Designing, Visualizing and Understanding Deep Neural Networks

Lecture 5: Convolutional Networks I

CS 194/294-129 Spring 2018
John Canny

Slides originated from Efros, Karpathy, Ransato, Seitz, and Palmer

Last Time - Gradient Descent

To reach a minimum of loss, we should follow the negative gradient.

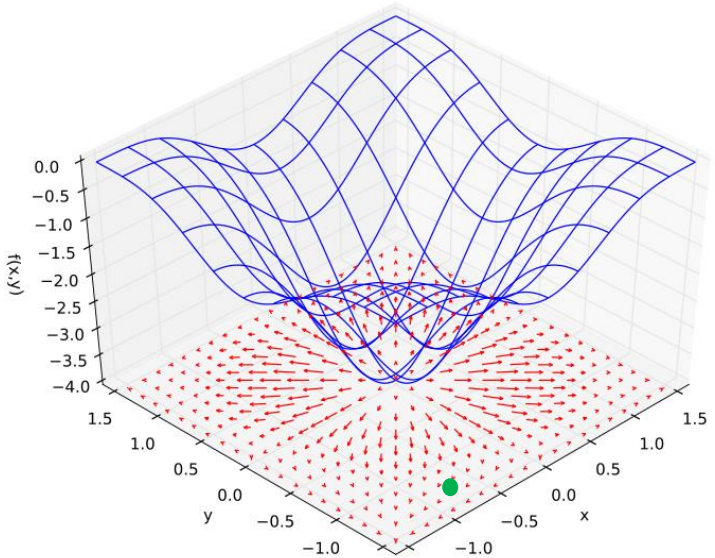
i.e. we should take small steps in direction

$$-\nabla_W L(W)$$

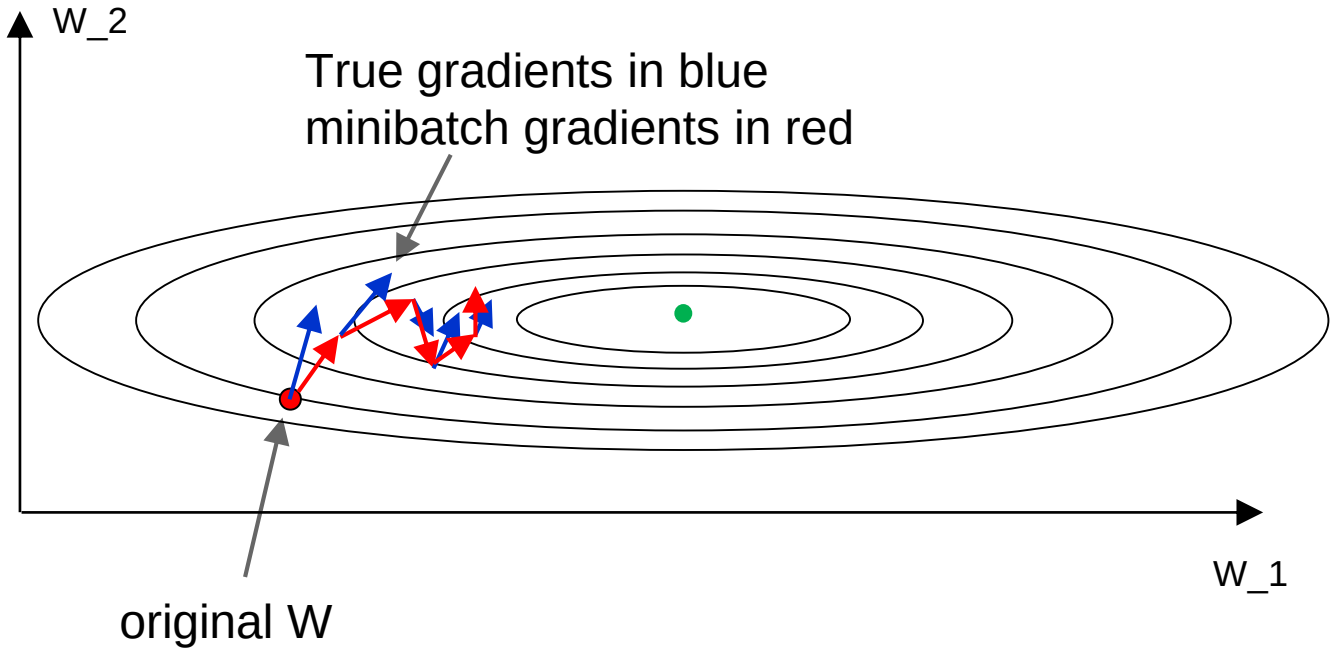
Let W^t denote the weights at step t of gradient descent. Then

$$W^{t+1} = W^t - \alpha \ \nabla_W L(W)$$

Where α is called the ***learning rate***.

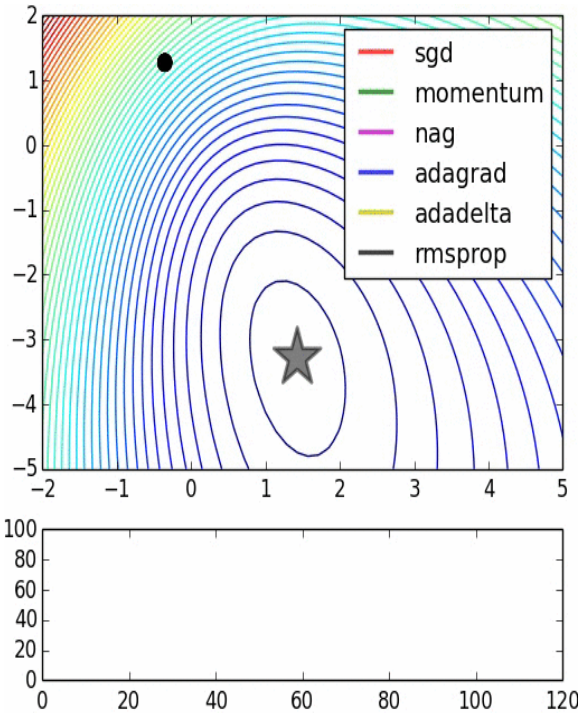


Last time: Minibatches and SGD



Gradients are noisy but still make good progress on average

Last Time: SGD refinements: Momentum, Nesterov Momentum, RMSprop, ADAGRAD



This Time: Backpropagation

So far we have been using gradient methods to minimize a loss over some parameters.

Our loss is of the form $L(f(x, W), y)$.

x is an input, y is a target, and W are the parameters.

To compute the gradient of L wrt W, we need the ***chain rule***. If f is single-valued, W a single parameter the chain rule is just:

$$\frac{dL}{dW} = \frac{dL}{df}\frac{df}{dW}$$

If W is a vector of parameters, then we have:

$$\nabla_W L = \frac{dL}{df}\nabla_W f$$

Which is really just the first rule applied to all the partial derivatives wrt elements of W.

Updates

- Project Proposals are due in two weeks.
- If you need help putting a team together, we suggest you post to Piazza in the “project_teams” folder.
- You can either post your own ideas on project topics, or respond to other groups looking to complete their team.

- Assignment 1 due on Feb 12, make sure at least that you’ve started by now…

The Chain Rule

If f is also vector-valued with k values is a vector of m parameters, then we can apply the chain rule parameter-wise and then sum over the contributions:

$$\frac{\partial L}{\partial W_j} = \sum_{i=1}^{k} \frac{\partial L}{\partial f_i} \frac{\partial f_i}{\partial W_j}$$

For $i = 1, \ldots, m$. This can be written as a matrix multiply

$$J_L(W) = J_L(f) J_f(W)$$

Where $J_f(W)$ is a Jacobian matrix.

$$J_f(W)_{ij} = \frac{\partial f_i}{\partial W_j}$$

Jacobians

The Jacobian generalizes the gradient of a scalar-valued function f to a k-valued function. Here we think of the function as a neural layer with m inputs and k outputs.

$$J_f(W) = \begin{bmatrix} \frac{\partial f_1}{\partial W_1} & \frac{\partial f_1}{\partial W_2} & \cdots & \frac{\partial f_1}{\partial W_m} \\ \frac{\partial f_2}{\partial W_1} & \frac{\partial f_2}{\partial W_2} & \cdots & \frac{\partial f_2}{\partial W_m} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_k}{\partial W_1} & \frac{\partial f_k}{\partial W_2} & \cdots & \frac{\partial f_k}{\partial W_m} \end{bmatrix}$$

The Jacobian has dimensions $k \times m$ which is $n_{outputs} \times n_{inputs}$.

N-step Chain Rule

Now suppose we have several vector-valued functions $A(.)$, $B(.)$, $C(.)$, ... composed in a chain (e.g. a deep network):

$$W \to A \to B \to C \to \cdots L$$

Algebraically, that looks like:

$$L(W) = L(K \cdots C(B(A(W))) \cdots)$$

Then we just multiply Jacobians (matrix multiply $*$) to get the gradient:

$$J_L(W) = J_L(K) * \cdots J_C(B) * J_B(A) * J_A(W)$$

And $J_L(W) = (\nabla_W L)^T$ which is the gradient we need to minimize loss over W.

Backpropagation

Now all the Jacobians are matrices, and matrix multiply is associative.

$$J_L(W) = J_L(K) * \cdots J_C(B) * J_B(A) * J_A(W)$$

So we could actually evaluate the product of Jacobians in any order, including left-to-right and right-to-left.

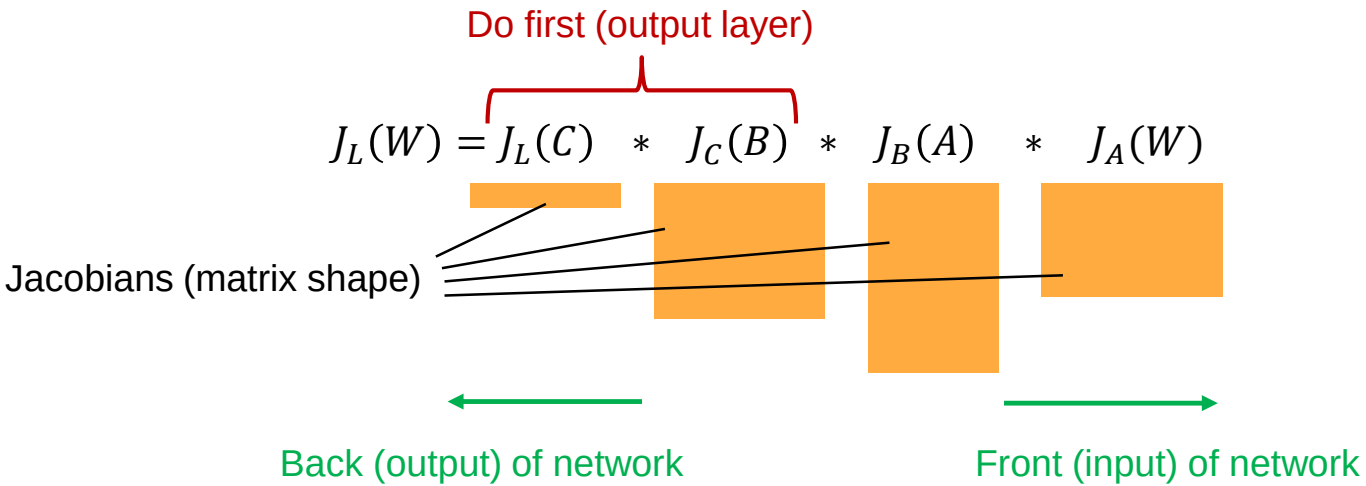
Backpropagation: evaluate the Jacobian product (loss gradient wrt params) left-to-right, i.e. from the output of the neural network toward its input.

Why not some other order?

Backpropagation

Reason 1: Efficiency

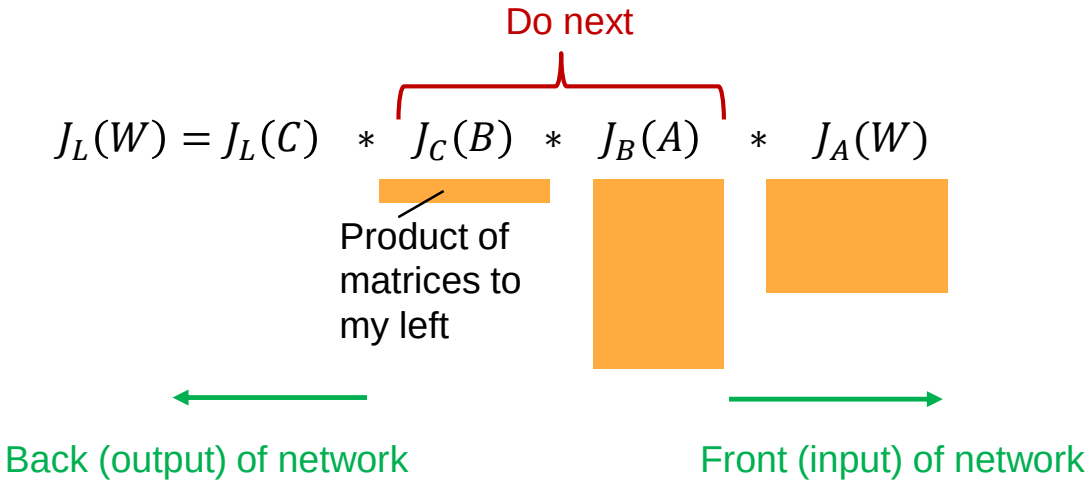
Output Jacobian is always a row vector (because loss is a scalar). Matrix-vector multiply is much less expensive than matrix-matrix multiply.



Backpropagation

Reason 1: Efficiency

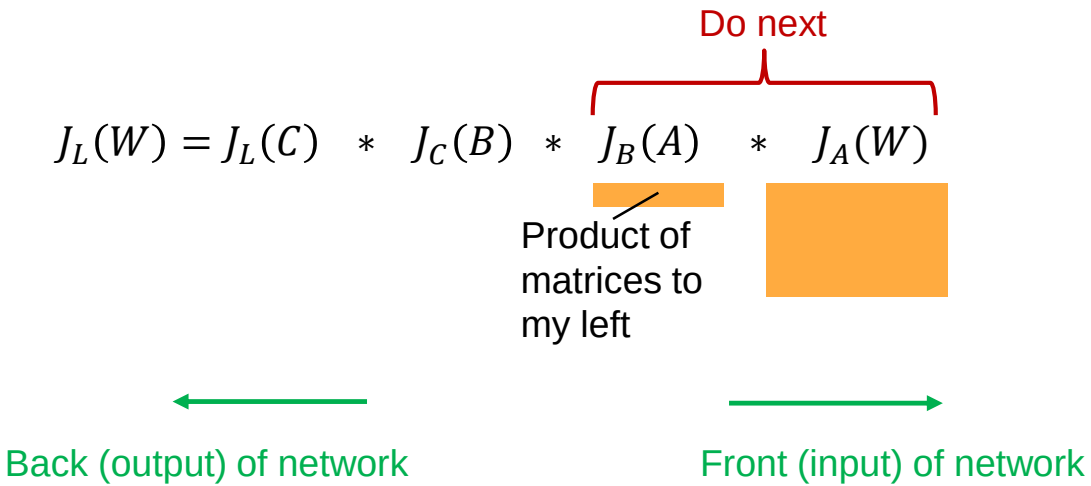
Output Jacobian is always a row vector. Matrix-vector multiply is much less expensive than matrix-matrix multiply.



Backpropagation

Reason 1: Efficiency

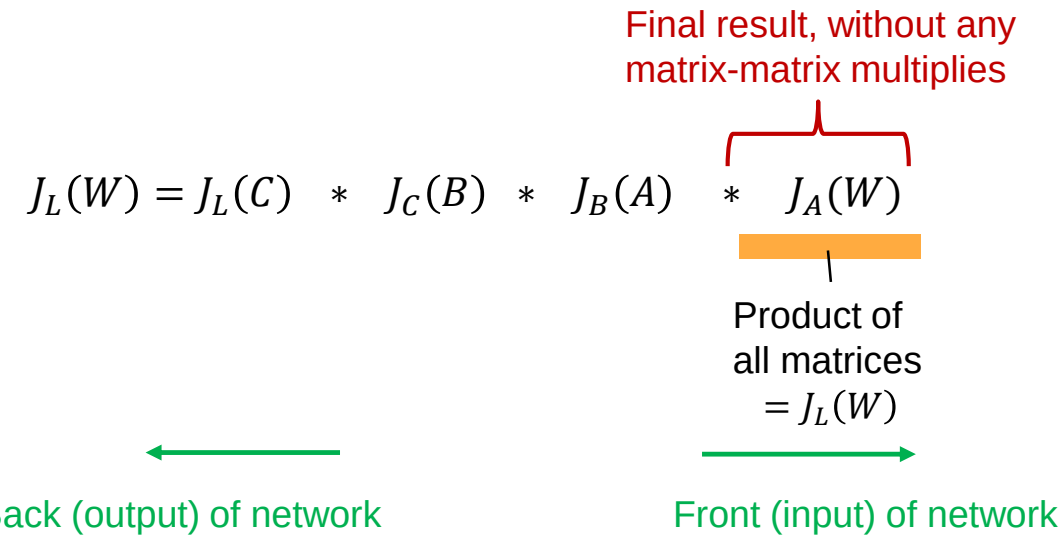
Output Jacobian is always a row vector. Matrix-vector multiply is much less expensive than matrix-matrix multiply.



Backpropagation

Reason 1: Efficiency

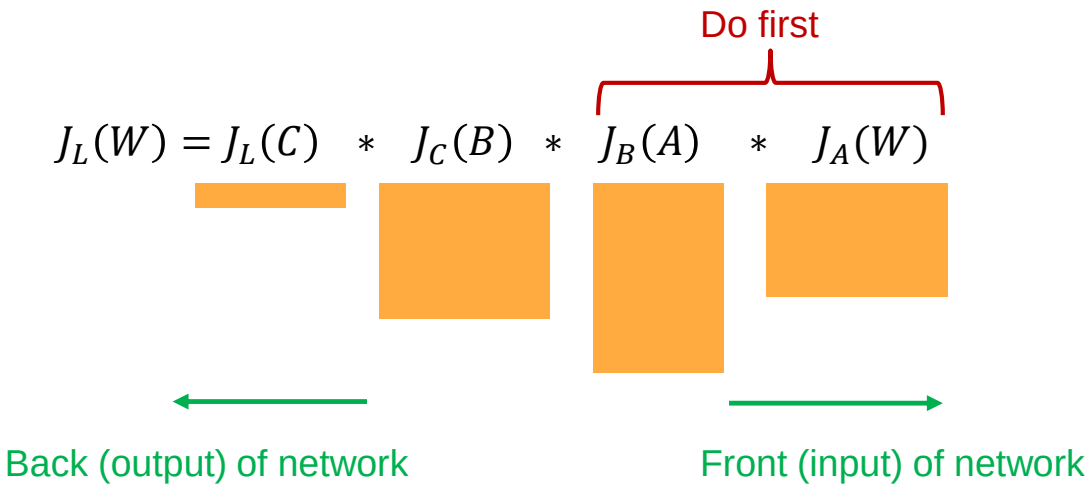
Output Jacobian is always a row vector. Matrix-vector multiply is much less expensive than matrix-matrix multiply.



For-propagation

Reason 1: Efficiency

By comparison, we could invent “for-propagation”:

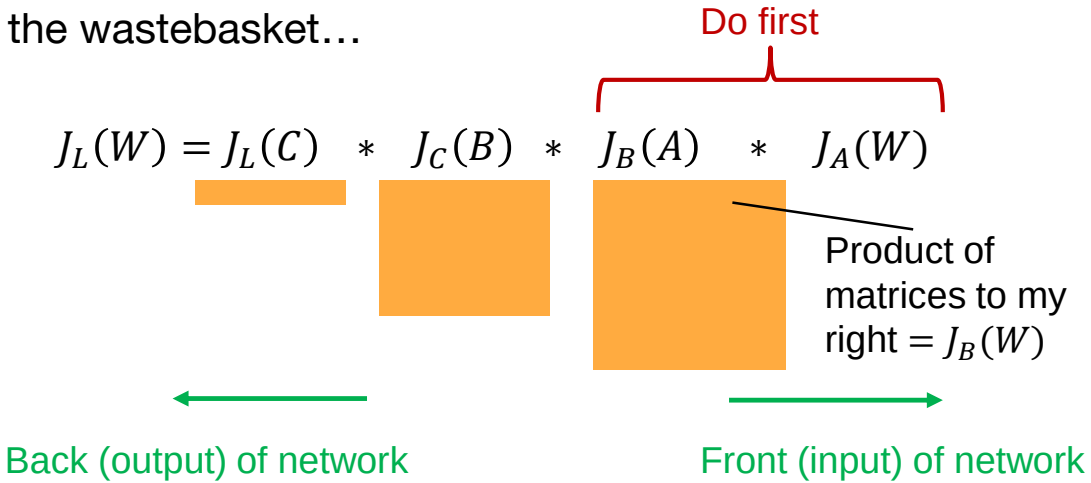


For-propagation

Reason 1: Efficiency

We could invent “for-propagation”: Oops, cost $O(n^3)$ instead of $O(n^2)$ for the first multiply and we still have another matrix-matrix multiply to do.

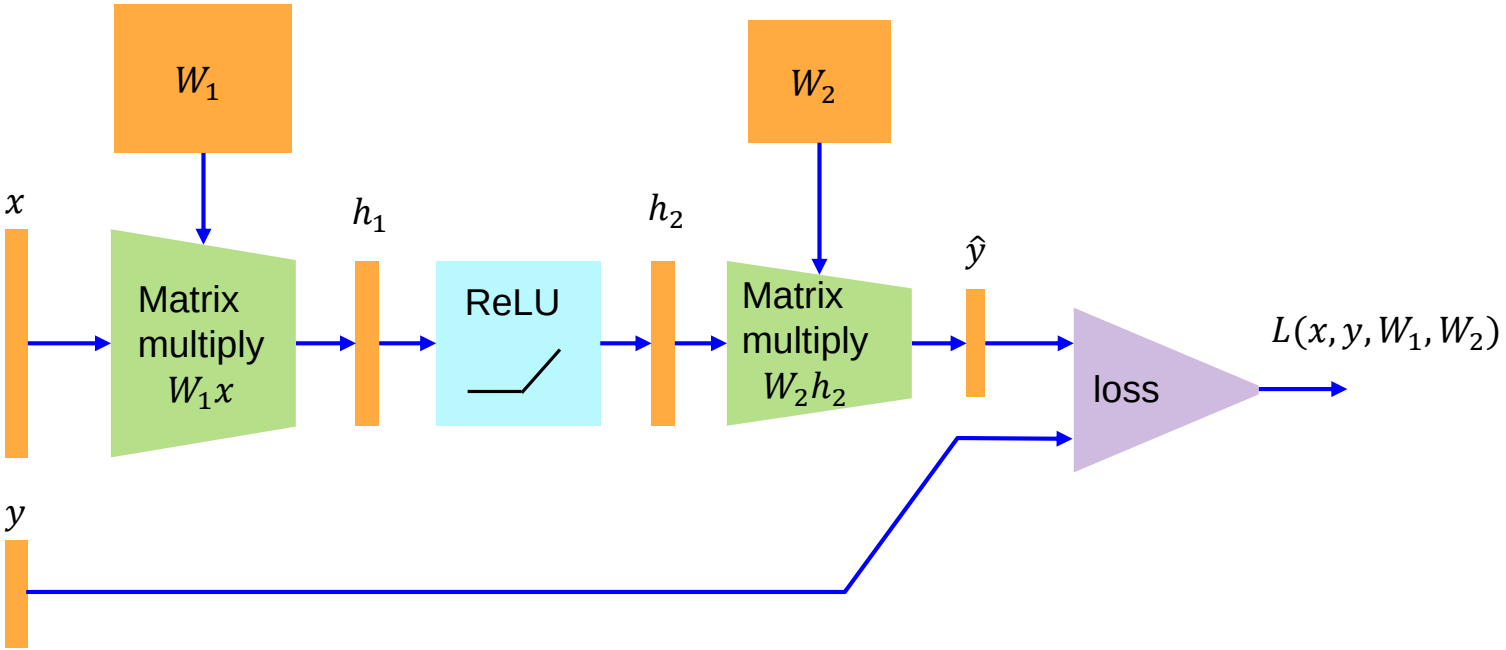
A good invention for the wastebasket…



Backpropagation

Reason 2 to use backpropagation: Common subexpressions.

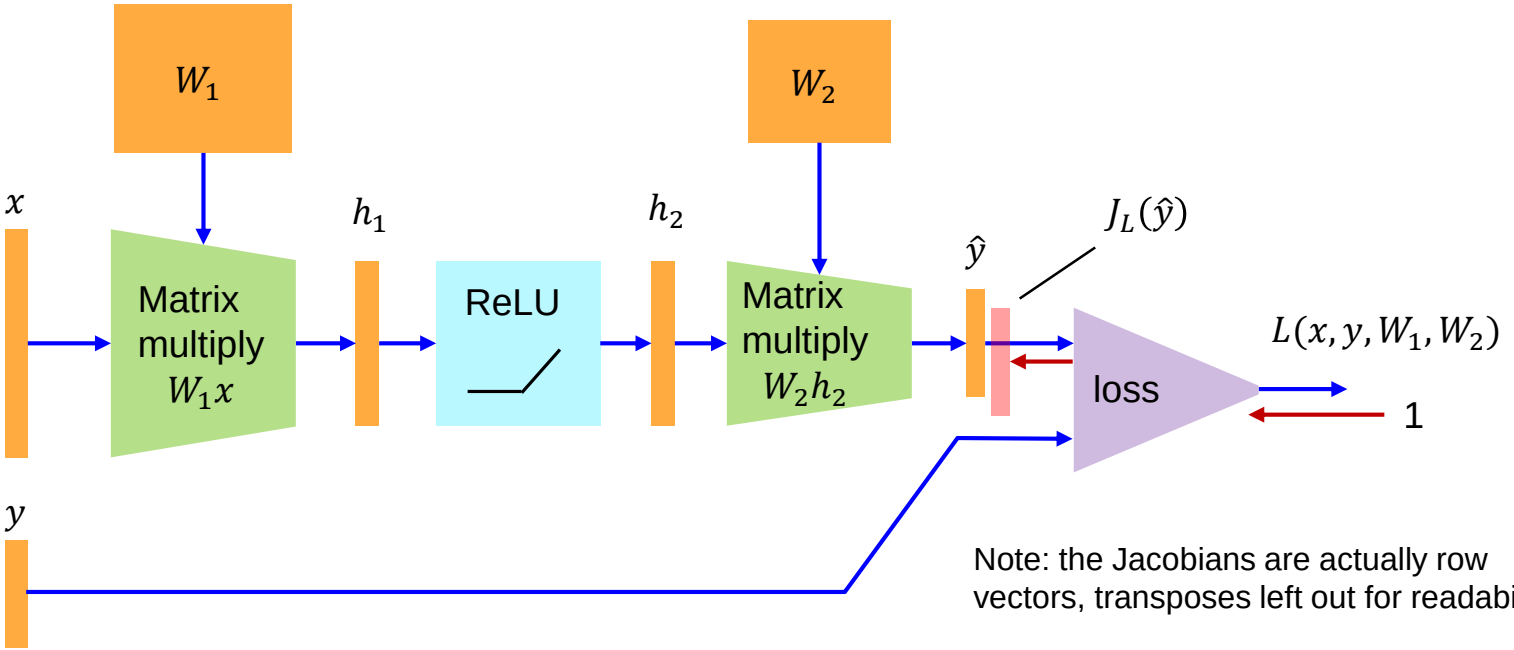
Let's build a real neural network (Tensorflow style):



Backpropagation

Reason 2 to use backpropagation: Common subexpressions.

