

Problem 1: Introduction to RStudio

- a) Change your working directory. Try the commands **help(c)** and **help(matrix)**.
- b) Calculate the affine transformation $\mathbf{y} = \mathbf{x}\mathbf{A}^{-1} + \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} 2 & 1 & 5 \\ -2 & 7 & 0 \\ 5 & -8 & -1 \end{pmatrix}, \quad \mathbf{x}^T = \begin{pmatrix} 8 \\ -4 \\ 2 \end{pmatrix}, \quad \mathbf{b}^T = \begin{pmatrix} 3 \\ 10 \\ -19 \end{pmatrix}.$$

- c) Install the package **mvtnorm** and load the corresponding functions to your workspace. Set the seed to 123 using the command **set.seed(123)**. Generate 100 observations from a two dimensional normal distribution with expected value $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. Visualize the observations.

$$\boldsymbol{\mu} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \quad \text{and} \quad \boldsymbol{\Sigma} = \begin{pmatrix} 4 & 1 \\ 1 & 2 \end{pmatrix}.$$

- d) Use the data from (c) and calculate the sample mean $\bar{\mathbf{x}}$ and the sample covariance matrix \mathbf{S}_x . Calculate the eigenvalues and eigenvectors from the matrix \mathbf{S}_x . Verify from the data, that the following equations hold: $\text{Tr}(\mathbf{S}_x) = \lambda_1 + \lambda_2 + \dots + \lambda_p$ and $\text{Det}(\mathbf{S}_x) = \lambda_1 \lambda_2 \dots \lambda_p$, where λ_i are the eigenvalues of \mathbf{S}_x .
- e) Calculate the affine transformation $\mathbf{y}_i = \mathbf{A}\mathbf{x}_i + \mathbf{b}$, where

$$\mathbf{b} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \quad \text{and} \quad \mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix},$$

verify that $\bar{\mathbf{y}} = \mathbf{A}\bar{\mathbf{x}} + \mathbf{b}$ and $\mathbf{S}_y = \mathbf{A}\mathbf{S}_x\mathbf{A}^T$. What does affine equivariance mean in practice?

- f) Upload the data from the file **Data1.txt** into your workspace. Create a function, that centers your data (removes the mean) and pairwise scatterplots the variables. Calculate the sample covariance and correlation matrices and the corresponding eigenvalues- and vectors.

Problem 2: The eigenvalues of a symmetric matrix

Show that the eigenvalues of a real valued symmetric matrix are always real valued.

Homework Assignment 1: Functions

Submit a pdf-file to MyCourses that contains carefully commented R-code and answer the following. Note that if you submit anything other than pdf-files to MyCourses, you will automatically get zero points.

- a) Create an R-function that takes a data matrix $\mathbf{X} \in \mathbb{R}^{n \times p}$, $n > p$, as an argument and returns the unbiased estimator of the covariance matrix. Do not use the built-in functions **cov()**, **cor()** or any additional R-packages.
- b) Create an R-function that takes a full-rank covariance matrix $\mathbf{A} \in \mathbb{R}^{p \times p}$ as an argument and returns the square root of the inverse matrix such that $\mathbf{A}^{-\frac{1}{2}}\mathbf{A}^{-\frac{1}{2}} = \mathbf{A}^{-1}$.
- c) Create an R-function that takes a full-rank covariance matrix \mathbf{A} as an argument and returns the corresponding correlation matrix.