

1. An example for which  $\Pr[\text{Enc}_k(m)=c] \neq \Pr[\text{Enc}_k(m')=c]$  could be:

Let  $m = \text{aaa}$ ,  $m' = \text{abc}$

$c = \text{zzz}$

Then  $\Pr[\text{Enc}_k(m)=c] > 0$  and

$\Pr[\text{Enc}_k(m')=c] = 0$ , since  $a, b, c$  can't be mapped to  $z$  at the same time by mono-alphabetic substitution cipher

Thus, mono-alphabetic substitution cipher is not perfectly secret

2. Since the ciphertext has 17 characters, and two days are in order, the plaintext can only be WEDNESDAYTHURSDAY or WEDNESDAYSATURDAY, which have exactly 17 characters

m W E D N E S D A Y

22 4 3 13 4 18 3 0 24

c Y U B C X G J R Y

24 21 1 2 23 6 9 17 24

k C R Y P T O G R A

2 17 24 15 19 14 6 17 0

By doing the math with first 9

characters of the ciphertext and

WEDNESDAY, we get the first 9

characters of the key to be

CRYPTOGRA

Thus, it is reasonable to guess the key starts with CRYPTOGRAPHY.

c H H R

7 7 17

Using PHY to decipher HHR, we get SAT.

k P H Y

15 7 24

Therefore, the plaintext must be WEDNESDAYSATURDAY.

m S A T

18 0 19

By comparing SATURDAY with HHRCJUL, we know the next part of the key is ISFUN, so the key is

CRYPTOGRAPHYISFUN

3. a

pqrs	15	16	17	18
mnp	12	13	14	15
svux	18	21	20	23
jml	9	12	11	14

Both pqrs and mnp have a form of  $k, k+1, k+2, k+3$  and both svux and jml have a form of  $k, k+3, k+2, k+1$ . Since the shift cipher shifts all characters by an equal amount, it is possible to tell whether the plaintext has a form of  $k, k+1, k+2, k+3$  or  $k, k+3, k+2, k+1$ . However, it is impossible to tell pqrs from mnp and tell svux from jml. For example, pqrs with a key 1 and mnp with a key 4 both gives qrst. Thus, it's impossible to determine the users password.

3, b. Period 2: In this case, the first and third characters will be shifted by the same amount. So do the second and fourth ones.

	3rd-1st	4th-2nd
mnp	$\boxed{2}$	$\boxed{2}$
jml	$\boxed{2}$	$\boxed{2}$

However, the differences are all 2. Thus, it's impossible.

Period 3: In this case, the first and fourth characters will be shifted by the same amount

	4th-1st
mnp	3
jml	5

The differences are different. So for the ciphertext, if this difference is 3, the plaintext is mnp, if this difference is 5 the plaintext is jml.



Period 4: For some ciphertext  $c$ , there exist keys  $K$  and  $K'$  such that  $\text{Enc}_K(\text{mnop}) = \text{Enc}_{K'}(\text{jml o}) = c$ . Thus, it's impossible.

4. a.  $\Pr[\text{Enc}_K(0)=0] = \frac{2}{5}$  (when  $k \in \{0, 5\}$ ).

$\Pr[\text{Enc}_K(1)=0] = \frac{1}{5}$  (when  $k=4$ )

$\Pr[\text{Enc}_K(0)=0] \neq \Pr[\text{Enc}_K(1)=0]$

Thus, it's not perfectly secret

4. b.  $|M| = |\{0, 1\}^{2n}| = 2^{2n}$ ,  $|K| = |\{0, 1\}^n| = 2^n$

$|K| < |M|$

According to Theorem 2.10, if  $|K| < |M|$ , then the encryption scheme is not perfectly secret.