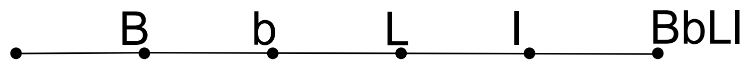
Q1: Guinea pigs

(a) What are the possible genotypes for the female? What are the possible genotypes for the male?

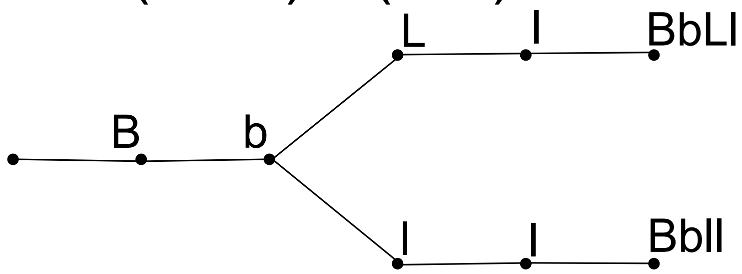
- Female: (BB/Bb) x (LL/Ll) = $(BBLL)$, $(BBLl)$, $(BbLL)$, $(BbLl)$
- Male: (bb) x (ll) = $(bbll)$

(b) For each possible genotype for the female construct a tree to represent the possible outcomes for the offspring.

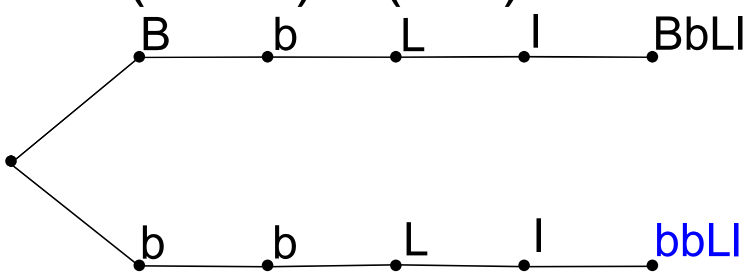
Case 1: (BBLL) x (bbll)



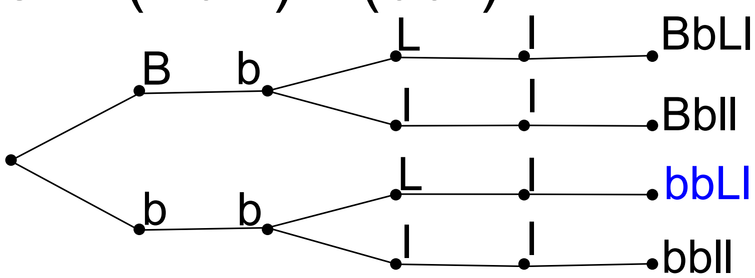
Case 2: (BBLl) x (bbll)



Case 3: (BbLL) x (bbll)



Case 4: (BbLl) x (bbll)



(c) Find the probability of obtaining an albino with short hair in each case.

The possible genotype for an albino with short hair is: $bbLL$ and $bbLl$. According to the above trees, we get

- Case 1: $(BBLL)$ x $(bbll)$, P(albino, short)=0
- Case 2: $(BBLl)$ x $(bbll)$, P(albino, short)=0
- Case 3: $(BbLL)$ x $(bbll)$, P(albino, short)=0.5

- Case 4: $(BbLl)$ x $(bbll)$, P(albino, short)=0.25

Q3: Binomial and Poisson

(a) In fruit flies, 4 sperms cells in every 10^5 carry a mutation for red eye to white eye, or vice versa.

How many mutations would you expect to occur in 200,000 sperm cells?

- The probability of mutation is $P = \frac{4}{10^5} = 0.00004$
- This problem can be described as a binomial distribution with $n = 200000$ and $p = 0.00004$
- The expected number of mutation to occur in 200,000 sperm cells is mean, $\mu = np = 200000 \times 0.00004 = 8$

What is the probability that at most 10 would occur?

