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A Generalized Time-Domain Signal Generation Algorithm for FSOC Turbulence Channel Simulation

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***Abstract*—To evaluate various methods and techniques proposed for mitigating the effects of atmospheric turbulence on optical communication performance, it is critical to generate optical channel states. Meanwhile, due to the complexity of the communication environment, traditional channel simulation methods are not always applicable. Based on the Gamma-Gamma turbulent channel, we propose a channel simulation method with an autoregressive model and rank matching method, which can simulate any channel condition described by autocorrelation function and probability density distribution. The method is simple to implement and its results are comparable to some more sophisticated methods. Although this paper discusses one-dimensional random signals, the method can also be extended to multiple dimensions, generating multiple random signals with mutual correlation numbers.**

Keywords—atmospheric turbulence, Channel Simulation, Random process, gamma-gamma distribution

I. INTRODUCTION

Recently, free-space optical (FSO) communications have been widely studied for their high interference immunity, license-free requirements, and security. However, due to random refractive index changes caused by atmospheric turbulence, light waves propagating through atmospheric channels experience random fading, limiting communication system performance. This fading is scintillation, which indicates turbulence-induced light intensity fluctuations[1,2]. An accurate and efficient simulation of the optical channel is an important basis for assessing and mitigating this effect. Because the state of atmospheric turbulence varies with time, the scintillation is also time-varying, and obtaining this time-varying property requires a covariance function or autocorrelation function of the light intensity fluctuations [3]. Studying from the perspective of statistical properties, researchers have developed different optical statistical distribution models[4, 5]. Due to the strong randomness of light intensity fluctuations at different moments, we need to use probabilistic statistics and stochastic process methods for optical channel simulation. The traditional channel simulation model will be limited in application for the increasingly complex channel environment, so the general channel simulation model becomes a hot research topic.

Some existing methods are summarized as follows. Bykhovsky proposed a stochastic differential equation (SDE) based simulator for weakly turbulent and strongly turbulent FSO channels[6,7], but the numerical solution of the first-order stochastic differential equation in this method is a smooth stochastic process with predefined exponential-form autocorrelation functions, which may not be effective for simulation under certain turbulent conditions. A generalized autoregressive (AR) modeling approach is used to generate exactly the correlated Rayleigh fading process, which is modeled as a complex Gaussian process[8]. The advantage of the AR approach is that it can generate variables with arbitrary second-order statistics as needed. Since optical channel simulation is similar to the problem of autocorrelated non-Gaussian process generation, some related methods are also worth considering. Liu and Munson proposed a general method that is an iteratively computed memory-free nonlinearity transform (ZMNL) with white Gaussian noise (WGN) as the source[9]. Jurado-Navas et al. used the Gaussian approximation of the Kolmogorov spectrum for filter design and ZMNL nonlinear processing to efficiently model log-normal FSO channels[10].

However, for non-specific probability density distributions (e.g., Gamma distribution), in-depth theoretical analysis with sufficiently long equations is required to improve the accuracy of the probability density function and the autocorrelation function. Filho proposed a method that uses the rank-matching method to convert the PDF to the desired non-Gaussian PDF[11]. This process of reordering sequences tries to simulate the effect of ZMNL and has the advantage of not requiring the use of analytic expressions for nonlinear transformations. Although the above-mentioned methods give more accurate results, they also have the limitations listed in the text.

Based on the above-discussed methods, a simple and general optical channel simulation method is proposed in this paper. Using the main idea of ZMNL, the fading channel with accurate statistical properties is generated in real-time based on the scalar autoregressive (AR) modeling method and rank matching method to generate the expected first-order statistics and second-order statistics, i.e., probability density distribution and autocorrelation function or power spectral density of the stochastic process. The AR model is less computationally intensive and statistically more accurate than traditional correlated Gaussian sequence simulation methods, such as the discrete Fourier inverse transform; for correlated non-Gaussian sequence methods, such as the zero memory nonlinear transformation method, in which the nonlinear transformation makes the correlation of the input sequence transformed, it is difficult to obtain the relational equation of the correlation function of the input and output sequences before and after the transformation, and currently only the relational equation of the correlation function after the nonlinear transformation of a few distributions is inferred, using the rank matching method can obtain the same effect and is simpler and more general. The method is applied to the Gamma-Gamma distribution model for free-space optical communication (FSOC) according to an arbitrarily specified autocorrelation function, and the simulation results are consistent with the expectation.