Let's build a real neural network (Tensorflow style):

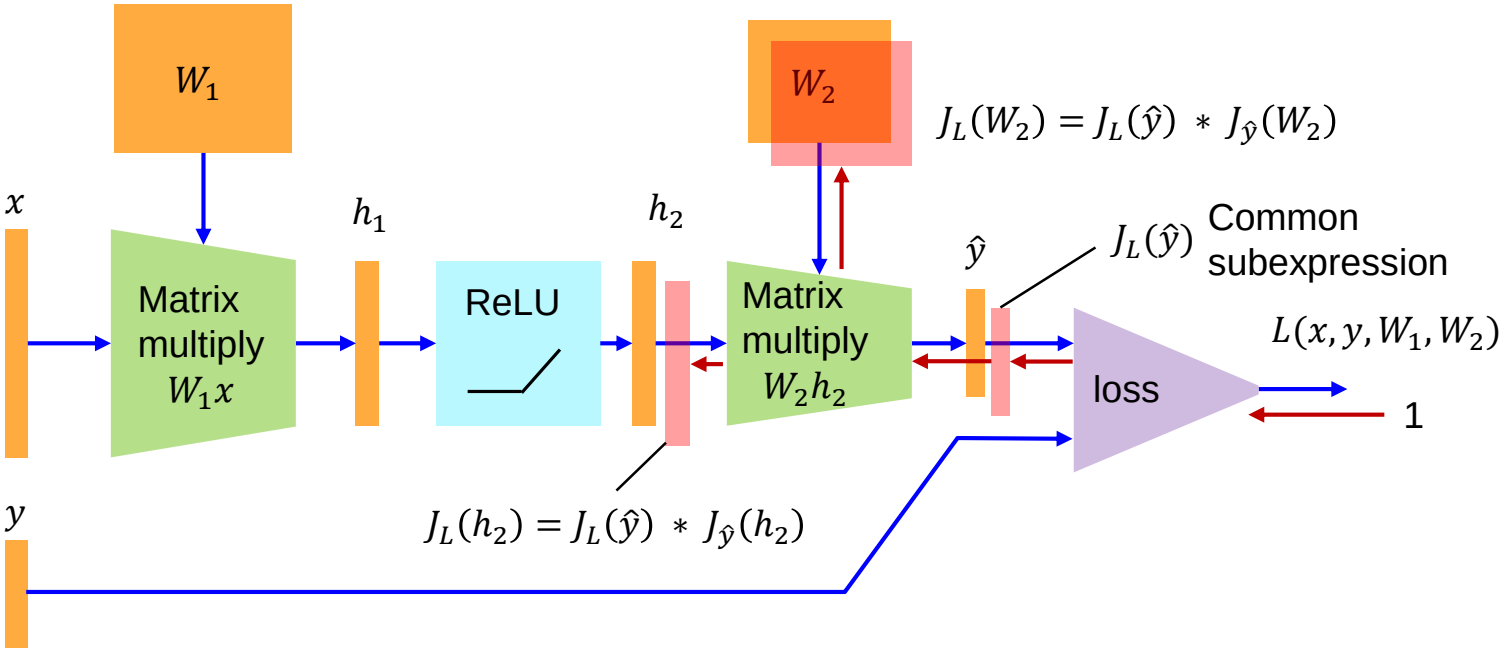


Note: the Jacobians are actually row vectors, transposes left out for readability.

Backpropagation

Reason 2 to use backpropagation: Common subexpressions.

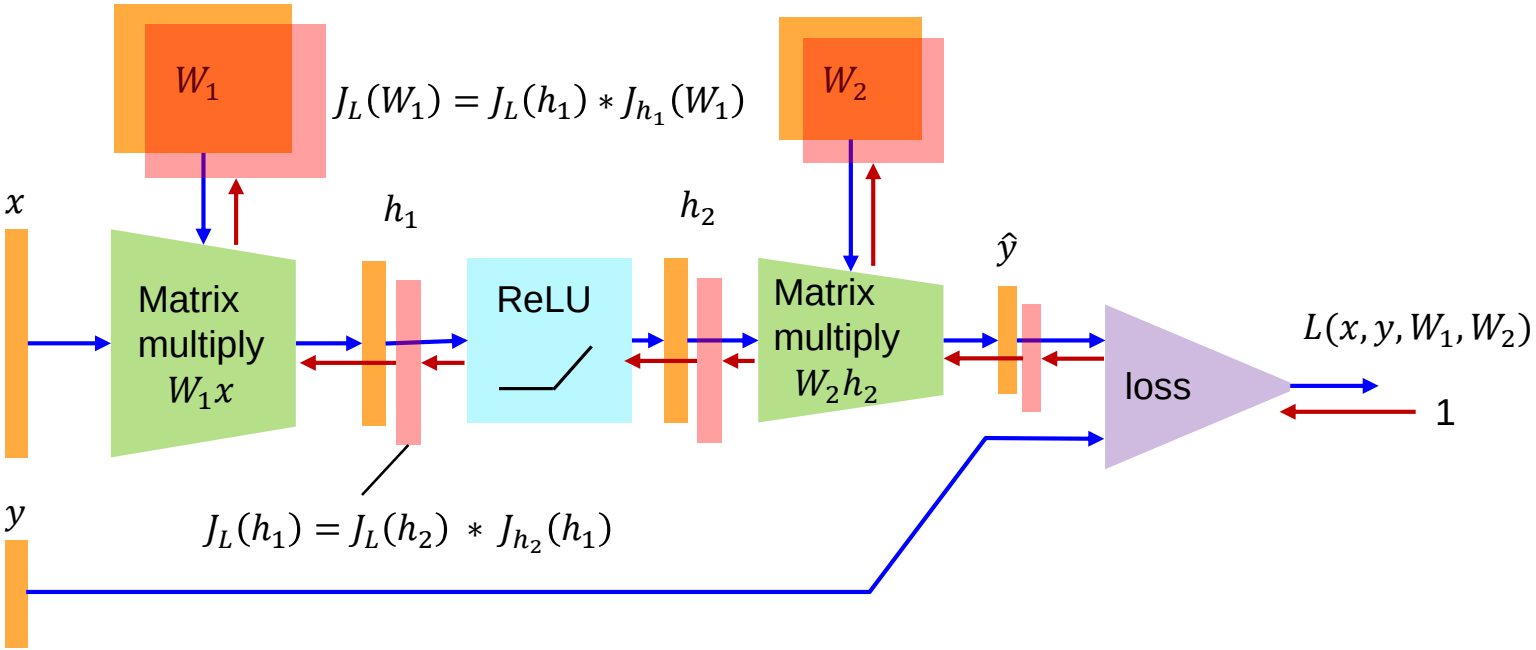
Let's build a real neural network (Tensorflow style):



Backpropagation

Reason 2 to use backpropagation: Common subexpressions.

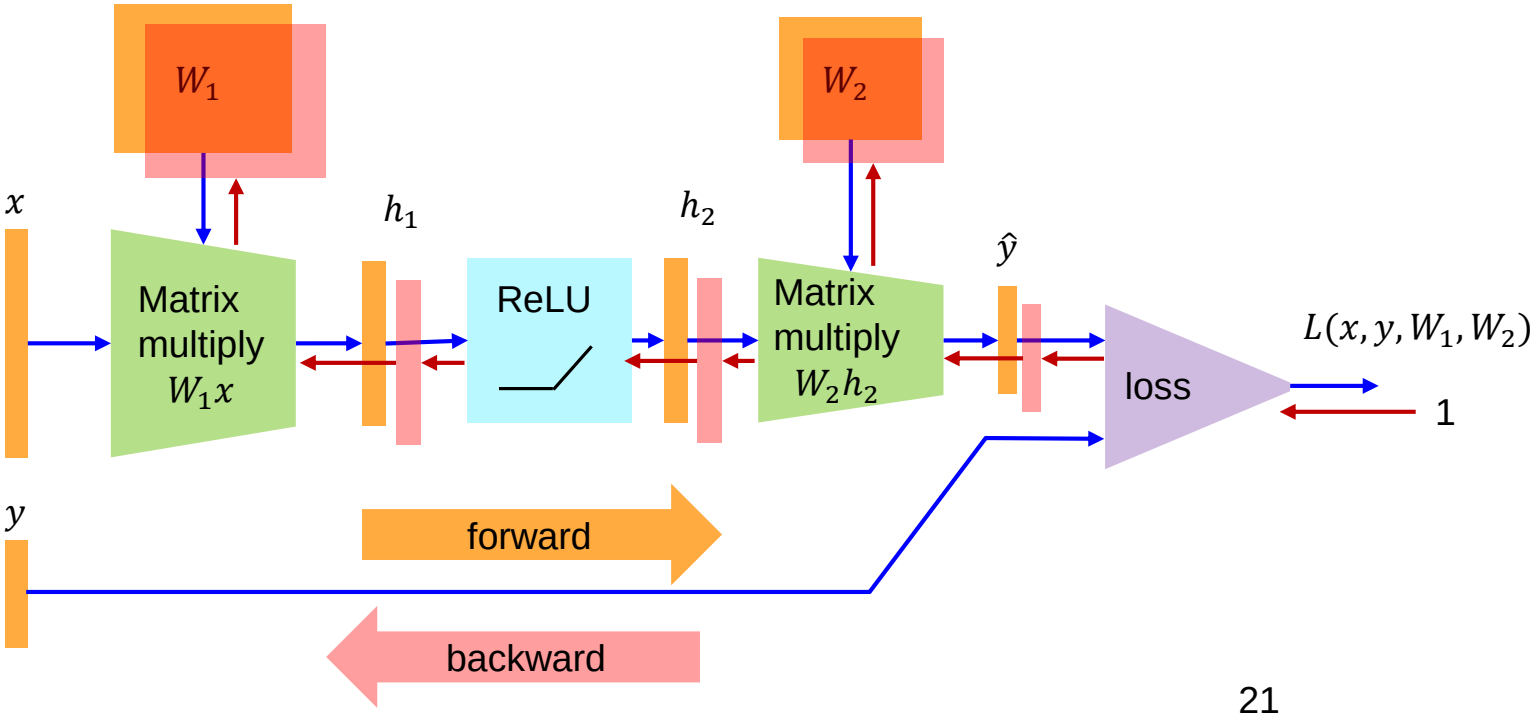
Let's build a real neural network (Tensorflow style):



Last Time: Backpropagation

Backprop Efficiency: matrix-vector multiply only and common subexpressions.

Example: a real neural network (Tensorflow style):



Backpropagation Recipes: Cross Entropy Loss

- This is just calculus: always check yourself:
- Cross-entropy loss (correct class label $y \in \{1, \ldots, n\}$ known exactly, input vector s):

$$L = -\log s_y$$

So

$$J_L(s)_i = \nabla_s L_i^T = \begin{cases} -\dfrac{1}{s_i} & \text{for } i = y \\ 0 & \text{otherwise} \end{cases}$$

In general if y is a distribution, $L = -\sum_{i=1}^{n} y_i \log s_i$, then

$$J_L(s)_i = \nabla_s L_i^T = -\frac{y_i}{s_i}$$

Backprop Recipes: Multiclass SVM Loss

- Assume label $y \in \{1, \ldots, n\}$ known exactly, input vector s:

$$L = \sum_{i=1}^{n} \max(0, 1 - s_y + s_i)$$

So

$$J_L(s)_i = \nabla_s L_i^T = \begin{cases} 1 \text{ if } s_y - s_i < 1 \text{ and } i \neq y \\ 0 \text{ otherwise} \end{cases}$$

Backprop Recipes: Multiclass Logistic Fn

Assume input vector s, output y of same dimension n.

$$y_i = \frac{\exp s_i}{\exp s_1 + \cdots + \exp s_n} = \frac{f}{g}$$

Then

$$\frac{\partial y_i}{\partial s_j} = \frac{f'g - g'f}{g^2} = \frac{\exp s_i \, g\delta_{ij} - \exp s_j \exp s_i}{g^2}$$

Simplifying:

$$\frac{\partial y_i}{\partial s_j} = y_i\delta_{ij} - y_i y_j$$

Where:

$$\delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$$

Backprop Recipes: Matrix Multiply (FC) Layer

Assume input vector s of dimension n, output y of dimension m, layer $y = Ws$

$$y_i = \sum_{j=1}^{n} W_{ij} s_j$$

Then

$$\frac{\partial y_i}{\partial s_j} = W_{ij}$$ and so $$J_L(s) = J_L(y)W$$

Also we want to backprop to the weight matrix and compute:

$$\frac{\partial y_i}{\partial W_{jk}}$$

Which is a tensor (ouch), but notice that its zero unless $i = j$

Backprop Recipes: Matrix Multiply (FC) Layer

We want: $\frac{\partial y_i}{\partial W_{jk}}$

Which is a tensor, but notice that its zero unless $i = j$

So it simplifies to

$$\frac{\partial y_i}{\partial W_{ik}} = s_k$$

And if we already know the output loss Jacobian (from backprop): $J_L(y)$, then

$$\frac{\partial L}{\partial W_{ik}} = \frac{\partial L}{\partial y_i}\frac{\partial y_i}{\partial W_{ik}} = J_L(y)_i s_k$$

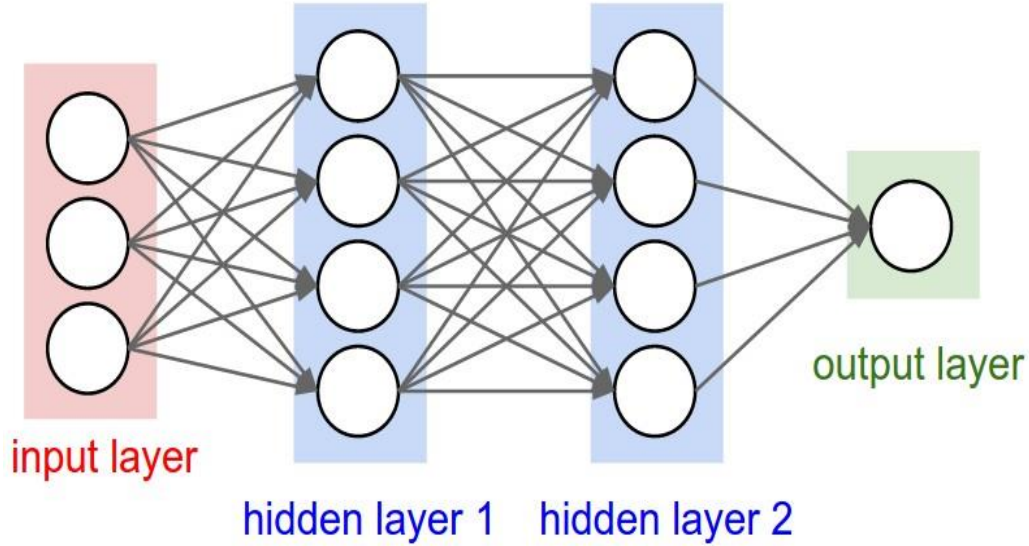
Or more compactly:

$$\frac{\partial L}{\partial W_{ik}} = \underbrace{J_L(y)^T}_{\text{Col vector}} \underbrace{s^T}_{\text{Row vector}}$$

Backpropagation

- Compute function values (activations) from the first layer to the last.
- Compute derivatives of the loss wrt other layers from the last layer to the first (backpropagation).
- This only requires matrix-vector multiplies.
- Paths from the loss layer to inner layers are re-used.

This time: Neural Networks for Visual Data

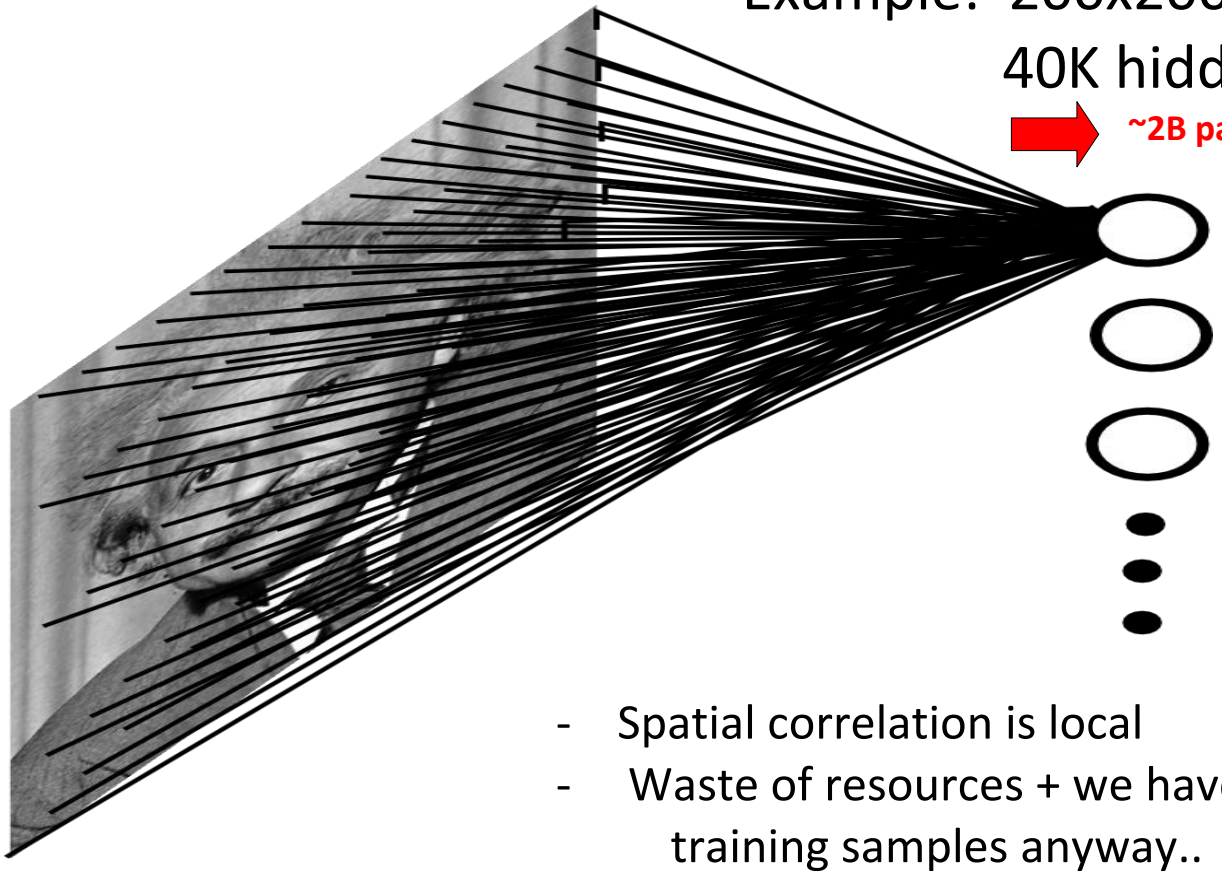


Fully Connected Layer

Example: 200x200 image

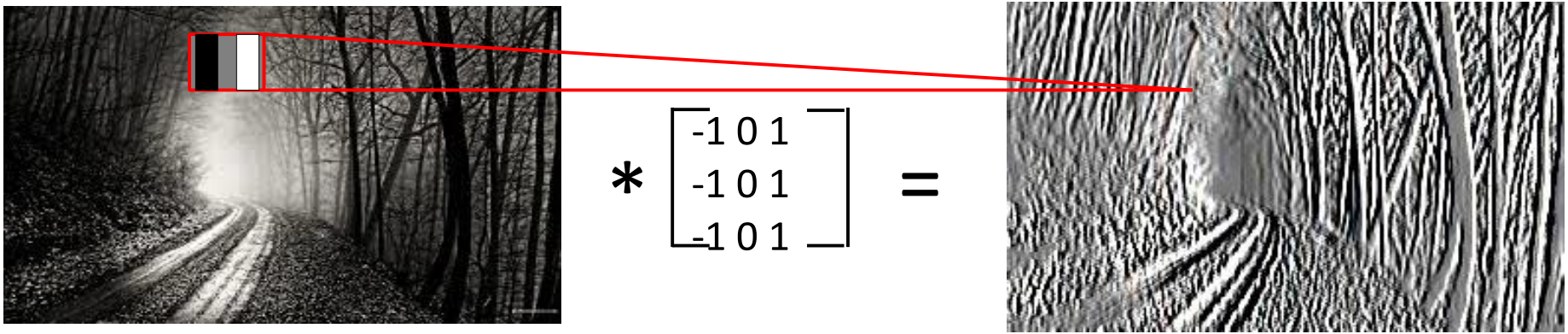
40K hidden units

~2B parameters!!!



- Spatial correlation is local
- Waste of resources + we have not enough training samples anyway..

Convolutional of Two Signals



$$g(x,y) = (h * f)(x,y) = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} h(i,j) f(x-i, y-j)$$

- elementwise multiplication and sum of a filter and the signal (image)

Convolutional of Two Signals

The convolution formula is:

$$(h * f)(x, y) = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} h(i, j) f(x - i, y - j)$$

It follows (by redefining indices) that convolution is commutative:

$$h * f = f * h$$

Finite filters: Typically one function, the "filter" (here its h) is defined over a finite support $[-w/2, w/2]^2$, then

$$(h * f)(x, y) = \sum_{i=-w/2}^{w/2} \sum_{j=-w/2}^{w/2} h(i, j) f(x - i, y - j)$$

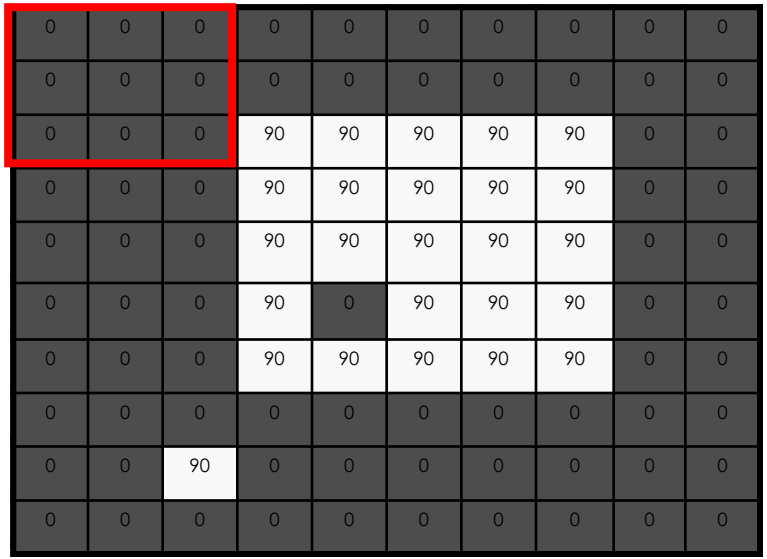
Image filtering

$$g = h * f$$

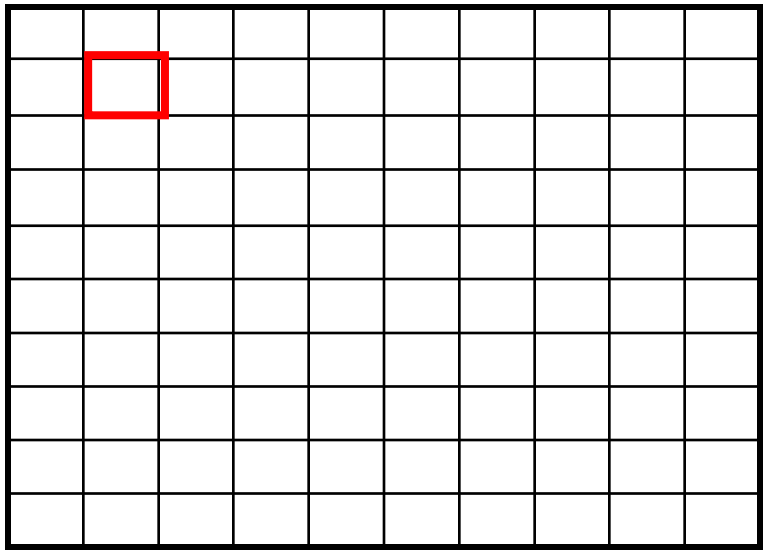
$$h[\cdot,\cdot] \; \frac{1}{9}$$

1	1	1
1	1	1
1	1	1

$f[.,.]$



$g[.,.]$



Credit: S. Seitz

Image filtering

$$g = h * f$$

$$h[\cdot,\cdot] \; \frac{1}{9}$$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[.,.]$

	0	10							

Credit: S. Seitz

Image filtering

$$g = h * f$$

$$h[\cdot,\cdot]\ \frac{1}{9}$$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[.,.]$

	0	10	20						

Credit: S. Seitz

Image filtering

$$g = h * f$$

$$h[\cdot,\cdot]\ \frac{1}{9}$$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[.,.]$

	0	10	20	30					

Credit: S. Seitz

Image filtering

$$g = h * f$$

$$h[\cdot,\cdot]\ \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[.,.]$

	0	10	20	30	30				

Credit: S. Seitz

Image filtering

$$g = h * f$$

$$h[\cdot,\cdot]\ \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[.,.]$

	0	10	20	30	30				
				?					

Credit: S. Seitz

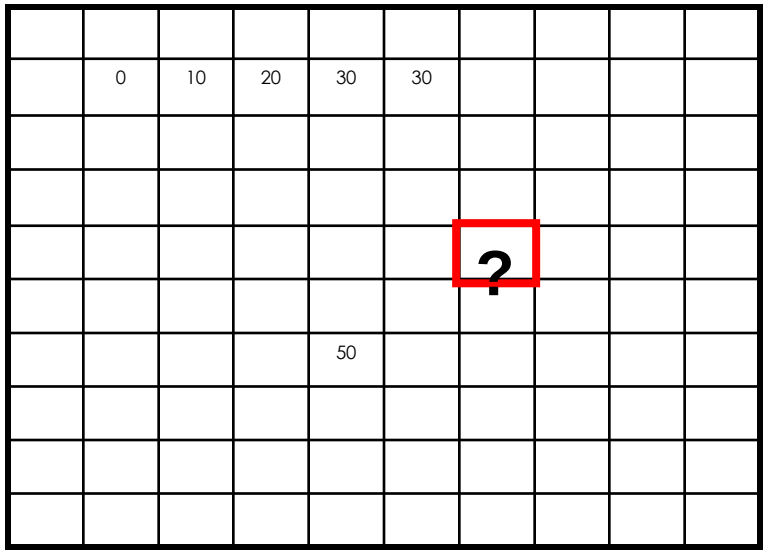
Image filtering

$$g = h * f$$

$$h[\cdot,\cdot]\ \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

$f[.,.]$

$g[.,.]$



Credit: S. Seitz

Image filtering

$g = h * f$ $h[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[.,.]$

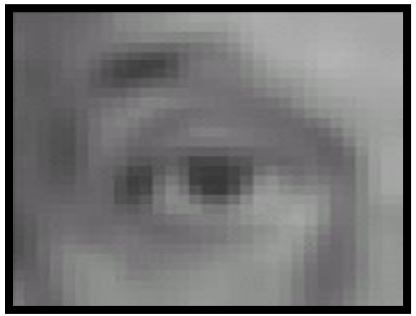
	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

Credit: S. Seitz

Linear filters: examples

Original

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} =$$



Blur (with a mean filter)

Source: D. Lowe

Practice with linear filters

Original

0	0	0
0	1	0
0	0	0

?

Source: D. Lowe

Practice with linear filters

Original

0	0	0
0	1	0
0	0	0

Filtered
(no change)

Source: D. Lowe

Practice with linear filters

Original

0	0	0
0	0	1
0	0	0

?

Source: D. Lowe

Practice with linear filters

Original

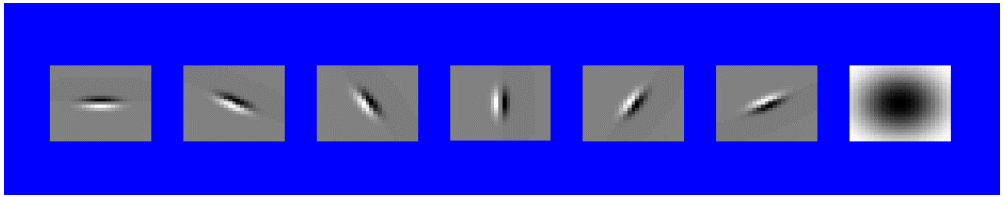
0	0	0
0	0	1
0	0	0

Shifted left
By 1 pixel

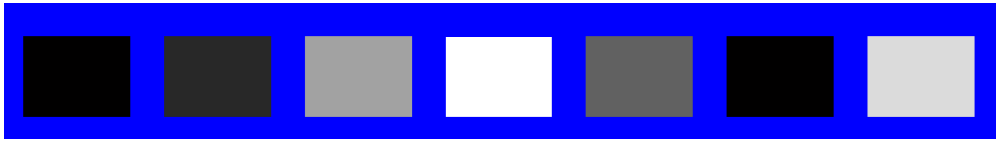
Source: D. Lowe

Can you match the texture to the response?

