Question 1: Getting into college

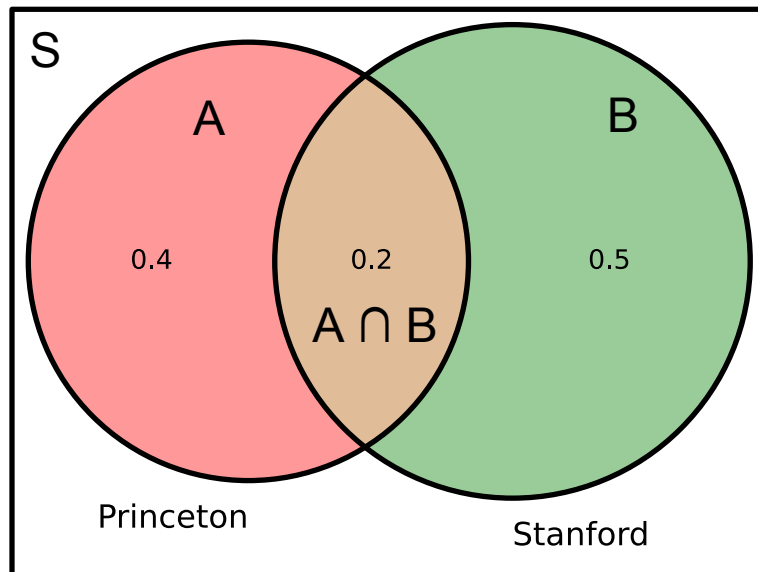We define two events A and B

- A = admitted by Princeton
- B = admitted by Stanford

Therefore

- $P(\text{A}) = 0.4$
- $P(\text{B}) = 0.5$
- $P(\text{A} \cap \text{B}) = 0.2$

(a) Make a Venn diagram with the probabilities given marked

(b) What is the probability that neither university admits Ramon?

From the Venn diagram, we know

- $A \cup B$ = At least one of university accepts him
- $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.5 - 0.2 = 0.7$

