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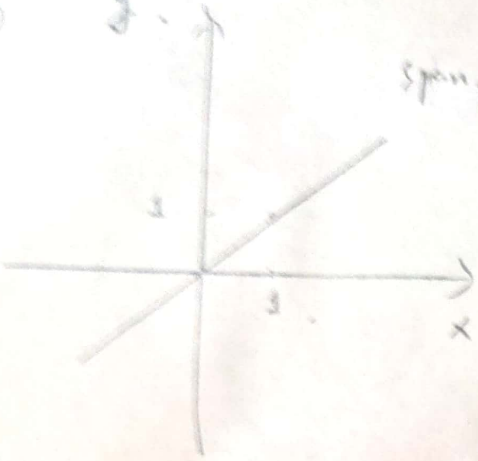
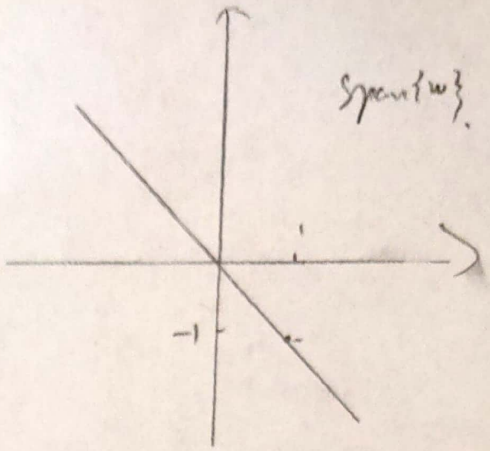
1. a)

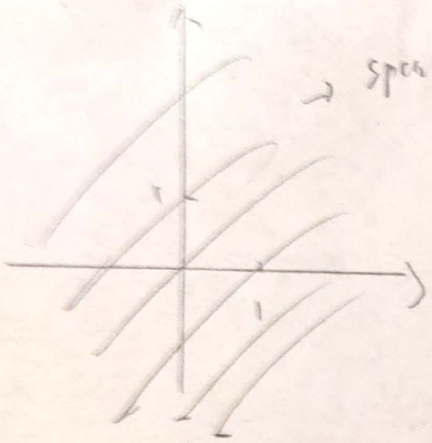




c) → span{z, w} ∎

b) $z^T w = 0$.

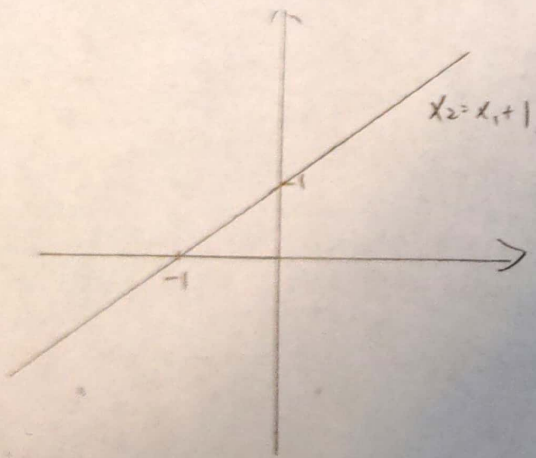
so they are orthogonal.



e) Not orthonormal bases, since not unit length.

$$z = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}, \quad w = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$$

2. a)



b) No.

(0,0) not included

3. a) $Rank\{X\} = 5$

b) $Dim\{X\}$ 5×7.

$Dim\{W\}$ $r \times 7$.

$Dim\{T\}$ $5 \times r$.

c) $w_{ij} = \frac{1}{\sqrt{5}} x_{:,j}$.

d) Avg of x's col are 6, 5.2, 3.6, 6.2, 7.2, 4.4, 2.6.

$w_1^T = \frac{1}{\sqrt{5}} [6 \quad 5.2 \quad 3.6 \quad 6.2 \quad 7.2 \quad 4.4 \quad 2.6]$.

$X = t_1 w_1^T = \frac{1}{\sqrt{5}} [1\ 1\ 1\ 1\ 1]^T [13\ 42\ 12\ 17 \quad 8.05\ 15.2\ 17.44\ 9.84\ 5.81]$

e) Ioan: Lightest

Jacinta: Lowest

f) $X - t_1 w_1^T = \begin{bmatrix} -2 & 1.2 & -1.6 & 1.2 & -2.2 & -0.4 & -0.6 \\ 3 & -2.8 & 1.4 & -0.8 & 2.2 & 0.6 & 2.4 \\ -2 & 2.2 & -0.6 & 0.2 & -1.8 & -2.4 & -1.6 \\ 3 & -3.8 & 2.4 & -1.8 & 1.2 & 0.6 & 1.4 \\ -2 & 3.2 & -1.6 & 1.2 & -0.8 & -0.4 & -1.6 \end{bmatrix}$

People who prefer "Pride & Prejudice" and "Sense & Sensibility" are likely to favor the rest of 3 movie genres.