Lecture 4 Probability

10/7/2020

Lecture 3: Two by two tables

- Association between two bivary variables
  - Relative Risk (相對風險)
  - Odds ratio (勝算比)
- Sampling scheme (不考)

Example: 研究所入學管道與性別的關係

交大電 X 所 (七個學年度)

	女	男	Total
甄試	256	879	1135
考試	43	708	751
Total	299	1587	1886

Pr(女生) = 0.1585, Pr(男生) = 0.8415 → 男多女少的研究所

Pr(甄試) = 0.60, Pr(考試) = 0.40 → 甄試名額較考試多

Pr(甄試|女) = 0.8562, Pr(考試|女) = 0.1438 → 該所女生有八成五透過甄試

Pr(甄試|男) = 0.5539, Pr(考試|男) = 0.4461→ 該所男生有五成五透過甄試

Odds (甄試:考試) for 全體$=\frac{1135}{751}=1.51$(全體學生:"甄試"是 "考試"人數的 1.51 倍)

Odds (甄試:考試) for 女生= $\frac{256}{43}=5.95$(女生:"甄試"是 "考試"人數的 5.95 倍)

Odds (甄試:考試) for 男生= $\frac{879}{708}=1.24$(男生:"甄試"是 "考試"人數的 1.24 倍)

Odds ratio (女:男) $=\frac{5.95}{1.24}=4.7953$

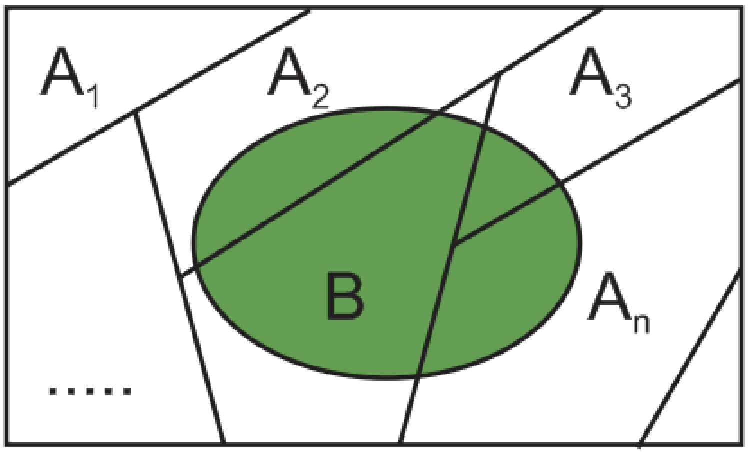
解讀:

- Odds ratio 是否等於 1 是判斷相關性的關鍵 (不等於 1 代表有相關)

- 女生較男生容易透過甄試管道入學 (GPA may be higher)

補充: 多數交大研究所都有 Odds ratio > 1 的現象，甄試對女生更有利

Bayes Theorem



Examples: Errors in testing the blood types

EXAMPLE 3.7.3. The blood type distribution in the United States at the time of World War II was thought to be type A, 41%; type B, 9%; type AB, 4%; and type O, 46%. It is estimated that during World War II, 4% of inductees with type O blood were typed as having type A; 88% of those with type A blood were correctly typed; 4% with type B blood were typed as A; and 10% with type AB were typed as A. A soldier was wounded and brought to surgery. He was typed as having type A blood. What is the probability that this was his true blood type?

Formulate the problem

Let

A_1: He has type A blood.
A_2: He has type B blood.
A_3: He has type AB blood.
A_4: He has type O blood.
B: He is typed as type A.

We want to find $P[A_1 \mid B]$. We are given that

$$P[A_1] = .41 \qquad P[B \mid A_1] = .88$$
$$P[A_2] = .09 \qquad P[B \mid A_2] = .04$$
$$P[A_3] = .04 \qquad P[B \mid A_3] = .10$$
$$P[A_4] = .46 \qquad P[B \mid A_4] = .04$$

Question:

$$P[A_1 \mid B] = \frac{P[B \mid A_1]P[A_1]}{\sum_{i=1}^{4} P[B \mid A_i]P[A_i]}$$
$$= \frac{(.88)(.41)}{(.88)(.41) + (.04)(.09) + (.10)(.04) + (.04)(.46)}$$
$$= .93$$

Example

In a certain city, 30% of the people are conservatives, 50% are liberals, and 20% are independents.

Records show that in a particular election, 65% of the conservatives voted, 82% of the liberals voted, and 50% of the independents voted.

