Statistical Mechanics Notes

Pratice and Examples

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[1] www.example.com

Dedicated to Calvin and Hobbes.

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Preface

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Structure of book

Each unit will focus on <SOMETHING>.

About the companion website

The website[1] for this file contains:

- A link to (freely downlodable) latest version of this document.
- Link to download LaTeX source for this document.
- Miscellaneous material (e.g. suggested readings etc).

Acknowledgements

- A special word of thanks goes to Professor Don Knuth[2] (for TEX) and Leslie Lamport[3] (for LATEX).
- I'll also like to thank Gummi[4] developers and LaTeXila[5] development team for their awesome LATEX editors.
- I'm deeply indebted my parents, colleagues and friends for their support and encouragement.

Amber Jain
http://amberj.devio.us/

[1] https://github.com/amberj/latex-book-template
[2] http://www-cs-faculty.stanford.edu/~uno/
[3] http://www.lamport.org/
[4] http://gummi.midnightcoding.org/
[5] http://projects.gnome.org/latexila/

1

Introductory Chapter

> *"Available energy is the main object at stake in the struggle for existence and the evolution of the world."*
>
> – Ludwig Boltzmann

1.1 Boltzmann's Distribution

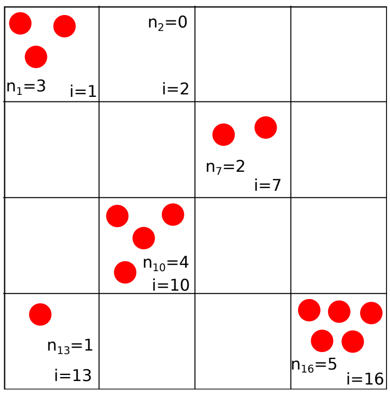
1.1.1 Gas Example

N Gas particles occupy small volumetric cells $i = 1, 2, 3, \cdots, s$ of phase space, with occupation number n_i. Particle number is conserved so

$$\sum_i n_i = N \tag{1.1}$$

The number of ways W that any particular distribution $\{n_1, n_2, ..., n_s\}$ of particles will fall within their respective volumetric cells in phase space is given by the multinomial formula

$$W = \frac{N!}{(n_1!n_2!...n_s!)} \tag{1.2}$$



Taking the logarithm of W and approximating the factorial for large N using Stirling's formula gives

$$\log W = \log N! - \sum_i \log n_i! \approx -N \sum_i \frac{n_i}{N} \log (\frac{n_i}{N}) = -N \sum_i p_i \log p_i \quad (1.3)$$

where $p_i = \frac{n_i}{N}$ is taken to be the probability that a particle is in cell i, provided N is sufficiently large. The entropy S is defined by

$$S = -k_B \sum_i p_i \log p_i \quad (1.4)$$

There are two constraints

$$\sum_i p_i = 1 \quad (1.5)$$

$$\sum_i p_i \epsilon_i = \bar{\epsilon} \quad (1.6)$$

where ϵ_i is the total energy of cell i, and $\bar{\epsilon}$ is the average energy per particle.

1.2 Shannon's information

Consider a signal that passes through a commnication channel Suppose the

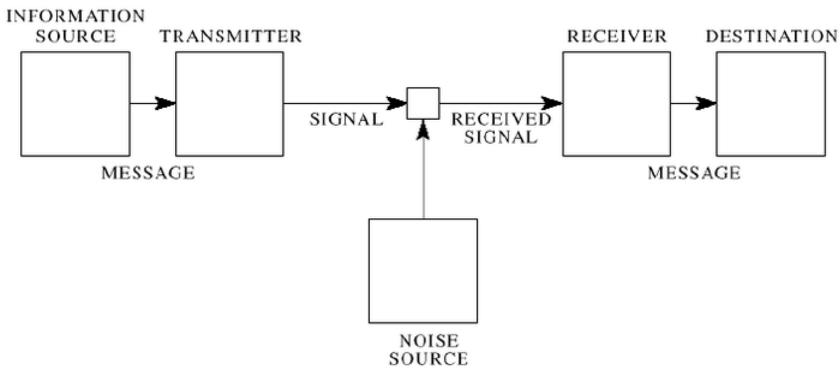


Figure 1.1: Shannon's channel

message is a linear string of symbols that is M characters long, where each character is drawn independently from an r-letter alphabet, with probability p_i. Let $m_i (i = 1, 2, 3, ..., r)$ represent the number of times that the ith type of character is observed in the message, as shown in Figure 1.2.

When M is large, the most likely M-letter message will have the composition $m_i = M p_i$, and this occurs with probability

$$P \equiv p_1^{m_1} \cdots p_r^{m_r} = p_1 M p_1 \cdots p_r M p_r \quad (1.7)$$

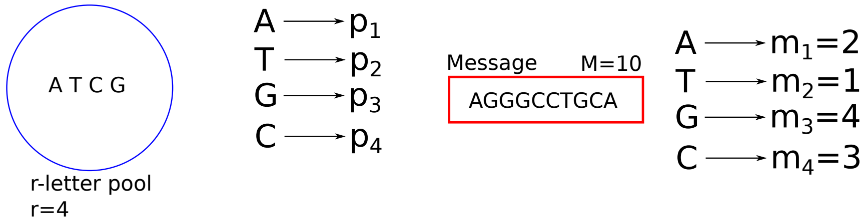


Figure 1.2: The illustration of message

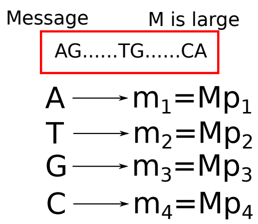


Figure 1.3: When M is large

In the limiting case of an alphabet having only a single letter, the probability of knwoing the message is

$$P = 1 \tag{1.8}$$

because there is only one possible string of characters. The larger the alphabet is(r becomes larger), the smaller the value of P and the greater the uncertainty of receiving a particular message. The logarithm of $1/P$

$$H = \log(1/P) = -M \sum_i p_i \log p_i \tag{1.9}$$

is sometimes called the uncertainty or missing information.

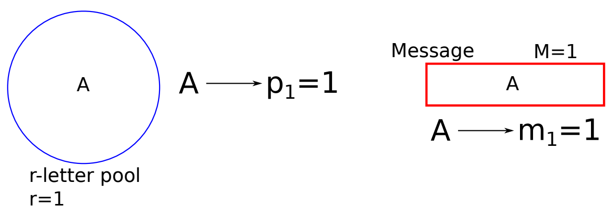


Figure 1.4: When the alphabet pool only has one letter.

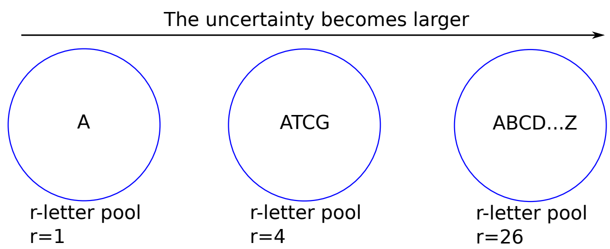


Figure 1.5: When the alphabet pool becomes larger, the larger the uncertainty