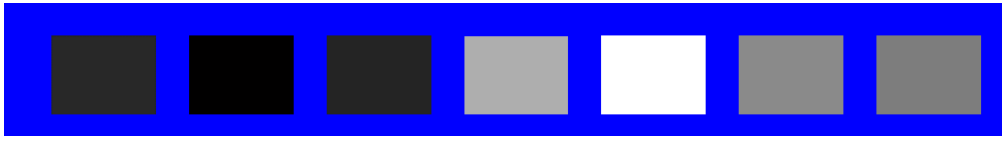
Filters



1



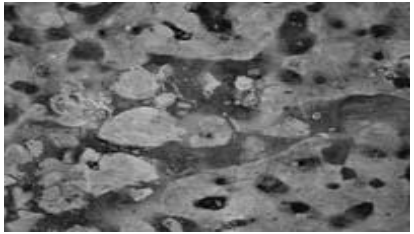
2



3

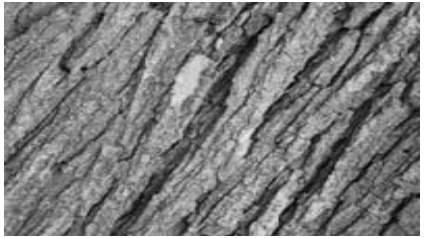
Mean abs responses

A



B

C

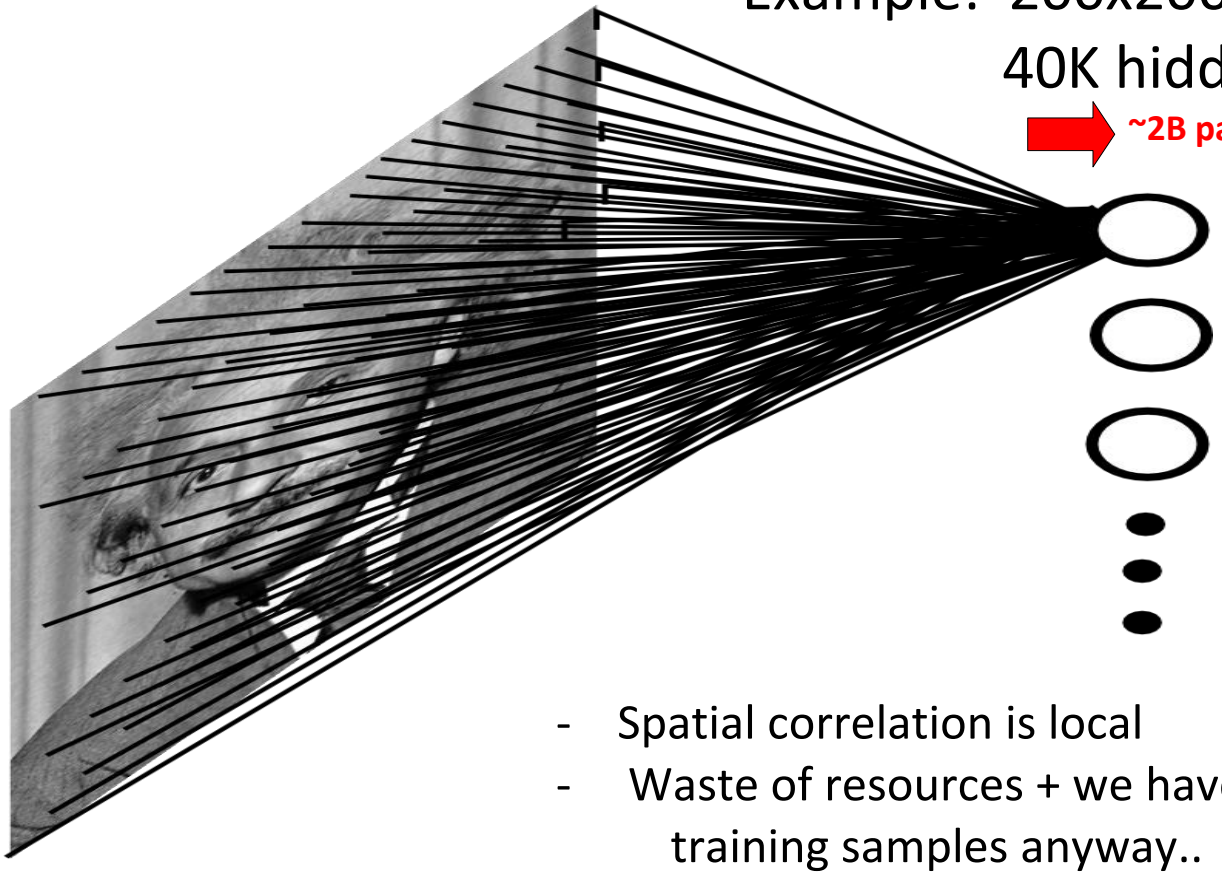


Fully Connected Layer

Example: 200x200 image

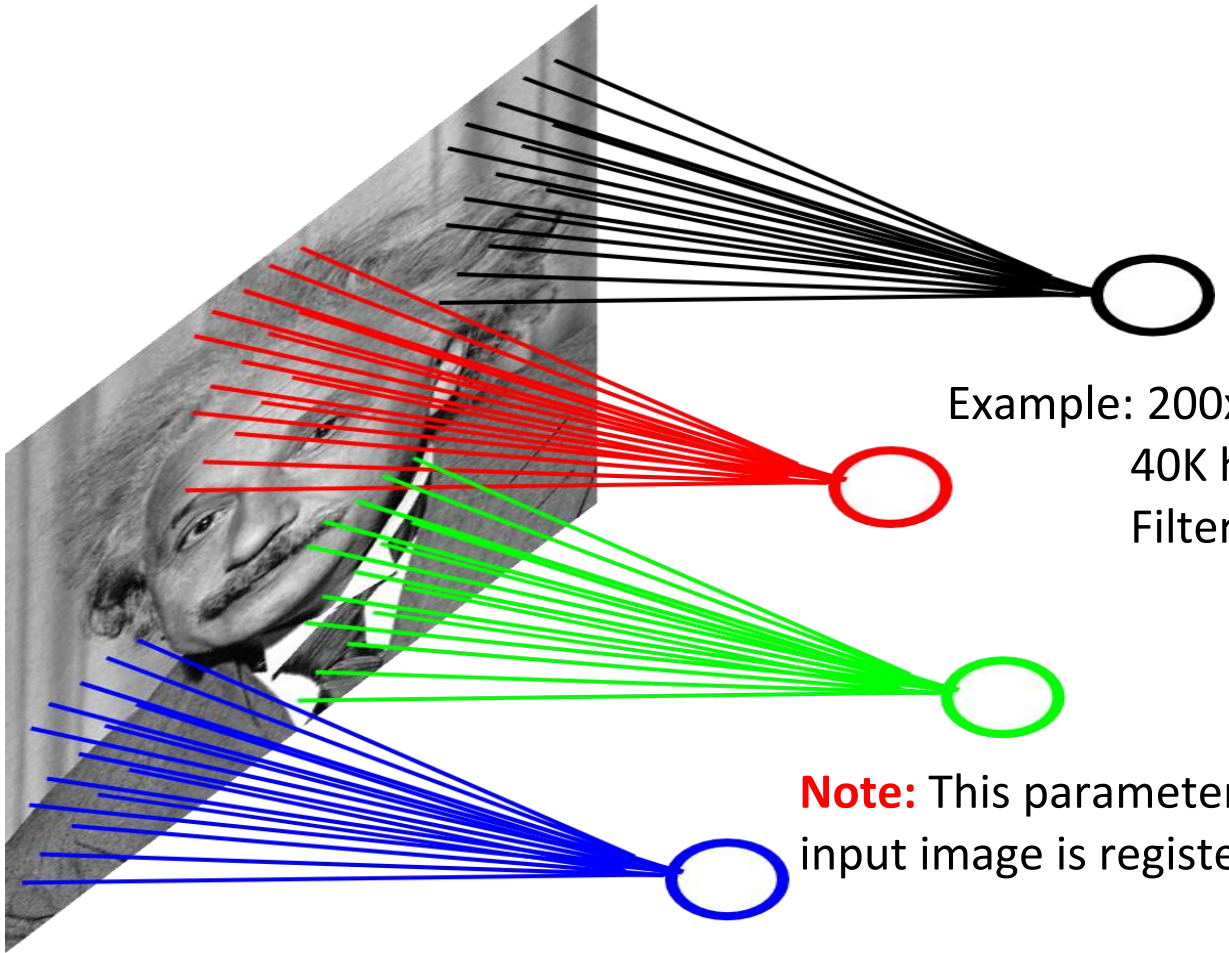
40K hidden units

~2B parameters!!!



- Spatial correlation is local
- Waste of resources + we have not enough training samples anyway..

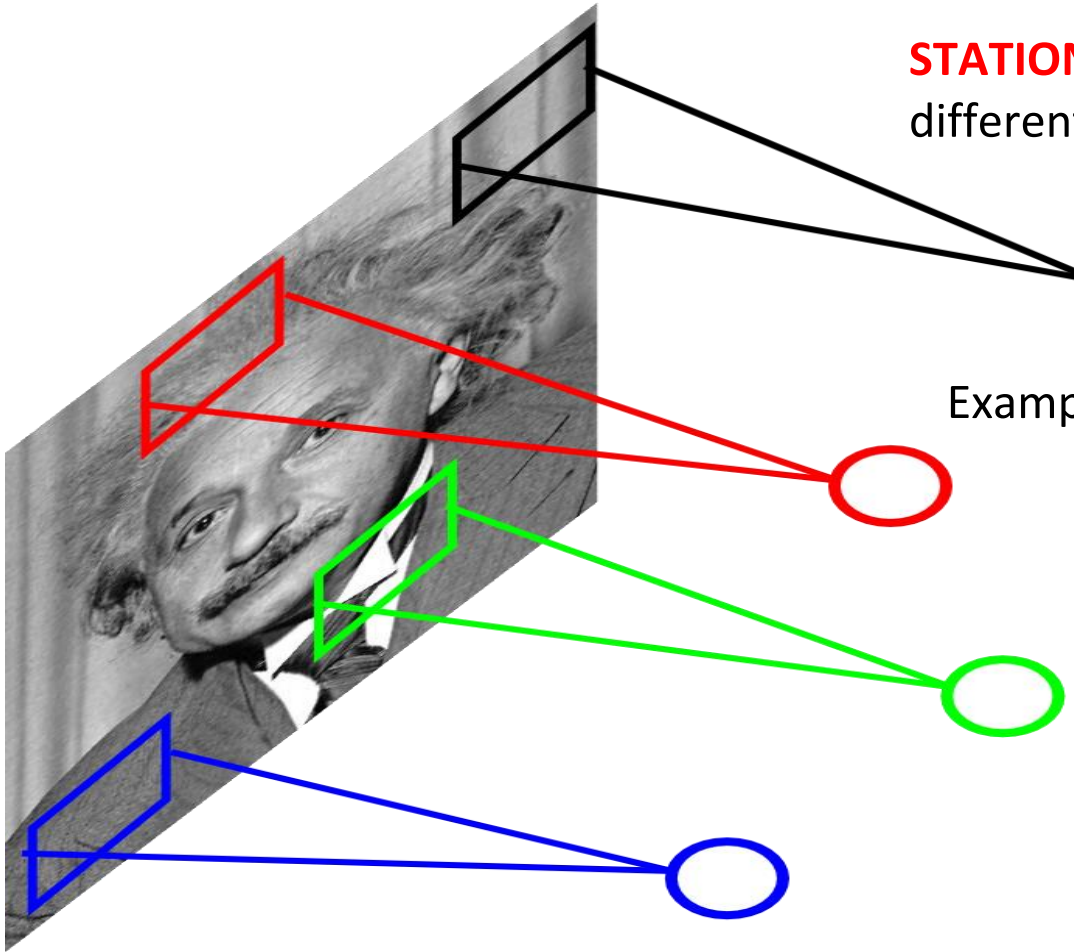
Locally Connected Layer



Example: 200x200 image
40K hidden units
Filter size: 10x10
4M parameters

Note: This parameterization is good when input image is registered (e.g., face recognition).

Locally Connected Layer



STATIONARITY? Statistics is similar at different locations

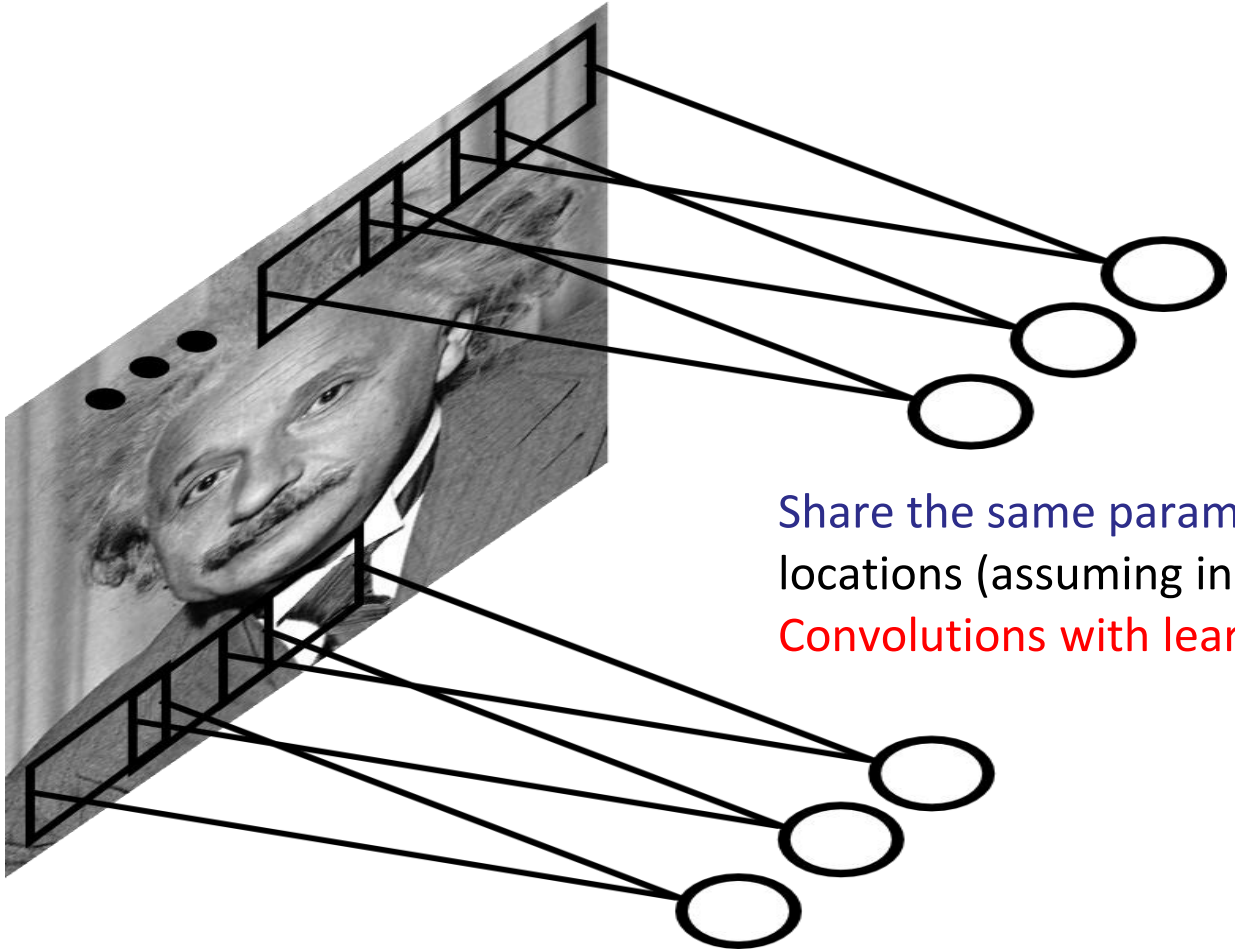
Example: 200x200 image

40K hidden units

Filter size: 10x10

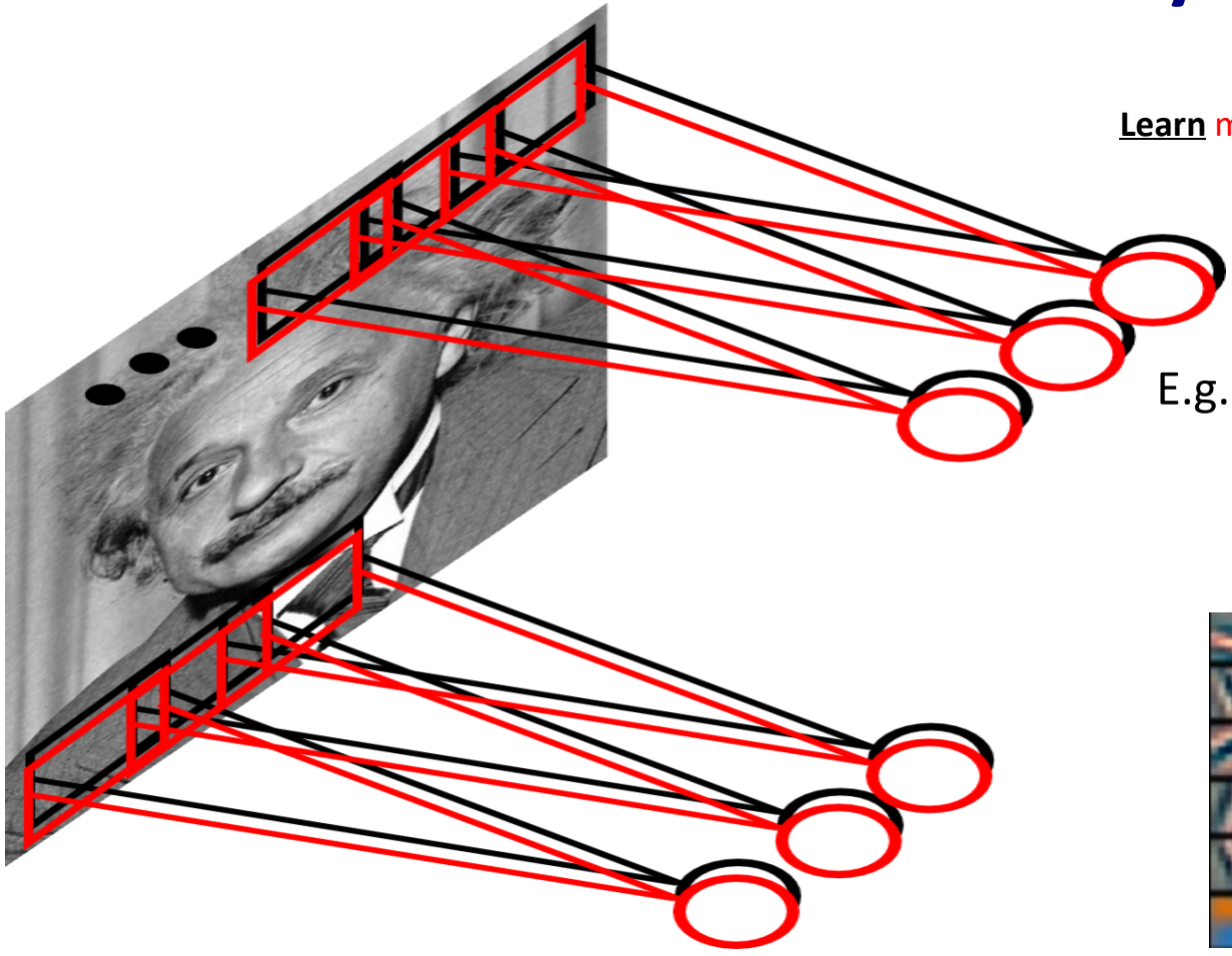
4M parameters

Convolutional Layer



Share the same parameters across different locations (assuming input is stationary):
Convolutions with learned kernels

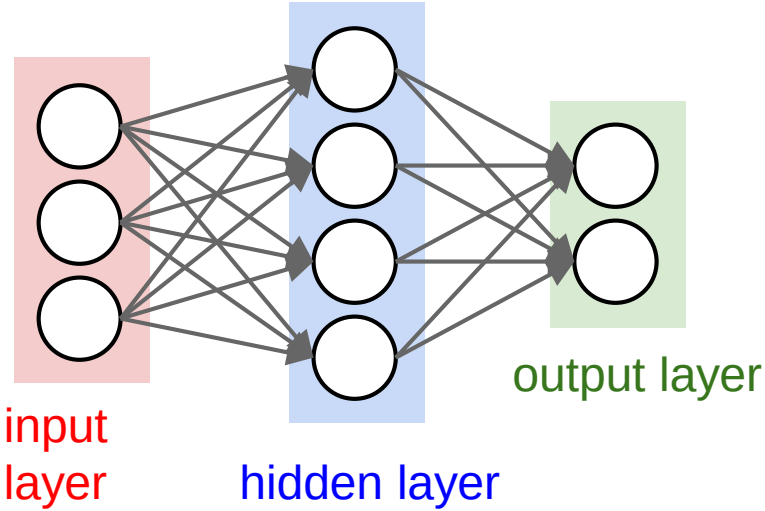
Convolutional Layer



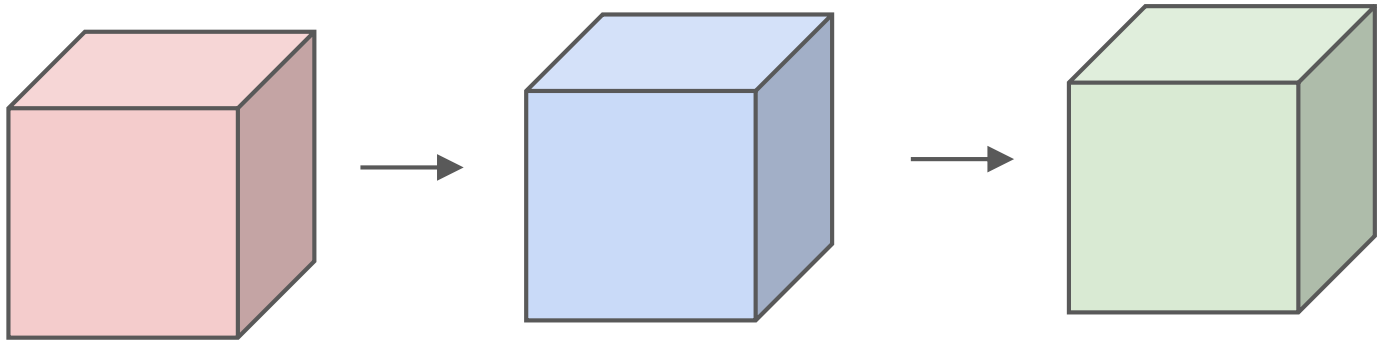
Learn multiple filters.

E.g.: 200x200 image
100 Filters
Filter size: 10x10
10K parameters

before:

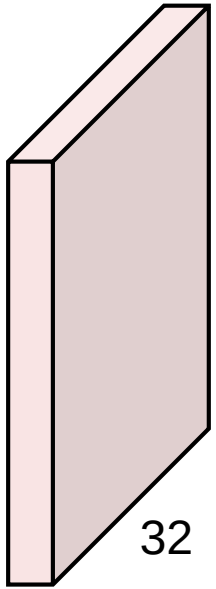


now:



Convolution Layer

32x32x3 image



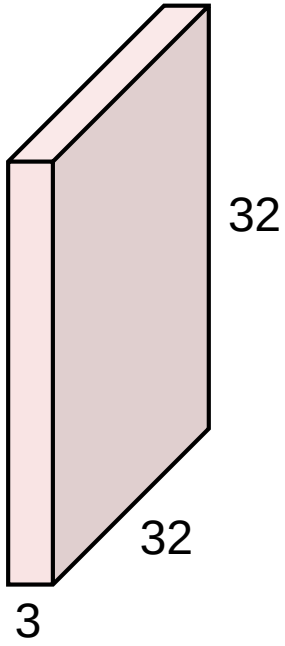
32 height

32 width

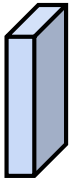
3 Depth (color)

Convolution Layer

32x32x3 image



5x5x3 filter

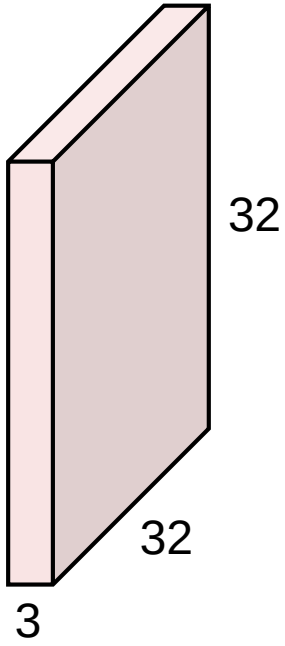


Convolve the filter with the image i.e. “slide over the image spatially, computing dot products”

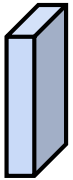
Convolution Layer

Filters always extend to the full depth of the input volume

32x32x3 image

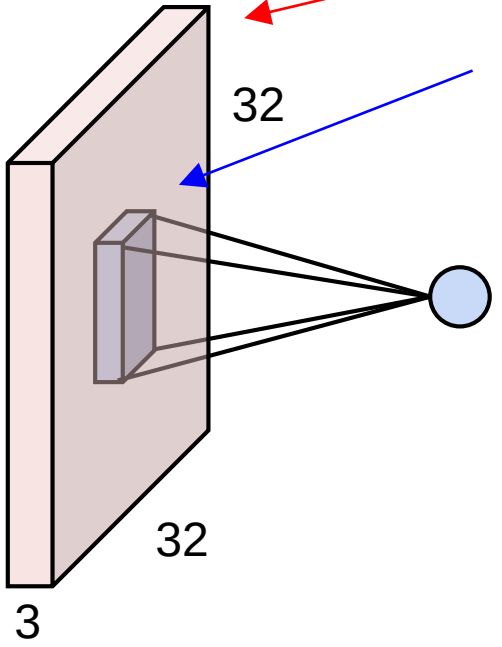


5x5x3 filter



Convolve the filter with the image i.e. “slide over the image spatially, computing dot products”

Convolution Layer



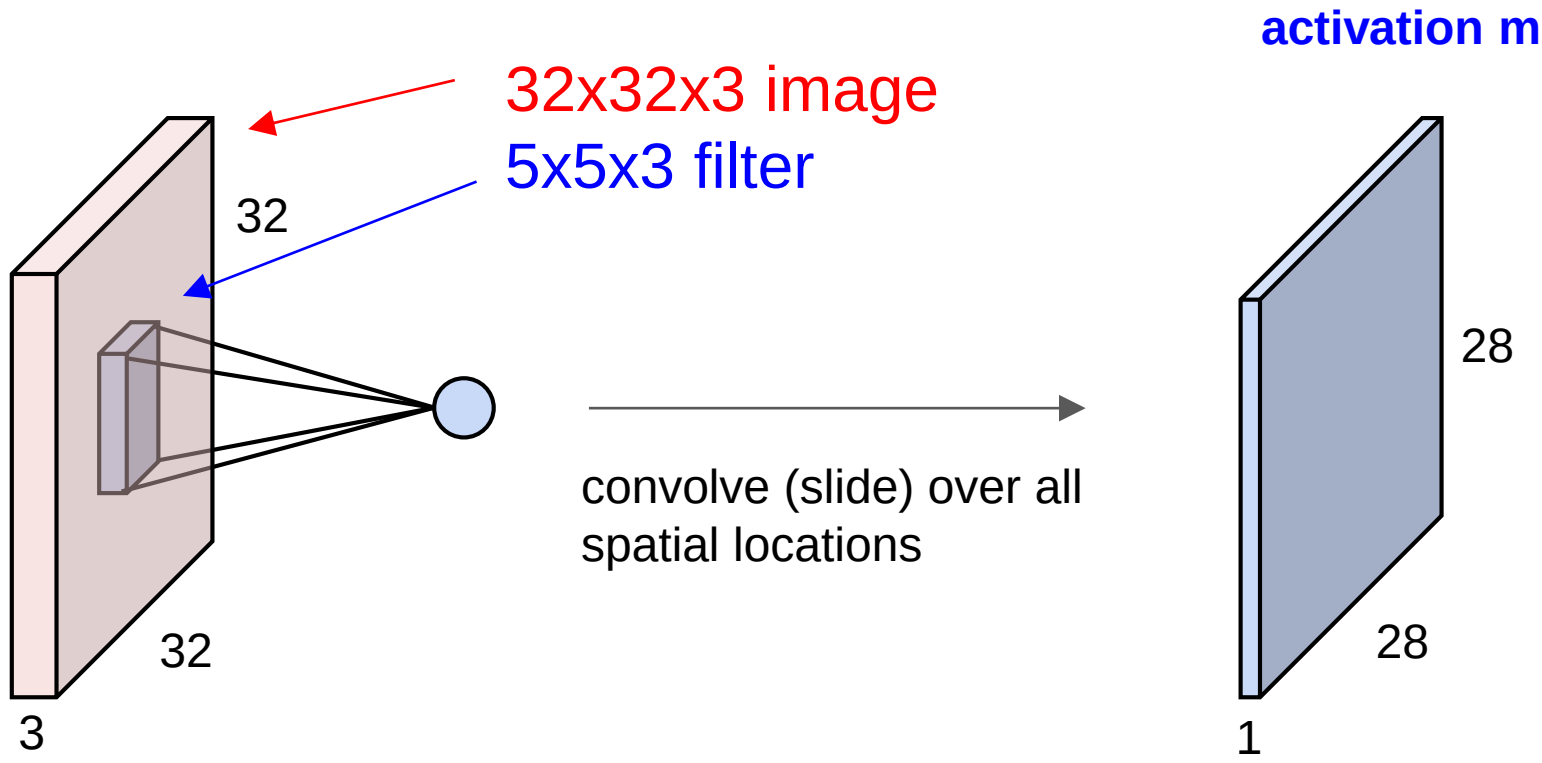
32x32x3 image

5x5x3 filter w

1 number:
the result of taking a dot product between the filter and a small 5x5x3 chunk of the image (i.e. 5*5*3 = 75-dimensional dot product + bias)