If a person in the city is selected at random and *it is learned that she did not vote in the last election*, what is the probability that she is a Liberal?

$\Pr(A_1) =$ P(Conservatives) = 0.3;

$\Pr(A_2) =$ Pr(Liberals) = 0.5;

$\Pr(A_3) =$ Pr(independent) = 0.2.

B = "vote"

$\Pr(B \mid A_1) =$ 0.65; $\Pr(B \mid A_2) =$ 0.82; $\Pr(B \mid A_3) =$ 0.5.

Question: $\Pr(A_2 \mid B^C) =$ Pr(liberal | not vote) = ?

Hint: (計算不確定是否正確，請自行檢查)

1. $\Pr(B \mid A_1)\Pr(A_1) = \Pr(B \cap A_1)$ = 0.65* 0.3= 0.195

   $\Pr(B \mid A_2)\Pr(A_2) = \Pr(B \cap A_2)$ = 0.82*0.5= 0.41

   $\Pr(B \mid A_3)\Pr(A_3) = \Pr(B \cap A_3)$ =0.5*0.2=0.1

2. $\Pr(B) = \Pr(B \cap A_2) + \Pr(B \cap A_2) + \Pr(B \cap A_3)$ = 0.705

3. $\Pr(B^C) = 1 - \Pr(B)$ = 0.295

4. $\Pr(A_2 \mid B^C) = \dfrac{\Pr(A_2 \cap B^C)}{\Pr(B^C)} = \dfrac{\Pr(A_2) - \Pr(A_2 \cap B)}{1 - \Pr(B)} = \dfrac{0.5 - 0.41}{0.295}$ =0.3051

Random variable (隨機變數)

Connection between set-based probability and random variables

➔ 從樣本空間找到可以量化 (測量或是分類) 的對應關係

Example: toss a coin three times

Two possible outcomes each time: H – (head, 正面) and T (tail, 反面)

Sample space: 8 elements $S = \{\omega_1, \ldots \omega_8\}$

$\omega_1 : HHH$, $\omega_2 : HHT$, $\omega_3 : HTH$, $\omega_4 : HTT$,

$\omega_5 : THH$, $\omega_6 : THT$, $\omega_7 : TTH$, $\omega_8 : TTT$

Define X = <u>number of heads obtained</u> in the three tosses

Possible values of X = 0, 1, 2, 3

$\{X = 0\} = \{(TTT)\}$.

$\{X = 1\} = \{(HTT), (THT), (TTH)\}$.

$\{X = 2\} = \{(HHT), (THH), (HTH)\}$.

$\{X = 3\} = \{(HHH)\}$.

Remarks:

- The value of X depends on the outcome of a random experiment.
- Thus we call X as a random variable.
- Formally, a random variable is a mapping from the sample space (the domain, 定義域) to the real line (range, 值域)

Example: $X(\omega_1) = 3$, $X(\omega_2) = 2$, $X(\omega_3) = 2$, $X(\omega_4) = 1$

$X(\omega_5) = 2$, $X(\omega_6) = 1$, $X(\omega_7) = 1$, $X(\omega_8) = 0$.

- Set-based probability focuses on the <u>domain</u> (sample space)

  while random-variable based probability focuses on the <u>range</u>.

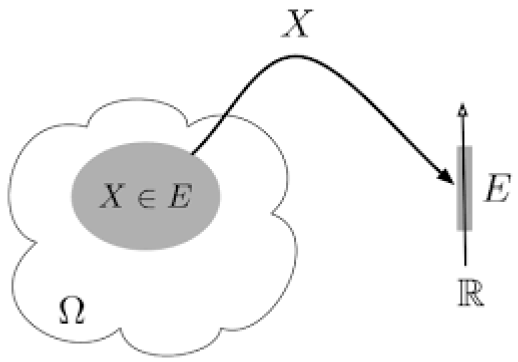
這個角度解釋了為何機率可以用集合討論 (以<u>定義域</u>的觀點), 或是以隨機變數來討論 (以<u>值域</u>的觀點).

Example: Pr(at least two heads) = $P(\{\omega_1,\omega_2,\omega_3,\omega_5\}) = \frac{4}{8}$

$$= P(X=2)+P(X=3) = \frac{3}{8}+\frac{1}{8}$$

Remarks:

- Probability based on random variables provides a <u>simplified and systematic framework</u> to study a phenomenon of interest.
- We can apply powerful mathematical skills to analyze real-world problems.



