1.

1. input : u_0 output : u_2

Let $x_1(t) = y(t) = u_2$

$x_2(t) = \dot{y}(t) = \dot{u}_2$

We have $\dot{x}_1 = x_2$ ①

$\dot{x}_2 = \ddot{u}_2$

$$\begin{cases} u_1 = R_2 C_2 \dot{u}_2 + u_2 \\ \dot{u}_1 = R_2 C_2 \ddot{u}_2 + \dot{u}_2 \end{cases}$$

$$u_0 + R_1 C_1 (R_2 C_2 \ddot{u}_2 + \dot{u}_2) + R_1 C_1 \dot{u}_2 + R_2 C_2 \dot{u}_2 + u_2 = 0$$

$$R_1 C_1 R_2 C_2 \ddot{u}_2 + 2 R_1 C_1 \dot{u}_2 + R_2 C_2 \dot{u}_2 + u_2 + u_0 = 0$$

$$\dot{x}_2 = \ddot{u}_2 = -\frac{2\dot{u}_2}{R_2 C_2} - \frac{\dot{u}_2}{R_1 C_1} - \frac{1}{R_1 C_1 R_2 C_2} u_2 + \frac{1}{R_1 C_1 R_2 C_2} u_0$$

$$\dot{x}_2 = -\frac{1}{R_1 C_1 R_2 C_2} x_1 - \frac{2R_1 C_1 + R_2 C_2}{R_1 C_1 R_2 C_2} x_2 + \frac{1}{R_1 C_1 R_2 C_2} u_0 \quad ②$$

So
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{R_1 C_1 R_2 C_2} & -\frac{2R_1 C_1 + R_2 C_2}{R_1 C_1 R_2 C_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{R_1 C_1 R_2 C_2} \end{bmatrix} u_0$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{1}{R_1 C_1 R_2 C_2} & -\frac{2R_1 C_1 + R_2 C_2}{R_1 C_1 R_2 C_2} \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ \frac{1}{R_1 C_1 R_2 C_2} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix} \qquad D = 0$$

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2.

2. $G(s) = C(sI - A)^{-1} B + D$

$$= \begin{bmatrix} 0 & 56.7 \end{bmatrix} \left(s \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -0.313 & 56.7 \\ -0.013 & -0.426 \end{bmatrix} \right)^{-1} \begin{bmatrix} 0.232 \\ 0.0203 \end{bmatrix} + 0$$

$$\cancel{= 11510100005 + 0.18925893}$$

$$= \frac{1.15101s + 0.18925893}{s^2 + 0.739s + 0.870438}$$

3.

3.(a) System output: x

System input: I

System constants: $R_c, \gamma, \phi, K, V, h, P$

Internal time-varying variables: x

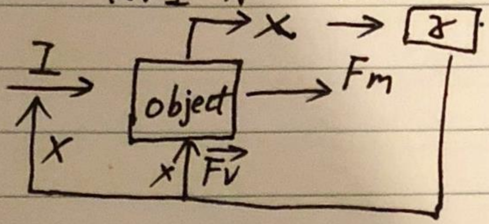
(b) According to Newton's second law:

$$\Sigma F = ma$$

$$m\ddot{x} = \vec{F_m} - \vec{T}\sin\alpha - \vec{F_v}\cos\alpha$$

$$= KVI^2x - h\dot{x}\cos\alpha - \gamma c\sin(\phi)\sin\alpha$$

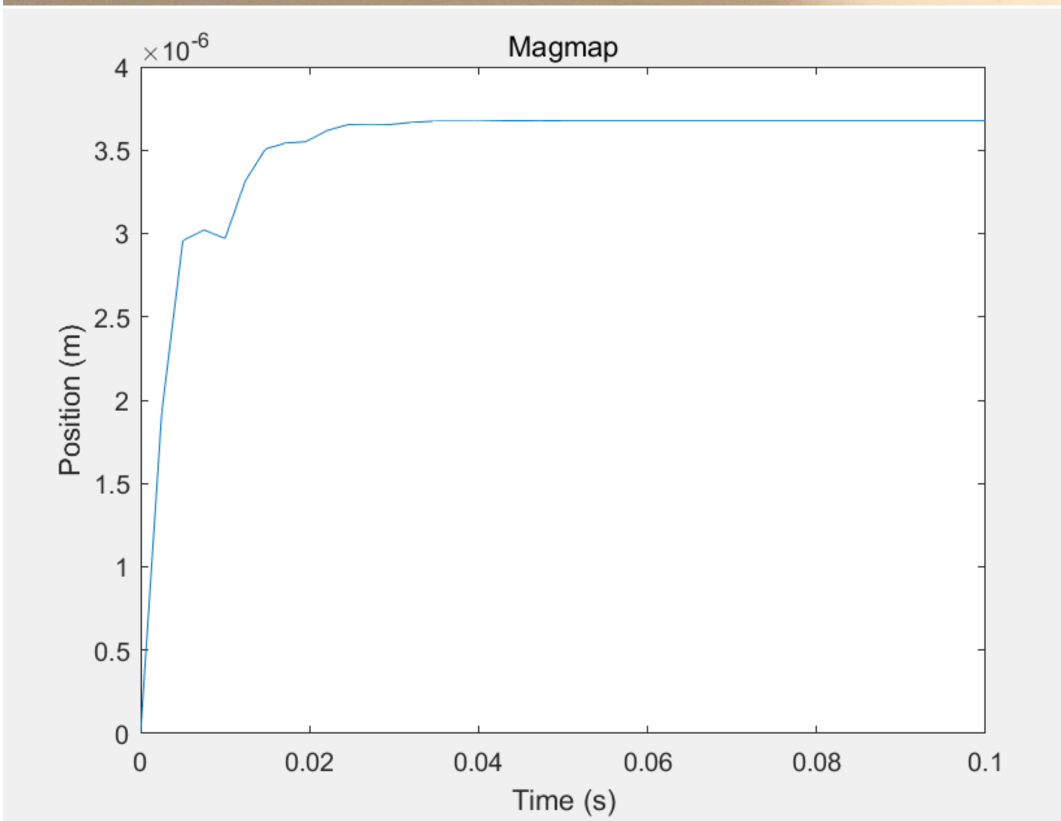
(c)



(d) inertia negligible $\quad m\ddot{x} = 0$

$\alpha << 1°\quad \sin\alpha \to \alpha \quad \cos\alpha \to 1$

$$KVI^2x - h\dot{x} - \gamma c\sin(\phi)\alpha = 0$$



4.

4. (a) system output: $A(t)$

System input: $A_i(t)$

System Constants: V q_0 q_i K_0 E_a R T

Internal time-varying Variables: $B(t)$

(b)
$$\begin{cases} \dot{A}(t) = \dfrac{A_i(t)q_i}{V} - \dfrac{A(t)q_0}{V} \\ \dot{B}(t) = \dfrac{KA(t)q_0}{V} - \dfrac{B(t)q_0}{V} \end{cases}$$

(c)
$$\begin{bmatrix} \dot{A}(t) \\ \dot{B}(t) \end{bmatrix} = \begin{bmatrix} -\frac{q_0}{V} & 0 \\ \frac{kq_0}{V} & -\frac{q_0}{V} \end{bmatrix} \begin{bmatrix} A(t) \\ B(t) \end{bmatrix} + \begin{bmatrix} \frac{q_i}{V} \\ 0 \end{bmatrix} A_i(t)$$

$$A(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} A(t) \\ B(t) \end{bmatrix} \qquad D = 0$$

(d)
$$G(s) = C(sI - A)^{-1}B + D$$
$$= \begin{bmatrix} 1 & 0 \end{bmatrix} \left(s \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -\frac{q_0}{V} & 0 \\ \frac{kq_0}{V} & -\frac{q_0}{V} \end{bmatrix} \right)^{-1} \begin{bmatrix} \frac{q_i}{V} \\ 0 \end{bmatrix} + 0$$
$$= \frac{q_i}{sV + q_0}$$