Activity 7 Yizhou Lu

1. a)

$$u_1 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ \phi \end{bmatrix}$$

$$u_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} - u_1\left(u_1^T \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1/2 \\ -1/2 \\ 2 \end{bmatrix}$$

$\rightarrow$ to unit length $u_2 = \begin{bmatrix} \sqrt{6}/6 \\ -\sqrt{6}/6 \\ \sqrt{6}/3 \end{bmatrix}$.

Geo of span $\{u_1, u_2\}$ is a plane.

b) i) Yes.

ii). $u_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} \\ \phi \\ 1/\sqrt{2} \end{bmatrix}$

Similar as a), we get $u_2 = \begin{bmatrix} \sqrt{6}/6 \\ \sqrt{6}/3 \\ -\sqrt{6}/6 \end{bmatrix}$.

span is the same plane as a).

iii) Not unique. Doesn't depend on order of cols.

2. a).

$$U = \begin{bmatrix} 1/\sqrt{2} & \sqrt{6}/6 \\ 1/\sqrt{2} & -\sqrt{6}/6 \\ \phi & \sqrt{6}/3 \end{bmatrix}$$

b).

$$U^T U = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_{2\times 2}$$

c). a_1 dim 2×2

a_1 contains the weight for Va_1 basis of orthonormal basis.

a_2 contains the weight for Va_2 basis of orthonormal basis.

d)

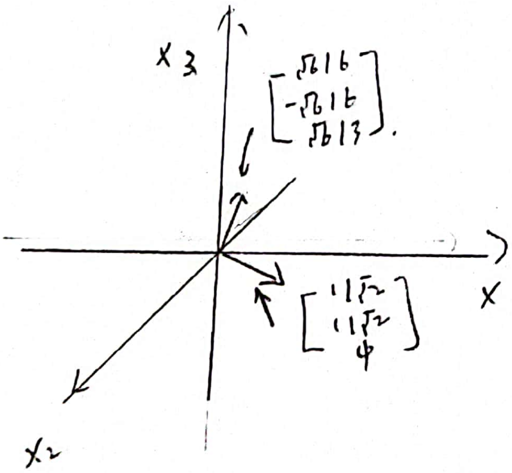
$$\hat{A} = U^T X = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & \phi \\ \sqrt{6}/6 & -\sqrt{6}/6 & \sqrt{6}/3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \sqrt{2} & \phi \\ \phi & \sqrt{6}/2 \end{bmatrix}.$$

3. a) T is a square matrix and all cols are linearly ind.

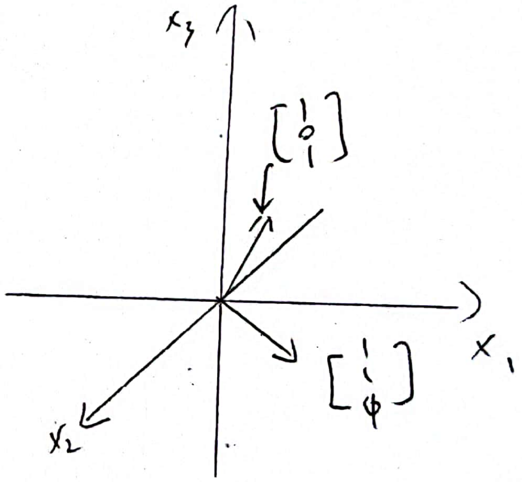
b).
$$P_X = X(X^TX)^{-1}X^T$$
$$= X X^{-1}(X^{-1})^{-1}X^T.$$
$$= UT(UT)^{-1}((UT)^T)^{-1}(UT)^T.$$
$$= U(U^TU)^{-1}U^T.$$
$$= P_U.$$

c) $P_U = U(U^TU)^{-1}U^T = UU^T.$

4. a)



$\begin{bmatrix} -\sqrt{6}/6 \\ -\sqrt{6}/6 \\ \sqrt{6}/3 \end{bmatrix}$, $\begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ \phi \end{bmatrix}$



$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \\ \phi \end{bmatrix}$

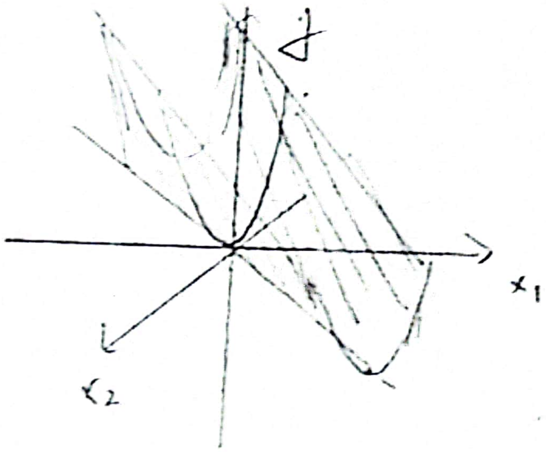
b).
$$\hat{b} = UU^Tb = \begin{bmatrix} 1/\sqrt{2} & \sqrt{6}/6 \\ 1/\sqrt{2} & -\sqrt{6}/6 \\ \phi & \sqrt{6}/3 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & \phi \\ \sqrt{6}/6 & -\sqrt{6}/6 & \sqrt{6}/3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5/3 \\ 4/3 \\ 1/3 \end{bmatrix}.$$

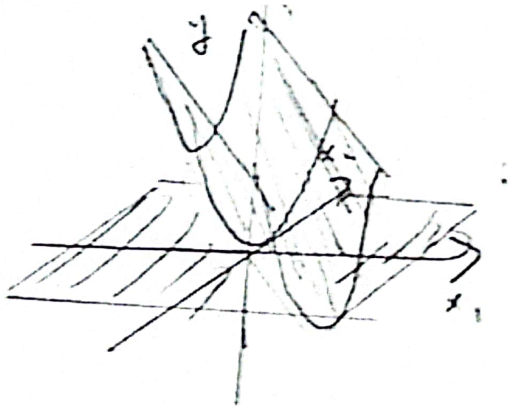
1. a) $Q = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$y = [x_1 \; x_2] \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = [x_1 + x_2 \;\; x_1 + x_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$= x_1^2 + x_1 x_2 + x_1 x_2 + x_2^2 = (x_1 + x_2)^2$



b)



c) Not the unique soln

d) $Q \geq 0$ but Q is not $> \phi$

since Q is positive semidefinite, not positive definite