Types of random variables

1. discrete type (離散型)

   X = number of heads obtained in three tosses,

   X = number of babies born in 2012 in Taiwan

2. continuous type (連續型)

   X = height, blood pressure, temperature

3. mixed type (混合)

Definitions for discrete random variables

1. *The probability mass function (機率質量函數):*

$$p(x) = \Pr(X = x)$$

property: $\sum_{\forall x} \Pr(X=x) = 1$ (i.e. $\Pr(S)=1$)

2. *The cumulative distribution of* X *(累積分佈函數):*

$$F(x) = \Pr(X \le x) = \sum_{a \le x} \Pr(X = a)$$

Properties:

- $F(x)$ is an increasing or non-decreasing function which jumps at “mass” point x with $\Pr(X=x)>0$.

Remarks:

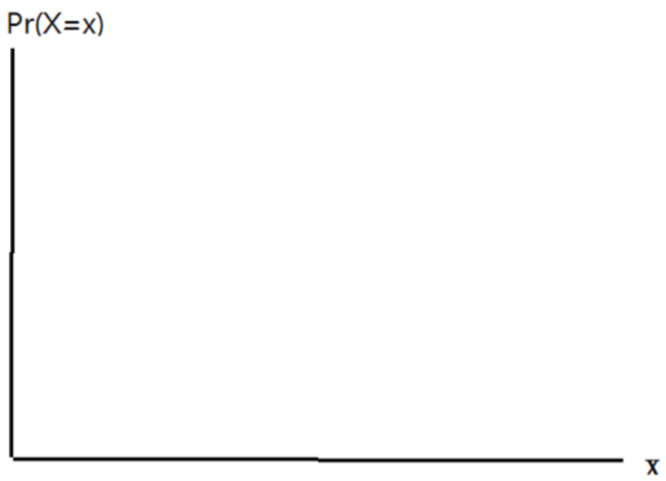
- $F(x)$ and $p(x)$ contain complete information of the random variable.
- 兩者可以互推，知道一個就可以推得另一個

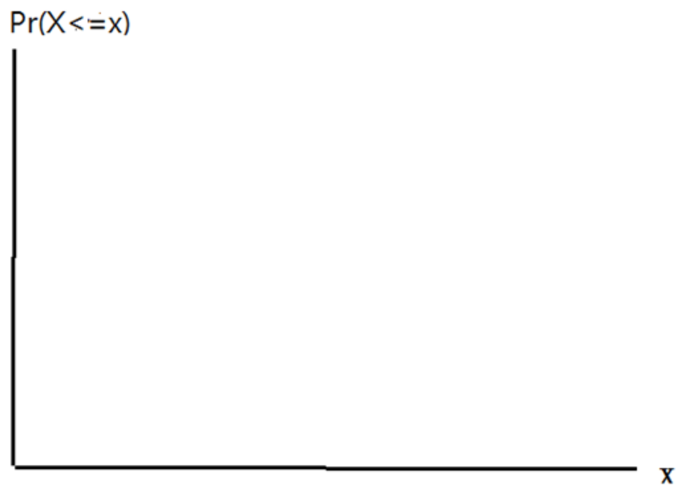
Example: Toss a coin three times, X = # of heads

$$\Pr(X=0)=\frac{1}{8},\ \Pr(X=1)=\frac{3}{8},\ \Pr(X=2)=\frac{3}{8},\ \Pr(X=3)=\frac{1}{8}$$

x value	Prob Pr(X=x)	Cumulative probability Pr(X <= x)
0	1/8	1/8
1	3/8	4/8
2	3/8	7/8
3	1/8	1

Draw Pr(X = x) and $F(x)$ (probability and distribution functions)





Exercise:

The following table shows the density for the random variable X, the number of wing beats per second of a species of large moth while in flight.

x	6	7	8	9	10
$f(x)$	.05	.1	.6	.15	?

(*a*) Find $f(10)$.
(*b*) Find $P[X \leq 8]$. Interpret this probability in the context of this problem.
(*c*) Find $P[X < 8]$.
(*d*) Find $P[X \geq 7]$.
(*e*) Find $P[X > 7]$.

Conventions of notations: (統計領域的符號使用習慣，目的是便於快速理解)

- Capital English letter (大寫英文字母): random variables
- Lower-case English letter (小寫英文字母):
  - realizations of random variables which are real numbers
- Greek letter (希臘字):
  - parameters which are constants
  - In probability calculations, parameter values are given; while in reality, parameters are unknown whose values can be estimated using statistical methods.