Therefore, the probability that neither university admits him is

- $1 - P(A \cup B) = 1 - 0.7 = 0.3$
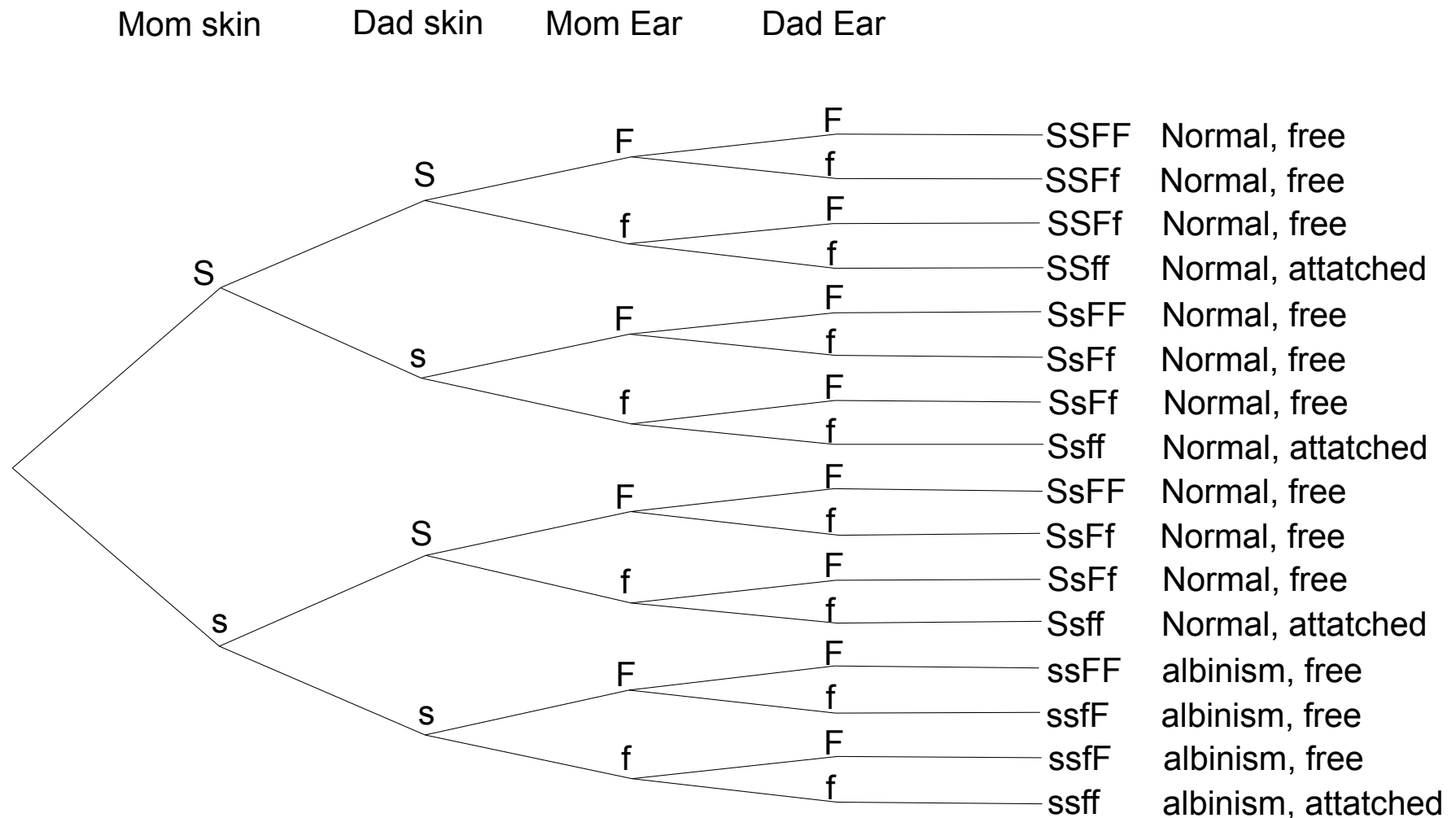
(c) What is the probability that he gets into Stanford but not Princeton?

The answer is

- $P(B) - P(A \cap B) = 0.5 - 0.2 = 0.3$

Question 2: Skin and Earlobes

(a) Draw the tree to represent the possible outcomes for the offspring.

Mom skin	Dad skin	Mom Ear	Dad Ear		

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                                              F
                                    F ─────────────── SSFF   Normal, free
                          S                   f
                                              ─────── SSFf   Normal, free
                                              F
                                    f ─────────────── SSFf   Normal, free
                 S                            f
                                              ─────── SSff   Normal, attatched
                                              F
                                    F ─────────────── SsFF   Normal, free
                          s                   f
                                              ─────── SsFf   Normal, free
                                              F
                                    f ─────────────── SsFf   Normal, free
                                              f
                                              ─────── Ssff   Normal, attatched
                                              F
                                    F ─────────────── SsFF   Normal, free
                          S                   f
                                              ─────── SsFf   Normal, free
                                              F
                                    f ─────────────── SsFf   Normal, free
                 s                            f
                                              ─────── Ssff   Normal, attatched
                                              F
                                    F ─────────────── ssFF   albinism, free
                          s                   f
                                              ─────── ssfF   albinism, free
                                              F
                                    f ─────────────── ssfF   albinism, free
                                              f
                                              ─────── ssff   albinism, attatched
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(b) Use the tree to find the probability that

As you can see, there are 16 possibilities totally.

