CS 577 Yizhou Lin lin773@wisc.edu HW10

1. We need to show ① LTFS is in NP
② find a mapping reduction from IS (Independent Set) to LTFS.
to prove that LTFS is NP-Complete.

①: Suppose we have a subset of size $\geq k$ for graph G, we can count every triple in the proved set. For each triple, if it meets the condition that it has at least 1 absent edge, then we can make sure that the subset is a triangle free subset and continue. If not, then it is not a triangle free subset. Since we can verify whether it's a triangle free subset or not in polynomial time, LTFS is in NP. □

②: → We need to construct a mapping from IS to LTFS in order to reduce IS to LTFS. To be specific, we have to build a mapping from an instance of $IS(G, k)$ to instance of $LTFS(G', k')$. Assuming we have an instance (G, k) of IS (i.e. $G=(V,E)$, $V=n$ (number of vertices), $E=m$ (number of edges)), we create a new vertex w_e for every $e=(u,v)\in E$ (call the set of vertices as W). For each vertex $w_e \in W$ ($e=(u,v)$), create 2 new edges (w_e, u) and (w_e, v) and we call this edge set E_W. Then we set G' to be graph G with added vertices and edges, ($G=(V\cup W, E\cup E_W)$). Let $k'=m+k$.

→ Then, the reduction substitute each edge $e=(u,v)$ in G by $\Delta\{u, w_e, v\}$.

We need to show if G' has a triangle-free subset of size at least $k'=m+k$, then G has an IS of size at least k.

Pick an IS S of size k in V, and let $S'=S\cup W$. Since $|S'|=m+k$, S' is triangle-free. In addition, the addition of any vertex in W won't create a Δ. (u and v cannot be both in S)

→ In this case, we need to show that if G' has a triangle-free subset of size at least $k'=m+k$, then G has an independent set of size at least k.

We can pick a triangle free subset S' of size $m+k$ from G'. If S' contains the entire set W, then S must be IS of G. (no 2 vertices u and v in S can share an edge e; if w_e, u, v all in S', S' is not triangle-free, contradiction exists) Thus, we may show that we can always find another triangle-free subset contain the entire W with the size of at least $m+k$. So, S is an IS of size k of G.

→ For each vertex $w_e \in W$, $e=(u,v)$ and w_e not in S', we have: a) if u & v both in S', remove u from S', add w_e to S'

b). if at least 1 of u, v not exist in S', add w_e to S'.

No matter for a) & b), we can add w_e into S' by not changing the size of S' or creating a new Δ. For each vertex $w_e \in W$, we perform this operation and get S'' with entire W and size of at least $m+k$.

→ In conclusion, we have shown that we make a reduction from IS to LTFS in polynomial time. Thus, LTSF is NP-Complete