機率的計算會假設參數值“已知”，統計會將參數視為“未知”，予以估計

Useful parameters of a (discrete) random variable

1. Expected value of X = the mean of X → information about the "center"

$$\mu = E(X) = \sum_x x \Pr(X = x)$$:“期望值” 或是 “平均數”

2. The variance of X (變異數) → information about the "dispersion"

$$\sigma^2 = Var(X) = \sum_x (x-\mu)^2 \Pr(X = x)$$

$$= E(X^2) - \mu^2$$ → easier to work with

Proof: $Var(X) = \sum_x (x-\mu)^2 \Pr(X = x) = \sum_x \{x^2 - 2\mu x + u^2\} \Pr(X = x)$

$$= \left[\sum_x x^2 \Pr(X = x)\right] - 2\mu^2 + \mu^2 = E(X^2) - \mu^2$$

3. The rth moment of X (X 的 r 階動差)

$$E(X^r) = \sum_x x^r \Pr(X = x)$$

$E(X) = \mu$ → the first moment is the mean

$E(X^2) = \mu^2 + \sigma^2$

→ the second moment contains the information of mean and variance

$E(X^3)$ → the third moment is related to skewness

$E(X^4)$ → the third moment is related to kurtosis (尾巴的厚度)

4. $E[g(X)]$ (X 函數的期望值)

$g(.)$ is a function

$$E[g(X)] = \sum_{\forall x} g(x) \Pr(X = x)$$

Example 1: $g(X) = X$: $\mu = E(X) = \sum_{\forall x} x \Pr(X = x)$ → mean

Example 2: $g(X) = (X - \mu)^2$

- $\sigma^2 = Var(X) = E[(X - \mu)^2] = \sum_{\forall x} (x - \mu)^2 \Pr(X = x)$

  $= E(X^2) - \mu^2$

Remark: Properties of expectation

X and Y are random variables, a, b, c are constants

- $E(a) = a$ and $Var(a) = 0$
- $E(aX + bY) = aE(X) + bE(Y)$
- $Var(aX) = a^2 Var(X)$
- Covariance → later

Example 3: $g(X) = e^{tX}$ (**Moment generating function, 動差母函數)**

- $E[e^{tX}] = \sum_{\forall x} e^{tx} \Pr(X = x) = M_X(t)$ $(t \in R)$ → function of t

- We can derive the moments of X from $M_X(t)$

  $\frac{\partial M_X(t)}{\partial t}\Big|_{t=0} = E(X)$ → 一次微分, t 代 0, 得一階動差

  $\frac{\partial^2 M_X(t)}{\partial t^2}\Big|_{t=0} = E(X^2)$ → 二次微分, t 代 0, 得二階動差

  …

- It is easier to take derivatives of a function that compute the moments directly. The former requires compute the sums or perform integrations; while the latter involves taking derivatives (微分比積分簡單)

Remark:

Calculations of "Average" of the sample and "Mean" of a discrete random variables

Note: 為何 資料分析裡的平均數與"機率"裡的 mean 計算公式乍看不同?

- In the sample (data), 所有資料點出現的可能性都一樣, 所以機率是 1/n.
- Under the framework of probability, $\Pr(X = x)$ is assumed to be known.

Example: $X =$ **the score of Essay Writing for the entrance exam (會考作文)**

= 0,1,…,6

Made-up example (Suppose there are 100 students taking the exam)

score	**0**	**1**	**2**	**3**	**4**	**5**	**6**	Total
number	0	15	18	25	24	16	2	100
probability	0	.15	.18	.25	.24	.16	0.02	1

Sample average

$$\overline{X} = \frac{\sum_{i=1}^{100} x_i}{100} = \frac{0*0+1*15+2*18+3*25+4*24+5*16+6*2}{100}$$

$$= \sum_{x=0}^{6} x \Pr(X = x) = 0*0+1*0.15+2*018+3*0.25+4*0.24+5*0.16+6*0.02$$

News (實際的數字): 105 年寫作測驗考「從陌生到熟悉」

6 級分人數共 4972 人（去年 3585 人），占整體 1.8%、

5 級分有 52563 人，占 19.08%、

4 級分人數最多有 173542 人，占 62.99%、

3 級分有 27112，占 9.84%、

2 級分有 7214 人，占 2.62%、

1 級分 2611 人，占 0.95%、

0 級分 7501 人 2.72%。

- Story of Dr. **Barbara McClintock**

  "Jumping genes" – 跳躍的基因 (書中提到她對計量方法的反思)