i. The child will be albino with attached earlobes

- $P = \frac{1}{16}$

ii. The child will have normal skin and free earlobes

- $P = \frac{9}{16}$

iii. The child will be albino

- $P = \frac{4}{16} = \frac{1}{4}$

iv. The child will have free earlobes
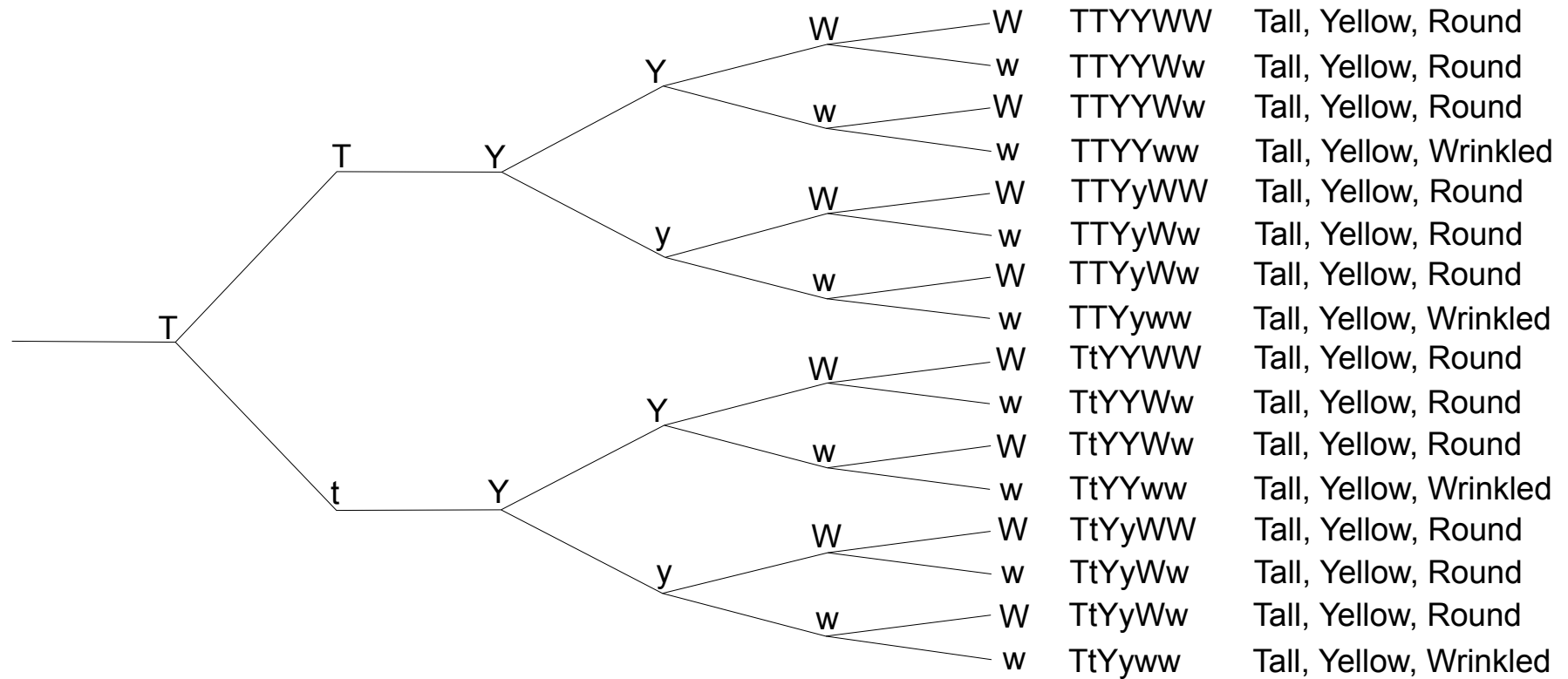
- $P = \frac{12}{16} = \frac{3}{4}$

Question 3: Pea plant

(a) Describe each of the parent plants relative to the three characteristics mentioned

- TTYYWw: Tall, yellow and round seeds
- TtYyWw: Tall, yellow and round seeds

(b) Draw a tree to represent the possible ways in which the cross-match can occur

