$\boxed{\Omega \vdash a :^{\ell} A}$ *(Typing)*

$$\textsc{T-Type}\ \frac{\mathbf{axiom}\ s_1\ s_2}{\Omega \vdash s_1 :^{\ell} s_2} \qquad \textsc{T-Conv}\ \frac{\Omega \vdash a :^{\ell} A \quad |\top \downarrow \Omega| \vdash A \equiv_{\ell_0} B \quad \top \downarrow \Omega \vdash B :^{\ell_0} s}{\Omega \vdash a :^{\ell} B} \qquad \textsc{T-Var}\ \frac{\ell_0 \leq \ell \qquad x :^{\ell_0} A \in \Omega}{\Omega \vdash x :^{\ell} A}$$

$$\textsc{T-Pi}\ \frac{\Omega \vdash A :^{\ell} s_1 \quad \Omega, x :^{\ell} A \vdash B :^{\ell} s_2 \quad \mathbf{rule\_pi}\ s_1\ s_2\ s_3}{\Omega \vdash \Pi x :^{\ell_0} A.B :^{\ell} s_3} \qquad \textsc{T-Abs}\ \frac{\Omega, x :^{\ell_0} A \vdash b :^{\ell} B \quad \top \downarrow \Omega \vdash (\Pi x :^{\ell_0} A.B) :^{\ell_1} s}{\Omega \vdash \lambda^{\ell_0} x : A.b :^{\ell} \Pi x :^{\ell_0} A.B}$$

$$\textsc{T-App}\ \frac{\Omega \vdash b :^{\ell} \Pi x :^{\ell_0} A.B \quad \ell \downarrow \Omega \vdash a :^{\ell_0} A}{\Omega \vdash b\ a^{\ell_0} :^{\ell} B\{a/x\}}$$

$$\textsc{T-WSigma}\ \frac{\Omega \vdash A :^{\ell} s_1 \quad \Omega, x :^{\ell} A \vdash B :^{\ell} s_2 \quad \mathbf{rule\_sig}\ s_1\ s_2\ s_3}{\Omega \vdash \Sigma x{:}^{\ell_0} A.B :^{\ell} s_3} \qquad \textsc{T-WPair}\ \frac{\top \downarrow \Omega \vdash (\Sigma x{:}^{\ell_0} A.B) :^{C} s \quad \ell \downarrow \Omega \vdash a :^{\ell_0} A \quad \Omega \vdash b :^{\ell} B\{a/x\}}{\Omega \vdash (a^{\ell_0}, b) :^{\ell} \Sigma x{:}^{\ell_0} A.B}$$

$$\textsc{T-LetPair}\ \frac{\top \downarrow \Omega \vdash (\Pi y :^{\ell} B.C) :^{\ell_1} s \quad \Omega \vdash a :^{\ell} (\Sigma x{:}^{\ell_0} A.B) \quad \Omega, x :^{\ell_0} A \vdash c :^{\ell} (\Pi y :^{k} B.C)}{\Omega \vdash \mathbf{let}\ (x^{\ell_0},) = a\ \mathbf{in}\ c :^{\ell} C}$$

$$\begin{aligned} \ell \downarrow \varnothing &= \varnothing \\ \ell \downarrow \Omega, x :^{\ell_0} A &= \ell \downarrow \Omega, x :^{\ell_0} A \text{ if } \neg(\ell_0 \leq \ell) \\ \ell \downarrow \Omega, x :^{\ell_0} A &= \ell \downarrow \Omega, x :^{\bot} A \text{ if } \ell_0 \leq \ell \end{aligned}$$

Figure 1: Typing