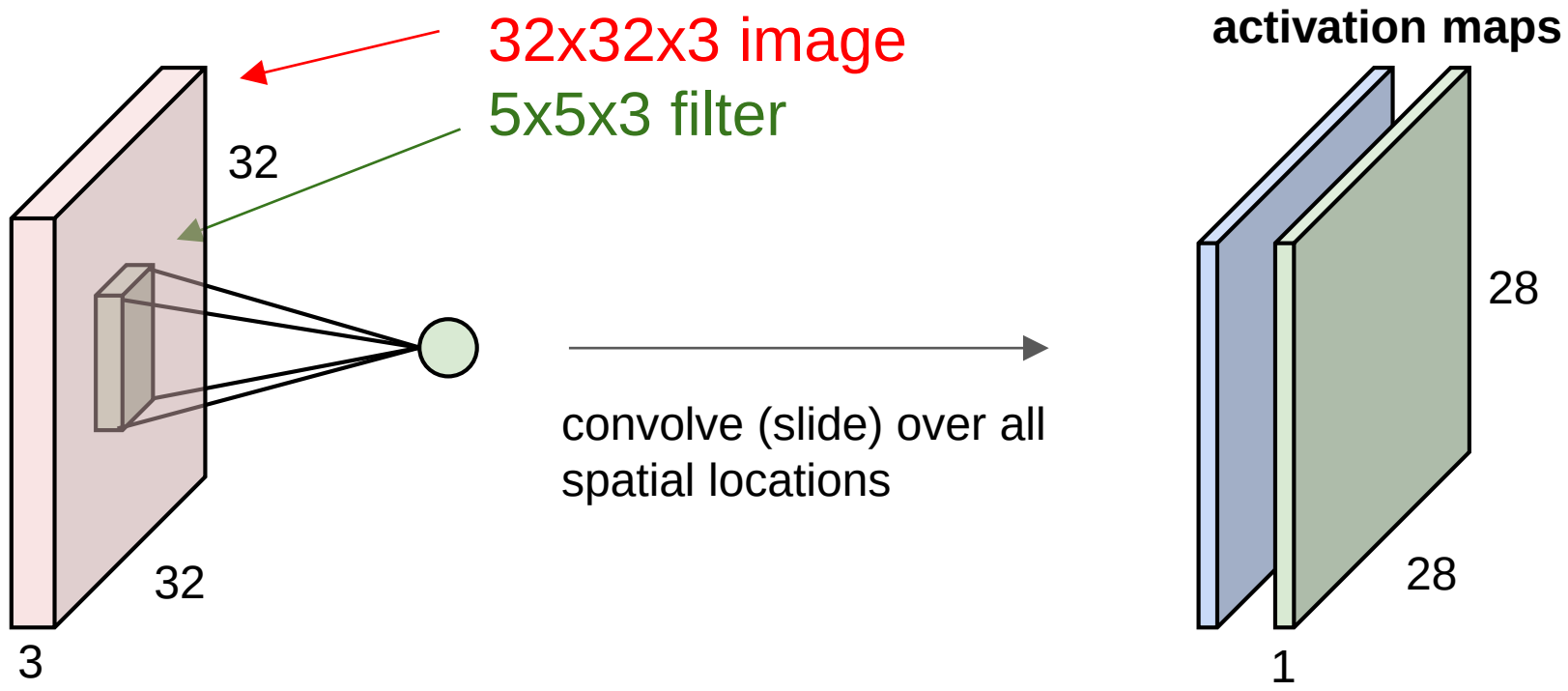
$$w^T x + b$$

Convolution Layer

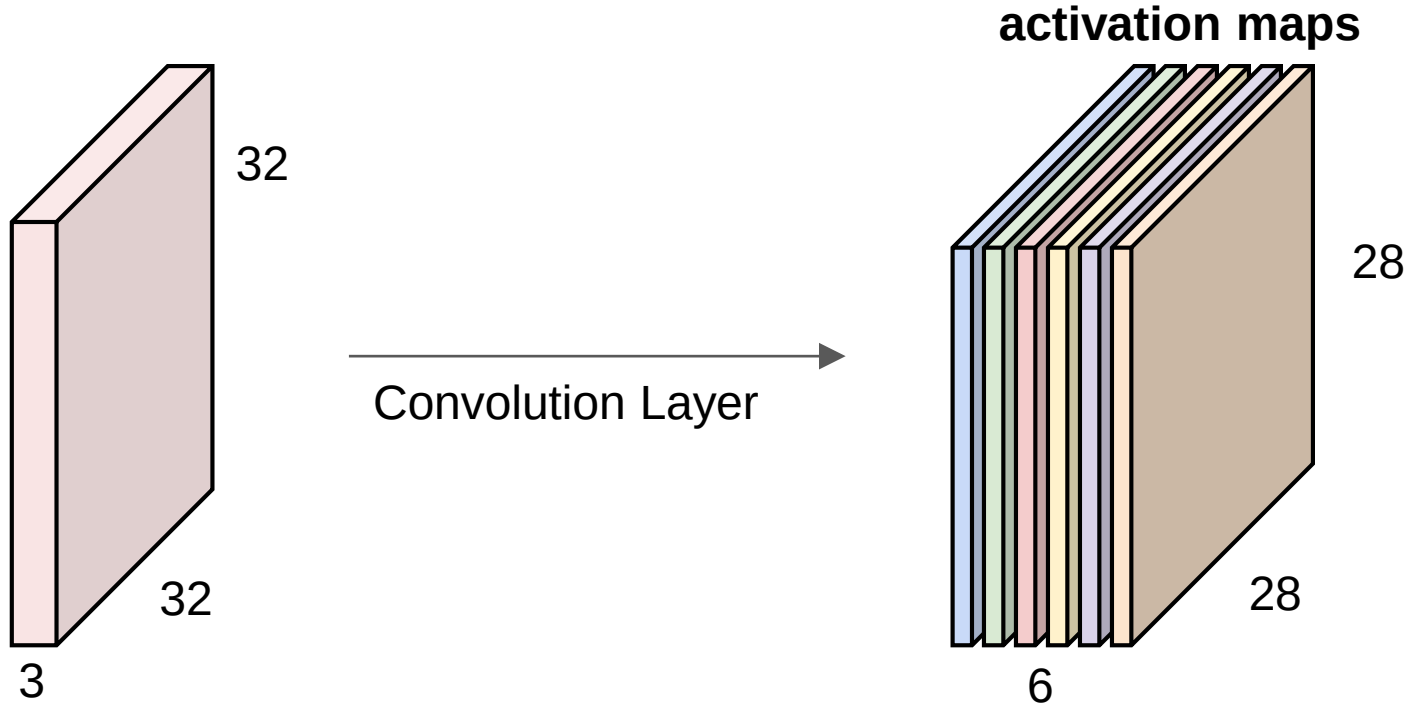


Convolution Layer

consider a second, green filter

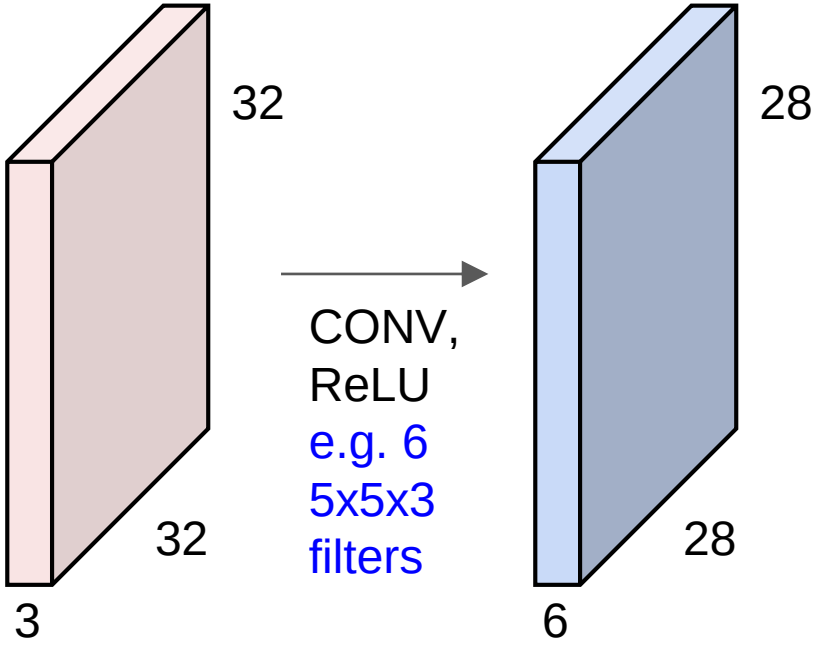


For example, if we had 6 5x5 filters, we'll get 6 separate activation maps:

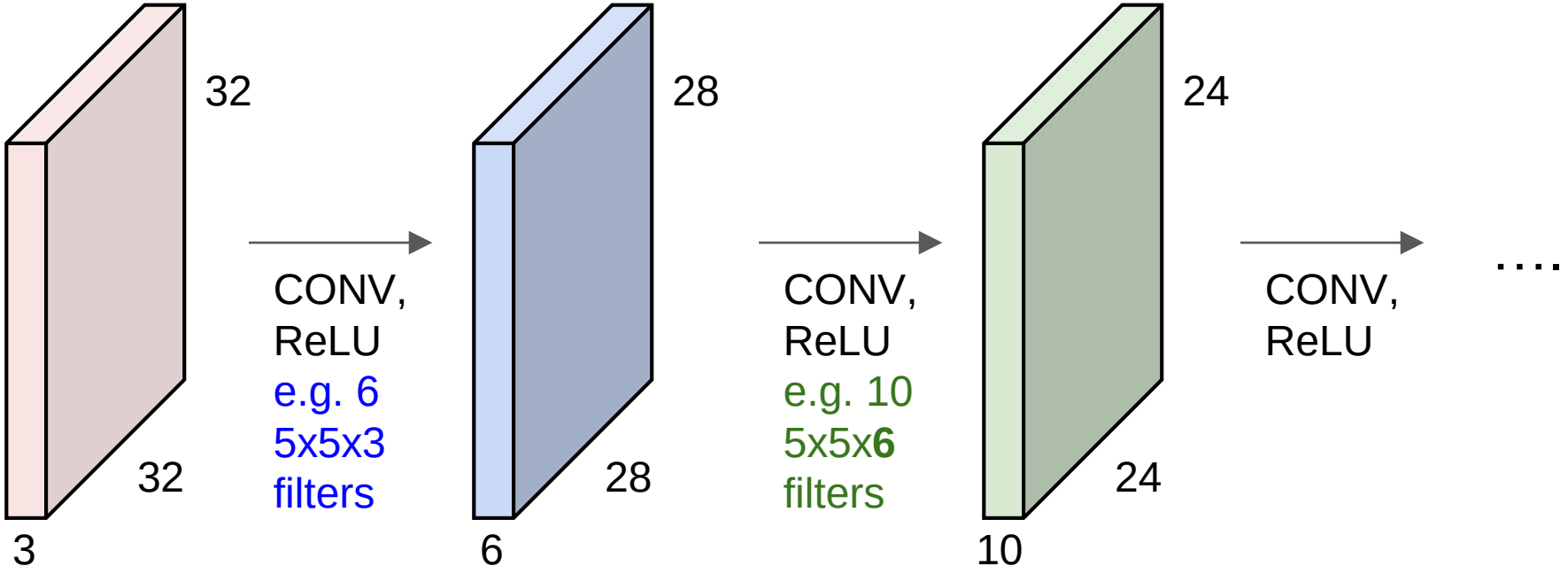


We stack these up to get a "new image" of size 28x28x6!

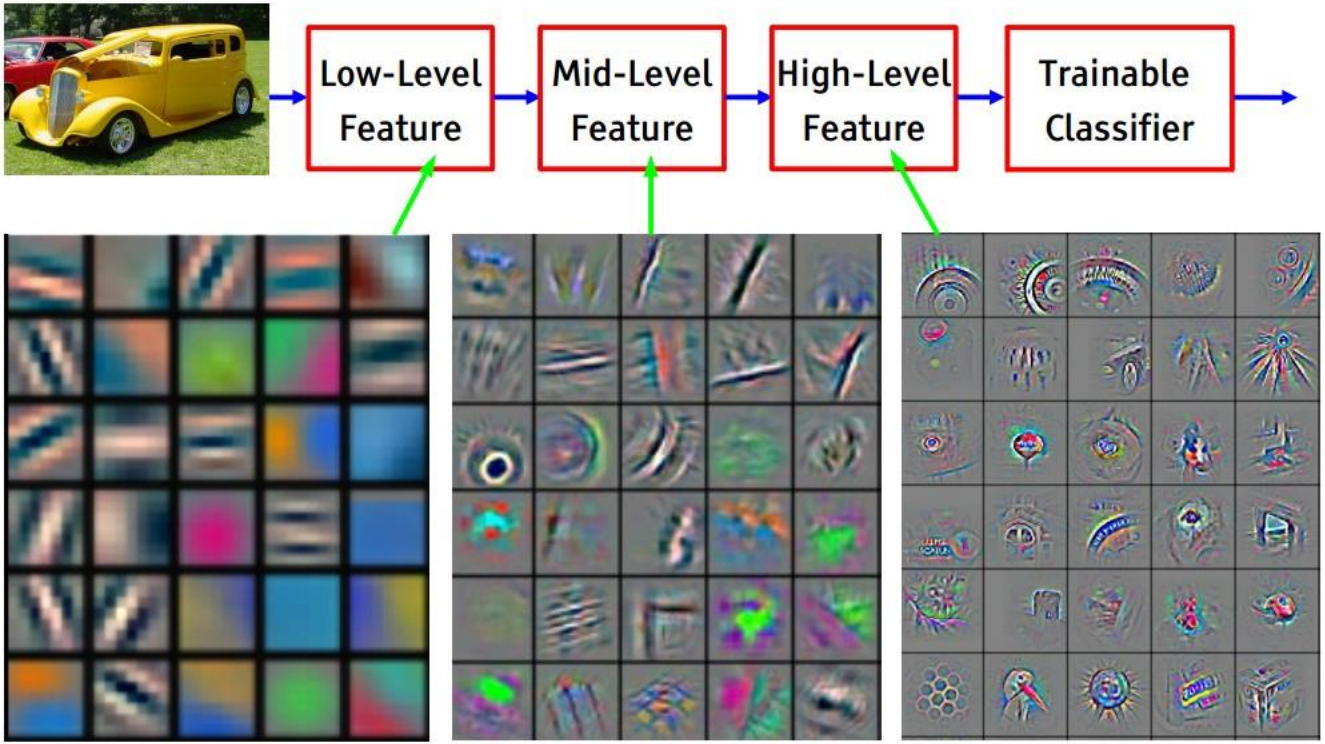
Preview: ConvNet is a sequence of Convolution Layers, interspersed with non-linear activation functions



Preview: ConvNet is a sequence of Convolution Layers, interspersed with non-linear activation functions



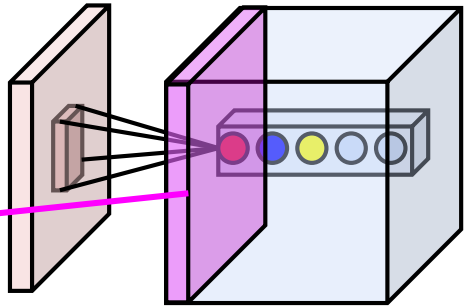
Recall



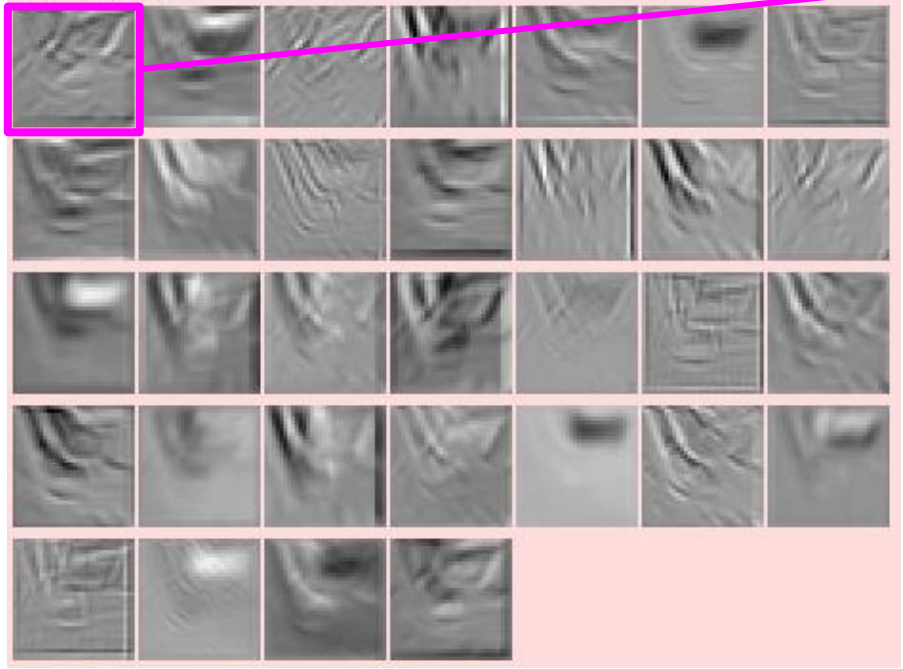
Feature visualization of convolutional net trained on ImageNet from [Zeiler & Fergus 2013]

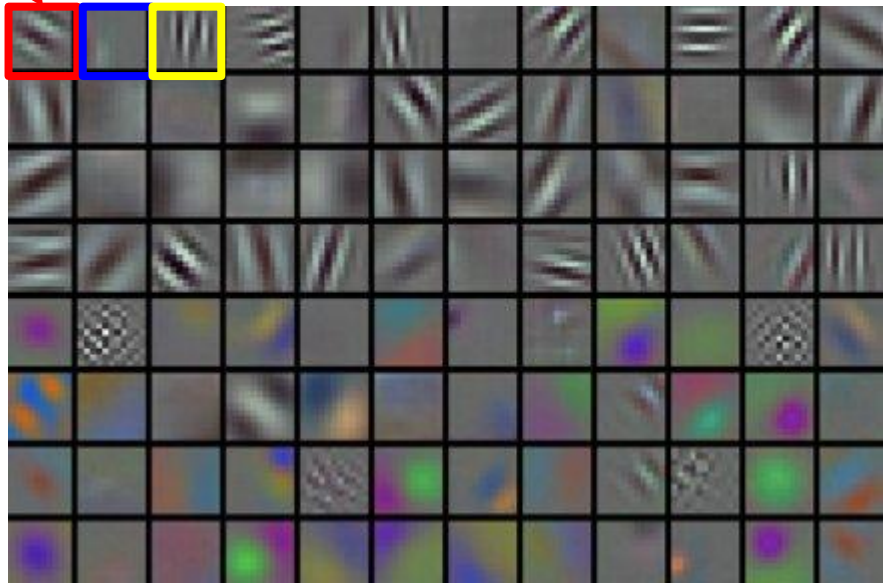
Activations:

convolving the first filter in the input gives the first slice of depth in output volume

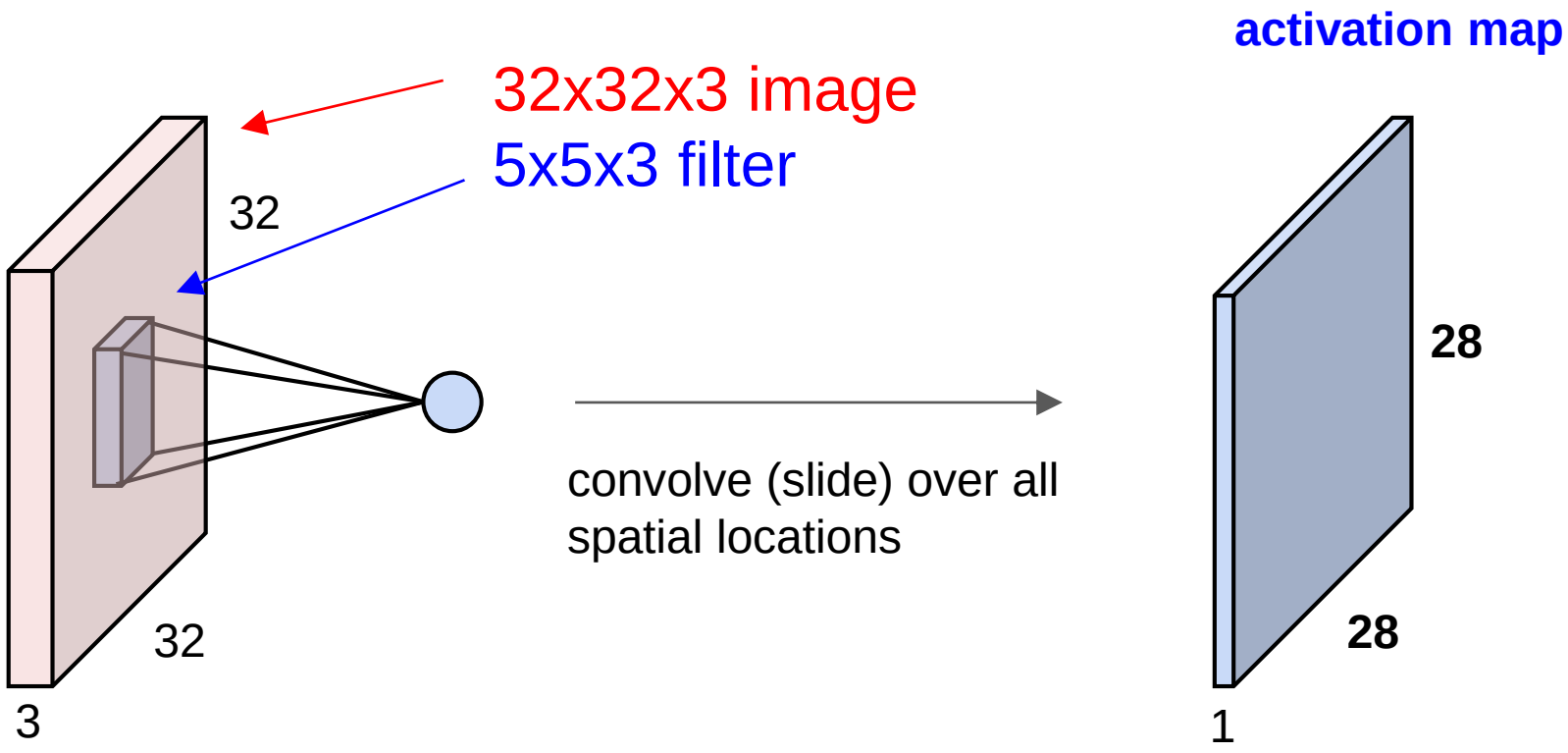


Activations:

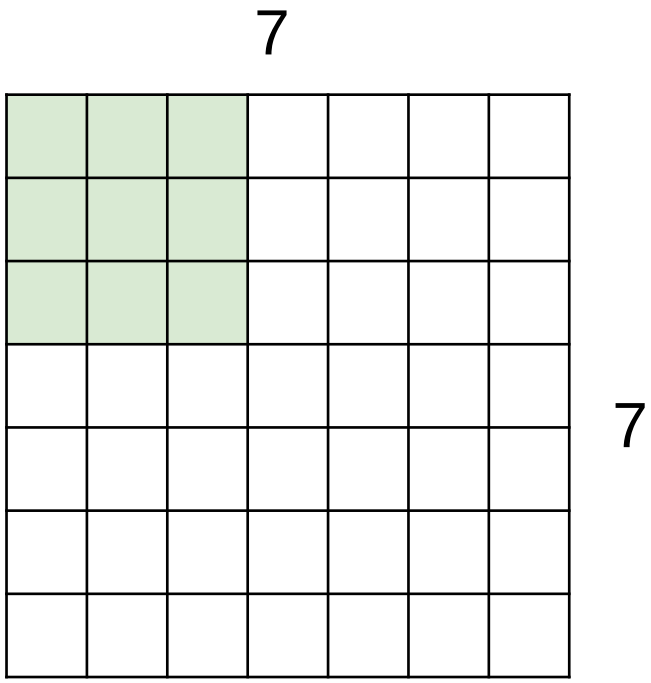




A closer look at spatial dimensions:

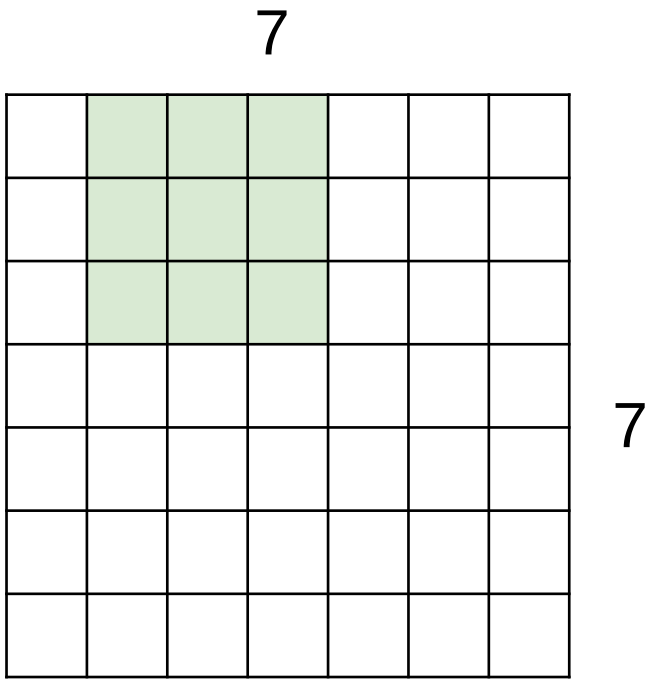


A closer look at spatial dimensions:



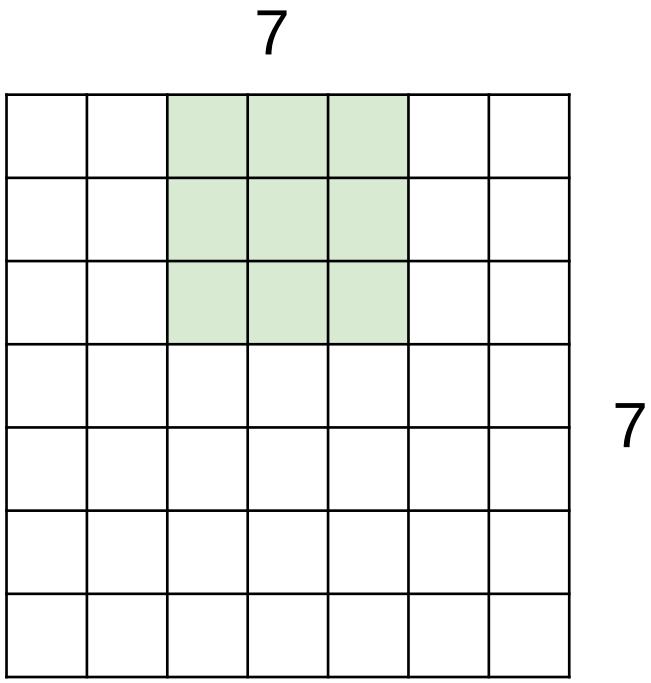
7x7 input (spatially)
assume 3x3 filter

A closer look at spatial dimensions:



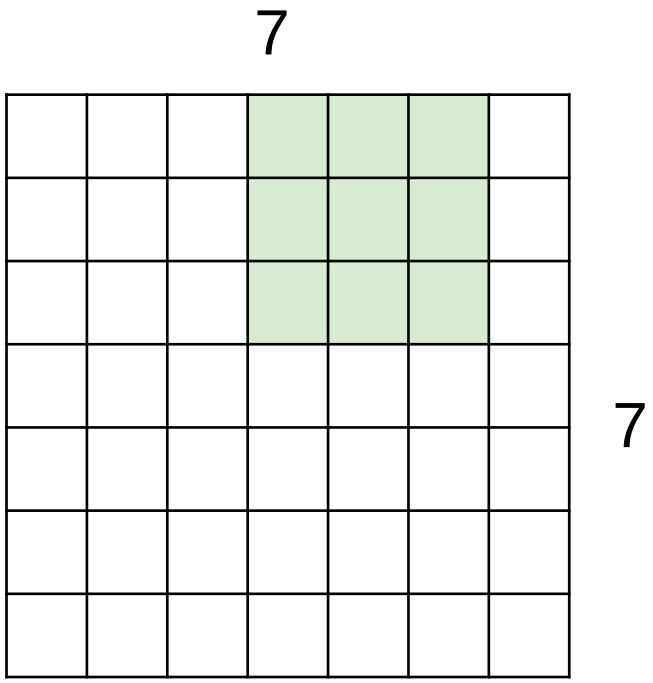
7x7 input (spatially)
assume 3x3 filter

A closer look at spatial dimensions:



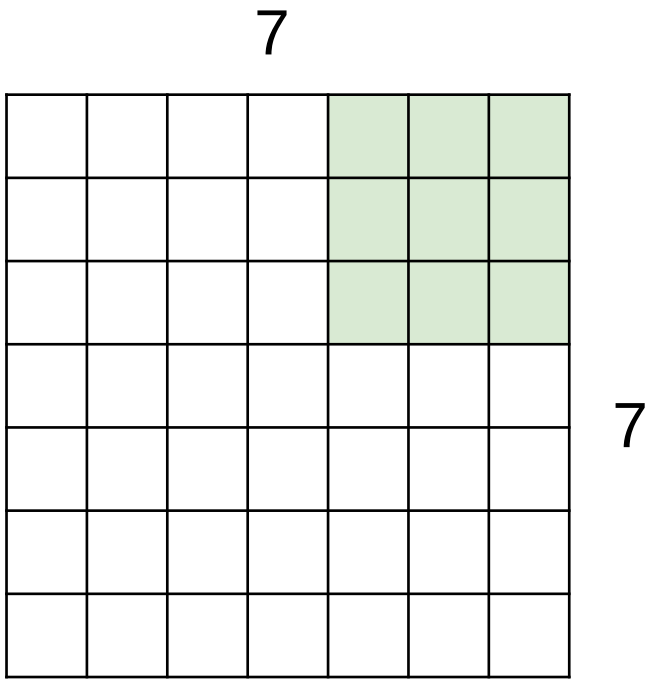
7x7 input (spatially)
assume 3x3 filter

A closer look at spatial dimensions:



7x7 input (spatially)
assume 3x3 filter

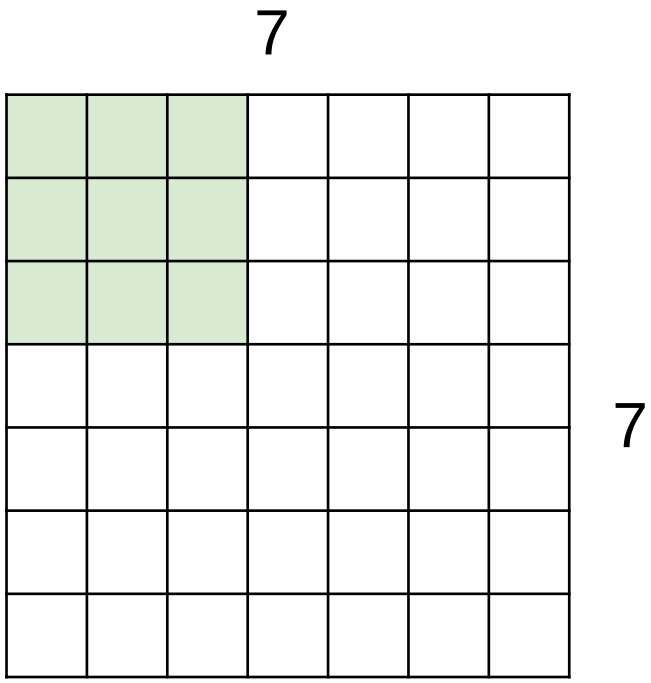
A closer look at spatial dimensions:



7x7 input (spatially)
assume 3x3 filter

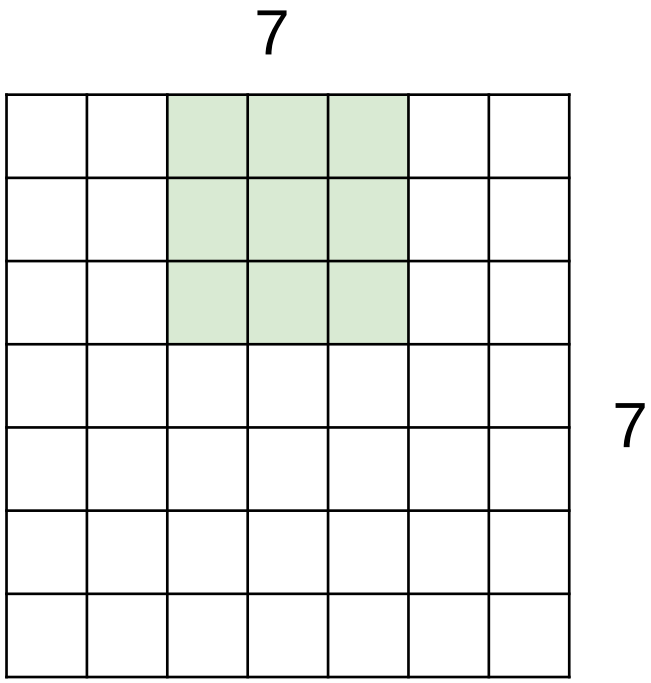
=> 5x5 output

A closer look at spatial dimensions:



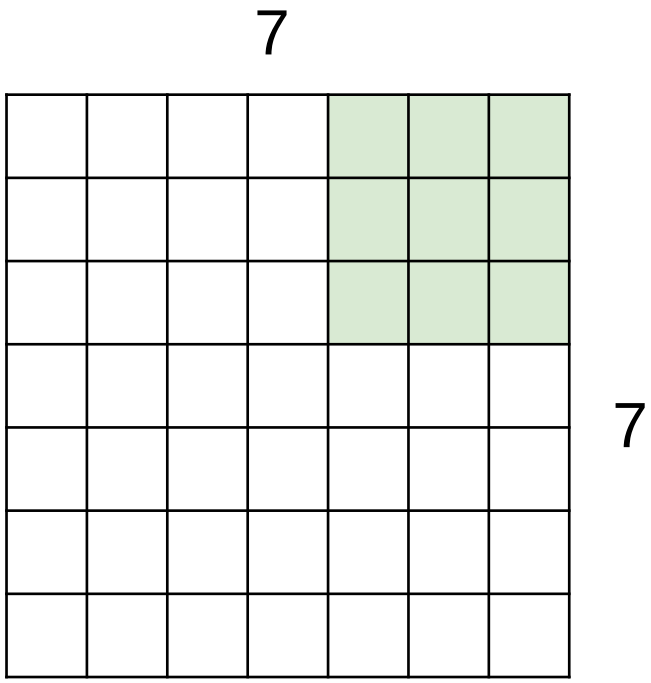
7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**

A closer look at spatial dimensions:



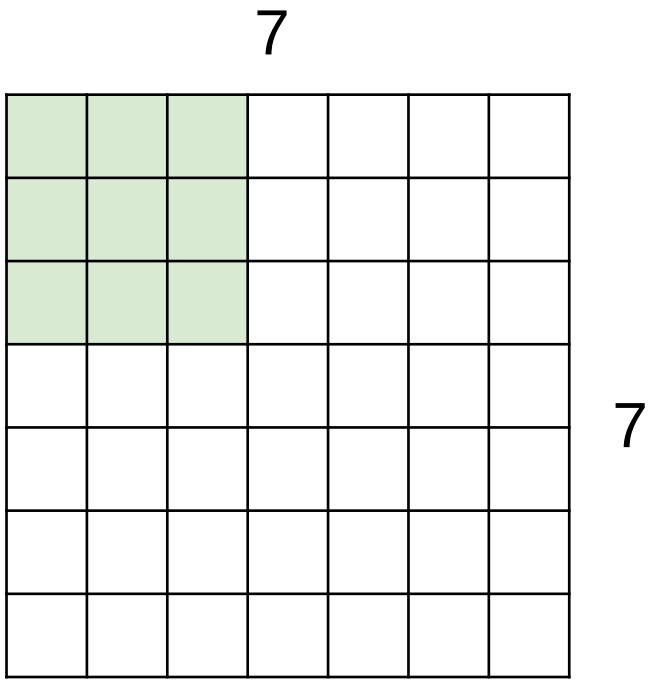
7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**

A closer look at spatial dimensions:



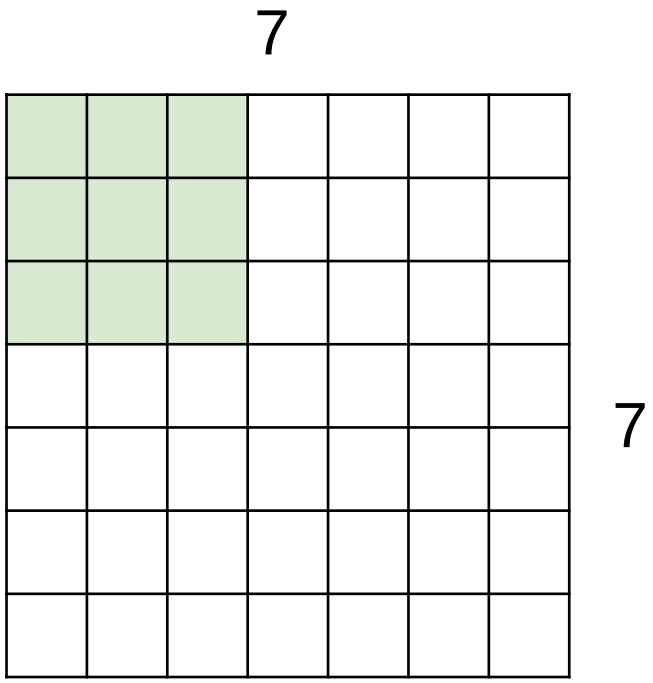
7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**
=> 3x3 output!

A closer look at spatial dimensions:



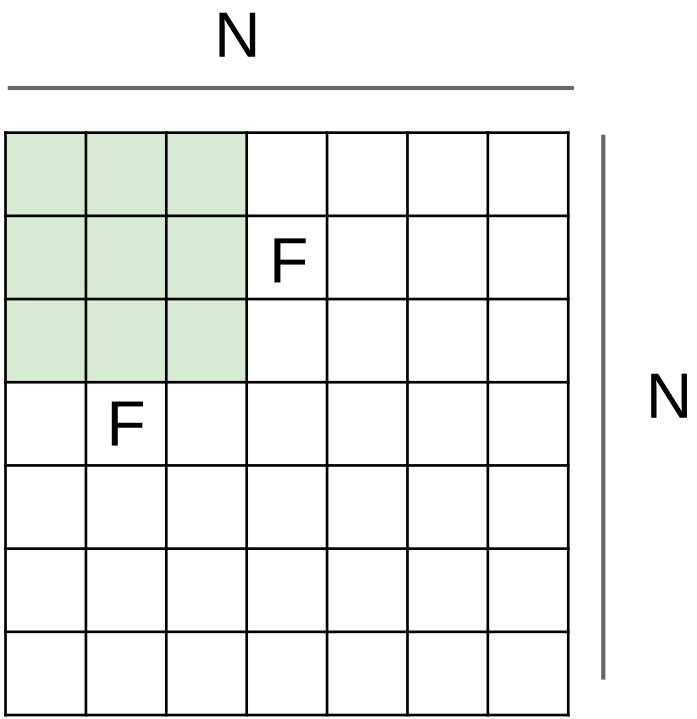
7x7 input (spatially)
assume 3x3 filter
applied **with stride 3?**

A closer look at spatial dimensions:



7x7 input (spatially)
assume 3x3 filter
applied **with stride 3?**

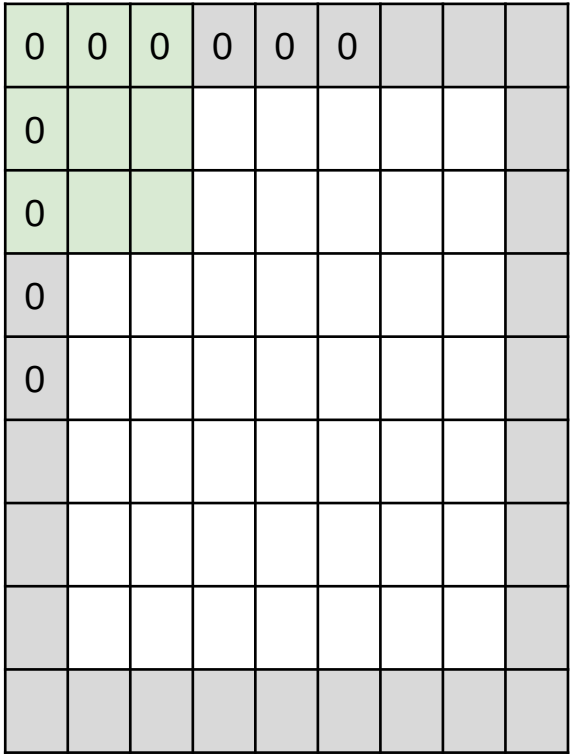
doesn't fit!
Wastes some input pixels.



Output size:
(N - F) / stride + 1

e.g. N = 7, F = 3:
stride 1 => (7 - 3)/1 + 1 = 5
stride 2 => (7 - 3)/2 + 1 = 3
stride 3 => (7 - 3)/3 + 1 = 2.33 :\

In practice: Common to zero pad the border

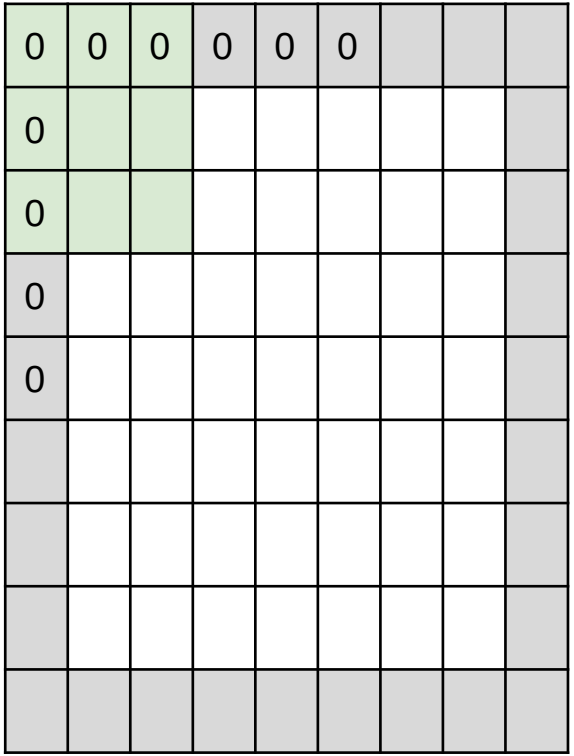


Aside: remove image mean first!

e.g. input 7x7
3x3 filter, applied with **stride 1**
pad with 1 pixel border => what is the output?

(recall:)
(N - F) / stride + 1

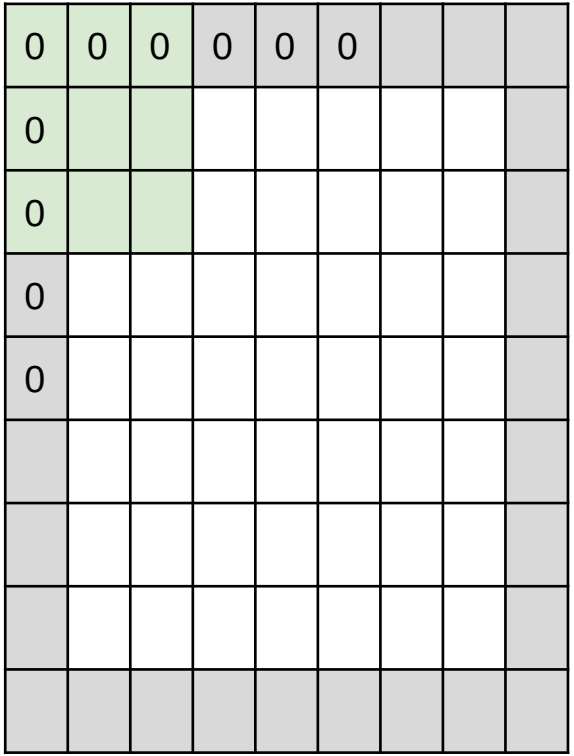
In practice: Common to zero pad the border



e.g. input 7x7
3x3 filter, applied with **stride 1**
pad with 1 pixel border => what is the output?

7x7 output!

In practice: Common to zero pad the border

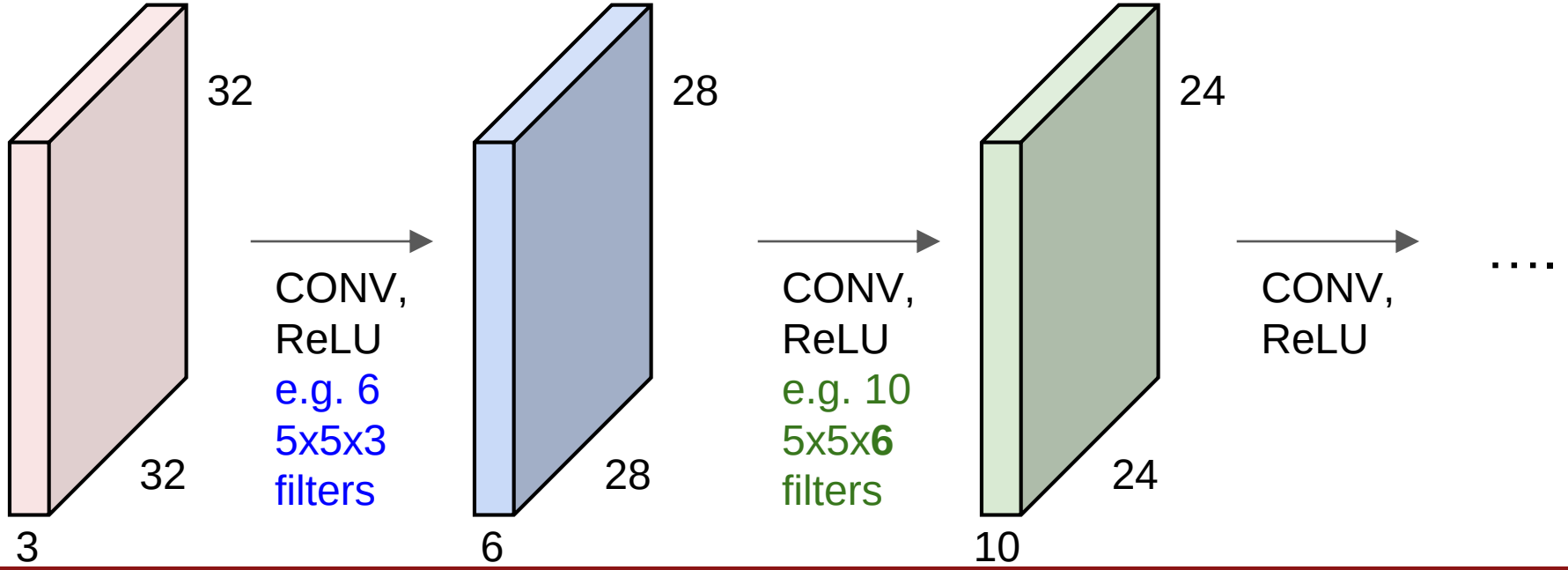


e.g. input 7x7
3x3 filter, applied with **stride 1**
pad with 1 pixel border => what is the output?

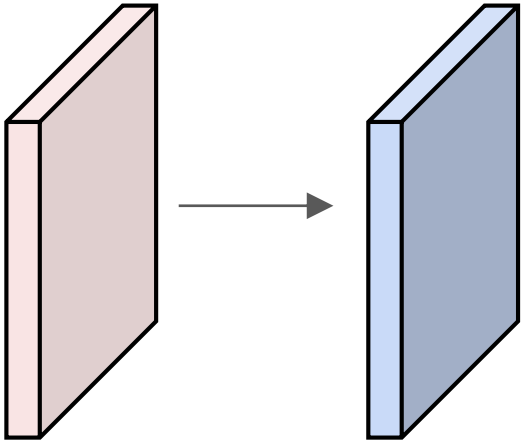
7x7 output!
in general, common to see CONV layers with stride 1, filters of size FxF, and zero-padding with (F-1)/2. (will preserve size spatially)
e.g. F = 3 => zero pad with 1
F = 5 => zero pad with 2
F = 7 => zero pad with 3

Remember back to…

E.g. 32x32 input convolved repeatedly with 5x5 filters shrinks volumes spatially! (32 -> 28 -> 24 ...). Shrinking too fast is not good, doesn't work well.



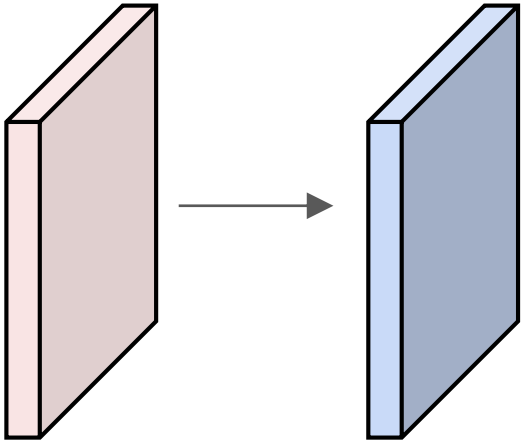
Examples time:



Input volume: **32x32x3**
10 5x5 filters with stride 1, pad 2

Output volume size: ?

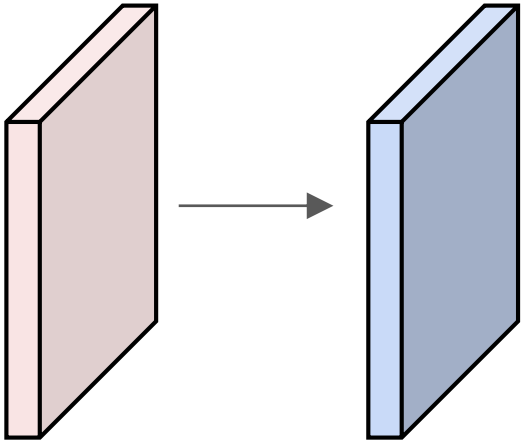
Examples time:



Input volume: **32x32x3**
10 5x5 filters with stride 1, pad 2

Output volume size:
(32+2*2-5)/1+1 = 32 spatially, so
32x32x10

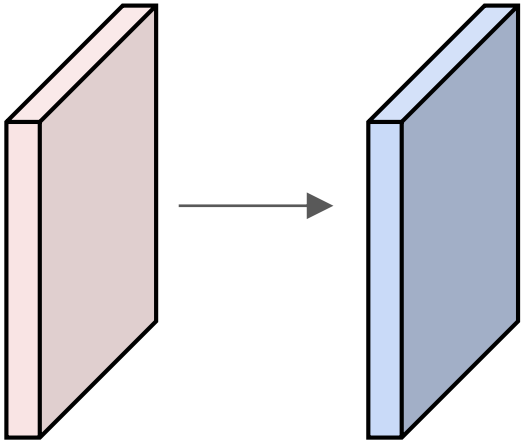
Examples time:



Input volume: **32x32x3**
10 5x5 filters with stride 1, pad 2

Number of parameters in this layer?

Examples time:



Input volume: **32x32x3**
10 5x5 filters with stride 1, pad 2

Number of parameters in this layer?
each filter has 5*5*3 + 1 = 76 params (+1 for bias)
=> 76*10 = **760**

Summary. To summarize, the Conv Layer:

- Accepts a volume of size $W_1 \times H_1 \times D_1$
- Requires four hyperparameters:
  - Number of filters K,
  - their spatial extent F,
  - the stride S,
  - the amount of zero padding P.
- Produces a volume of size $W_2 \times H_2 \times D_2$ where:
  - $W_2 = (W_1 - F + 2P)/S + 1$
  - $H_2 = (H_1 - F + 2P)/S + 1$ (i.e. width and height are computed equally by symmetry)
  - $D_2 = K$
- With parameter sharing, it introduces $F \cdot F \cdot D_1$ weights per filter, for a total of $(F \cdot F \cdot D_1) \cdot K$ weights and K biases.
- In the output volume, the d-th depth slice (of size $W_2 \times H_2$) is the result of performing a valid convolution of the d-th filter over the input volume with a stride of S, and then offset by d-th bias.