(c) Describe the plant associated with each path through the tree

			W	W	TTYYWW	Tall, Yellow, Round
		Y		w	TTYYWw	Tall, Yellow, Round
			w	W	TTYYWw	Tall, Yellow, Round
	T	Y		w	TTYYww	Tall, Yellow, Wrinkled
			W	W	TTYyWW	Tall, Yellow, Round
		y		w	TTYyWw	Tall, Yellow, Round
			w	W	TTYyWw	Tall, Yellow, Round
T				w	TTYyww	Tall, Yellow, Wrinkled
			W	W	TtYYWW	Tall, Yellow, Round
		Y		w	TtYYWw	Tall, Yellow, Round
			w	W	TtYYWw	Tall, Yellow, Round
	t	Y		w	TtYYww	Tall, Yellow, Wrinkled
			W	W	TtYyWW	Tall, Yellow, Round
		y		w	TtYyWw	Tall, Yellow, Round
			w	W	TtYyWw	Tall, Yellow, Round
				w	TtYyww	Tall, Yellow, Wrinkled

(d) Use the tree to find the probability that the cross-match will result in a tall pea plant.

All pea plants are tall.

- $P = 1$

(e) What is the probability that the cross-match will result in a tall, yellow, wrinkled pea plant?

- $P = \frac{4}{16} = \frac{1}{4}$

(e) What is the probability that the cross-match will produce a green pea plant?

There will no green plant.

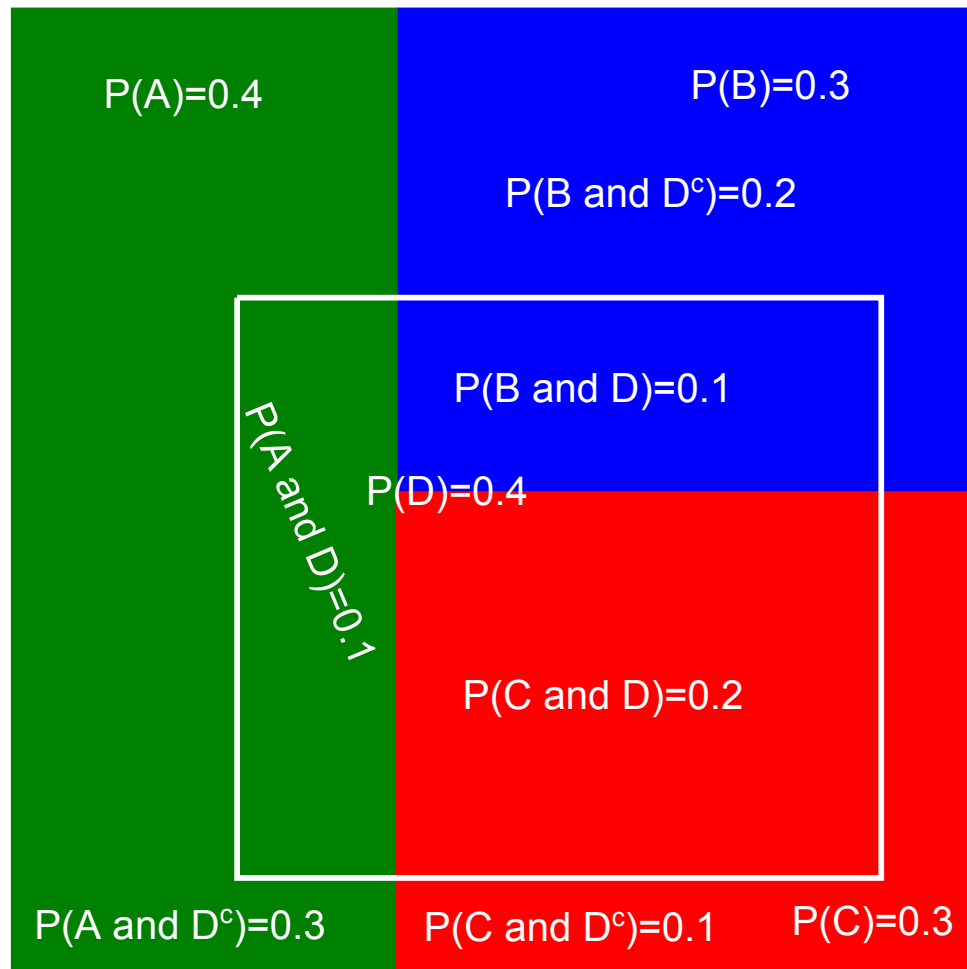## Question 4: Preparing for the GMAT.

