

$$\boxed{\Omega \vdash a :^\ell A}$$

(Typing)

$$\begin{array}{c}
\text{T-TYPE} \\
\frac{\text{axiom } s_1 \ s_2}{\Omega \vdash s_1 :^\ell s_2}
\end{array}
\quad
\begin{array}{c}
\text{T-CONV} \\
\frac{\Omega \vdash a :^\ell A \quad |\top \downarrow \Omega| \vdash A \equiv_{\ell_0} B \quad \top \downarrow \Omega \vdash B :^{\ell_0} s}{\Omega \vdash a :^\ell B}
\end{array}
\quad
\begin{array}{c}
\text{T-VAR} \\
\frac{\ell_0 \leq \ell \quad x :^{\ell_0} A \in \Omega}{\Omega \vdash x :^\ell A}
\end{array}$$

$$\begin{array}{c}
\text{T-PI} \\
\frac{\Omega \vdash A :^\ell s_1 \quad \Omega, x :^\ell A \vdash B :^\ell s_2 \quad \text{rule_pi } s_1 \ s_2 \ s_3}{\Omega \vdash \Pi x :^{\ell_0} A. B :^\ell s_3}
\end{array}
\quad
\begin{array}{c}
\text{T-ABS} \\
\frac{\Omega, x :^{\ell_0} A \vdash b :^\ell B \quad \top \downarrow \Omega \vdash (\Pi x :^{\ell_0} A. B) :^{\ell_1} s}{\Omega \vdash \lambda^{\ell_0} x : A. b :^\ell \Pi x :^{\ell_0} A. B}
\end{array}
\quad
\begin{array}{c}
\text{T-APP} \\
\frac{\Omega \vdash b :^\ell \Pi x :^{\ell_0} A. B \quad \ell \downarrow \Omega \vdash a :^{\ell_0} A}{\Omega \vdash b \ a^{\ell_0} :^\ell B\{a/x\}}
\end{array}$$

$$\begin{array}{c}
\text{T-WSIGMA} \\
\frac{\Omega \vdash A :^\ell s_1 \quad \Omega, x :^\ell A \vdash B :^\ell s_2 \quad \text{rule_sig } s_1 \ s_2 \ s_3}{\Omega \vdash \Sigma x :^{\ell_0} A. B :^\ell s_3}
\end{array}
\quad
\begin{array}{c}
\text{T-WPAIR} \\
\frac{\top \downarrow \Omega \vdash (\Sigma x :^{\ell_0} A. B) :^C s \quad \ell \downarrow \Omega \vdash a :^{\ell_0} A \quad \Omega \vdash b :^\ell B\{a/x\}}{\Omega \vdash (a^{\ell_0}, b) :^\ell \Sigma x :^{\ell_0} A. B}
\end{array}$$

$$\begin{array}{c}
\text{T-LETPAIR} \\
\frac{\top \downarrow \Omega \vdash (\Pi y :^\ell B. C) :^{\ell_1} s \quad \Omega \vdash a :^\ell (\Sigma x :^{\ell_0} A. B) \quad \Omega, x :^{\ell_0} A \vdash c :^\ell (\Pi y :^k B. C)}{\Omega \vdash \text{let } (x^{\ell_0},) = a \text{ in } c :^\ell C}
\end{array}$$

$$\begin{aligned}
&\ell \downarrow \emptyset = \emptyset \\
&\ell \downarrow \Omega, x :^{\ell_0} A = \ell \downarrow \Omega, x :^{\ell_0} A \text{ if } \neg(\ell_0 \leq \ell) \\
&\ell \downarrow \Omega, x :^{\ell_0} A = \ell \downarrow \Omega, x :^\perp A \text{ if } \ell_0 \leq \ell
\end{aligned}$$

Figure 1: Typing