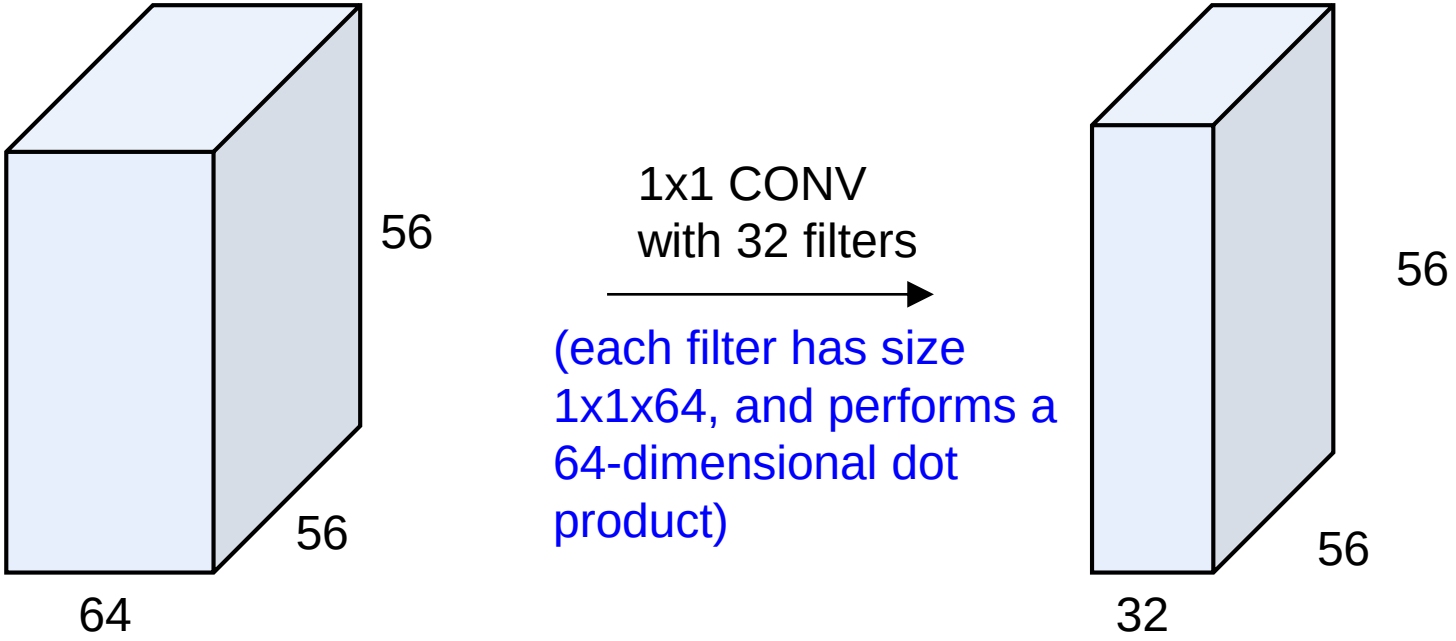
Common settings:

K = (powers of 2, e.g. 32, 64, 128, 512)
- F = 3, S = 1, P = 1
- F = 5, S = 1, P = 2
- F = 5, S = 2, P = ? (whatever fits)
- F = 1, S = 1, P = 0

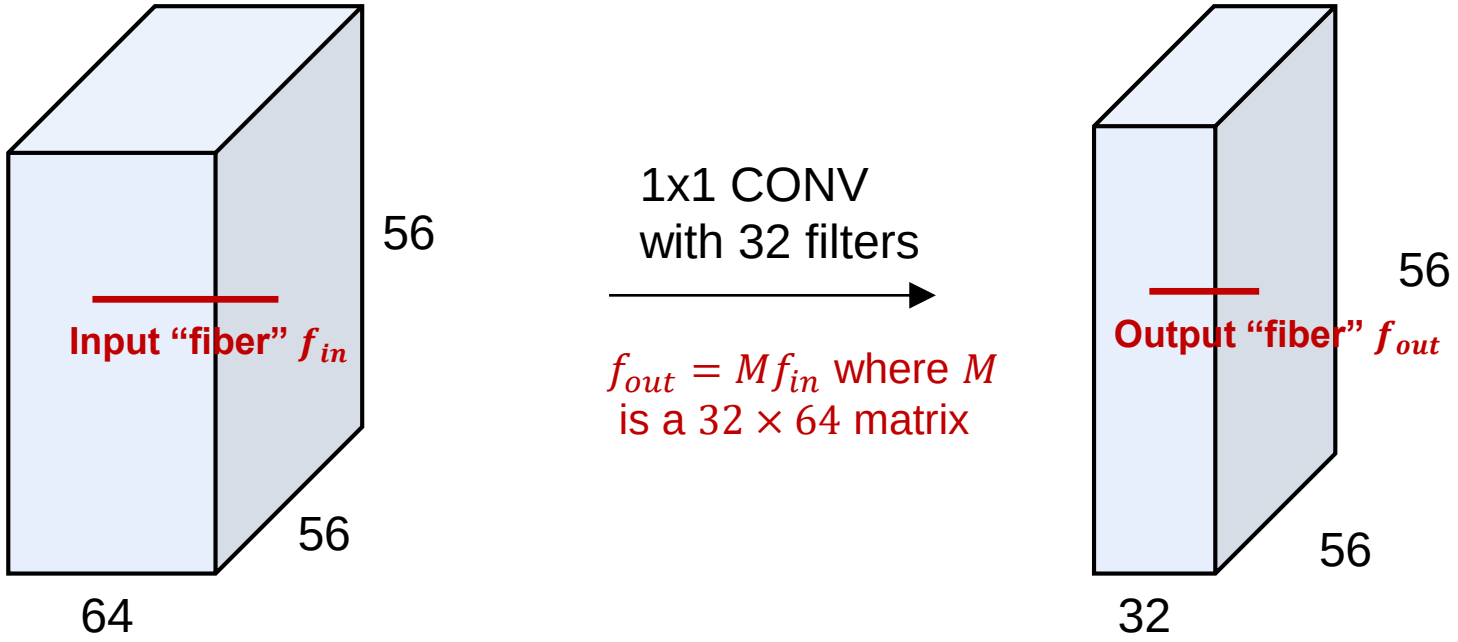
Summary. To summarize, the Conv Layer:

- Accepts a volume of size $W_1 \times H_1 \times D_1$
- Requires four hyperparameters:
  - Number of filters K,
  - their spatial extent F,
  - the stride S,
  - the amount of zero padding P.
- Produces a volume of size $W_2 \times H_2 \times D_2$ where:
  - $W_2 = (W_1 - F + 2P)/S + 1$
  - $H_2 = (H_1 - F + 2P)/S + 1$ (i.e. width and height are computed equally by symmetry)
  - $D_2 = K$
- With parameter sharing, it introduces $F \cdot F \cdot D_1$ weights per filter, for a total of $(F \cdot F \cdot D_1) \cdot K$ weights and K biases.
- In the output volume, the d-th depth slice (of size $W_2 \times H_2$) is the result of performing a valid convolution of the d-th filter over the input volume with a stride of S, and then offset by d-th bias.

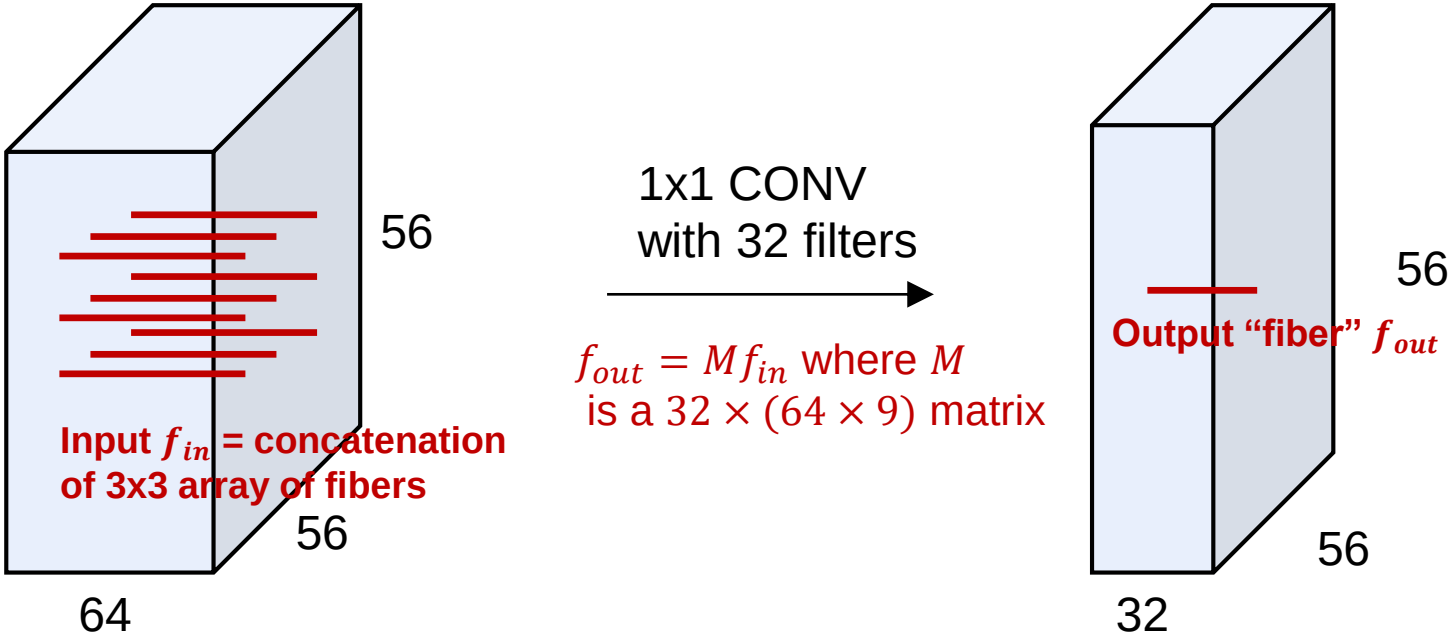
(btw, 1x1 convolution layers make perfect sense)



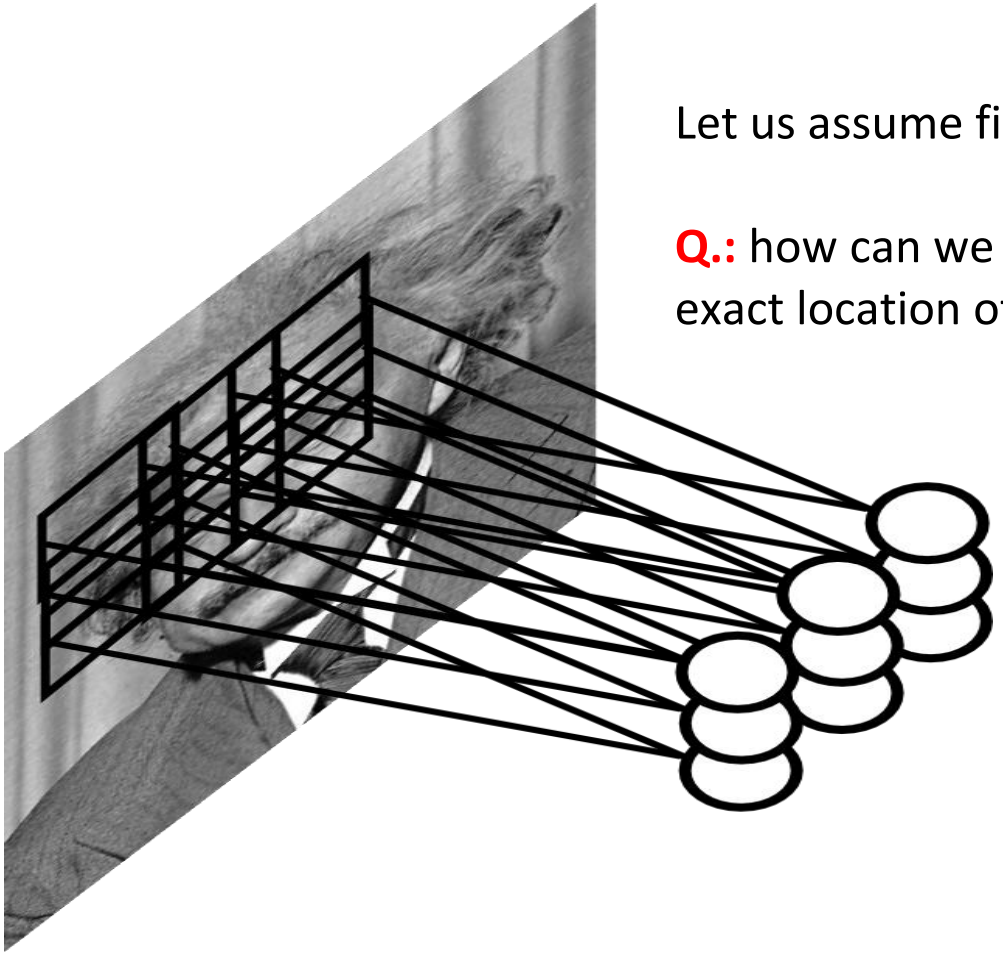
(btw, 1x1 convolution layers make perfect sense)



Aside: convolution via matrix multiply: im2col



Pooling Layer

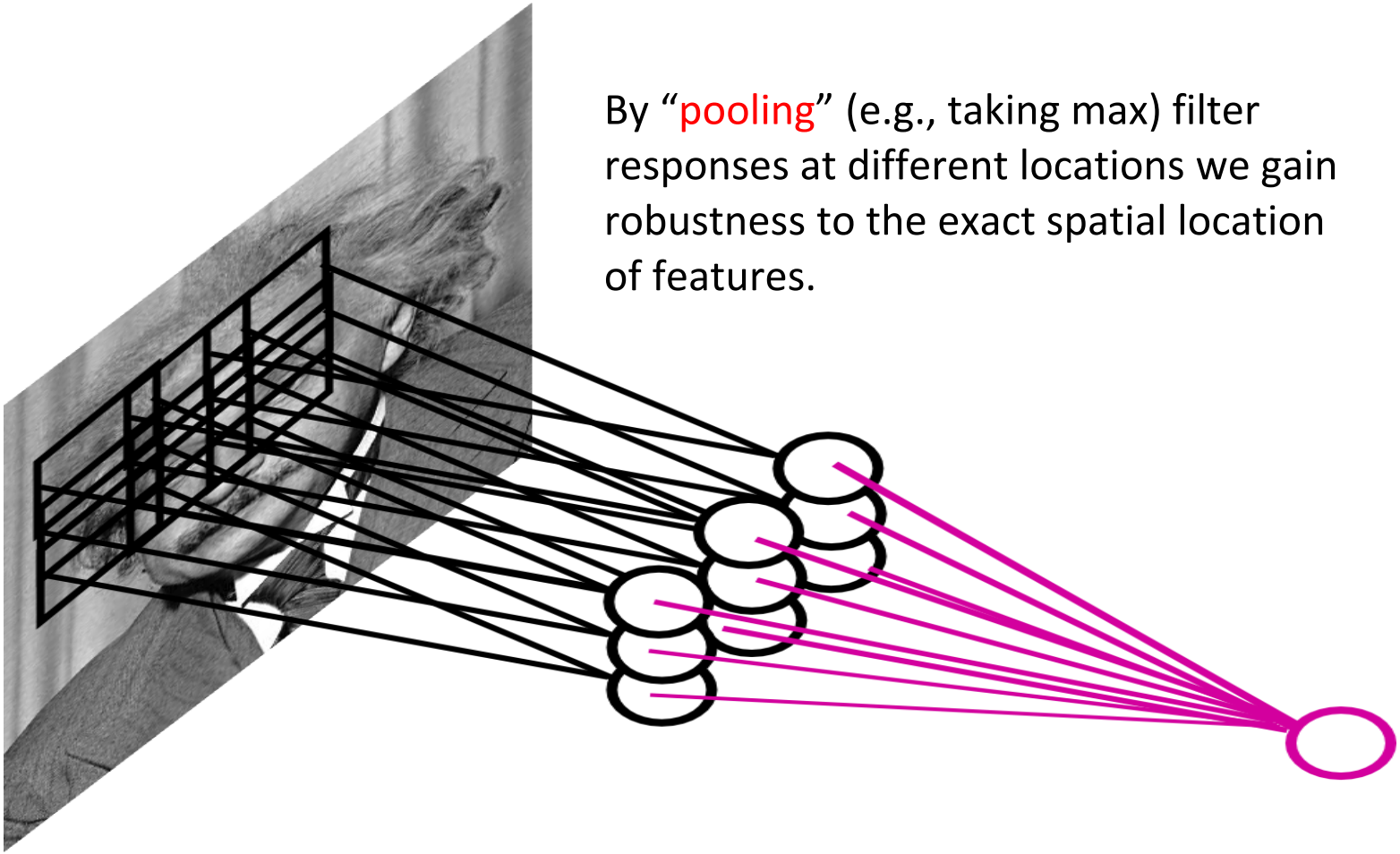


Let us assume filter is an “eye” detector.

Q.: how can we make the detection robust to the exact location of the eye?

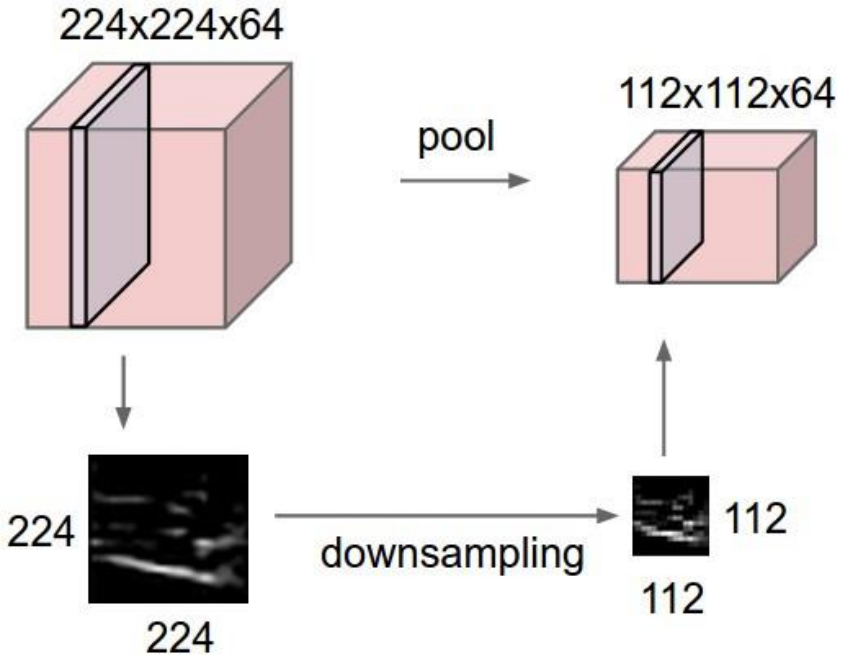
Pooling Layer

By “pooling” (e.g., taking max) filter responses at different locations we gain robustness to the exact spatial location of features.

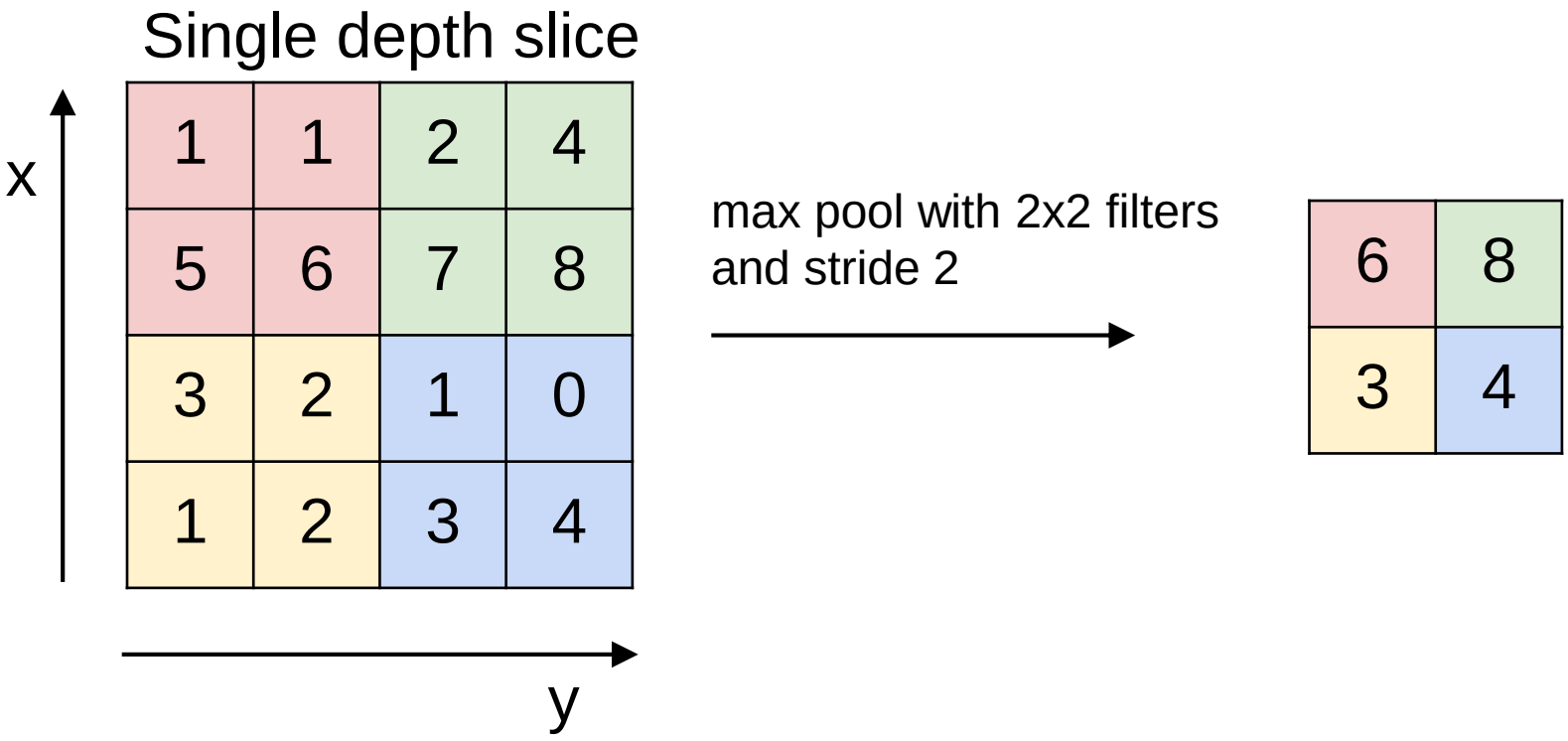


Pooling layer

- makes the representations smaller and more manageable
- operates over each activation map independently:

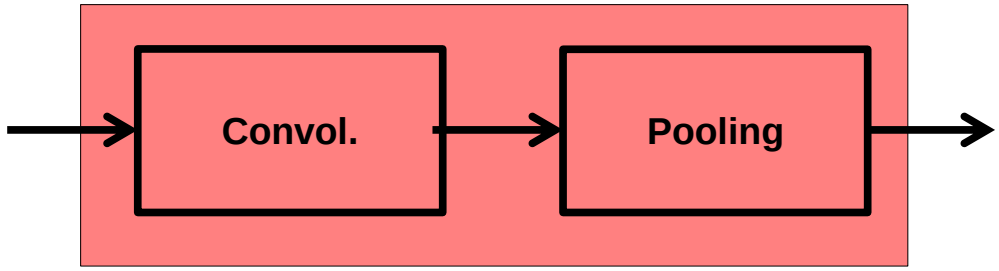


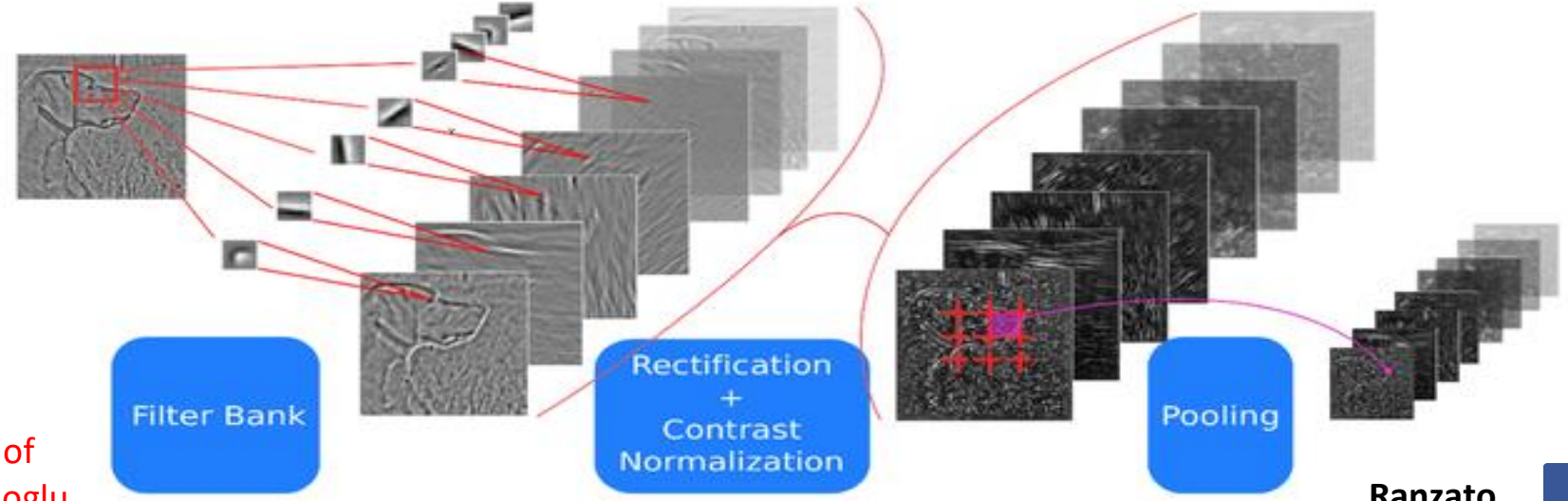
MAX POOLING


Single depth slice
x
y
1 1 2 4
5 6 7 8
3 2 1 0
1 2 3 4
max pool with 2x2 filters
and stride 2
6 8
3 4

ConvNets: Typical Stage

One stage (zoom)





courtesy of
K. Kavukcuoglu

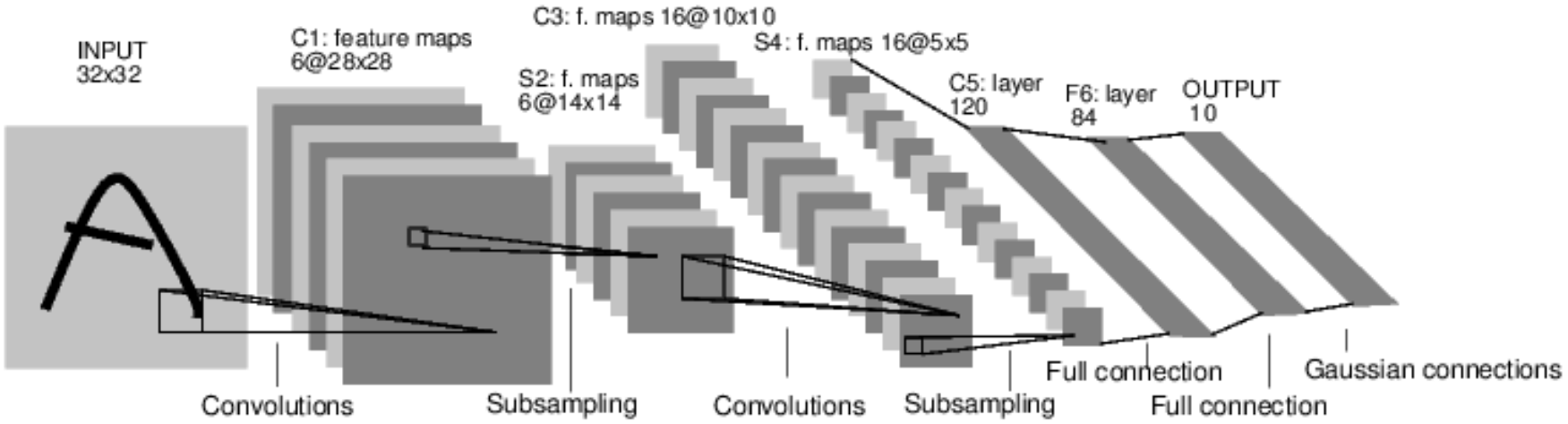
Ranzato

[ConvNetJS demo: training on CIFAR-10]

http://cs.stanford.edu/people/karpathy/convnetjs/demo/cifar10.html

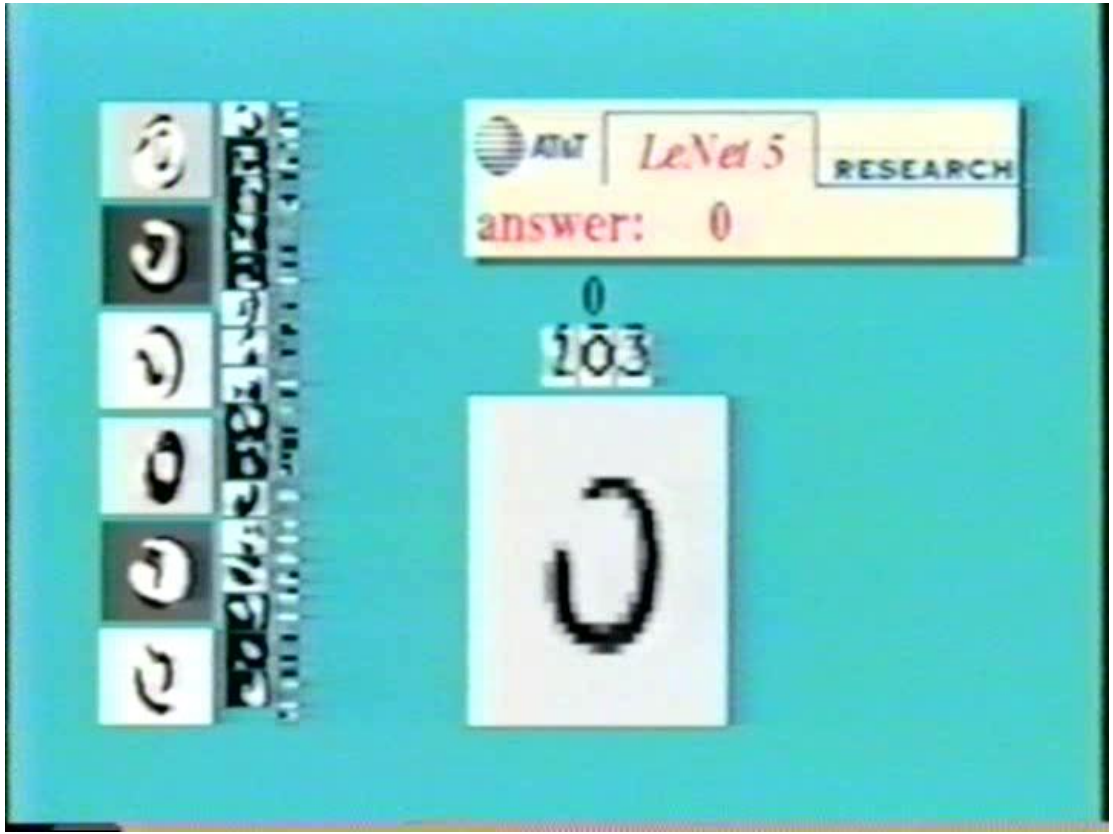
Case Study: LeNet-5

[LeCun et al., 1998]



