Q1: Use Z table and find the values of $p = \Pr(Z > k)$

(1) If $\Pr(Z > k) = 0.2$, $k =$?

From Z table, we get

$$\Pr(Z \leq 0.84) = 0.7995$$
$$\Pr(Z \leq 0.85) = 0.8023$$

so, it is reasonable to assume

$$\Pr(Z \leq 0.845) \approx 0.80 \tag{1}$$

and

$$\Pr(Z > 0.845) = 1 - \Pr(Z \leq 0.845) \approx 1 - 0.80 = 0.2 \tag{2}$$

Therefore

$$k = 0.845 \tag{3}$$

(2) If $\Pr(Z > k) = 0.1$, $k =$?

From Z table, we get

$$\Pr(Z \leq 1.28) = 0.8997$$
$$\Pr(Z \leq 1.29) = 0.9015$$

so, it is reasonable to assume

$$\Pr(Z \leq 1.285) \approx 0.9 \tag{4}$$

and

$$\Pr(Z > 1.285) = 1 - \Pr(Z \leq 1.285) \approx 1 - 0.90 = 0.1 \tag{5}$$

Therefore

$$k = 1.285 \tag{6}$$

(3) If $\Pr(Z > k) = 0.05$, $k =$?

From Z table, we get

$$\Pr(Z \leq 1.64) = 0.9495$$
$$\Pr(Z \leq 1.65) = 0.9505$$

so, it is reasonable to assume

$$\Pr(Z \leq 1.645) \approx 0.95 \tag{7}$$

and

$$\Pr(Z > 1.645) = 1 - \Pr(Z \leq 1.645) \approx 1 - 0.95 = 0.1 \tag{8}$$

Therefore

$$k = 1.645 \tag{9}$$

(4) If $\Pr(Z \leq k) = 0.2$, $k =?$

We have already known

$$\Pr(Z \leq 0.845) \approx 0.80 \tag{10}$$

Because of the symmetry of Normal distribution

$$\Pr(Z \leq k) = 1 - \Pr(Z \leq 0.845) = 0.2$$

Therefore

$$k = -0.845 \tag{11}$$

(5) If $\Pr(Z \leq k) = 0.1$, $k =?$

We have already known

$$\Pr(Z \leq 1.285) \approx 0.9 \tag{12}$$

Because of the symmetry of Normal distribution

$$\Pr(Z \leq k) = 1 - \Pr(Z \leq 1.285) = 0.1$$

Therefore

$$k = -1.285 \tag{13}$$

(6) If $\Pr(Z \leq k) = 0.05$, $k =?$

We have already known

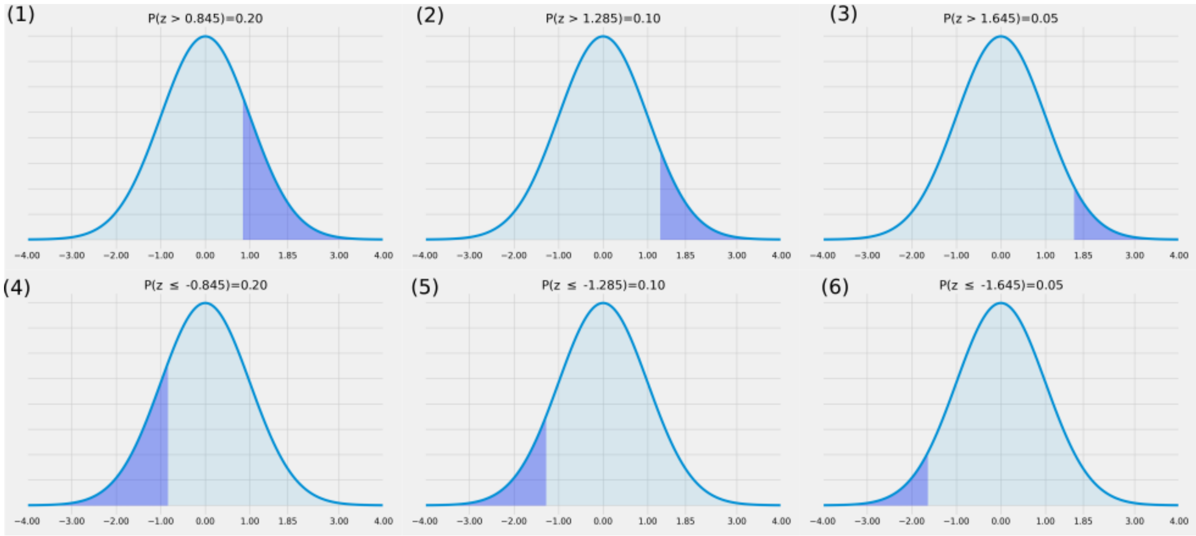
$$\Pr(Z \leq 1.645) \approx 0.95 \tag{14}$$

Because of the symmetry of Normal distribution

$$\Pr(Z \leq k) = 1 - \Pr(Z \leq 1.645) = 0.05$$

Therefore

$$k = -1.645 \tag{15}$$



Q2

Deer mice are small rodents native to North America. Their body lengths (excluding tail) are known to vary approximately Normally with mean $\mu = 86$ mm and standard deviation $\sigma = 8$ mm. Deer mice are found in diverse habitats and exhibit different adaptations to their environment. A random sample of 14 deer mice in a rich forest habitat gives an average body length of $\bar{x} = 91.1$ mm. Assume that the standard deviation σ of all deer mice in this area is also 8 mm.

The population distribution is

$$N(\mu = 86, \sigma = 8) \tag{16}$$

(a) What is the standard deviation of the mean length of $\bar{x}$

If individual observations have the $N(\mu, \sigma)$ dsitribution, then the sample mean $\bar{x}$ of an SRS of size n has the $N(\mu, \frac{\sigma}{\sqrt{n}})$ distribution.

Therefore, the standard deviation of the mean length of $\bar{x}$</h4> is

$$\frac{\sigma}{\sqrt{n}} = \frac{8}{\sqrt{14}} = 2.138 \tag{17}$$

(b) What critical value do you need to use in order to compute a 95% confidence interval for the mean μ?

The critical value is

$$1.96 \times 2.138 = 4.19 \tag{18}$$

(c) Give a 95% confidence interval for the mean body length of all deer mice in the forest habitat.

95% confidence interval for μ is

$$91.1 \pm 4.19 \tag{19}$$

Q3

The 14 deer mice described in the previous exercise had average body length of $\bar{x}$=91.1 mm. Assume that the standard deviation of body length in the population of all deer mice in the forest habitat is the same as the $\sigma = 8$ mm for the general deer mouse population.

(a) Following your approach in the previous exercise, now give a 90% confidence interval for the mean body length of all deer mice in the forest habitat.

The critical value is

$$1.645 \times 2.138 = 3.52 \tag{20}$$

95% confidence interval for μ is

$$91.1 \pm 3.52 \tag{21}$$

(b) This confidence interval is shorter than your interval in the previous exercise, even though the intervals come from the same sample. Why does the second interval have a smaller margin of error?

The confidence is the probability that the interval will capture μ. When we decrease the confidence, it means that the probability that the confidence interval will capture μ decreases. The case happens when the interval becomes shorter.

As you can see in the following figure, the green line(95% C-interval) has higher chance to capture μ than the magenta line(90% C-interval).