Irradiance Distribution and Performance Analysis of Vector Vortex Beams in Atmospheric Turbulence

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ABSTRACT
Free space optical communication (FSOC) offers secure and high data transmission. However, atmospheric turbulence poses a significant bottleneck for FSOC. These limitations can be effectively addressed with vector vortex beams (VVBs), which leverage phase and polarization singularities, exhibiting resilience to turbulence. Despite this potential, existing studies have not explored the irradiance (intensity) distribution of the VVB in the turbulent channel. This paper derives the irradiance distribution for C-point and V-point beams, which are prominent VVBs known in the literature for their increased resilience to turbulence. Additionally, an in-depth analysis of the bit error rate (BER) performance of VVBs using on-off keying (OOK) modulation has been conducted. We tested the model using a scalar Gaussian beam to validate the obtained results and compared it with the results available in the literature.
Keywords: FSOC, VVBs, bit error rate, polarization singularity, atmospheric turbulence.

1. INTRODUCTION

In an age of increasing data requirements and the constant search for faster communication technologies, free space optical communication (FSOC) is a potential solution due to its high-speed transmission capacities [1]. However, atmospheric turbulence restricts the achievable data rate and reduces the link availability [1].

Meanwhile, beams carrying orbital angular momentum (OAM) have evolved as an effective approach to improve the spectral efficiency of FSOC by leveraging its orthogonality [2]. OAM beams are a specific type of structured light with a spiral phase front. Unfortunately, the turbulence causes severe degradation due to the helical phasefront of the OAM beams [2].

Conversely, vector vortex beams (VVBs) have surfaced as a potential breakthrough for secure and high-capacity FSOC systems. Their unique properties, including helical phase fronts characterized by an integer topological charge (TC) and polarization singularity on the axis, provide potential benefits compared to traditional Gaussian beams and scalar OAM beams [3]. However, atmospheric turbulence continues to pose a substantial obstacle to optical communication by deteriorating signal quality and restricting transmission capabilities.

Using established theoretical frameworks, this study analyzes the impact of atmospheric turbulence on VVBs, with a focus on intensity distribution. Moreover, we compare the performance of VVBs with Gaussian beams in terms of bit error rate (BER) for a specific energy per bit (E_b). This paper demonstrates the superiority of vector beams over scalar Gaussian beams. Furthermore, V-point beams have better bit error rate (BER) performance under low turbulence settings, whereas C-point beams show better performance under high turbulence situations. Additionally, VVBs outperform Gaussian beams under all turbulence conditions because of their inherent diversity and encoding characteristics.

The subsequent sections are organized as follows: section 2 outlines the methods for implementing our channel model, including the simulation conditions and parameters. Section 3 encompasses the findings and examination. Ultimately, we bring our paper to a conclusion in section 4.

2. MATHEMATICAL REPRESENTATION OF VVBs AND CHANNEL

Laguerre-Gaussian (LG) beams [4] are the most used examples of scalar OAM beams. VVBs introduce a new dimension through their spatially varying polarisation states, which enhances their susceptibility to turbulence. These vector vortices can be represented as the superposition of two scalar OAM beams [4], as shown in Eq. 1.

$$|\psi_{l_1,l_2}\rangle = \left(\alpha_R^{l_1} e^{i(l_1\theta+\delta_1)}|R\rangle + \alpha_L^{l_2} e^{i(l_2\theta+\delta_2)}|L\rangle\right) exp\left(-\frac{r^2}{\omega_o^2}\right), \quad (1)$$

here, $|R\rangle$ and $|L\rangle$ denote the left circular polarization (LCP) and right circular polarization (RCP) basis, respectively. $\alpha_R^{l_1}$ and $\alpha_L^{l_2}$ represent the complex amplitude, l_1 and l_2 are the TCs, δ_1 and δ_2 denote the initial phases of LCP and RCP components of the electric field, respectively, ω_o is waist radius, and θ represents the azimuthal angle.