- Because large n and small p, we can use Possion distribution to approximate.
- $P(Y = k) = \frac{e^{-np}(np)^k}{k!}$
- This problem is to ask $P(X \leq 10)$
- By Poisson distribution, $P(Y \leq 10) = \sum_{k=0}^{10} P(Y = k) = \sum_{k=0}^{10} \frac{e^{-np}(np)^k}{k!}$
- $\sum_{k=0}^{10} \frac{e^{-8}(8)^k}{k!} = e^{-8}[\frac{(8)^0}{0!} + \frac{(8)^1}{1!} + \cdots + \frac{(8)^{10}}{10!}] = 0.816$
- the probability that at most 10 would occur is 0.816

What is the probability that between 6 and 10, inclusive, would occur?

- This problem is to ask $P(6 \leq X \leq 10)$
- By Poisson distribution, $P(6 \leq Y \leq 10) = P(6) + P(7) + P(8) + P(9) + P(10) = \sum_{k=6}^{10} \frac{e^{-np}(np)^k}{k!}$
- $\sum_{k=6}^{10} \frac{e^{-8}(8)^k}{k!} = e^{-8}[\frac{(8)^6}{6!} + \frac{(8)^7}{7!} + \frac{(8)^8}{8!} + \frac{(8)^9}{9!} + \frac{(8)^{10}}{10!}] = 0.625$
- the probability that between 6 and 10, inclusive, would occur is 0.625

(b) In human beings, mutations for Huntington's disease occur in about 5 of every 10^6 gametes. What is the probability that in 2 million gametes there will be at least one mutation?

- First, let us use Binomial to describe
- $X =$ number of mutation
- $X \sim Binomial(n = 2 \times 10^6, p = 5 \times 10^{-6})$
- Because large n and small p, we can use Possion distribution to approximate.
- The problem is to ask $P(X \geq 1) = 1 - P(X = 0)$
- By Poisson distribution, $P(Y = 0) = \frac{e^{-np}(np)^0}{0!} = e^{-10} \approx P(X = 0)$
- Therefore, $P(X \geq 1) = 1 - P(X = 0) \approx 1 - e^{-10} = 0.99995$
- the probability that in 2 million gametes there will be at least one mutation is 0.99995

(b) It is estimated that only 1 in every 50 parrots captured in the Amazon Basin for use as household pets will survive the transition.

- $X =$ number of survive
- The probability of survive is $p = \frac{1}{50} = 0.02$

During the course of a day, 700 birds are captrued. What is the expected number of survivors?

- $X \sim Binomial(n = 700, p = 0.02)$
- $\mu = np = 14$
- the expected number of survivors is 14

What is the probability that at most 10 birds will survive?

- For $n = 700$ and $p = 0.02$, Poisson is still a good approximation.
- This problem is to ask $P(X \leq 10)$
- By Poisson distribution, $P(Y \leq 10) = \sum_{k=0}^{10} P(Y = k) = \sum_{k=0}^{10} \frac{e^{-np}(np)^k}{k!}$
- $\sum_{k=0}^{10} \frac{e^{-14}(14)^k}{k!} = e^{-14}[\frac{(14)^0}{0!} + \frac{(14)^1}{1!} + \cdots + \frac{(14)^{10}}{10!}] = 0.176$
- the probability that at most 10 birds will survive is 0.176

During a given 3-day period, 700 birds are captrued each day. What is the probability that on each of the 3 days at most 10 birds will survive?

- For this question, we need to change the setting of the original binomial.
- $X \sim Binomial(n = 3 \times 700, p = 0.02) = Binomial(n = 2100, p = 0.02)$
- $\mu = np = 42$
- This problem is still to ask $P(X \leq 10)$
- By Poisson distribution, $P(Y \leq 10) = \sum_{k=0}^{10} P(Y = k) = \sum_{k=0}^{10} \frac{e^{-np}(np)^k}{k!}$
- $\sum_{k=0}^{10} \frac{e^{-42}(42)^k}{k!} = e^{-42}[\frac{(42)^0}{0!} + \frac{(42)^1}{1!} + \cdots + \frac{(42)^{10}}{10!}] \approx 0$
- the probability that on each of the 3 days at most 10 birds will survive is 0