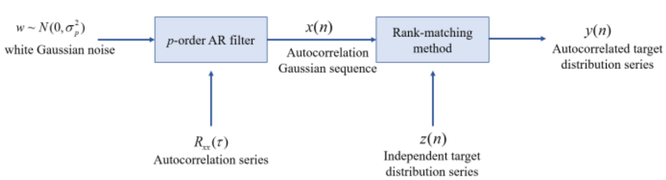


Fig. 1. Algorithm design diagram

II. FSOC Atmospheric Turbulence Gamma-Gamma Time Domain Signal Generation Model

The main idea of ZMNL is to first generate an autocorrelated Gaussian random process by linear filtering and then obtain the required autocorrelated random sequence by some nonlinear transformation. Since the nonlinear transformation is performed to generate the random sequence of the specified distribution while maintaining the autocorrelation of the random sequence, it greatly increases the complexity of the simulation method. Therefore, we use the rank matching method to simulate the nonlinear transformation, which can generate the random sequence obeying the specified distribution alone, then use the AR model method to generate the required autocorrelation, and finally combine the two by the rank matching method.

As shown in Fig.1, our proposed method is divided into the following two steps: (1) generating the required autocorrelation Gaussian sequences based on the scalar AR model, and all autocorrelations in this paper are normalized autocorrelations; (2) reordering the independent target distribution sequences according to the size order of the relevant Gaussian sequences by using the rank matching method.

A. Gaussian Autoregressive Process

The properties of Gaussian AR have been extensively studied in [8, 12, 13] and one of the useful properties is the Yule-Walker equation, which represents the relationship between the autocorrelation and the parameters of the Gaussian AR process. In addition, the AR filter has the correlation matching property that the first p autocorrelation function values of the AR(p) process are exactly the same as the input theoretical autocorrelation sequence. In addition, the Gaussian process does not affect its probability distribution when linear filtering is performed. Therefore, we choose a p-order Gaussian AR process to represent the smoothly correlated Gaussian stochastic process with specified autocorrelation. An AR(p) process can be generated by time-domain recursion.

$$x(n) = -\sum_{k=1}^{p} a_k x(n-k) + w(n) \tag{1}$$

Where, a_k, k=1,2,…,p, are the coefficients of AR(p) and p is the order of AR(p). $w(n)$ is a Gaussian white noise sequence with mean 0 and variance σ_p^2. The AR(p) parameters are related to the specified autocorrelation sequence $R_{xx}(\tau)$ as follows:

$$\begin{bmatrix} R_{xx}(0) & R_{xx}(-1) & \cdots & R_{xx}(-p+1) \\ R_{xx}(1) & R_{xx}(0) & \cdots & R_{xx}(-p+2) \\ \vdots & \vdots & \ddots & \vdots \\ R_{xx}(p-1) & R_{xx}(p-2) & \cdots & R_{xx}(0) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{bmatrix} = -\begin{bmatrix} R_{xx}(1) \\ R_{xx}(2) \\ \vdots \\ R_{xx}(p) \end{bmatrix} \tag{2}$$

The Yule-Walker equation of Eq. (2) could be solved quickly using the Levinson-Durbin recursive method[14]. The initial value of the recursive algorithm is expressed as

$$a_1[1] = -\frac{R_{xx}[1]}{R_{xx}[0]} \tag{3}$$

$$\rho_1 = \left(1 - \left|a_1[1]\right|^2\right) R_{xx}[0] \tag{4}$$

For k = 2, 3, ..., p, the recursion is

$$a_k[k] = -\frac{R_{xx}[k] + \sum_{l=1}^{k-1} a_{k-1}[l] R_{xx}[0]}{\rho_{k-1}} \tag{5}$$