Special discrete variables (因為有用，所以會予以命名)

- They are useful to describe the real world phenomenon
- Thus their properties are thoroughly discussed
- Given their names, you know what they are.
- 對學生來說，最困難的是遇到實際問題，不知道使用哪個分配去描述

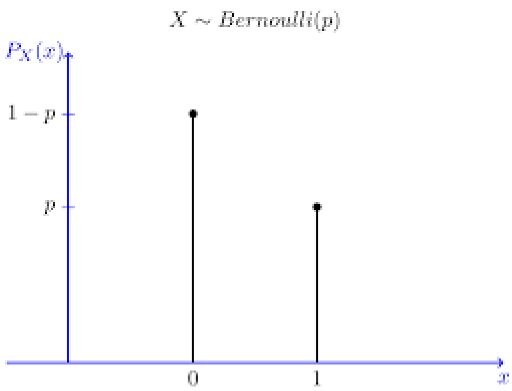
1. Bernoulli random variable (白努力隨機變數)

Bernoulli trial: a trial which contains two possible outcomes (success or failure)

Define

$Y = 1$ if "success" occurs

$= 0$ if "failure" occurs



Probability function of $Y \sim \text{Bernoulli}(p)$

- $\Pr(Y = 1) = p$ and $\Pr(Y = 0) = 1 - p$
- Write the above two formula in one equation:

$$\Pr(Y = y) = p^y (1 - p)^{1-y} \quad \text{for} \quad y = 0,1$$

Derivations:

$$E(Y) = 1 \times \Pr(Y=1) + 0 \times \Pr(Y=1) = p$$

$$E(Y^2) = 1^2 \times \Pr(Y=1) + 0^2 \times \Pr(Y=1) = p$$

$$Var(Y) = E(Y^2) - [E(Y)]^2 = p - p^2 = p(1-p)$$

Remarks:

- $Var(Y)$ achieves its maximum if $p = 1 - p = \frac{1}{2}$

  （旗鼓相當的比賽，結果最難預料）

- Odds = $\frac{p}{1-p}$

2. Binomial random variable (二項式隨機變數)

滿足 Binomial 分配的基本要素 (Binomial Setting)

THE BINOMIAL SETTING

1. There are a fixed number n of observations.
2. The n observations are all **independent.** That is, knowing the result of one observation tells you nothing about the other observations.
3. Each observation falls into one of just two categories, which for convenience we call "success" and "failure."
4. The probability of a success, call it p, is the same for each observation.

Determine whether the random variable defined in each example is a Binomial random variable.

Example: A couple have 3 children, X = # of boys

Ans: YES

Example: A couple decides to continue to have children until their first girl is born.

X = # of children

Ans: NO, n is not fixed.

Example: Randomly select 10 persons from a sample consisting of 20 males and 20 females. X = # of males

Ans: NO, the trials are not independent; the success probability is different in each selection. (每次機率不同，與前面結果有關)

Example: Randomly select 10 persons from a sample consisting of 5000 males and 5000 females. X = # of males
Ans: X "approximately" follows a Binomial distribution . (每次機率不同，與前面結果有關，但因為總數很大，影響很小)

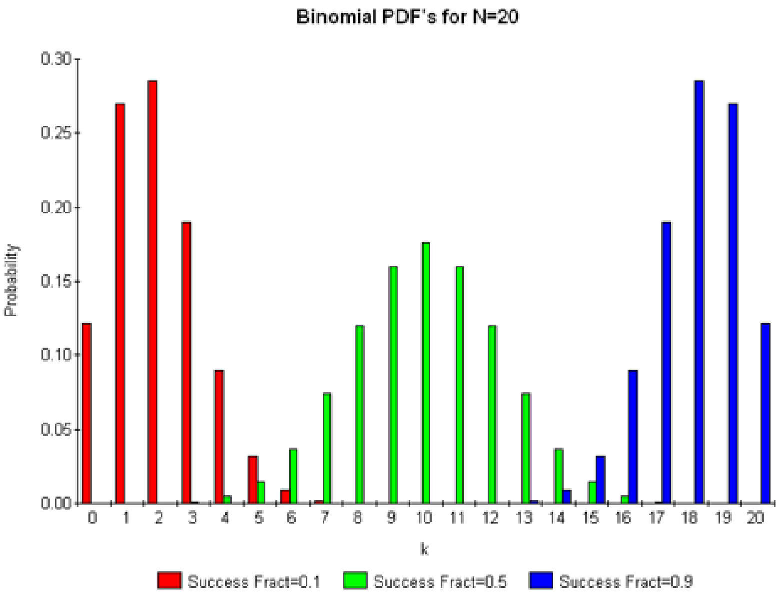
Probability function for $X \sim Binomial(n, p)$:

$$\Pr(X = x) = \binom{n}{x} p^x (1-p)^{n-x} \quad \text{for} \quad x = 0,1,\ldots,n$$