Conv filters were 5x5, applied at stride 1
Subsampling (Pooling) layers were 2x2 applied at stride 2
i.e. architecture is [CONV-POOL-CONV-POOL-CONV-FC]

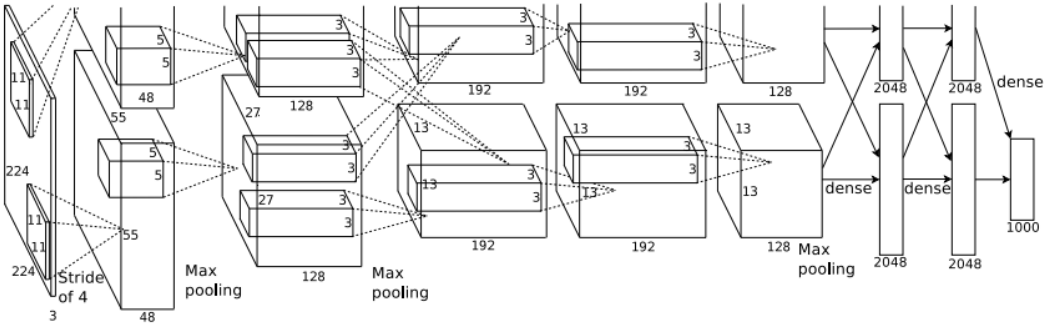
Handwritten digit classification



[Courtesy of Yann LeCun]

Case Study: AlexNet

[Krizhevsky et al. 2012]



Input: 227x227x3 images

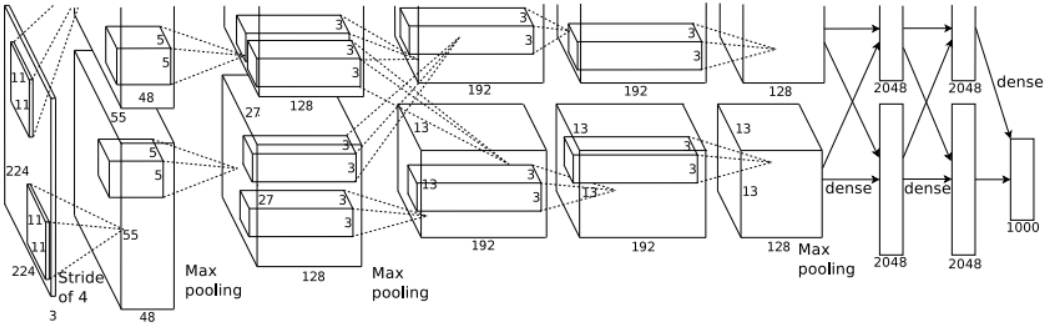
First layer (CONV1): 96 11x11 filters applied at stride 4

=>

Q: what is the output volume size? Hint: (227-11)/4+1 = 55

Case Study: AlexNet

[Krizhevsky et al. 2012]



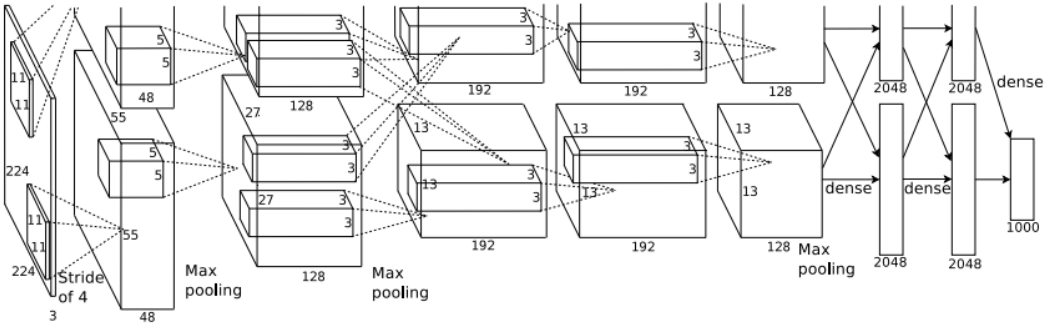
Input: 227x227x3 images

First layer (CONV1): 96 11x11 filters applied at stride 4
=>
Output volume **[55x55x96]**

Q: What is the total number of parameters in this layer?

Case Study: AlexNet

[Krizhevsky et al. 2012]

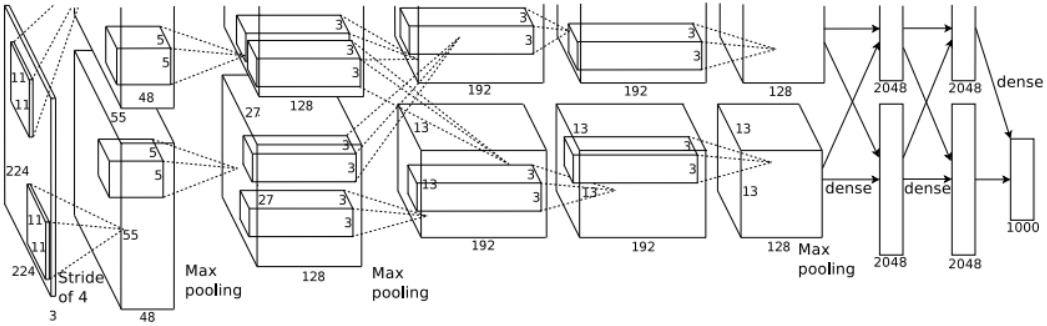


Input: 227x227x3 images

First layer (CONV1): 96 11x11 filters applied at stride 4
=>
Output volume **[55x55x96]**
Parameters: (11*11*3)*96 = **35K**

Case Study: AlexNet

[Krizhevsky et al. 2012]



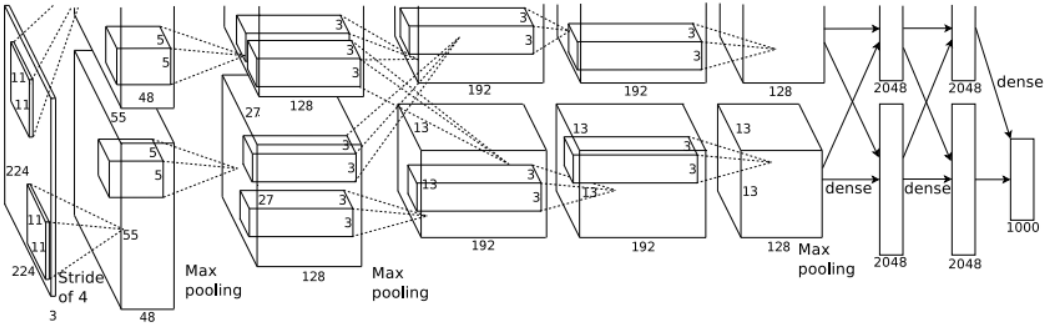
Input: 227x227x3 images
After CONV1: 55x55x96

Second layer (POOL1): 3x3 filters applied at stride 2

Q: what is the output volume size? Hint: (55-3)/2+1 = 27

Case Study: AlexNet

[Krizhevsky et al. 2012]



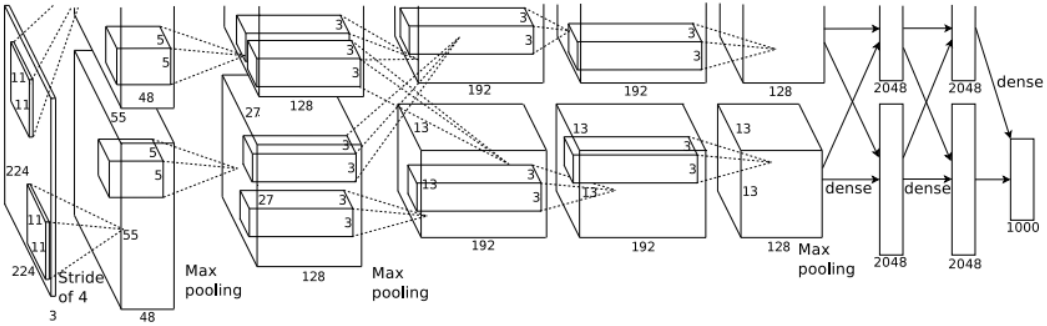
Input: 227x227x3 images
After CONV1: 55x55x96

Second layer (POOL1): 3x3 filters applied at stride 2
Output volume: 27x27x96

Q: what is the number of parameters in this layer?

Case Study: AlexNet

[Krizhevsky et al. 2012]

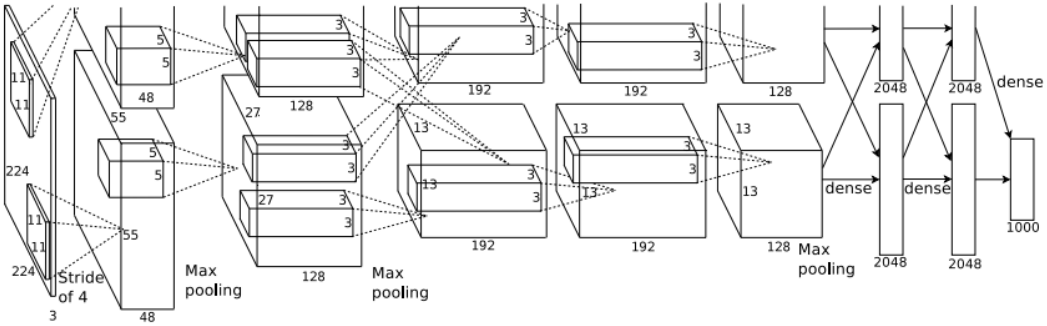


Input: 227x227x3 images
After CONV1: 55x55x96

Second layer (POOL1): 3x3 filters applied at stride 2
Output volume: 27x27x96
Parameters: 0!

Case Study: AlexNet

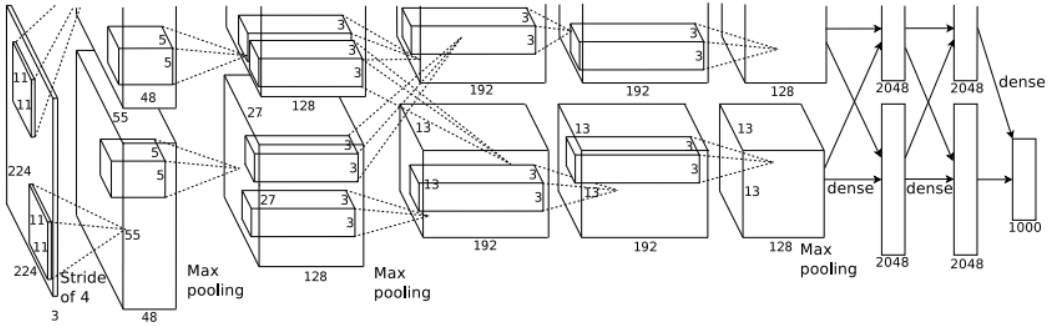
[Krizhevsky et al. 2012]



Input: 227x227x3 images
After CONV1: 55x55x96
After POOL1: 27x27x96
...

Case Study: AlexNet

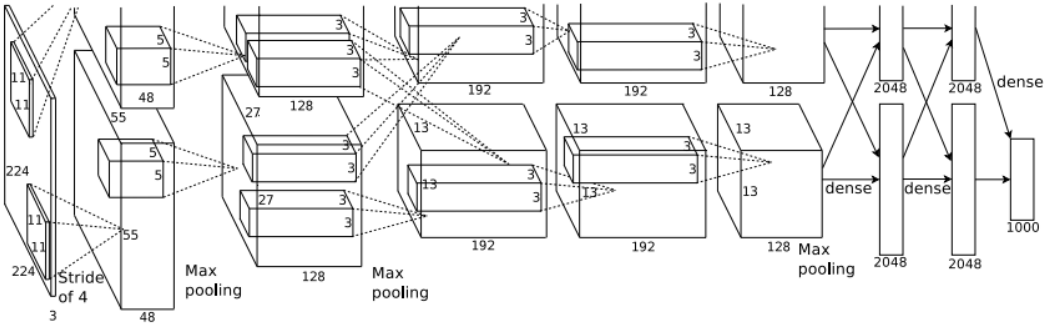
[Krizhevsky et al. 2012]



Full (simplified) AlexNet architecture:
[227x227x3] INPUT
[55x55x96] CONV1: 96 11x11 filters at stride 4, pad 0
[27x27x96] MAX POOL1: 3x3 filters at stride 2
[27x27x96] NORM1: Normalization layer
[27x27x256] CONV2: 256 5x5 filters at stride 1, pad 2
[13x13x256] MAX POOL2: 3x3 filters at stride 2
[13x13x256] NORM2: Normalization layer
[13x13x384] CONV3: 384 3x3 filters at stride 1, pad 1
[13x13x384] CONV4: 384 3x3 filters at stride 1, pad 1
[13x13x256] CONV5: 256 3x3 filters at stride 1, pad 1
[6x6x256] MAX POOL3: 3x3 filters at stride 2
[4096] FC6: 4096 neurons
[4096] FC7: 4096 neurons
[1000] FC8: 1000 neurons (class scores)

Case Study: AlexNet

[Krizhevsky et al. 2012]



Full (simplified) AlexNet architecture:
[227x227x3] INPUT
[55x55x96] CONV1: 96 11x11 filters at stride 4, pad 0
[27x27x96] MAX POOL1: 3x3 filters at stride 2
[27x27x96] NORM1: Normalization layer
[27x27x256] CONV2: 256 5x5 filters at stride 1, pad 2
[13x13x256] MAX POOL2: 3x3 filters at stride 2
[13x13x256] NORM2: Normalization layer
[13x13x384] CONV3: 384 3x3 filters at stride 1, pad 1
[13x13x384] CONV4: 384 3x3 filters at stride 1, pad 1
[13x13x256] CONV5: 256 3x3 filters at stride 1, pad 1
[6x6x256] MAX POOL3: 3x3 filters at stride 2
[4096] FC6: 4096 neurons
[4096] FC7: 4096 neurons
[1000] FC8: 1000 neurons (class scores)

Details/Retrospectives:
- first use of ReLU
- used Norm layers (not common anymore)
- heavy data augmentation
- dropout 0.5
- batch size 128
- SGD Momentum 0.9
- Learning rate 1e-2, reduced by 10 manually when val accuracy plateaus
- L2 weight decay 5e-4
- 7 CNN ensemble: 18.2% -> 15.4%

“You need a lot of a data if you want to train/use CNNs”

Transfer Learning

“You need a lot of a data if you want to train/use CNNs”

Transfer Learning with CNNs

1. Train on Imagenet

Transfer Learning with CNNs

1. Train on Imagenet

2. If small dataset: fix all weights (treat CNN as fixed feature extractor), retrain only the classifier

i.e. swap the Softmax layer at the end

Transfer Learning with CNNs

1. Train on Imagenet

2. If small dataset: fix all weights (treat CNN as fixed feature extractor), retrain only the classifier

i.e. swap the Softmax layer at the end

3. If you have medium sized dataset, **"finetune"** instead: use the old weights as initialization, train the full network or only some of the higher layers

retrain bigger portion of the network, or even all of it.

Transfer Learning with CNNs

1. Train on Imagenet

2. If small dataset: fix all weights (treat CNN as fixed feature extractor), retrain only the classifier

i.e. swap the Softmax layer at the end

3. If you have medium sized dataset, **"finetune"** instead: use the old weights as initialization, train the full network or only some of the higher layers

retrain bigger portion of the network, or even all of it.

tip: use only ~1/10th of the original learning rate in finetuning to player, and ~1/100th on intermediate layers

Case Study: VGGNet

[Simonyan and Zisserman, 2014]

Only 3x3 CONV stride 1, pad 1
and 2x2 MAX POOL stride 2

best model

11.2% top 5 error in ILSVRC 2013
->
7.3% top 5 error

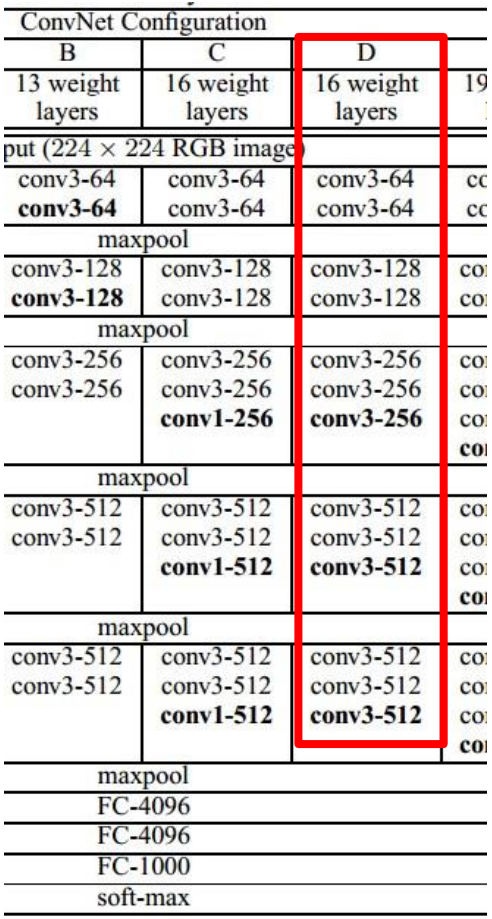
ConvNet Configuration					
A	A-LRN	B	C	D	E
11 weight layers	11 weight layers	13 weight layers	16 weight layers	16 weight layers	19 weight layers
input (224 × 224 RGB image)					
conv3-64	conv3-64 **LRN**	conv3-64 **conv3-64**	conv3-64 conv3-64	conv3-64 conv3-64	conv3-64 conv3-64
maxpool					
conv3-128	conv3-128	conv3-128 **conv3-128**	conv3-128 conv3-128	conv3-128 conv3-128	conv3-128 conv3-128
maxpool					
conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 **conv1-256**	conv3-256 conv3-256 **conv3-256**	conv3-256 conv3-256 conv3-256 **conv3-256**
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 **conv1-512**	conv3-512 conv3-512 **conv3-512**	conv3-512 conv3-512 conv3-512 **conv3-512**
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 **conv1-512**	conv3-512 conv3-512 **conv3-512**	conv3-512 conv3-512 conv3-512 **conv3-512**
maxpool					
FC-4096					
FC-4096					
FC-1000					
soft-max					

Table 2: **Number of parameters** (in millions).

Network	A,A-LRN	B	C	D	E
Number of parameters	133	133	134	138	144

(not counting biases)

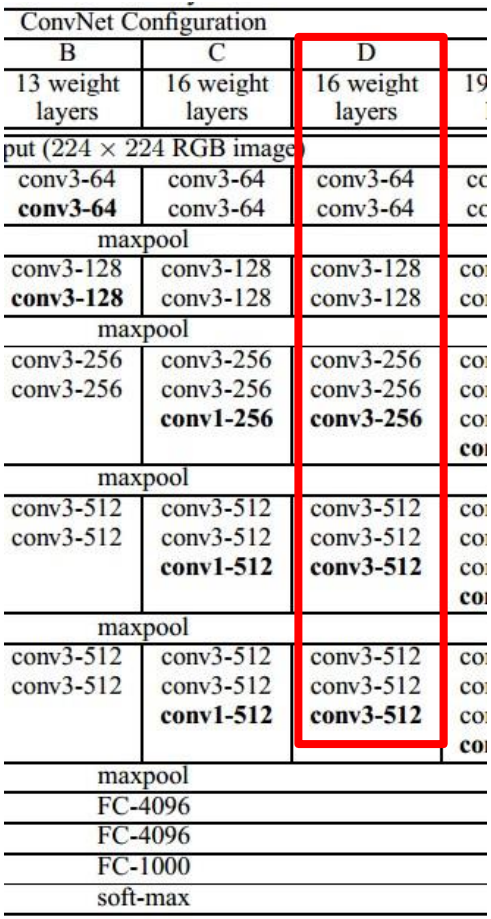
INPUT: [224x224x3] memory: 224*224*3=150K params: 0
CONV3-64: [224x224x64] memory: 224*224*64=3.2M params: (3*3*3)*64 = 1,728
CONV3-64: [224x224x64] memory: 224*224*64=3.2M params: (3*3*64)*64 = 36,864
POOL2: [112x112x64] memory: 112*112*64=800K params: 0
CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*64)*128 = 73,728
CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*128)*128 = 147,456
POOL2: [56x56x128] memory: 56*56*128=400K params: 0
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*128)*256 = 294,912
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589,824
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589,824
POOL2: [28x28x256] memory: 28*28*256=200K params: 0
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*256)*512 = 1,179,648
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296
POOL2: [14x14x512] memory: 14*14*512=100K params: 0
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
POOL2: [7x7x512] memory: 7*7*512=25K params: 0
FC: [1x1x4096] memory: 4096 params: 7*7*512*4096 = 102,760,448
FC: [1x1x4096] memory: 4096 params: 4096*4096 = 16,777,216
FC: [1x1x1000] memory: 1000 params: 4096*1000 = 4,096,000



(not counting biases)

INPUT: [224x224x3] memory: 224*224*3=150K params: 0
CONV3-64: [224x224x64] memory: 224*224*64=3.2M params: (3*3*3)*64 = 1,728
CONV3-64: [224x224x64] memory: 224*224*64=3.2M params: (3*3*64)*64 = 36,864
POOL2: [112x112x64] memory: 112*112*64=800K params: 0
CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*64)*128 = 73,728
CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*128)*128 = 147,456
POOL2: [56x56x128] memory: 56*56*128=400K params: 0
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*128)*256 = 294,912
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589,824
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589,824
POOL2: [28x28x256] memory: 28*28*256=200K params: 0
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*256)*512 = 1,179,648
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296
POOL2: [14x14x512] memory: 14*14*512=100K params: 0
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
POOL2: [7x7x512] memory: 7*7*512=25K params: 0
FC: [1x1x4096] memory: 4096 params: 7*7*512*4096 = 102,760,448
FC: [1x1x4096] memory: 4096 params: 4096*4096 = 16,777,216
FC: [1x1x1000] memory: 1000 params: 4096*1000 = 4,096,000

TOTAL memory: 24M * 4 bytes ~= 93MB / image (only forward! ~*2 for bwd)
TOTAL params: 138M parameters



(not counting biases)

INPUT: [224x224x3] memory: 224*224*3=150K params: 0
CONV3-64: [224x224x64] memory: **224*224*64=3.2M** params: (3*3*3)*64 = 1,728
CONV3-64: [224x224x64] memory: **224*224*64=3.2M** params: (3*3*64)*64 = 36,864
POOL2: [112x112x64] memory: 112*112*64=800K params: 0
CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*64)*128 = 73,728
CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*128)*128 = 147,456
POOL2: [56x56x128] memory: 56*56*128=400K params: 0
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*128)*256 = 294,912
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589,824
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589,824
POOL2: [28x28x256] memory: 28*28*256=200K params: 0
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*256)*512 = 1,179,648
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296
POOL2: [14x14x512] memory: 14*14*512=100K params: 0
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
POOL2: [7x7x512] memory: 7*7*512=25K params: 0
FC: [1x1x4096] memory: 4096 params: 7*7*512*4096 = **102,760,448**
FC: [1x1x4096] memory: 4096 params: 4096*4096 = 16,777,216
FC: [1x1x1000] memory: 1000 params: 4096*1000 = 4,096,000

Note:

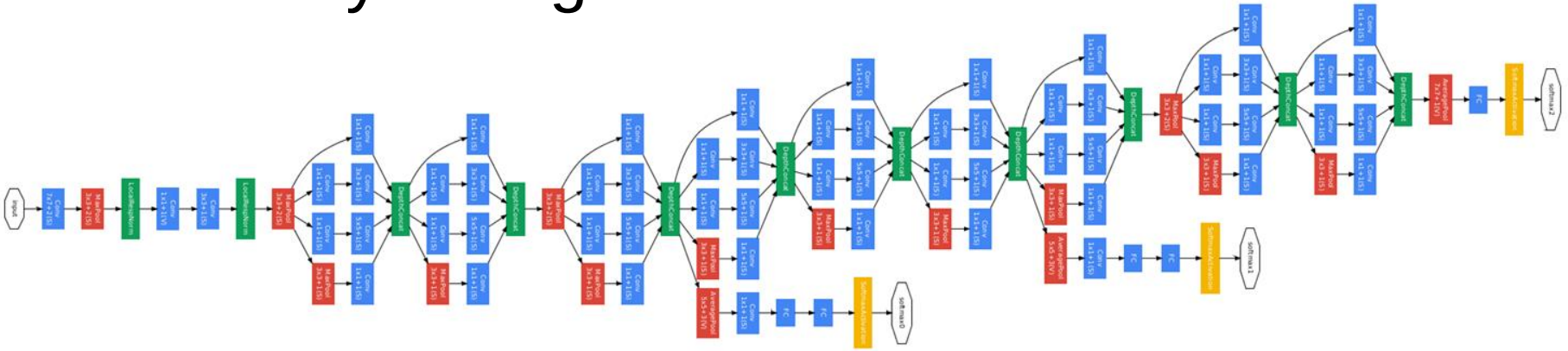
Most memory is in early CONV

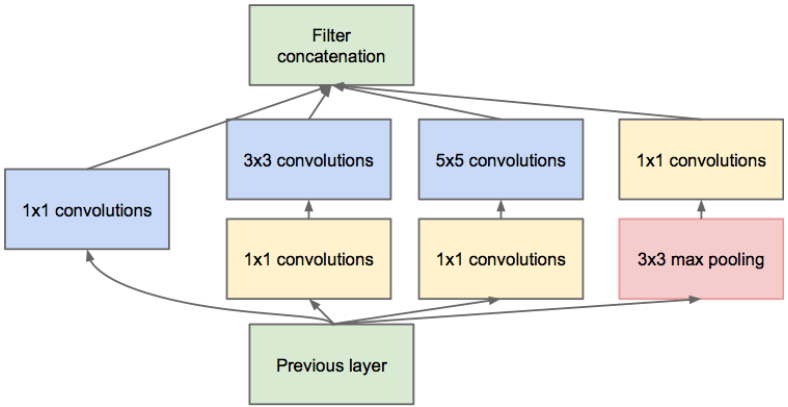
Most params are in late FC

TOTAL memory: 24M * 4 bytes ~= 93MB / image (only forward! ~*2 for bwd)
TOTAL params: 138M parameters

Case Study: GoogLeNet

[Szegedy et al., 2014]





Inception module

ILSVRC 2014 winner (6.7% top 5 error)

Case Study: GoogLeNet

type	patch size/ stride	output size	depth	#1×1	#3×3 reduce	#3×3	#5×5 reduce	#5×5	pool proj	params	ops
convolution	7×7/2	112×112×64	1							2.7K	34M
max pool	3×3/2	56×56×64	0								
convolution	3×3/1	56×56×192	2		64	192				112K	360M
max pool	3×3/2	28×28×192	0								
inception (3a)		28×28×256	2	64	96	128	16	32	32	159K	128M
inception (3b)		28×28×480	2	128	128	192	32	96	64	380K	304M
max pool	3×3/2	14×14×480	0								
inception (4a)		14×14×512	2	192	96	208	16	48	64	364K	73M
inception (4b)		14×14×512	2	160	112	224	24	64	64	437K	88M
inception (4c)		14×14×512	2	128	128	256	24	64	64	463K	100M
inception (4d)		14×14×528	2	112	144	288	32	64	64	580K	119M
inception (4e)		14×14×832	2	256	160	320	32	128	128	840K	170M
max pool	3×3/2	7×7×832	0								
inception (5a)		7×7×832	2	256	160	320	32	128	128	1072K	54M
inception (5b)		7×7×1024	2	384	192	384	48	128	128	1388K	71M
avg pool	7×7/1	1×1×1024	0								
dropout (40%)		1×1×1024	0								
linear		1×1×1000	1							1000K	1M
softmax		1×1×1000	0								