$$a_k[i] = a_{k-1}[i] + a_k[k] a_{k-1}^*[k-i], i = 1, 2, \ldots, k-1 \tag{6}$$

$$\rho_k = \left(1 - \left|a_k[k]\right|^2\right) \rho_{k-1} \tag{7}$$

where $k_k = a_k[k]$ is the reflection coefficient and ρ is the prediction error variance. The set of coefficients $\{a_1[1], \rho_1\}$, $\{a_2[1], a_2[2], \rho_2\}$, …, $\{a_p[1], a_p[2], \ldots, a_p[p], \rho_p\}$ are obtained. If $x(n)$ is an AR(p) process, then $a_p[i] = a_p$, $i = 1, 2, \ldots, p$, $\rho_p = \sigma_p^2$. It is important to note that when performing Levinson-Durbin recursion, if $\rho \le 0$ is found, the recursion should be stopped immediately, which is a basic requirement for a smooth AR model. However, some autocorrelation functions will be in this situation when the recurrence is low, and the autocorrelation matrix **R** of such autocorrelation functions is generally a singular matrix. When stopping the recursion, p is too small and the AR process cannot obtain the target correlation, resulting in low simulation accuracy, which seriously limits the performance of the AR model. To avoid this problem, we use the regularization method [8] to transform the matrix **R** into a non-singular matrix, i.e., $R_{xx}(0) = R_{xx}(0) + \varepsilon$, with ε being a very small number.

Although the Levinson-Durbin recurrence relation equation gives a method for estimating the parameters of the AR model with increasing order, it does not give a method for determining the order. In this paper, the Akaike criterion (AIC)[14] is chosen to determine the order p of the AR model, defined as

$$\text{AIC}(k) = N \ln \sigma_k^2 + 2k \tag{8}$$

Select the order that minimizes AIC.

In addition, to eliminate the onset transient phenomenon due to the absence of input sequence before k=0, the initial condition of the AR filter[14] is specified in this paper so that the output of the filter is in a stable state at k=0.

B. Autocorrelated Non-Gaussian Process

the autocorrelation function is used to measure the degree of association of samples at different time points, so the rank is used to represent the position relationship between different data in the sequence [11]. The model based on the rank matching method is shown in Fig. 2, where the Gaussian sequences of generated autocorrelations are first sorted to obtain their sorted position indexes; then random sequences obeying the target light intensity undulation distribution are generated and sorted; and finally, they are reordered according to the indexes in the first step, which in turn match the target autocorrelations. If the corresponding random number generator does not exist in the second step, we choose the rejection sampling method to generate the random numbers obeying the target distribution. The reordering in the last step tries to simulate the nonlinear transformation of ZMNL without specifying the functional form of ZMNL. This method essentially meets the probability density distribution requirement, since any rearrangement does not affect the distribution of the samples. The model based on the rank-matching method is shown in Fig. 2.

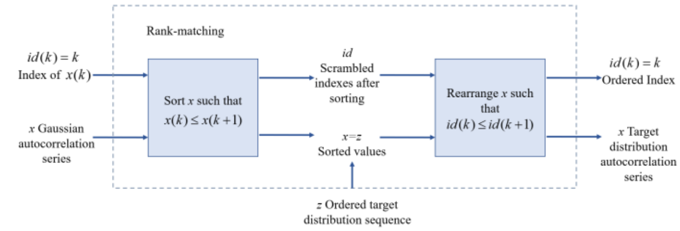


Fig. 2. Rank matching method

III. Time Domain Gamma-Gamma Atmospheric Turbulence Channel

The gamma-gamma statistic can describe the free-space optical communication scintillation phenomenon in a wide range of turbulent conditions, providing a probability density function (PDF) of the received light intensity (I) in the following form.

$$p(I) = \frac{2(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)} I^{\{[(\alpha+\beta)/2]-1\}} \mathrm{K}_{\alpha-\beta}\left(2\sqrt{\alpha\beta I}\right) \tag{9}$$

where Γ(x) is the gamma function and K(x) is the type II modified Bessel function. α and β represent the effective number of large- and small-scale scattering elements, respectively, related to the large and small scale undulation variance. gamma-gamma processes require two gamma distributions whose probability density functions are

$$p_x(x) = \frac{\alpha(\alpha x)^{\alpha-1}}{\Gamma(\alpha)} \exp(-\alpha x) \tag{10}$$

$$p_y(y) = \frac{\beta(\beta y)^{\beta-1}}{\Gamma(\beta)} \exp(-\beta y) \tag{11}$$

When two gamma distribution processes are given, the two can be multiplied to generate an independent gamma-gamma process, i.e., $I=XY$.