We have used two types of VVBs, namely C-point and V-point beams. The superposition of scalar OAM beams with unequal topological charge, i.e. $|l_1| \neq |l_2|$ yields C-point beams, and these beams have non-zero OAM. For the C-point beams, we use two scalar LG beams of TCs 0 and 1 with LCP and RCP basis, respectively. V-point beams [4] have net zero OAM with equal and opposite TCs (i.e. $l_1 = -l_2$). For V-point beams, we use 1 and -1 TCs with LCP and RCP, respectively. The behaviour of different beams within identical channels can vary significantly, depending upon the spatial distribution of their amplitude, phase, and polarisation.

In order to mimic the channel's behaviour, various models for power spectral density of refractive-index fluctuation have been proposed in the literature, including Kolmogorov, von Karman, and others. Among these, the modified atmospheric spectrum is a widely adopted power spectrum to simulate the physical channel [5] and is given below:

$$\Phi_n(k) = 0.033 C_n^2 \left[1 + 1.802\left(\frac{k}{k_l}\right) - 0.254\left(\frac{k}{k_l}\right)^{7/6}\right] \frac{e^{-k^2/k_l^2}}{\left(k^2+k_o^2\right)^{11/6}}, \quad (2)$$

where C_n^2 represents the refractive-index structure parameter, $k_l = 3.3/l_o$, $k_o = 1/L_0$, l_o and L_0 are the inner and outer scales, respectively. In the next section, we will explore the simulation techniques to evaluate the performance of the FSOC channel using the abovementioned channel model.

3. METHODOLOGY AND SIMULATION PARAMETERS

We construct a channel model utilizing the split-step method to replicate atmospheric turbulence [4] depending on the turbulence regime. We generate a collimated beam at the source using a 512 by 512 matrix of pixel size 1 mm. To get the irradiance fluctuations, we capture the on-axis intensity [5] of C-point beams at the focal plane, whereas we measure the irradiance fluctuation of x – polarized and y – polarized components separately for V-point beam [3]. This is done using a photodetector with an active area diameter of 5 μm, which matches the size of the pixels at the receiver. In addition, we assume to operate in a shot noise limited regime, supposing a high background noise. As a result, the noise is treated as additive. We employ on-off keying (OOK) and direct detection to analyse the BER performance of each beam.

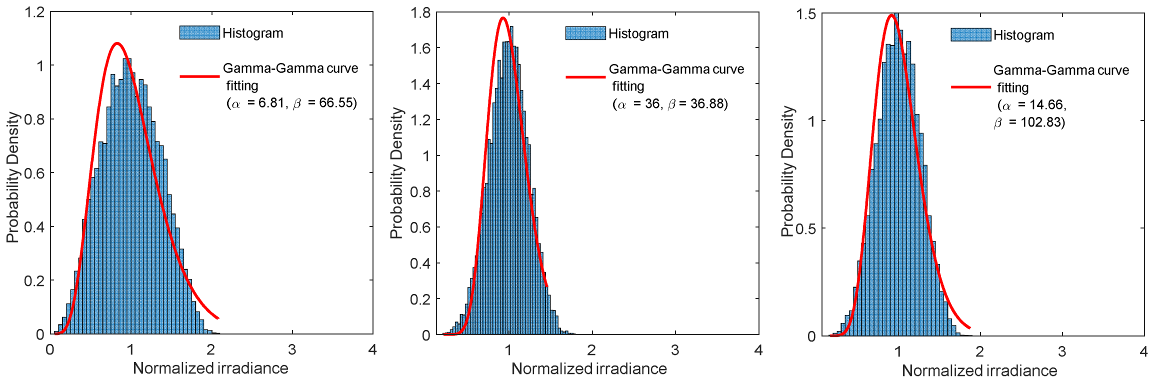
We measure turbulence intensity by C_n^2, which ranges from $10^{-17}\ m^{-2/3}$ to $10^{-13}\ m^{-2/3}$, for weak to high turbulence [6]. We assume untracked horizontal FSO links. The beam's and channel's physical parameters chosen are well within the practical regime tabulated below,

Table 1. Simulation parameters used for the modelling of turbulence

Parameters	Values
Link length	5 km
Collimated beam waist at source	5 cm
Wavelength	1550 nm
Focal length	30 cm
Grid size	50 cm

4. RESULTS AND DISCUSSION

To produce the channel behaviour, we generate a histogram of normalized irradiance fluctuation. The variations in the received optical power consistently follow the well-established gamma-gamma distribution, regardless of the level of turbulence in the atmosphere. Fig. 1 illustrates the characteristics of various beams under different turbulence levels with histograms of irradiance fluctuation and their corresponding gamma-gamma probability density functions (PDFs). Each row corresponds to a specific turbulence regime, ranging from weakest (top) to strongest (bottom), and each column represents a particular beam type, namely Gaussian, C-point, and V-point, arranged from left to right, allowing for comparison of their performance under different turbulence conditions.



