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1. a) Yes. Since the linear combination of these 2 cols equal to ϕ iff the coefficients are ϕ

b). Yes. Since the linear combination of these 3 cols equal to ϕ iff the coefficients are ϕ

c) No, This time, we can have non-zero coefficients making linear combination sums ϕ.

d) Rank(A) = 2.

e). It depends. $A^TA = \begin{bmatrix} 75 & -10 \\ -10 & 12 \end{bmatrix}$, easy to see it's invertible.

Therefore, $(A^TA)w = d$, we write $[A^TA \; d]$ (i.e, augmented matrix).

① if rank $(A^TA) \neq$ rank (augmented matrix), then no solution

② if rank (A^TA) = rank (augmented matrix); then unique solution & rank (A^TA) = # of unknowns

③ if rank of both matrix are the same but rank$(A^TA) <$ # of unknown, then infinite # so

2. a) Given that $\|\cdot\|_a$ & $\|\cdot\|_b$ are norms on $\mathbb{R}^n$,

then ① $\|x\|_a + \|x\|_b \geq \phi$. ② $\|x\|_a + \|x\|_b = \phi$ iff $x = \phi$.

③ $\|kx\|_a + \|kx\|_b = k \cdot \|x\|_a + k\|x\|_b = k(\|x\|_a + \|x\|_b)$.

given $k \in \mathbb{R}$, $x \in \mathbb{R}^n$, ④ $f(x+y) = \|x+y\|_a + \|x+y\|_b =$

$\|x\|_a + \|y\|_a + \|x\|_b + \|y\|_b = f(x) + f(y)$,

By ①, ②, ③, ④, $f(x) = \|x\|_a + \|x\|_b$ is a norm in $\mathbb{R}^n$.

b).