CS 577: Introduction to Algorithms — Homework 6

Out: 3/10/20 — **Due: 3/24/20**

Ground Rules

- The homework is worth 2 points (out of a total of 18 points you can accumulate in this course).
- The homework is to be done and submitted individually. You may discuss the homework with others in either section but you must write up the solution *on your own*
- You are not allowed to consult any material outside of assigned textbooks and material the instructors post on the course websites. In particular, consulting the internet will be considered plagiarism and penalized appropriately.
- The homework is due at 11:59 PM on the due date. No extensions to the due date will be given under any circumstances.
- Homework must be submitted electronically on canvas. We will not accept paper submissions. Canvas accepts files in pdf/doc/jpeg formats.

Problem (page limit: 4 pages)

After running the teleportation delivery company *Algo Express* for many years, you discover the power of dynamic programming. You leave the company to start a new venture (*DPAlgo Express*) that can process very big delivery orders. In particular, each order now takes several days for the teleportation machine to complete.

Suppose on a certain day, n customers give you packages to deliver. Each delivery i should be made within d_i days, takes t_i days to deliver, and the customer pays you p_i dollars for doing it on time (if you don't do it on time, you get paid 0 dollars). On-time delivery means that if package i is due within $d_i = k$ days, the delivery should be completed on or before day k to be on time (that is, it should start on or before day $k - t_i$). As before, your teleportation machine can only make one delivery at a time.

Input: A set of n deliveries with due dates $d_i \in \mathbb{N}, d_i \geq 1$, number of days needed for delivery $t_i \in \mathbb{N}, t_i \geq 1$ and payments $p_i > 0$ for each delivery $i \in \{1, \cdots, n\}$.

1. Describe and analyze an efficient algorithm to determine which deliveries to make and in what order so as to maximize your profit. (Note: unlike the previous version of the problem, deliveries may now take more than one day). Your algorithm should have a pseudo-polynomial running time – running time polynomial in n and T, where T is the latest deadline among all deliveries.
2. Unfortunately your "greedy" competitor from homework 4, *Algo Express*, is catching up to your lead in the very big delivery business. You decide to focus on maximizing your market share to beat out the competition. To do this, you have decided to neglect profits and only maximize the number of deliveries that you make (remember - deliveries may still take more than 1 day each). Describe and analyze a polynomial time algorithm to determine which deliveries to make and in what order so as to maximize the number of deliveries you make. The input to the problem is the same as before, but this time your algorithm should run in time polynomial in n alone.

We recommend using dynamic programming for each of the above parts. Please provide a brief proof of correctness for your recursive equations.