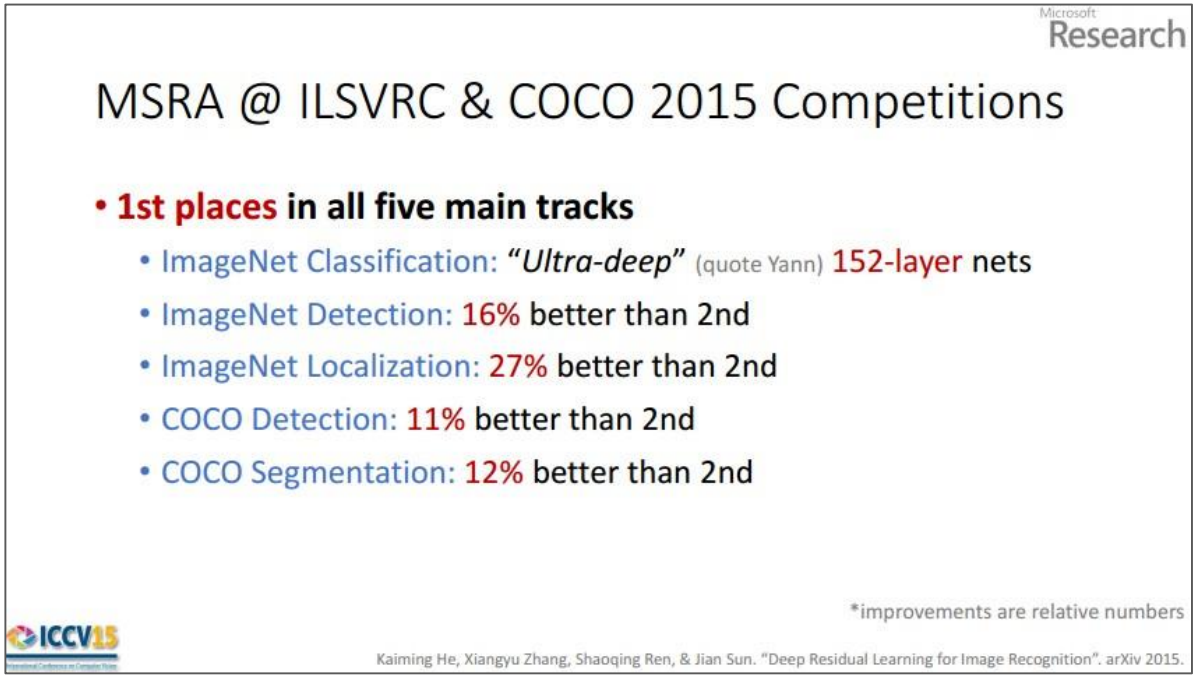
Fun features:

- Only 5 million params! (Removes FC layers completely)

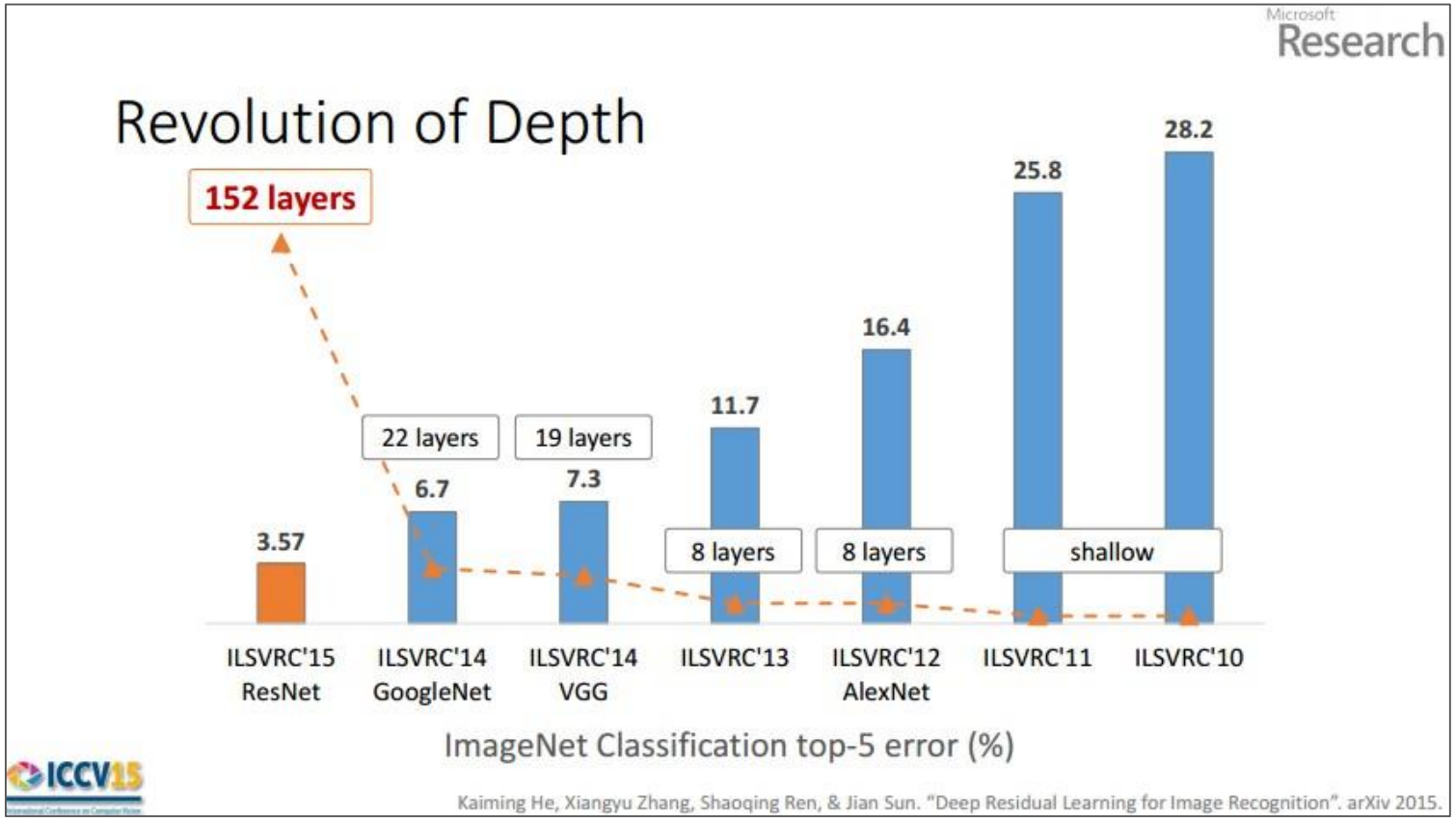
Compared to AlexNet:
- 12X less params
- 2x more compute
- 6.67% (vs. 16.4%)

Case Study: ResNet *[He et al., 2015]*

ILSVRC 2015 winner (3.6% top 5 error)

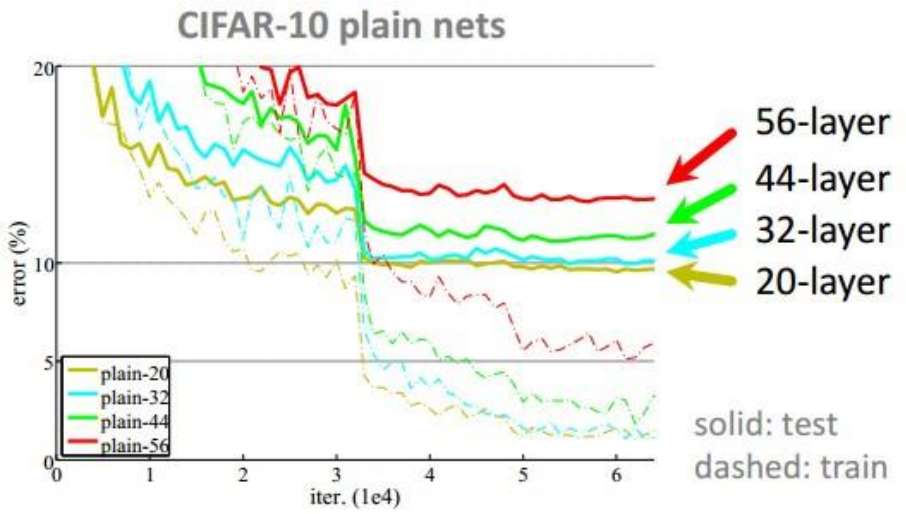


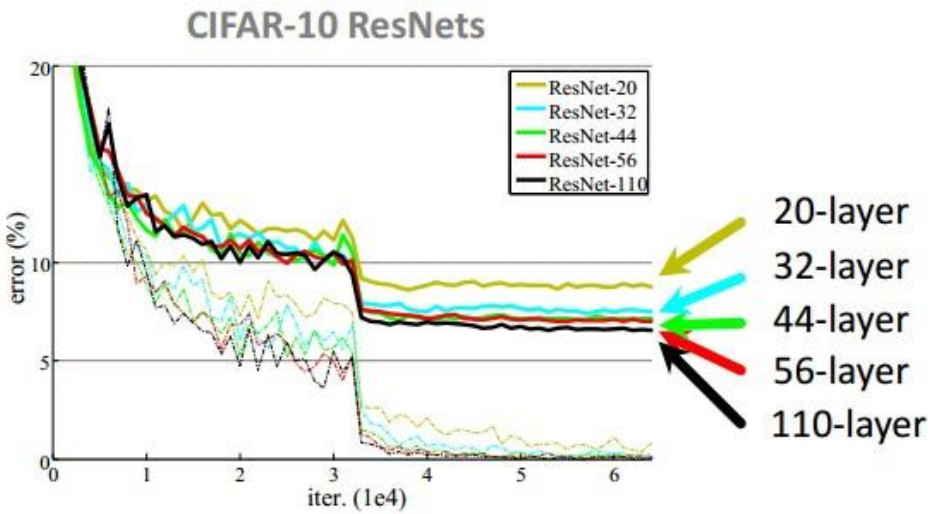
Slide from Kaiming He's recent presentation https://www.youtube.com/watch?v=1PGLj-uKT1w



(slide from Kaiming He's recent presentation)

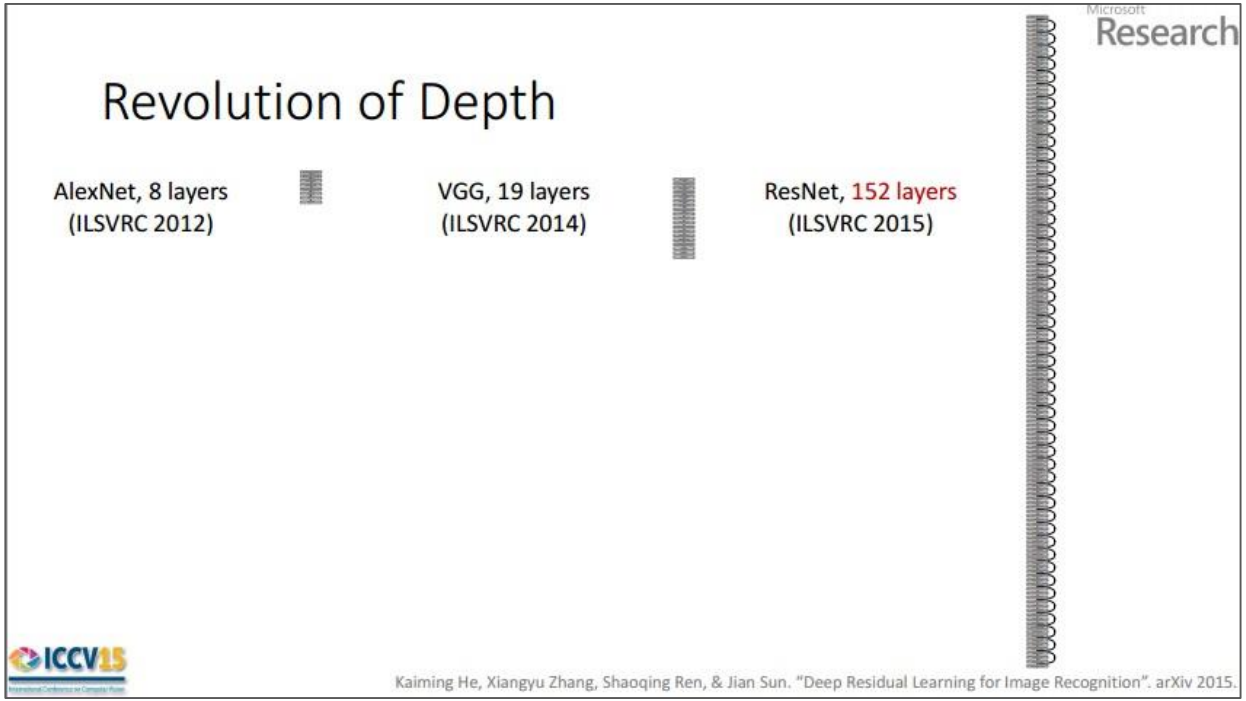
CIFAR-10 experiments





Case Study: ResNet *[He et al., 2015]*

ILSVRC 2015 winner (3.6% top 5 error)



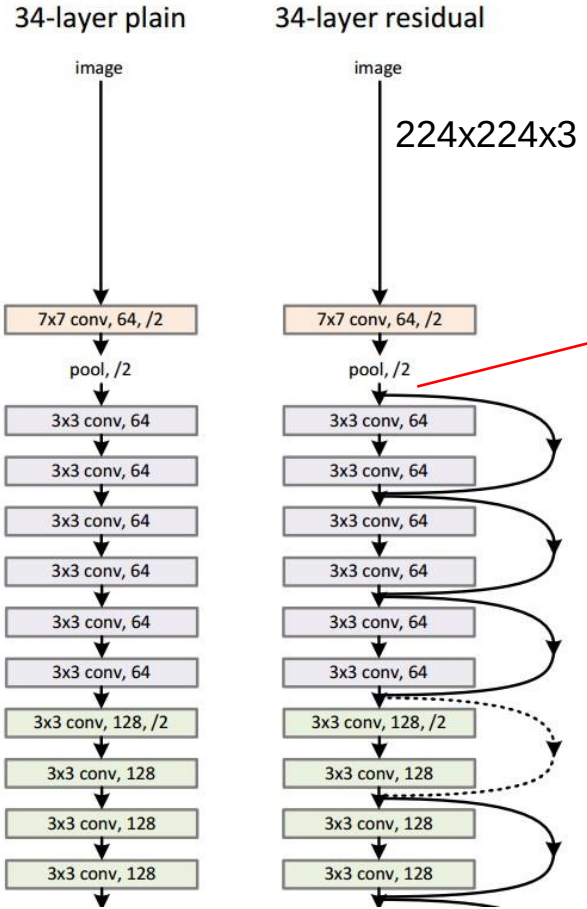
2-3 weeks of training on 8 GPU machine

at runtime: faster than a VGGNet! (even though it has 8x more layers)

(slide from Kaiming He's recent presentation)

Case Study: ResNet

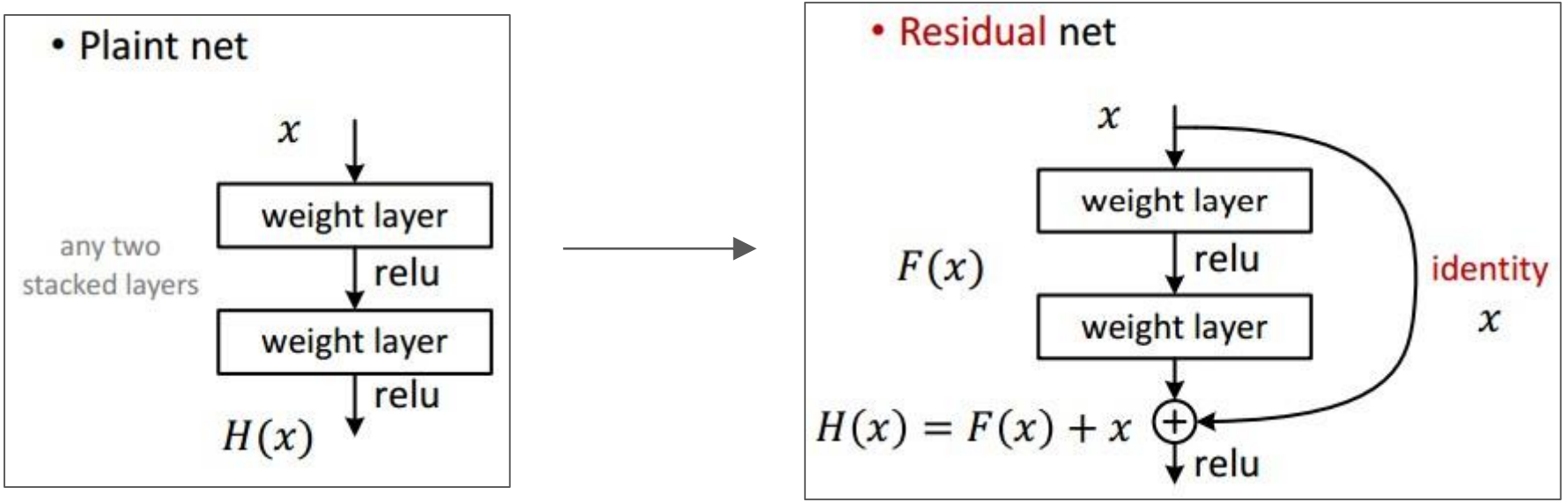
[He et al., 2015]



spatial dimension only 56x56!

Case Study: ResNet

[He et al., 2015]

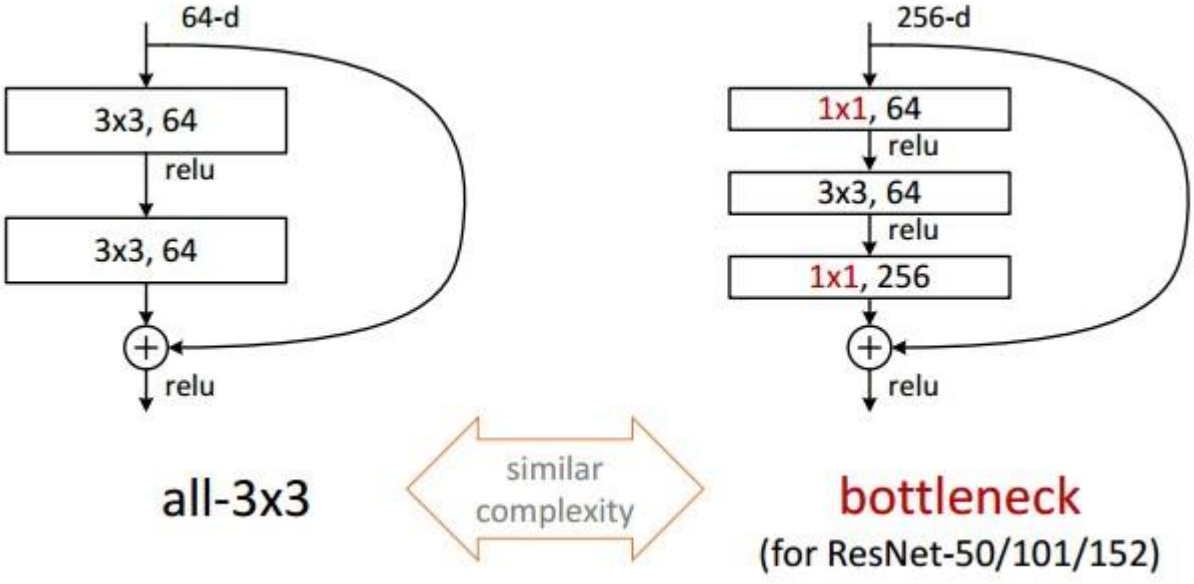


Case Study: ResNet *[He et al., 2015]*

- Batch Normalization after every CONV layer
- Xavier/2 initialization from He et al.
- SGD + Momentum (0.9)
- Learning rate: 0.1, divided by 10 when validation error plateaus
- Mini-batch size 256
- Weight decay of 1e-5
- No dropout used

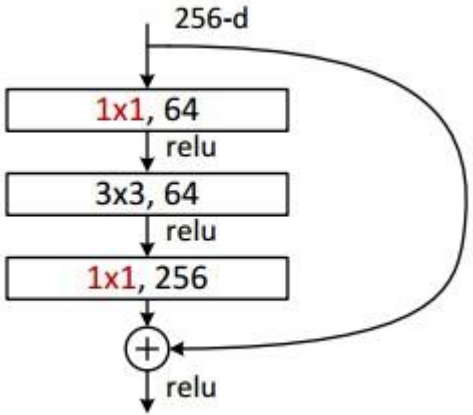
Case Study: ResNet

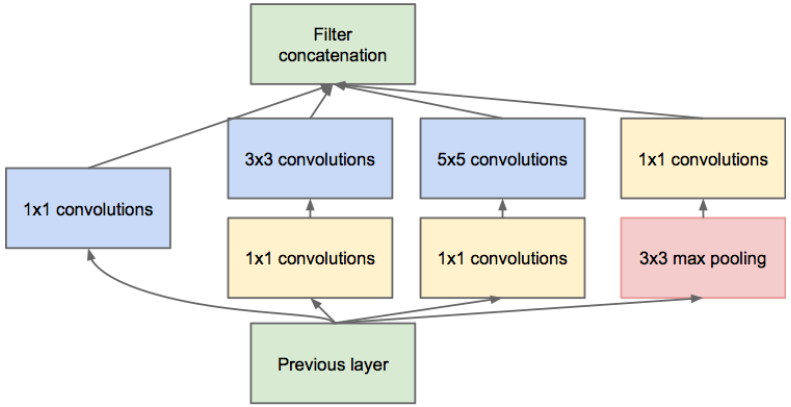
[He et al., 2015]



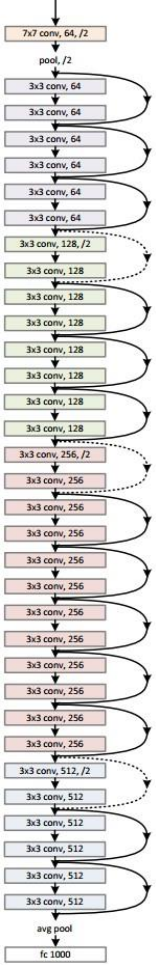
Case Study: ResNet

[He et al., 2015]





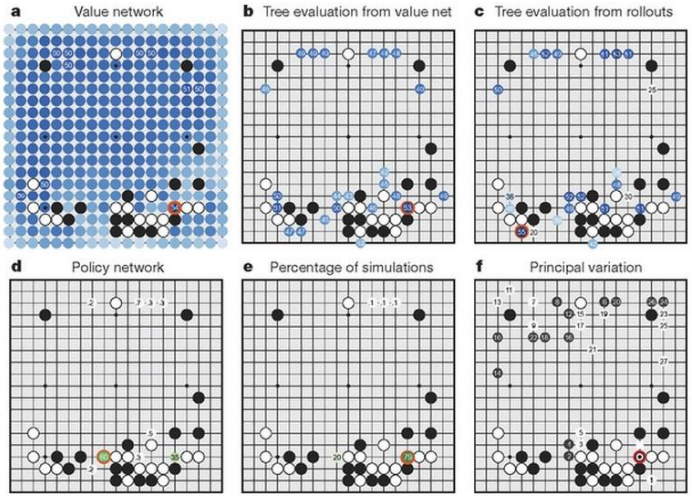
(this trick is also used in GoogLeNet)



Case Study: ResNet *[He et al., 2015]*

layer name	output size	18-layer	34-layer	50-layer	101-layer	152-layer
conv1	112×112	7×7, 64, stride 2				
conv2_x	56×56	3×3 max pool, stride 2				
		$\begin{bmatrix} 3\times3, 64 \\ 3\times3, 64 \end{bmatrix}\times2$	$\begin{bmatrix} 3\times3, 64 \\ 3\times3, 64 \end{bmatrix}\times3$	$\begin{bmatrix} 1\times1, 64 \\ 3\times3, 64 \\ 1\times1, 256 \end{bmatrix}\times3$	$\begin{bmatrix} 1\times1, 64 \\ 3\times3, 64 \\ 1\times1, 256 \end{bmatrix}\times3$	$\begin{bmatrix} 1\times1, 64 \\ 3\times3, 64 \\ 1\times1, 256 \end{bmatrix}\times3$
conv3_x	28×28	$\begin{bmatrix} 3\times3, 128 \\ 3\times3, 128 \end{bmatrix}\times2$	$\begin{bmatrix} 3\times3, 128 \\ 3\times3, 128 \end{bmatrix}\times4$	$\begin{bmatrix} 1\times1, 128 \\ 3\times3, 128 \\ 1\times1, 512 \end{bmatrix}\times4$	$\begin{bmatrix} 1\times1, 128 \\ 3\times3, 128 \\ 1\times1, 512 \end{bmatrix}\times4$	$\begin{bmatrix} 1\times1, 128 \\ 3\times3, 128 \\ 1\times1, 512 \end{bmatrix}\times8$
conv4_x	14×14	$\begin{bmatrix} 3\times3, 256 \\ 3\times3, 256 \end{bmatrix}\times2$	$\begin{bmatrix} 3\times3, 256 \\ 3\times3, 256 \end{bmatrix}\times6$	$\begin{bmatrix} 1\times1, 256 \\ 3\times3, 256 \\ 1\times1, 1024 \end{bmatrix}\times6$	$\begin{bmatrix} 1\times1, 256 \\ 3\times3, 256 \\ 1\times1, 1024 \end{bmatrix}\times23$	$\begin{bmatrix} 1\times1, 256 \\ 3\times3, 256 \\ 1\times1, 1024 \end{bmatrix}\times36$
conv5_x	7×7	$\begin{bmatrix} 3\times3, 512 \\ 3\times3, 512 \end{bmatrix}\times2$	$\begin{bmatrix} 3\times3, 512 \\ 3\times3, 512 \end{bmatrix}\times3$	$\begin{bmatrix} 1\times1, 512 \\ 3\times3, 512 \\ 1\times1, 2048 \end{bmatrix}\times3$	$\begin{bmatrix} 1\times1, 512 \\ 3\times3, 512 \\ 1\times1, 2048 \end{bmatrix}\times3$	$\begin{bmatrix} 1\times1, 512 \\ 3\times3, 512 \\ 1\times1, 2048 \end{bmatrix}\times3$
	1×1	average pool, 1000-d fc, softmax				
FLOPs		1.8×10^9	3.6×10^9	3.8×10^9	7.6×10^9	11.3×10^9

Case Study Bonus: DeepMind's AlphaGo



The input to the policy network is a 19 × 19 × 48 image stack consisting of 48 feature planes. The first hidden layer zero pads the input into a 23 × 23 image, then convolves *k* filters of kernel size 5 × 5 with stride 1 with the input image and applies a rectifier nonlinearity. Each of the subsequent hidden layers 2 to 12 zero pads the respective previous hidden layer into a 21 × 21 image, then convolves *k* filters of kernel size 3 × 3 with stride 1, again followed by a rectifier nonlinearity. The final layer convolves 1 filter of kernel size 1 × 1 with stride 1, with a different bias for each position, and applies a softmax function. The match version of AlphaGo used *k* = 192 filters; Fig. 2b and Extended Data Table 3 additionally show the results of training with *k* = 128, 256 and 384 filters.

policy network:
[19x19x48] Input
CONV1: 192 5x5 filters , stride 1, pad 2 => [19x19x192]
CONV2..12: 192 3x3 filters, stride 1, pad 1 => [19x19x192]
CONV: 1 1x1 filter, stride 1, pad 0 => [19x19] *(probability map of promising moves)*

The Chain Rule

- ConvNets stack CONV,ReLU,POOL,FC layers
- Trend towards smaller filters and deeper architectures
- Trend towards getting rid of POOL/FC layers (just CONV)
- Early architectures look like

  [(CONV-RELU)*N-POOL?]*M-(FC-RELU)*K,SOFTMAX
- but recent advances such as ResNet/GoogLeNet use only Conv-ReLU, 1x1 convolutions and Softmax