1. **Rewrite the following differential equation to state-space formulation (input: $u_0$, output: $u_2$)**

$$\begin{cases} u_0 + R_1C_1\dot{u}_1 + R_1C_1\dot{u}_2 + u_1 = 0 \\ -u_1 + R_2C_2\dot{u}_2 + u_2 = 0 \end{cases}$$

2. **Write the transfer function of the following system (input: $\delta$ output: $\dot{\theta}$)**

$$\begin{cases} \dot{\alpha} = -0.313\alpha + 56.7q + 0.232\delta \\ \dot{q} = -0.013\alpha - 0.426q + 0.0203\delta \\ \dot{\theta} = 56.7q \end{cases}$$

3. **Problem 1**

**Consider the planar magnetic device represented in Fig.1. This system is composed of four electromagnets that are used to move a magnetic micro-object located at the air-liquid interface.**

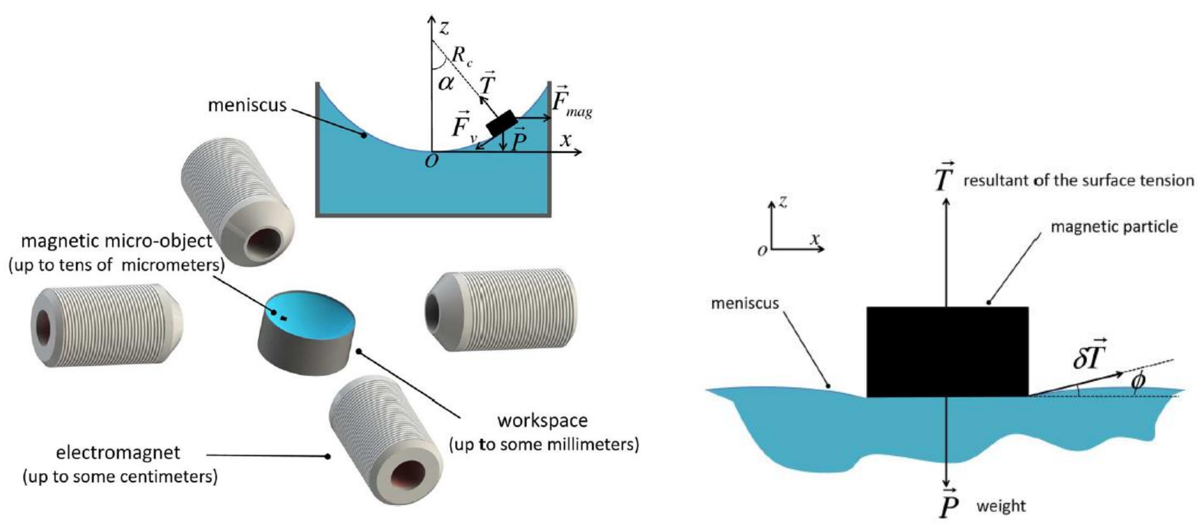


Figure 1 – Magnetic micro-manipulator

**Because the workspace is limited, a meniscus appears on the reservoir. The shape of the meniscus can be defined by $z = f(x)$. We consider that the meniscus is spherical (radius $R_c$) and thus the following equation can be used:**

$$\begin{cases} x = R_c sin(\alpha) \\ z = R_c(1 - cos(\alpha)) \end{cases}$$

**The micro-object is affected by various forces:**

- **The weight of the micro-object.**
- **The resultant force of the surface tension: $\vec{T}$ where $\vec{T} = \gamma C sin(\phi)$ (see Fig.1). $\gamma$ represents the surface tension, $C$ the perimeter of the object and $\phi$ the contact angle.**
- **The magnetic force given by: $\overrightarrow{F_m} = kVI^2\vec{x}$ where V is the volume of the micro-object, $I$ the current supplied to the electromagnets and k a coefficient related to the magnetic properties of the object.**
- **The viscous force created by the fluid surrounding the object: $\vec{F}_v = -h\vec{v}$, where h is the viscosity of the fluid and v the velocity of the micro-object.**

a) Based on the description of the system, structure the problem. You are supposed to answer these sorts of questions: What signal(s) are the output(s) of the system? What signal(s) are the input(s) of the system? What are the constants of the system? What are the internal time-varying variables of the system?

b) Set up the basic equations of the system. You are supposed to get one differential equation that represent the system (for the x axis).

c) Represent the block diagram of this system.

d) Assume that the motion of the particle takes place around the center of the meniscus (i.e. $\alpha \ll 1°$) and that the size of the object makes the inertia negligible, rewrite the differential equation.

e) Write a MATLAB script named “MagMap.m” that solves the differential equations. We are interested to check the response of the system to the following input and parameters. Plot the position of the particle for 100 milliseconds.

f) Feel free to test the model with different parameters.

