生物統計 Homework 6

1. **Making auto parts.** An automatic grinding machine in an auto parts plant prepares axles with a target diameter $\mu = 40.125$ millimeters (mm). The machine has some variability, so the standard deviation of the diameters is $\sigma = 0.002$ mm. A sample of 4 axles is inspected each hour for process control purposes, and records are kept of the sample mean diameter. What will be the mean and standard deviation of the numbers recorded?

2. **Dust in coal mines.** A laboratory weighs filters from a coal mine to measure the amount of dust in the mine atmosphere. Repeated measurements of the weight of dust on the same filter vary Normally with standard deviation $\sigma = 0.08$ milligram (mg) because the weighing is not perfectly precise. The dust on a particular filter actually weighs 123 mg. Repeated weighings will then have the Normal distribution with mean 123 mg and standard deviation 0.08 mg.

   (a) The laboratory reports the mean of 3 weighings. What is the distribution of this mean? $\bar{x} = (X_1 + X_2 + X_3)/3$

   (b) What is the probability that the laboratory reports a weight of 124 mg or higher for this filter?

3. **Pollutants in auto exhausts.** The level of nitrogen oxides (NOX) in the exhaust of a particular car model varies with mean 0.9 grams per mile (g/mi) and standard deviation 0.15 g/mi. A company has 125 cars of this model in its fleet.

   (a) What is the approximate distribution of the mean NOX emission level $\bar{x}$ for these cars?

   (b) What is the level L such that the probability that $\bar{x}$ is greater than L is only 0.01? (Hint: This requires a backward Normal calculation.

4. **How many people in a car?** A study of rush-hour traffic in San Francisco counts the number of people in each car entering a freeway at a suburban interchange. Suppose that this count has mean 1.5 and standard deviation 0.75 in the population of all cars that enter at this interchange during rush hours.

   (a) Could the exact distribution of the count be Normal? Why or why not?

   (b) Traffic engineers estimate that the capacity of the interchange is 700 cars per hour. According to the central limit theorem, what is the approximate distribution of the mean number of persons $\bar{x}$ in 700 randomly selected cars at this interchange?

   (c) What is the probability that 700 cars will carry more than 1075 people? (Hint: Restate this event in terms of the mean number of people $\bar{x}$ per car.)

5. **Random-digit dialing.** When an opinion poll or telemarketer calls residential telephone numbers at random, 20% of the calls reach a live person. You watch the random dialing machine make 15 calls. The number that reach a person has the binomial distribution with $n = 15$ and $p = 0.2$.

   (a) What is the probability that exactly 3 calls reach a person?

   (b) What is the probability that 3 or fewer calls reach a person?

6. **Multiple-choice tests.** Here is a simple probability model for multiple-choice tests. Suppose that each student has probability p of correctly answering a question chosen at random from a universe of possible questions. (A strong student has a higher p than a weak student.) Answers to different questions are independent. Jodi is a good student for whom $p = 0.75$.

(a) Use the Normal approximation to find the probability that Jodi scores 70% or lower on a 100-question test.

(b) If the test contains 250 questions, what is the probability that Jodi will score 70% or lower?

7. **Reaching dropouts.** High school dropouts make up 13% of all Americans aged 18 to 24. A vocational school that wants to attract dropouts mails an advertising flyer to 25,000 persons between the ages of 18 and 24.

(a) If the mailing list can be considered a random sample of the population, what is the mean number of high school dropouts who will receive the flyer?

(b) What is the probability that at least 3500 dropouts will receive the flyer?

8.1 *Chebyshev's inequality.* This inequality points out another useful property of the standard deviation. In particular, it states that "The probability that any random variable X falls within k standard deviations of its mean is at least $1 - 1/k^2$." For example, if we know that X has mean 3 and standard deviation 1, then we can conclude that the probability that X lies between 1 and 5 ($k = 2$ standard deviations from the mean) is at least $1 - 1/2^2 = .75$.

(*a*) Let X denote the amount of rainfall received per week in a region. Assume that $\mu = 1.00$ inch and $\sigma = .25$ inch. Would it be unusual for this region to receive more than 2 inches of rain in a given week? Explain on the basis of Chebyshev's inequality.

(*b*) Let X denote the number of cases of rabies reported in a given state per week. Assume that $\mu = \frac{1}{2}$ and $\sigma^2 = \frac{1}{25}$. Would it be unusual to observe two cases in a given week? Explain on the basis of Chebyshev's inequality.

8.2 各學校聯合舉辦模疑考，有 4000 人參加，自然平均分數為 60 分，標準差為 7 分，

1. 求在 46 分與 74 分之間的人數有多少人
2. 求最多佔全部 1/9 的分數範圍