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1. a) rank = 2

b). linearly independent cols: $\begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\\0\end{bmatrix}$.

total: 5 sets

c). $$w_1\begin{bmatrix}1\\1\\0\end{bmatrix} + w_2\begin{bmatrix}0\\1\\1\end{bmatrix} = \begin{bmatrix}a\\b\\-1\end{bmatrix}$$ linear combination

So, we get $\begin{bmatrix}w_1\\w_1+w_2\\w_2\end{bmatrix} = \begin{bmatrix}a\\b\\-1\end{bmatrix}$

Then, $a = b+1$ make rank{A} = 2

2. a) $$\begin{bmatrix}1&0\\1&1\\0&1\end{bmatrix}x = \begin{bmatrix}8\\6\\-2\end{bmatrix} \rightarrow \begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}8\\-2\end{bmatrix}. \quad x = \begin{bmatrix}8\\-2\end{bmatrix}$$

3×2 $\quad 2\times1.$ $\quad 3\times1.$

b). $$\begin{bmatrix}1&0\\1&1\\0&1\end{bmatrix}x = \begin{bmatrix}4\\6\\1\end{bmatrix}$$ → no solution, since we cannot satisfy $x_1 = 4$, $x_1+x_2 = 6$, $x_2 = 1$ at the same time.

c) rank{A} = rank{[A b]} means that col b in span A and we ~~can~~ know that it can be expressed as a linear combination of col vectors in A. linear combinations means that there must be at least 1 solution exist.

for $Ax=b$.

3. a) i) Yes.

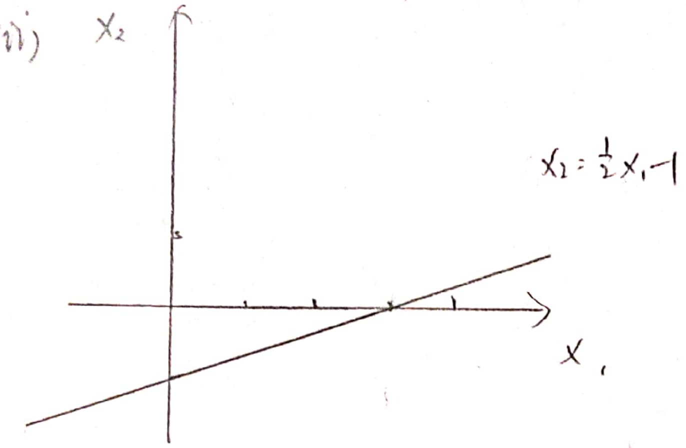
ii) It's non-unique

rank(A) = 1 < 2,

rank(A) = rank(A|b) = 1

So there's more than 1 solution

iii)



b). Given $A(x+w) = b$, A's dim is $N \times M$,

Since $Ax = A(x+w) = b$, we get $Aw = \phi$

So we can conclude that A's col vector is linearly dependent, rank $A < M$.

Given A's dim as $N \times M$,

we conclude that $\text{rank}\{A\} \leq \min\{N, M\}$ & $\text{rank}\{A\} < M$

and can use this eqn. to determine the solution existence.