| Parameter | Value | Units | Remark |
|---|---|---|---|
| $\boldsymbol{g}$ | 9.81 | $m.s^2$ | Acceleration of the gravity |
| $\boldsymbol{\gamma}$ | $72.10^{-3}$ | $N.m^{-1}$ | Surface tension |
| $\boldsymbol{\phi}$ | $7.2 \times 10^{-4}$ | $rad$ | Contact angle |
| $\boldsymbol{\rho}$ | 8902 | $kg.m^{-3}$ | Density of the micro-object |
| $\boldsymbol{V}$ | $100 \times 90 \times 25$ | $\mu m^3$ | Dimension of the micro-object |
| $I$ | 0.40 | $A$ | Coil current |
| $\boldsymbol{h}$ | $3.39 \times 10^{-7}$ | $kg.s^{-1}$ | Viscous coefficient |
| $\boldsymbol{k}$ | 3750 | $T/Am^2$ | Coefficient related to the magnetic properties |
| $\boldsymbol{R_c}$ | 400 | $\mu m$ | Radius of meniscus |

## 4. Problem 2

**Consider the chemical reactor represented in Fig.2. This system is composed of the following parts:**

- **A tank in which the chemical reaction takes place. This tank has the following parameters: a constant volume $V$, a homogeneous temperature $T$, a concentration in reactant $A$ and a concentration in production $B$,**
- **An inflow used to bring the reactant inside the tank. The concentration of the reactant in the inflow is noted $A_i$ and the volumetric flow is noted $q_i$,**
- **An outflow used to gather the product of the reaction. The volumetric flow is noted $q_o$.**

**Inside the tank, reactant A is transformed into product B at the following rate:**

$$r = kA$$

**where $r$ is the reaction rate expressed in $mol.s^{-1}.L^{-1}$ and $k = k_0 exp(-E_a/RT)$ the rate constant given by Arrhenius' equation.**

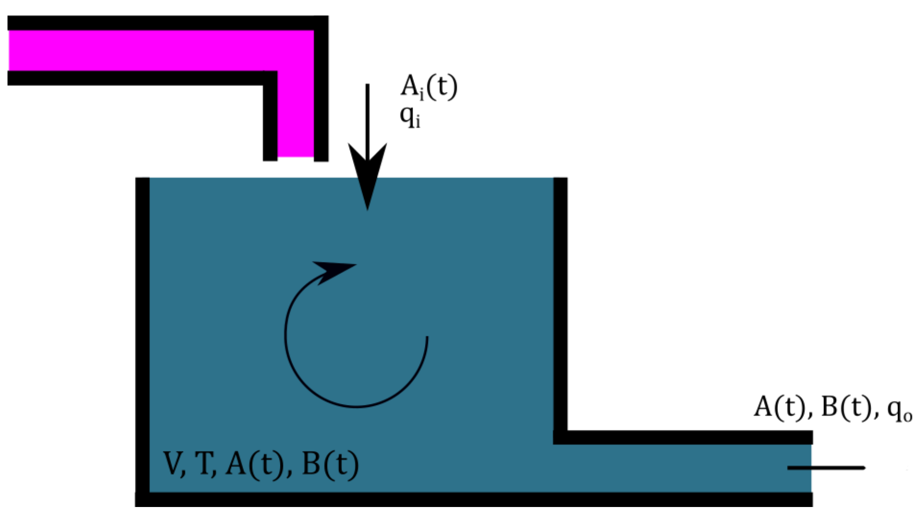


Figure 2 – Chemical reactor

a) Based on the description of the system, structure the problem. You are supposed to answer these sort of questions: What signal(s) are the output(s) of the system? What signal(s) are the input(s) of the system? What are the constants of the system? What are the internal time-varying variables of the system?

b) Set up the basic equations of the system. You are supposed to get two differential equations that represent the system. These equations describe the evolution of the quantity of the reactant and product inside the tank.

**Remark**: you need to consider input, output and transformed quantity of reactant and product.

c) What are the state variable(s) of the system? Form the state-space model of the system.

d) Write the transfer function of this system.

e) Write a MATLAB script named "ChemReactor.m" that solves the differential equations using the function ode45 and the state-space formulation. Do the same using the transfer function formulation.

We are interested to check the response of the system to the following input and parameters. Plot the concentration of reactant and product for 20 seconds.

| Parameter | Value | Units | Remark |
|---|---|---|---|
| $\boldsymbol{V}$ | 10 | $L$ | Volume of the tank |
| $q_o, \boldsymbol{q_i}$ | 0.25 | $L.s^{-1}$ | Volumetric flow |
| $\boldsymbol{A_i(t)}$ | $\frac{1}{0.3\sqrt{\pi}} e^{-\left(\frac{t}{0.3}\right)^2}$ | $mol.L^{-1}$ | Concentration of reactant |
| $k_0$ | 9.4 | $s^{-1}$ | Frequency of collisions |
| $\boldsymbol{E_a}$ | 2500 | $J.mol^{-1}$ | Activation energy |
| $R$ | 8.31 | $J.k^{-1}mol^{-1}$ | Gas constant |
| $T$ | 293 | $K$ | Temperature |

f) For these parameters, at what time the concentration of reactant becomes negligible in the outlet? When is the tank completely free of both reactant and product?

g) Feel free to test how the system behave when different input function $\boldsymbol{A_i(t)}$ are used.

# What to return?

You are supposed to submit your assignment to the related link for assignment 1 in MyCourses. Your submission should include one zip file "Assign01_student number.zip" consisting of a pdf file "Assign01_student number.pdf", and two MATLAB files "ChemReactor.m", and "MagMap.m".

The hard deadline for submission of this assignment is 23.09.2018 at 23.59.