1. $\begin{cases} x \equiv c \pmod p \\ x \equiv c \pmod q \\ x \equiv c \pmod r \end{cases}$

Since $c < \min\{p, q, r\}$, clearly, $x = c$ is a solution to the equations above

CRT implies the solutions to the above equations are congruent modulo $N = pqr$

Therefore $x \equiv c \pmod N$

2. $\begin{cases} x \equiv 1 \pmod 5 \\ x \equiv 3 \pmod 7 \\ x \equiv 3 \pmod{11} \end{cases}$

$M_1 = 77 \quad M_2 = 55 \quad M_3 = 35$

$N_1 = 3 \quad N_2 = 6 \quad N_3 = 6$

$77 \times 3 \times 1 + 55 \times 6 \times 3 + 35 \times 6 \times 3 = 1851.$

Since $x \equiv 1851 \pmod{385}$ and $x < 1500$

$x = 1466$. Therefore, there are 34 deserters

3. The CRT does not apply because $\gcd(10, 25) = 5 \neq 1$

$x \equiv 3 \pmod{10} \Rightarrow \begin{cases} x \equiv 3 \pmod 2 \\ x \equiv 3 \pmod 5 \end{cases}$

$x \equiv 3 \pmod{25} \Rightarrow x \equiv 3 \pmod 5$

$x \equiv 3 \pmod 2 \Rightarrow x \equiv 1 \pmod 2$

The system is equivalent to

$$\begin{cases} x \equiv 3 \pmod 7 \\ x \equiv 1 \pmod 2 \\ x \equiv 3 \pmod{25} \end{cases}$$

$M_1 = 350$ $M_2 = 175$ $M_3 = 14$

$N_1 = 1$ $N_2 = 1$ $N_3 = 9$

$350 \times 1 \times 3 + 175 \times 1 \times 1 + 14 \times 9 \times 3 = 703$

$703 \bmod 350 = 3$

Therefore the solution is $x \equiv 3 \pmod{350}$

4. If $m \notin \mathbb{Z}_N^*$, then $\gcd(m, N) \neq 1$

Since $N = pq$ where p, q are two large primes, $\gcd(m, N) = p$ or $\gcd(m, N) = q$

$\gcd(m, N)$ can be computed efficiently using the Euclidean algorithm.

Therefore, Oscar can factor N