The tree diagram is

Event A		Event B	P(A and B)

Event A

Event B

P(A and B)

GMAT >= 600 _____ 0.2

0.5

Undergraduate

0.5

GMAT < 600 _____ 0.2

0.4

0.6

GMAT >= 600 _____ 0.42

0.7

Graduate

0.3
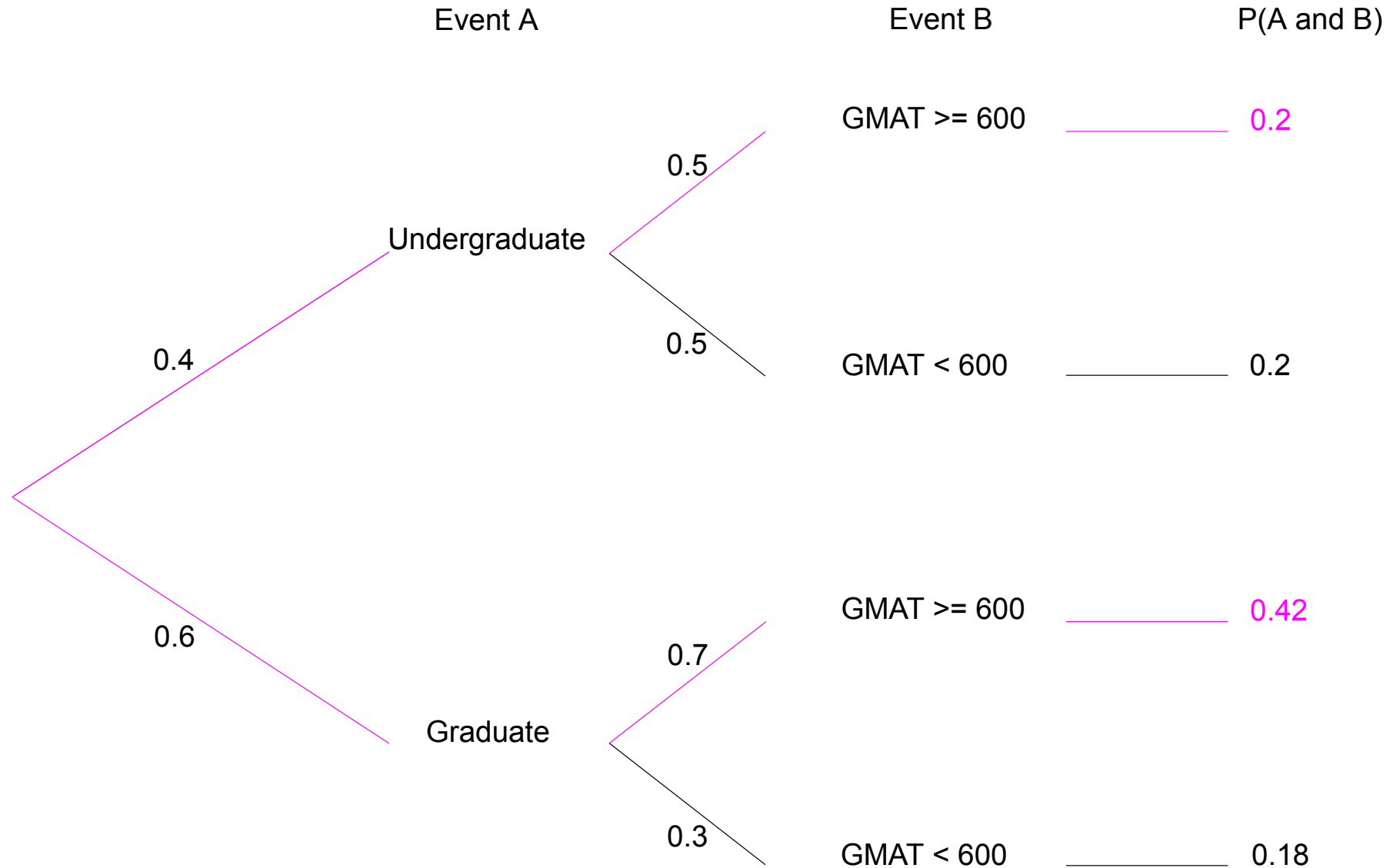
GMAT < 600 _____ 0.18

As you can see in the above diagram, $P(\text{GMAT} >= 600) = 0.2 + 0.42 = 0.62$

- Therefore, 62% of all customers score at least 600 on the GMAT.

Question 5: Screening job applicants.

Venn diagram as the following:

- A is green.
- B is blue.
- C is red.
- D is the inner white rectangle.
- $P(D) = P(A \text{ and } D) + P(B \text{ and } D) + P(C \text{ and } D) = 0.4$

P(A)=0.4

P(B)=0.3

P(B and D^c)=0.2