Then we choose the normalized autocorrelation function given by the following equation

$$R_{xx}(\tau) = \exp(-\frac{\tau}{\tau_c}) \tag{12}$$

where the correlation time τ_c is used for the correlation function characterization and is defined as

$$R_{xx}(\tau_c) = \exp(-1) \tag{13}$$

That is, when the correlation coefficient decays from 1 to 1/e, it is approximated that the random signals are no longer correlated.

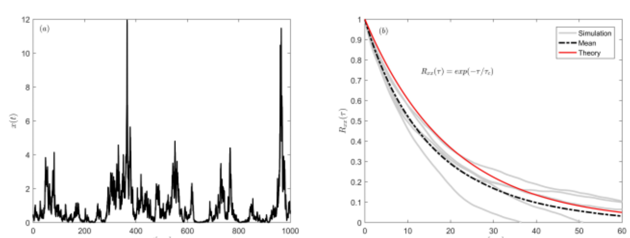
We choose τ_c=20ms and set the AR model order p to 1 using the ACI criterion of Eq. (4). Because the autocorrelation function of Eq. (12), the prediction error variance in Eq. (7) does not change with increasing p after performing one Levinson-

Durbin recursion. Fig. 3(a) shows the autocorrelated gamma-gamma time domain signal with α = 5 and β = 1.05. Fig. 3(c) shows the probability density function of the simulated sequence consistent with the gamma-gamma distribution. Autocorrelation is a common property of the channel, and we generated 100 random signals, and any five of them were selected to draw their own and their mean autocorrelation functions for comparison with the theoretical autocorrelation function of Eq. (4), as shown in Fig. 3(b). The results show that the autocorrelation functions are very similar. To quantify the accuracy of the simulation, we take the deviation of the simulated autocorrelation estimates of the random signals from the value of the target autocorrelation function as the standard,

$$E = \frac{1}{N}\sum_{k=0}^{N}\left|\overline{R}(k) - R(k)\right| \tag{14}$$

where $\overline{R}(k)$ and $R(k)$ are the autocorrelation function values of the mean of the random signal and the target autocorrelation function values, respectively. The autocorrelation function deviation E_1 is 0.0511 under this condition. In addition, the same process is simulated for α=6 and β=5, and the corresponding time signals are shown in Fig. 4(a). Fig. 4(c) shows that the probability density function of the simulated signal is consistent with the theory. To verify the effectiveness of the method, we arbitrarily choose τ_c=5*ms* for the simulation. Fig. 4(b) shows the autocorrelation functions of two different correlation times τ_c=20*ms* and 5*ms* signals, it can be seen that both simulated correlation functions are similar to the theoretical settings, and the deviations of autocorrelation functions E_2 and E_3 are 0.0298 and 0.0208 respectively. when τc=20ms, the autocorrelation function with α=6 and β=5 fits better than the one with α=5 and β=1.05 in Fig. 3(b) , and this conclusion can also be obtained from $E_{2<} E_1$.

As the correlation time is used to characterize the autocorrelation function, to quantify the simulation performance, the standard deviation of τ_c was calculated for 100 different realizations, and linear interpolation was applied to obtain the time delay when the value of the correlation function was 1/e. The standard deviation was 1.6971 for τ_c=20*ms*, which was 8.5% of the predefined one, and 0.2465 for τ_c=5*ms*, which was 4.9% of the predefined correlation time. As shown in Fig. 5, when the predefined τ_c is small, the correlation time of 100 random signals deviates less from the theory, which may be the reason for the high ratio of random signal length to correlation time.