(a) (b) (c)

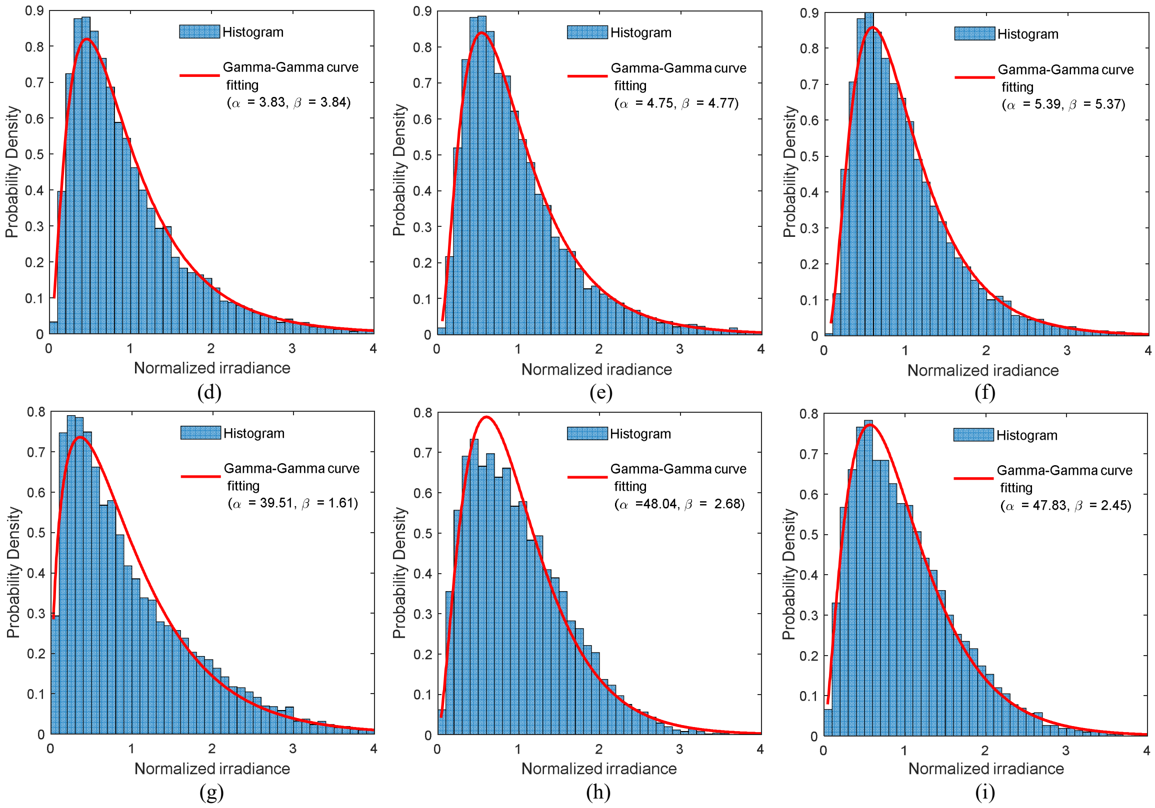


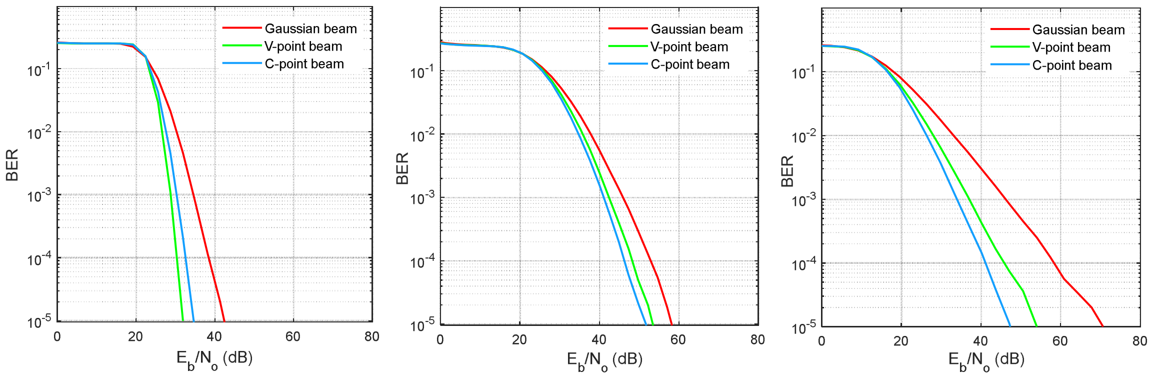
Figure 1. Histogram of normalized irradiance fluctuation at the focal plane along with the fitted gamma-gamma pdf for (a)-(c) weak, (d)-(f) moderate, and (g)-(i) high turbulence for Gaussian ((a),(d), and (g)), C-point ((b), (e), and (h)), and V-point ((c), (f), and (i)) beams respectively.

The observed differences in beam behaviour, as highlighted in the previous plots, can be attributed to three key characteristics: amplitude, phase, and polarization. Hence, the parameter of the gamma-gamma distribution varies when exposed to the same turbulence level. The data shown in Table 2 supports the results depicted in Fig. 1, where we have documented the scintillation index for every beam type under various turbulence conditions. The table demonstrates that the scintillation index increases in direct correlation with the turbulence intensity. In addition, we have examined the BER performance for a specific E_b in each scenario, as shown in Fig. 2.

Table 2. Scintillation index of different beams in different turbulence regimes

Types of Beams	Weak turbulence ($C_n^2 = 10^{-15}$)	Moderate turbulence ($C_n^2 = 10^{-14}$)	Strong turbulence ($C_n^2 = 10^{-13}$)
Gaussian beam	0.13	0.63	0.68
V-point beam	0.05	0.39	0.52
C-point beam	0.08	0.37	0.45

In Fig. 2, we analyze the performance of the V-point and C-point beams. To determine the E_b/N_o gain, we fix the desired BER to 10^{-5} and use a Gaussian beam for reference. When there is weak turbulence (as seen in Fig. 2(a)), we can see that the V-point beam outperforms the C-point beam. The V-point beam achieves a gain of 9 dB, whereas the C-point beam achieves a gain of 6 dB. Within the range of moderate turbulence (as shown in Fig. 2(b)), it is evident that both the V-point and C-point beams exhibit nearly equal performance, with