Derivations:

- the probability of getting $(S, S, \ldots, S, F, F, \ldots, F)$ → $p^x (1-p)^{n-x}$
- the probability of getting $(S, F, \ldots, S, S, F, \ldots, F)$ → $p^x (1-p)^{n-x}$
- In n trials, the number of combinations for x successes and $(n-x)$ failure → $\binom{n}{x}$

- probability of the union of the $\binom{n}{x}$ disjoint events = the sum of each probability ($p^x (1-p)^{n-x}$)
- Left: $X \sim Binomial(n = 20, p = 0.1)$
- Middle: $X \sim Binomial(n = 20, p = 0.5)$ → symmetric
- Right: $X \sim Binomial(n = 20, p = 0.9)$



C. Important properties of X ~ Binomial(n,p)

$$E(X)=\sum_{x=0}^{x=n} x\cdot\binom{n}{x}p^{x}(1-p)^{n-x}=n\cdot p$$

$$Var(X)=\sum_{x=0}^{x=n} x^{2}\cdot\binom{n}{x}p^{x}(1-p)^{n-x}-(n\cdot p)^{2}=n\cdot p\cdot(1-p)$$

$$F(a)=\sum_{x=0}^{x=a} x\cdot\binom{n}{x}p^{x}(1-p)^{n-x}\quad \text{(no explicit form)}$$

skill: $\Pr(X\geq 1)=1-\Pr(X=0)$ (右式比較好算)

Proof: (2 ways)

a. direct proof → not easy to calculate the sum of a complicated series

(數學上求級數和，並不容易，往往沒有 closed-form)

b. From its relationship with Bernoulli random variables → easy

令 $X=Y_1+\ldots+Y_n$, where $Y_i\sim Bernoulli(p)$.

know: $E(Y_i)=1\cdot p+0\cdot(1-p)=p$

$$Var(Y_i)=1^2\cdot p+0^2\cdot(1-p)-p^2=p(1-p)$$

→ $E(X)=E(Y_1+\ldots+Y_n)=n\cdot p$

→ $Var(X)=Var(Y_1+\ldots+Y_n)=\sum_{i=1}^{n}Var(Y_i)+\sum\sum_{i\neq j}Cov(Y_i,Y_j)$

$=n\cdot p(1-p)+0$ (note Y_i & Y_j are independent, second term = 0)

Useful properties for expectations (revisited)

1.

$E(X+Y)=E(X)+E(Y)$

$E(c)=c$ (常數的期望值)

2. When c is a constant, $Var(c)=0$

Supplementary Knowledge: Two random variables (will not be on the exam)

$$Cov(X,Y) = E[(X-\mu_x)(Y-\mu_Y)]$$

$$= \sum_x \sum_y (x-\mu_x)(y-\mu_y)\Pr(X=x, Y=y)$$

Correlation coefficient:

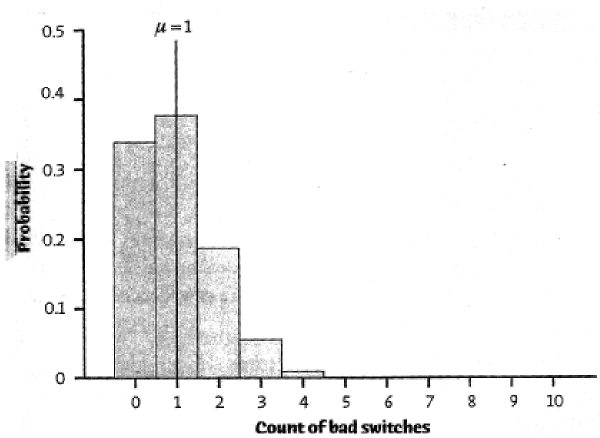
$$\rho_{X,Y} = \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}} = \frac{E[(X-\mu_x)(Y-\mu_Y)]}{\sqrt{E[(X-\mu_x)^2]E[(Y-\mu_Y)^2]}}$$

Remarks:

- If X and Y are independent → $Cov(X,Y)=0$ but not vice versa.
- $Var(X) = Cov(X,X)$

Exercises:

Binomial(n=10,p=0.1)



Example: Inspecting switches

Continuing Example 12.5, the count X of bad switches is binomial with $n = 10$ and $p = 0.1$. The histogram in Figure 12.2 displays this probability distribution. (Because probabilities are long-run proportions, using probabilities as the heights of the bars shows what the distribution of X would be in very many repetitions.) The distribution is strongly skewed. Although X can take any whole-number value from 0 to 10, the probabilities of values larger than 5 are so small that they do not appear in the histogram.

