begin{array}{l} \top \downarrow \Omega \vdash (\Pi y :^\ell B. C) :^{\ell_1} s \\ \Omega \vdash a :^\ell (\Sigma x :^{\ell_0} A. B) \\ \Omega, x :^{\ell_0} A \vdash c :^\ell (\Pi y :^k B. C) \end{array}}{\Omega \vdash \text{let } (x^{\ell_0},) = a \text{ in } c :^\ell C}$	$\frac{\text{T-SSIGMA} \quad \begin{array}{l} \Omega \vdash A :^\ell s_1 \\ \Omega, x :^\ell A \vdash B :^\ell s_2 \\ \text{rule\_sig } s_1 \ s_2 \ s_3 \end{array}}{\Omega \vdash (x :^{\ell_0} A) \& B :^\ell s_3}$	
$\frac{\text{T-SPAIR} \quad \begin{array}{l} \top \downarrow \Omega \vdash (x :^{\ell_0} A) \& B :^{\ell_1} s \\ \ell \downarrow \Omega \vdash a :^{\ell_0} A \\ \Omega \vdash b :^\ell B\{a/x\} \end{array}}{\Omega \vdash (a^{\ell_0} \& b) :^\ell (x :^{\ell_0} A) \& B}$	$\frac{\text{T-PROJ1} \quad \begin{array}{l} \Omega \vdash a :^\ell (x :^{\ell_0} A) \& B \\ \ell_0 \leq \ell \end{array}}{\Omega \vdash \pi_1^{\ell_0} a :^\ell A}$	

$$\frac{\text{T-PROJ2} \quad \Omega \vdash a :^\ell (x :^{\ell_0} A) \& B}{\Omega \vdash \pi_2^{\ell_0} a :^\ell B\{\pi_1^{\ell_0} a/x\}}$$