P(A and D)=0.1

P(B and D)=0.1

P(D)=0.4

P(C and D)=0.2

P(A and D^c)=0.3

P(C and D^c)=0.1

P(C)=0.3

Question 6: Randomized response method of getting honest anwers to sensitive questions

(a) Use the fact that the event "answer yes" is the event "anser yes and answered A" or "answer yes and answered B" to show that $P[\text{answer yes}|\text{answered B}]$ equals $\dfrac{P[\text{answer yes}] - P[\text{answer yes}|\text{answered A}]P[\text{answered A}]}{P[\text{answered B}]}$
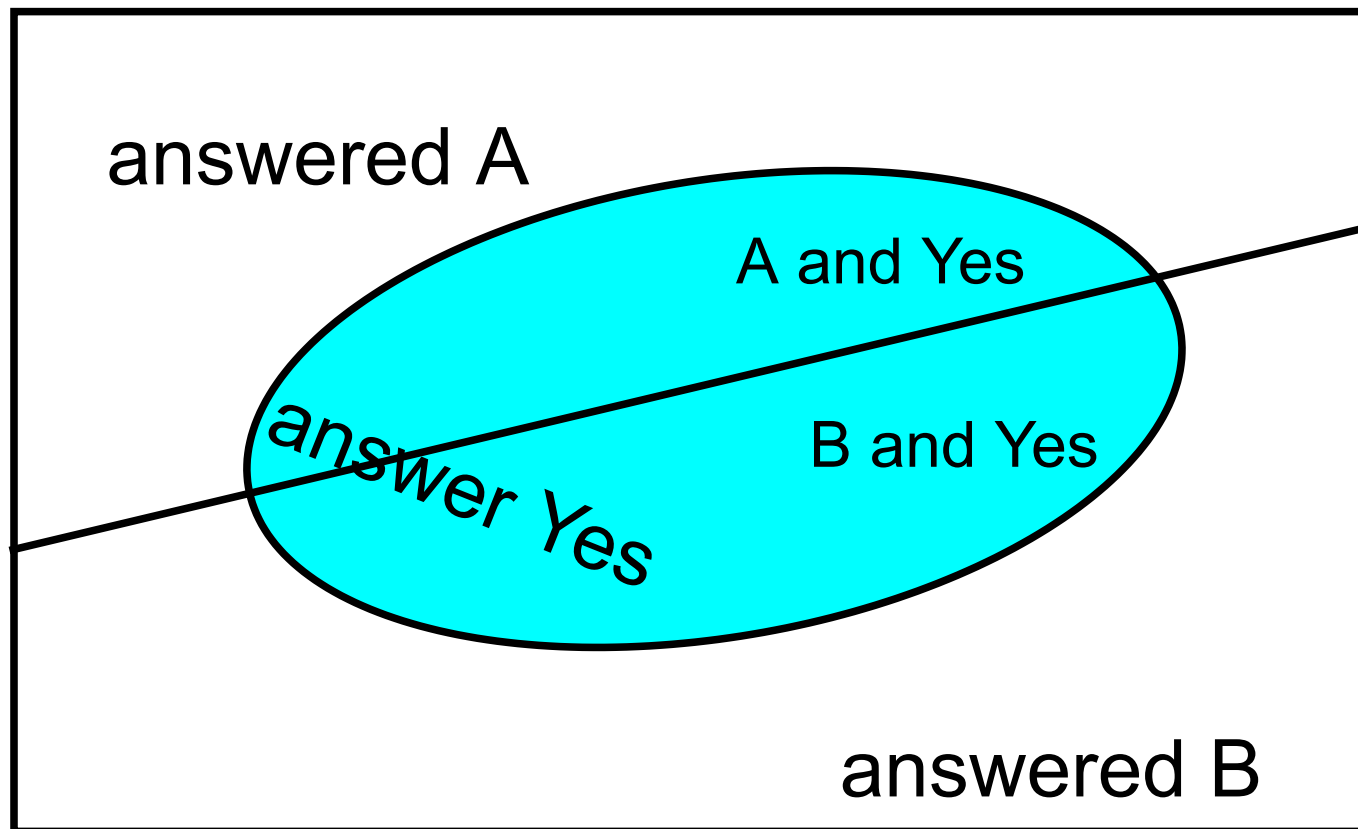
1. According to the general multiplication rule: $P(A \; and \; B) = P(A)P(B|A)$, we get $P(B|A) = \frac{P(A \; and \; B)}{P(A)}$. Therefore, we want to find

