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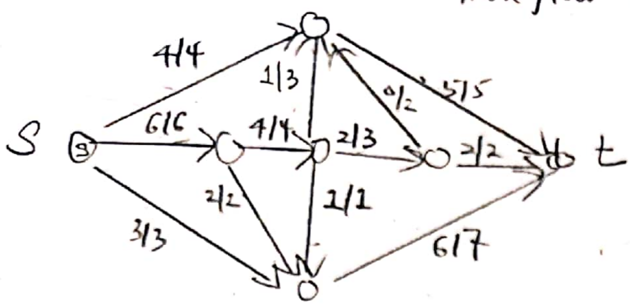
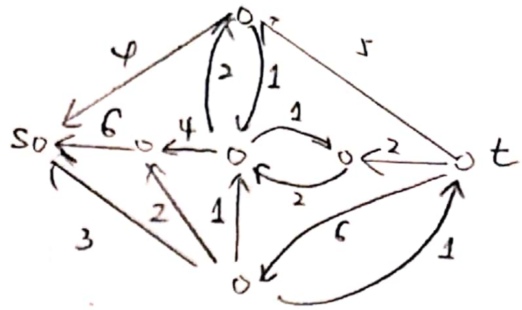
1. a) max s-t flow f^*.

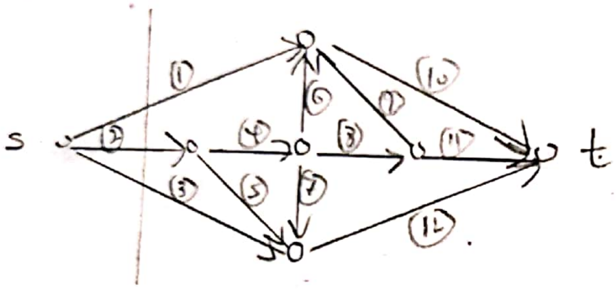
max flow = 13



b) Residual Network G_{f^*}.



c) min s-t cut



Legend: ① represent edge 1,
② represent edge 2, etc.

2. Upper Binding: ③
Lower Binding: ②, ①, ⑫, ⑩, ⑤, ⑪, ③, ⑦

3. Algo:

- We can build residual network G_{f^*} by max s-t flow f^*
- Perform DFS on G, get vertices reachable to source and get vertices reachable to the sink.
- We get all the edges based on these vertices.
- For all these edges:
  if it is the only saturated edge from source to sink, add it to set P.
- Return set P

Program Correctness:

We first get the residual graph, which gives us info. about how many edges left we can use to send flow to sink. By Ford-Fulkerson algorithm, we've already found the max flow graph G. By our algorithm, we continue to find all the edges containing source (call it S) and all the edges containing the sink (call it T). If we add 1 capacity to the edge (that connects set S and T), we can make sure it reach the sink and increase the max flow for the network. Thus, we get the upper-binding edges.

Time Complexity: $O(m+n)$

Get the residual graph takes $O(m+n)$, DFS takes $O(m+n)$, the for loop requires $O(m)$.

So, the total time needed $O(2(m+n)) = O(m+n)$

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