The mean and standard deviation of the binomial distribution in Figure 12.2 are

$$\mu = np = (10)(0.1) = 1$$

$$\sigma = \sqrt{np(1-p)}$$

$$= \sqrt{(10)(0.1)(0.9)} = \sqrt{0.9} \doteq 0.9487$$

EXAMPLE 12.1 Blood types

Genetics says that children receive genes from their parents independently. Each child of a particular pair of parents has probability 0.25 of having type O blood. If these parents have 5 children, the number who have type O blood is the count X of successes in 5 independent trials with probability 0.25 of a success on each trial. So X has the binomial distribution with $n = 5$ and $p = 0.25$.

Pr(<u>None</u> of the children have blood type O) $= \left(\frac{3}{4}\right)^5$

Pr(<u>Exactly</u> one child has blood type O) = $\binom{5}{1}\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^4$

Pr(<u>At least</u> two children have blood type O) = $1 - \mathrm{P}(X=0) - P(X=1)$

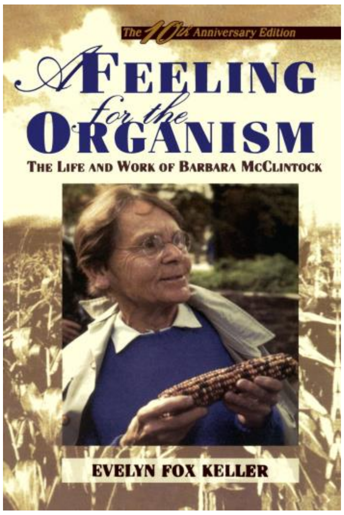
$$\mathrm{P}(X=0) = \binom{5}{0}\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^5;\quad \mathrm{P}(X=1) = \binom{5}{1}\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^4$$

Pr(<u>At most</u> two children have blood type O) = $\mathrm{P}(X=0) + P(X=1) + P(X=2)$

$$\mathrm{P}(X=2) = \binom{5}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3$$

玉米田的先知: 異類遺傳學家麥克林托克 (Dr. Barbara McClintock)

- 現下通行的分類制度,以及對統計數字的迷信,都在
- 鼓勵研究人員忽略差異,“只把它歸類為是一項例外,異常或是一種汙染”
- 以上這種態度,“無論巨細, 他們都會錯過真相”



Barbara McClintock (1902- 1992, Huntington, New York)

- American scientist whose discovery in the 1940s and ’50s of mobile genetic elements, or “jumping genes,” won her the Nobel Prize for Physiology or Medicine in 1983.
- After attending high school, she enrolled as a biology major at Cornell University in 1919. She received a B.S. in 1923, a master’s degree two years later, and, having specialized in cytology, genetics, and zoology, a Ph.D. in 1927. During graduate school she began the work that would occupy her entire professional life: the chromosomal analysis of corn (maize). She used a microscope and a staining technique that allowed her to examine, identify, and describe individual corn chromosomes.
- In 1931 she and a colleague, Harriet Creighton, published “A Correlation of Cytological and Genetical Crossing-over in Zea mays,” a paper that established that chromosomes formed the basis of genetics. Based on her experiments and publications during the 1930s, McClintock was elected vice president of the Genetics Society of America in 1939 and president of the Genetics Society in 1944. She received a Guggenheim Fellowship in 1933 to study in Germany, but she left early because of the rise of Nazism.
- When she returned to Cornell, her alma mater, she found that the university would not hire a female professor. The Rockefeller Foundation funded her research at Cornell (1934–36) until she was hired by the University of Missouri (1936–41).
- In 1941 McClintock moved to Long Island, New York, to work at the Cold Spring Harbor Laboratory, where she spent the rest of her professional life.
- In the 1940s, by observing and experimenting with variations in the coloration of kernels of corn, she discovered that genetic information is not stationary. By tracing pigmentation changes in corn and using a microscope to examine that plant’s large chromosomes, she isolated two genes that she called “controlling elements.” These genes controlled the genes that were actually responsible for pigmentation. McClintock found that the controlling elements could move along the chromosome to a different site, and that these changes affected the behaviour of neighbouring genes. She suggested that these transposable elements were responsible for new mutations in pigmentation or other characteristics.
- **McClintock’s work was ahead of its time and was for many years considered too radical—or was simply ignored—by her fellow scientists.** Deeply disappointed with her colleagues, she stopped publishing the results of her work and ceased giving lectures, though she continued doing research. Not until the late 1960s and ’70s, after biologists had determined that the genetic material was DNA, did members of the scientific community begin to verify her early findings. When recognition finally came, McClintock was inundated with awards and honours, most notably the 1983 Nobel Prize for Physiology or Medicine. She was the first woman to be the sole winner of this award.

