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1. a)

$\min_w \|d - Aw\|_2$ → minimize euclidean distance

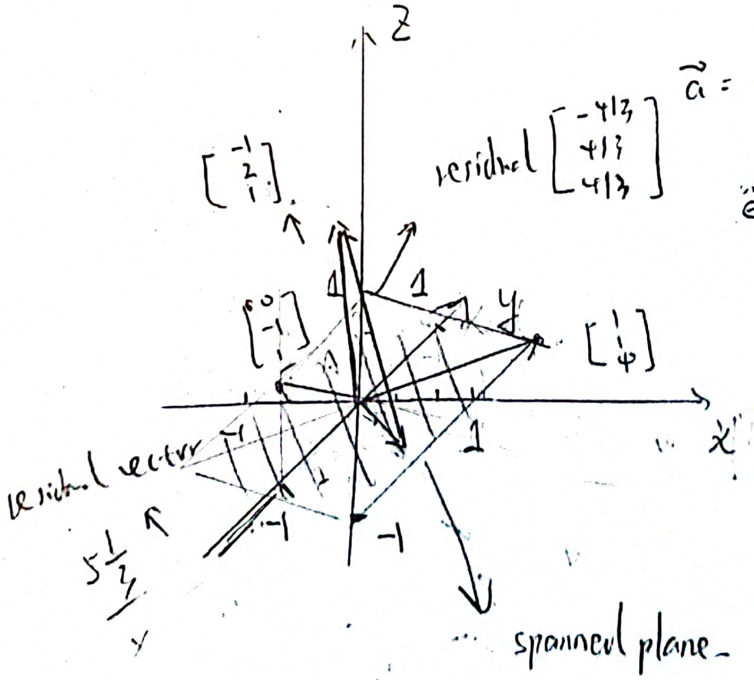
$$\min\left(\underset{3\times 1}{\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}} - \underset{3\times 2}{\begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix}} \underset{2\times 1}{w} \right)$$

$$= \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 - w_1 \\ 2 - w_1 + w_2 \\ 1 - w_2 \end{bmatrix}$$

$$\vec{w} = (A^T A)^{-1} A^T d \Rightarrow w = \begin{bmatrix} \frac{1}{3} \\ -\frac{1}{3} \end{bmatrix}$$

b).



$$\vec{a} = \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0.33 \\ -0.33 \end{bmatrix} = \begin{bmatrix} 1/3 \\ 2/3 \\ -1/3 \end{bmatrix}$$

$$\vec{e} = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} - \vec{a} = \begin{bmatrix} -4/3 \\ 4/3 \\ 4/3 \end{bmatrix}$$

2. a)

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import numpy as np.
from scipy.linalg import solve

X = solve (A, b)
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Solution $x = \begin{bmatrix} 1.25 \\ 17.5 \\ 3.75 \end{bmatrix}$

b). i) No. Since matrix A is singular.

ii) No. Since matrix A is singular.

iii)

$$A^T(A\vec{x} - \vec{b}) = \phi$$

$$\vec{x} = (A^TA)^{-1}A^Tb = \begin{bmatrix} 4 \\ 4 \\ 9 \\ 4 \end{bmatrix}$$

$$\vec{b}' = A\vec{x} = \begin{bmatrix} 104 \\ 97 \\ 193 \\ 132 \\ 174 \end{bmatrix}$$

$$\|A\vec{x} - \vec{b}\|_2^2 = 1.46 \times 10^{-12}$$

3. a)

$$A = TW^T = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & -0.5 \\ 0.5 & -0.5 \\ 0.5 & 0.5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -0.5 & 1 \\ 0 & 1.5 & 0 \\ 0 & 1.5 & 0 \\ 1 & -0.5 & 1 \end{bmatrix}$$

4×2 $\quad 2 \times 3$

$\text{rank}(A) = 2$

b). Dim 2, t_1 and t_2 are linearly independent

c) No. Since Q is not invertible

d) infinite number of soln. Rank(A) = 2 < number of A's feature.

e). $$Ax = TW^T W \tilde{x} = \begin{bmatrix} 1.5 & 3 \\ 1.5 & -3 \\ 1.5 & -3 \\ 1.5 & 3 \end{bmatrix} \tilde{x}.$$

$$\min_x \|b - Ax\|_2^2 = \phi.$$

$$\rightarrow \quad (TW^TW)^T (b - TW^TW\tilde{x}) = \phi$$

$$\tilde{x} = \begin{bmatrix} 1 \\ 0.167 \end{bmatrix}.$$