Assignment 6 · Yizhuo Lan lin773@wisc.edu

1. We need to substitute A to power iteration

Since A is rank 1, n×n symmetric, $f_2 \cdots f_n = 0$

$A = f_1 v_1 v_1^T$. $A^T = A$. $f_1 v_1 v_1^T = f_1 v_1 v_1^T \rightarrow u_1 = v_1$

$Ab_0 = f_1 v_1 v_1^T \frac{1}{\sqrt{n}} \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} = \frac{f_1 \sum_{i=1}^{n} u_1}{\sqrt{n}} v_1 = v_1$

$b_0' = \frac{Ab_0}{\|Ab_0\|_2} = \frac{cv_1}{c} = v_1$, so, 1 iteration can lead our guess to v_1.

2. a) No. In this plot, the data doesn't contain $(0,0,0)$

b) We can use feature projection, e.g., PCA.

c) Yes, $(0,0,0)$ is included

d) 1-D subspace is good approximation since data points maximize the largest variation by first principal component.

e) $X = XT$, $X_2 = V_1 G_1 U_1^T$.

By Eckart Young Theorem, best rank-1 approximation to X_2 is $G_1 z_1 v_1^T$, so, $x_{2i} \simeq G_1 u_1 v_1^T$.

w_i is $G_1 u_i V$, where G_1 is biggest singular value in S,

u_i is the col of u_1.

f) $X^T = U \Sigma U^T = \sum_{i=1}^{3} G_i v_i u_i^T = v_1 \cdot G_1 u_1^T + (G_2 v_2 u_2^T + G_3 v_3 u_3^T)$.

Since $x_i = a w_i + b$, $b = G_2 v_2 u_{2i} + G_3 v_3 u_{3i}$.

g) $\|E\|_F^2 = \sigma_2^2 + \sigma_3^2$.

h. See Screenshot

i) $X^T = U \Sigma V^T = U_1 \sigma_1 V_1^T + U_2 \sigma_2 V_2^T$

So we get $x_{2i} = a_i w_{1i} + a_1 w_{1i} + b$, where $a_i = v_{i1}$, $a_1 = v_{11}$, $w_{1i} = \sigma_1 u_{1i}$

→ rank 2 approx. does fit in a plane, and it captures

j) σ_3^2

k) Rank 1 approx. $\|E\|_F^2 = 626.69$

Rank 2 approx $\|E\|_F^2 = 152.94$

3. a) the avg error rate is 8.14%,

b) the avg error rate is 4.62%.

See screenshot for details.