對照下，莫納德和傑哥布的研究精神則完全受分子生物學及中心教條的影響。結果他們卻和麥克林托克一樣，堅信在遺傳作用裡，存在著調節性的機制，而其中又牽涉到兩種不同的調節性基因。

麥克林托克不僅是在另一個大陸上進行研究工作，研究的對象也是另一種遠較細菌複雜的生物；我們更可以說，她所屬的生物學世界和莫納德與傑哥布的世界完全是兩回事。後者是分子生物學家，專心研究的是大腸桿菌；麥氏則是傳統生物學家，研究的是玉米。

莫納德與傑哥布利用生化分析法決定基因交換的效果，他們尋找的是一種分子的機制；麥克林托克卻運用自然學者熟悉的研究技巧，觀察玉米葉及玉米粒上在染色後形成的標記及圖案，以及玉米染色體在顯微鏡下的排列形式。她尋找的是一種概念性的架構，這架構植基於所推論出的結果，與其功能的互相吻合。

本書最重要的一項企圖，是要描述這位科學家個人的「風格」。

風格，一部分來自學習，一部分則是自我陶冶的結果。而麥克林托克的風格，在現代生物研究界，當屬極端的異類。她執著於個別的、不同於常的現象，她曾經說過：

「最重要的，就是要訓練自己發展出一種能力，能夠看出那一粒與眾不同的玉米粒，然後追根究柢。」「如果某樣東西異於常類，一定有它的道理，你就應該把它搞清楚。」麥克林托克認爲現下通行的分類制度，以及對統計數字的迷信，都在鼓勵研究人員忽略差異，「只把它歸納成是一項例外、異常或是一種污染。」她認爲這種態度會造成極嚴重的後果，「無論巨細，他們都會錯過真象。」

從她還在當研究生的時候開始，對於研究工作裡努力的部分，她一向親自動手，無論是多麼繁重或枯燥的事，都不假他人之手。所有科學家都是這樣起步的，但大多數的

或許這個問題的答案，就在於她對每一株玉米親密且徹底的了解。她有一位同儕曾經說過，麥克林托克可以爲她研究過的每一株玉米寫「傳記」。

她尊重玉米複雜的運作方式，就像她尊重人類心智奧祕難解的運作方式一樣。但她也很確信，只要多留心注意，她就可以信任自己的直覺。在往後的歲月裡，這一份自信成爲她生命力最大的泉源。

早期遺傳學與演化學的關係，和遺傳學與胚胎學的關係如出一轍。第一代的孟德爾學派認爲天擇根本不足以解釋演化上的變化，一直等到三〇年代，才出現一項成功地把遺傳學及演化學結合在一起的理論。

該理論強調演化是族羣的現象，因此在研究演化的改變時，焦點應該放在遺傳特徵在族羣中的分布。本著這項認知，英國遺傳學家霍登（J. B. S Haldane, 1892–1964）、費雪（Sir Ronald Fisher, 1890–1962）、與美國統計學者暨遺傳學家萊特（Sewall Wright, 1889–1988）等人共同發展出一套族羣遺傳學的數學理論，成爲將過去各自獨立的遺傳學、生物統計學、古生物學以及分類學綜合起來的基礎。

因此麥克林托克對結合遺傳學與演化學的族羣遺傳學懷疑更深，她認爲那一整套分析系統都架構在不牢靠的觀念上，族羣遺傳學「所討論的實體是一堆符號，這些符號以那種方式處理有不足之處。」

與眾不同的玉米仁，然後追根究柢。」她覺得很多同儕常常只熱中於「計算」，而忽略了那一顆異常的玉米仁。她除了這一點與眾不同之外，還很早就開始對胚胎學產生興趣，而且從未稍減。