a respective gain of 5 dB and 6 dB. When the turbulence is strong (as seen in Fig. 2(c)), the C-point beam performs better than the V-point beam. A gain of 20 dB is achieved in the C-point, while a gain of 12 dB is achieved in the V-point beam. Based on the data above, we see variations in the E_b/N_o gain between C-point and V-point beams. When using a C-point beam, the greater the turbulence, the more significant the enhancement in terms of E_b/N_o gain.



(a) (b) (c)

Figure 2. BER against bit energy per noise spectral density (E_b/N_o) for Gaussian, C-point, and V-point beams in (a) weak, (b) moderate, and (c) high turbulence regimes at the focal plane.

5. CONCLUSIONS

This study presents a distribution of irradiance fluctuation for the FSO channel under various turbulence conditions, specifically focusing on the Gaussian beams and VVBs. We analyze the BER performance for various values of E_b/N_o. Based on our analysis, we determine that the C-point beam provides the most significant enhancement in a high-turbulence environment, resulting in a 20 dB E_b/N_o gain. In contrast, it yields a gain of 6 dB in a low turbulence scenario. Meanwhile, the V-point exhibits the most enhancement in the low turbulence regime, resulting in a gain of 9 dB in E_b/N_o. Both beams exhibit satisfactory performance under conditions of moderate turbulence. Therefore, the VVBs have the potential to serve as an alternative to the Gaussian beam and provide a greater data rate for wireless optical communication. Although all parameters fall within the feasible limits based on our current understanding, thoroughly verifying the results through experimentation is crucial. Subsequent phases of our research will encompass thorough mathematical scrutiny and experimental validation on hardware to affirm the credibility of our findings.

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