$$s_n^t = s_n^t$$

$$m_n^t = \text{Meet}\left(\{s_v^t \,|\, v \in N(n)\}, \{x_{(v,n)}^{\text{edge-type}} \,|\, v \in N(n)\}, x_{\mathcal{T}}^{\text{may/must}}, x_{\mathcal{T}}^{\text{direction}}\right)$$

$$s_n^{t+1} = \text{Transfer}\left(m_n^t, x_n^{\text{gen}}, x_n^{\text{kill}}\right)$$

(a) DFA

$$h_n^t = \text{Linear}_{\text{state}}(s_n^t)$$

$$\text{gate}_{(v,n)} = \text{Linear}_{\text{gate}}\left(\sum_{i \in \{x_{(v,n)}^{\text{edge-type}}, x_{\mathcal{T}}^{\text{may/must}}, x_{\mathcal{T}}^{\text{direction}}\}}(\{f_i\})\right)$$

$$m_n^t = \text{Max}\left(\{\text{gate}_{(v,n)} * h_v^t \,|\, v \in N(v)\}\right)$$

$$h_n^{t+1} = \text{Linear}_{\text{update}}\left(m_n^t + \sum_{i \in \{x_n^{\text{gen}}, x_n^{\text{kill}}\}}(\{f_i\})\right)$$

(b) DFA-GNN$^+$

$$h_n^t = \text{Linear}_{\text{state}}(s_n^t) + \sum_{i \in \{x_{\mathcal{T}}^{\text{may/must}}, x_{\mathcal{T}}^{\text{direction}}, x_n^{\text{gen}}, x_n^{kill}\}}(\{f_i\})$$

$$\text{gate}_{(v,n)} = \text{Linear}_{\text{gate}}\left(f_{x_{(v,n)}^{\text{edge-type}}}\right)$$

$$m_n^t = \text{Max}\left(\{\text{gate}_{(v,n)} * h_v^t \,|\, v \in N(n)\}\right)$$

$$h_n^{t+1} = \text{Linear}_{\text{update}}(m_n^t)$$

(c) DFA-GNN

$$h_n^t = \text{Linear}_{\text{state}}(s_n^t) + \sum_{i \in \{x_{\mathcal{T}}^{\text{may/must}}, x_{\mathcal{T}}^{\text{direction}}, x_n^{\text{gen}}, x_n^{kill}\}}(\{f_i\})$$

$$\text{gate}_{(v,n)} = \text{Linear}_{\text{gate}}\left(f_{x_{(v,n)}^{\text{edge-type}}}\right)$$

$$m_n^t = \text{Max}\left(\{\text{gate}_{(v,n)} * h_v^t \,|\, v \in N(n)\}\right)$$

$$Q = \text{Linear}_{\text{Q}}(M^t); K = \text{Linear}_{\text{K}}(M^t); V = \text{Linear}_{\text{V}}(M^t)$$

$$H^{t+1} = \text{Softmax}\left(\frac{QK^T}{\sqrt{d_k}}\right)V$$

(d) DFA-GNN$^-$