   - $P[\text{answer yes}|\text{answered B}] = \frac{P[\text{answer yes } and \text{ answered B}]}{P[\text{answered B}]}$

2. We also know that $P[\text{answer yes}] = P[\text{answer yes } and \text{ answered A}] + P[\text{answer yes } and \text{ answered B}]$, so

   - $P[\text{answer yes } and \text{ answered B}] = P[\text{answer yes}] - P[\text{answer yes } and \text{ answered A}]$

3. By using the general multiplication rule again,

   - $P[\text{answer yes } and \text{ answered A}] = P[\text{answer yes}|\text{answered A}]P[\text{answered A}]$

4. Substitute (4) into (3), we get

   - $P[\text{answer yes } and \text{ answered B}] = P[\text{answer yes}] - P[\text{answer yes}|\text{answered A}]P[\text{answered A}]$

5. Substitute (5) into (1), we finally get

- $P[\text{answer yes}|\text{answered B}] = \dfrac{P[\text{answer yes}] - P[\text{answer yes}|\text{answered } A]P[\text{answered } A]}{P[\text{answered B}]}$

(b) If this technique is tried on 100 subjects and 60 answered yes, find the approximate probability that a person randomly selected from the group has intentionally filed a fraudulent claim.

1. This question is to ask $P[\text{answer yes}|\text{answered B}]$
2. We already know that $P[\text{answer yes}|\text{answered } A] = 0.5$
3. From flipping coin, we know $P[\text{answered A}] = 0.5$ and $P[\text{answered B}] = 0.5$
4. From the description in (b), we know that $P[\text{answer yes}] = \dfrac{60}{100} = 0.6$
5. Therefore, $P[\text{answer yes}|\text{answered B}] = \dfrac{P[\text{answer yes}] - P[\text{answer yes}|\text{answered } A]P[\text{answered } A]}{P[\text{answered B}]} = \dfrac{0.6 - 0.5 \times 0.5}{0.5} = \dfrac{0.6 - 0.25}{0.5} = 0.7$
6. The probability that a person randomly selected from the group has intentionally filed a fraudulent claim is 0.7