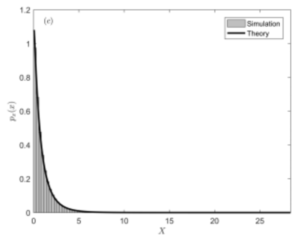


Fig. 3. Example of autocorrelation gamma-gamma process (α=5， β=1.05， τ_c=20*ms*). (a) time domain signal fragments (b) normalized autocorrelation functions of the five simulated sequences and their mean sums, and the target autocorrelation function (c) empirical PDF and theoretical PDF of the gamma-gamma process

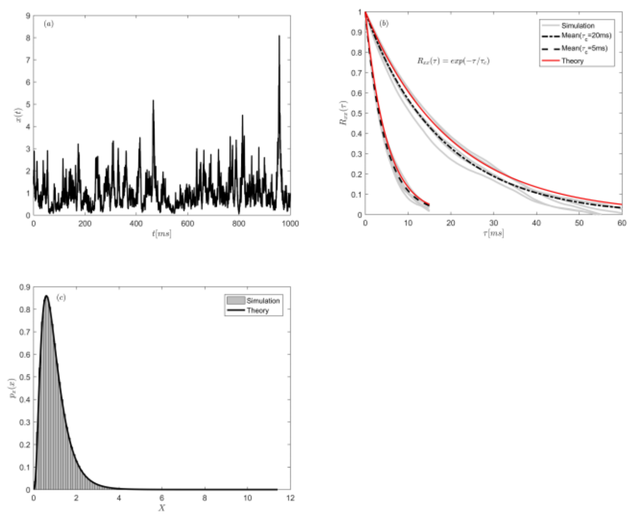


Fig. 4. Example of autocorrelation gamma-gamma process (α=6， β=5， τ_c=5*ms*). (a) Normalized autocorrelation functions for (b) the five simulated sequences with τ_c=20*ms* and 5*ms* and their mean sums, and the target autocorrelation function for the time domain signal fragments. (c) Empirical PDF and theoretical PDF of the gamma-gamma process.

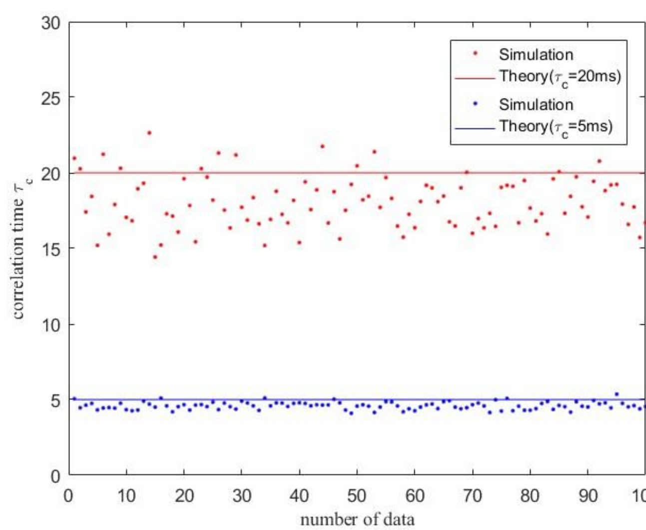


Fig. 5. Comparison of theoretical correlation time and correlation time of 100 simulations.

For the autocorrelation function in the exponential form used in this paper, the accuracy of autocorrelation simulation slightly decreases when the ratio of signal length to correlation time τ_c is bottom. better simulation accuracy is obtained when both α and β of Gamma-Gamma distribution are greater than 5, and the influence of β on this is larger.

IV. CONCLUTION

In this paper, a simple and effective method is proposed for the simulation of turbulent channels under different conditions. These conditions are described by the turbulent channel distribution function and the autocorrelation function. The Gaussian time-domain signal is generated in real-time by filtering with an autoregressive model, and then the rank-matching method passes the time correlation to the independent signals of the specified turbulent channel distribution, which in turn stimulates the optical channel state. Since the rank-matching method involves only ranking, it is relatively easy to implement, and the AR model can accurately simulate the smoothness and second-order statistical properties of the channel as needed. The channel states generated by the simulation are similar to the theory, demonstrating the effectiveness and generality of the method. The algorithm also requires the generation of samples based on the target probability distribution, and in cases where standard methods are not applicable, simple Monte Carlo rejection sampling technique is used to provide the required samples in this paper.

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