Q4: Adult females of howler monkeys

(a)-(i) Find $P(X \leq 3)$

- $P(X \leq 3) = P(1) + P(2) + P(3) = 0.1 + 0.15 + 0.5 = 0.75$

(a)-(ii) Find $P(X > 1)$

- $P(X > 1) = 1 - P(1) = 1 - 0.1 = 0.9$

(a)-(iii) Find $P(2 \leq X \leq 4)$

- $P(2 \leq X \leq 4) = P(2) + P(3) + P(4) = 0.15 + 0.5 + 0.15 = 0.8$

(b) Find the average number of adult females per band

- $\mu = E(X) = \sum_{\forall x} xP(X = x)$
- $\mu = 1 \times 0.1 + 2 \times 0.15 + 3 \times 0.5 + 4 \times 0.15 + 5 \times 0.1 = 3$
- The average number is 3

(c) Find σ^2

- $\sigma^2 = Var(X) = E[(X - \mu)^2] = \sum_{\forall x}(x - \mu)^2 P(X = x)$
- $\sigma^2 = (1-3)^2 \times 0.1 + (2-3)^2 \times 0.15 + (3-3)^2 \times 0.5 + (4-3)^2 \times 0.15 + (5-3)^2 \times 0.1 = 1.1$
- σ^2 is 1.1

(d) Find $E(e^x)$

- $E(e^x) = \sum_{\forall x} e^x P(X = x)$
- $E(e^x) = e^1 \times 0.1 + e^2 \times 0.15 + e^3 \times 0.5 + e^4 \times 0.15 + e^5 \times 0.1 = 34.454$
- $E(e^x)$ is 34.454

(e) Find $E(\sqrt{x})$

- $E(\sqrt{x}) = \sum_{\forall x} \sqrt{x} P(X = x)$
- $E(\sqrt{x}) = \sqrt{1} \times 0.1 + \sqrt{2} \times 0.15 + \sqrt{3} \times 0.5 + \sqrt{4} \times 0.15 + \sqrt{5} \times 0.1 = 1.702$
- $E(\sqrt{x})$ is 1.702

Q5: Nuclear power plant and Albino rats

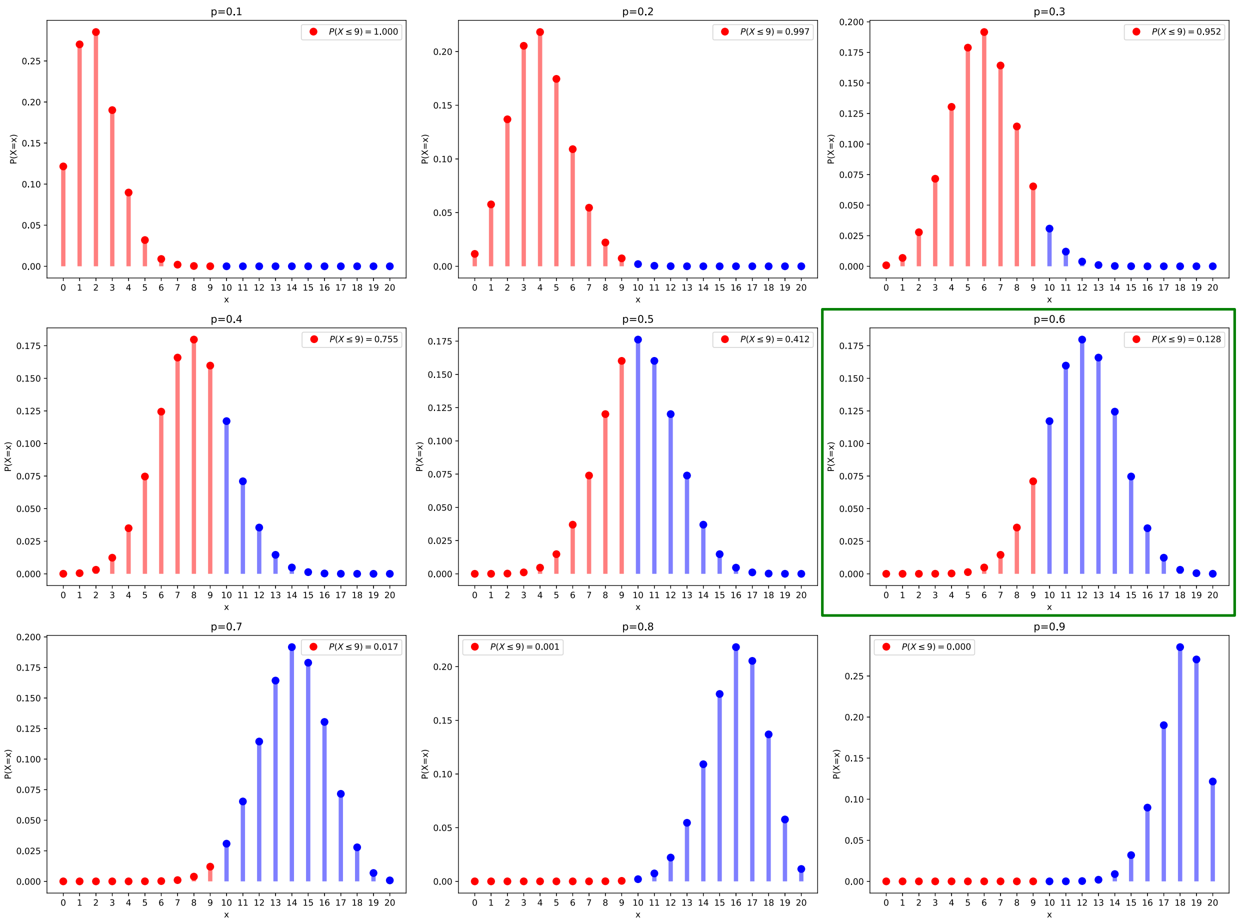
(a) A nuclear power plant is to be built.....How many would you expect to express a favorable opinion?

- First, let us use Binomial to describe
- $X =$ the number of people who express a favorable opinion
- $X \sim Binomial(n = 20, p = 0.6)$
- The problem is to ask $E(X)$
- $E(X) = np = 20 \times 0.6 = 12$
- The number of people who express a favorable opinion is expected to be 12.

If nine or fewer express such an opinion, do you think that there is strong reason to suspect the 60%

figure?

- The problem is to ask $P(X \leq 9)$
- $P(X \leq 9) = P(0) + P(1) + P(2) + \cdots + P(9)$
- As you can see in the following figure, when p=0.6, $P(X \leq 9) = 0.128$, it means that the probability that nine or fewer people express the favaorable opinion is 0.128. It further says that the 60% assumption is not so good.
- On the other hand, if we assume the 50%, 40%, 30%, 20%, even 10% figure to express favorable opinion, it is more possible to get nine or fewer exress from the investigation of 20 people.



(b) Albino rats ...If 10 animals are treated with the drug, what is the probability that at least 8 will be alive at the end of the experiment?

- First, let us use Binomial to describe
- $X =$ the number of rats which die from the drug before the experiment is over
- Usually, 4 out of 20 rats die from the drug before the experiment is over. That is, $p(\text{die}) = 4/20 = 0.2$
- $X \sim Binomial(n = 10, p = 0.2)$
- At least 8 will be alive, which is eqaul to at most 2 will die. Therefore, the problem is to ask $P(X \leq 2)$
- $$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2) = \binom{10}{0} p^0 (1-p)^{10} + \binom{10}{1} p^1 (1-p)^9 + \binom{10}{2} p^2 (1-p)^8$$
- $P(X \leq 2) = 0.107 + 0.268 + 0.302 = 0.678$
- The probability that at least 8 will be alive at the end of the experiment is 0.678