Value-based RL algorithms

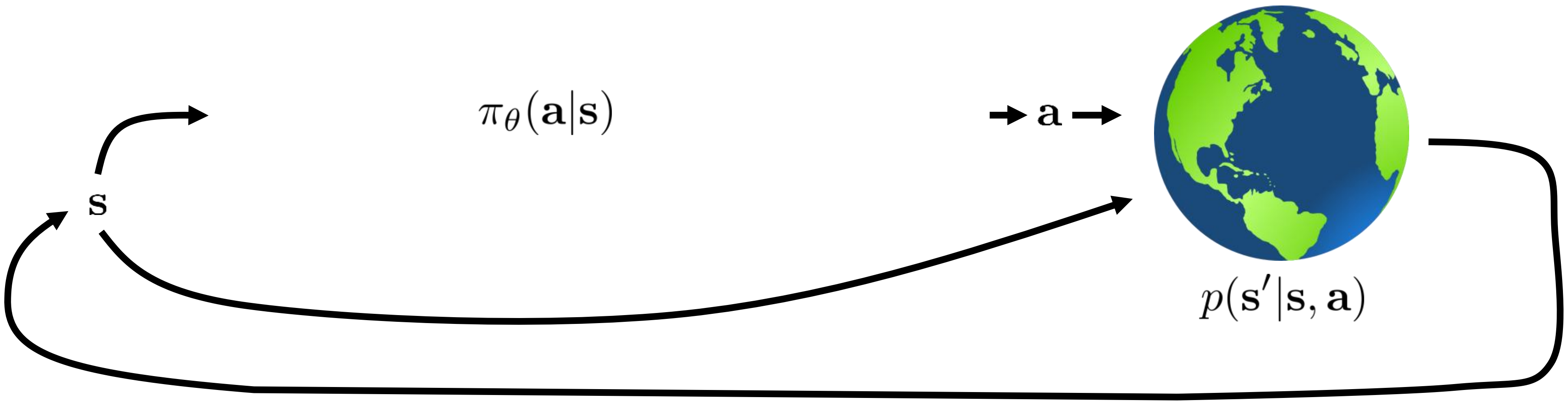
Carlos Florensa

Spring 2018

Lecture 21 of CS194/294-129: Designing, Visualizing and Understanding Deep Neural Networks

Most slides borrowed from S. Levine et al. “Deep Reinforcement Learning”

Recap: The goal of reinforcement learning



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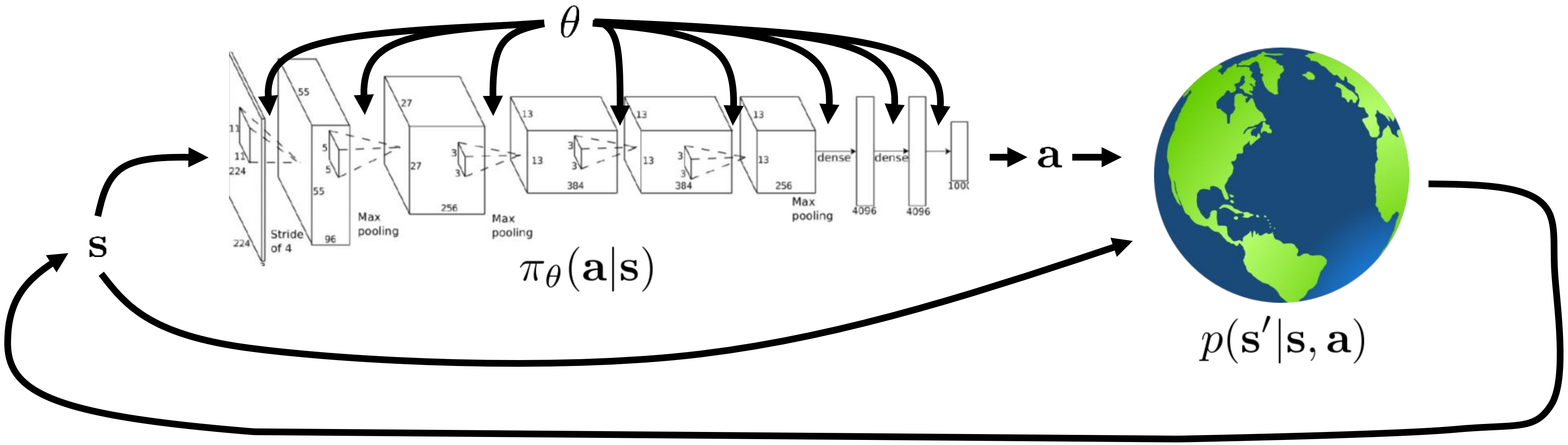

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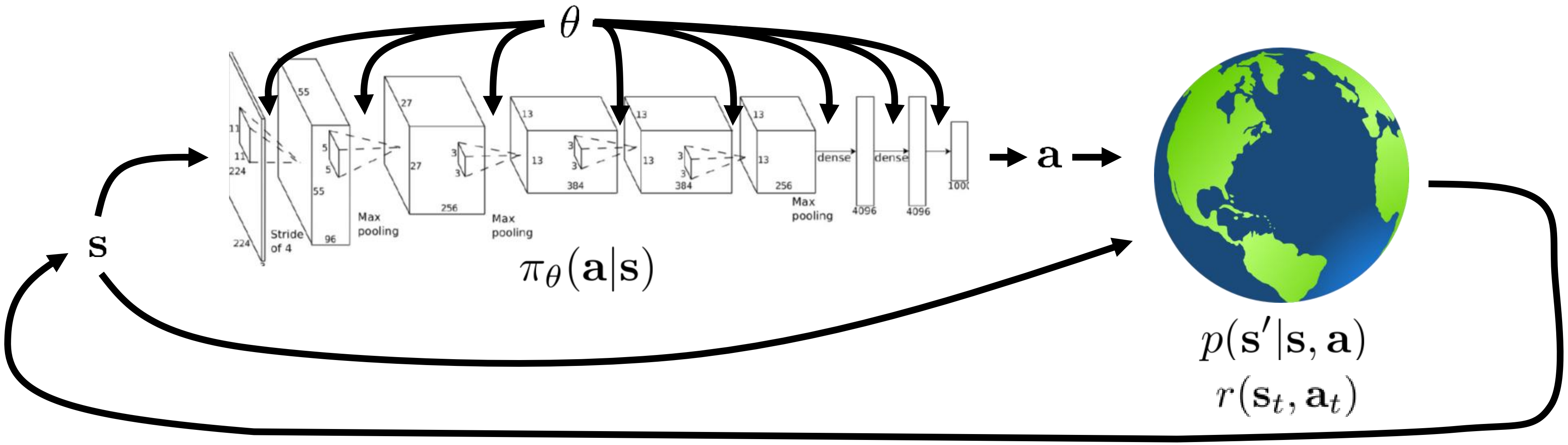

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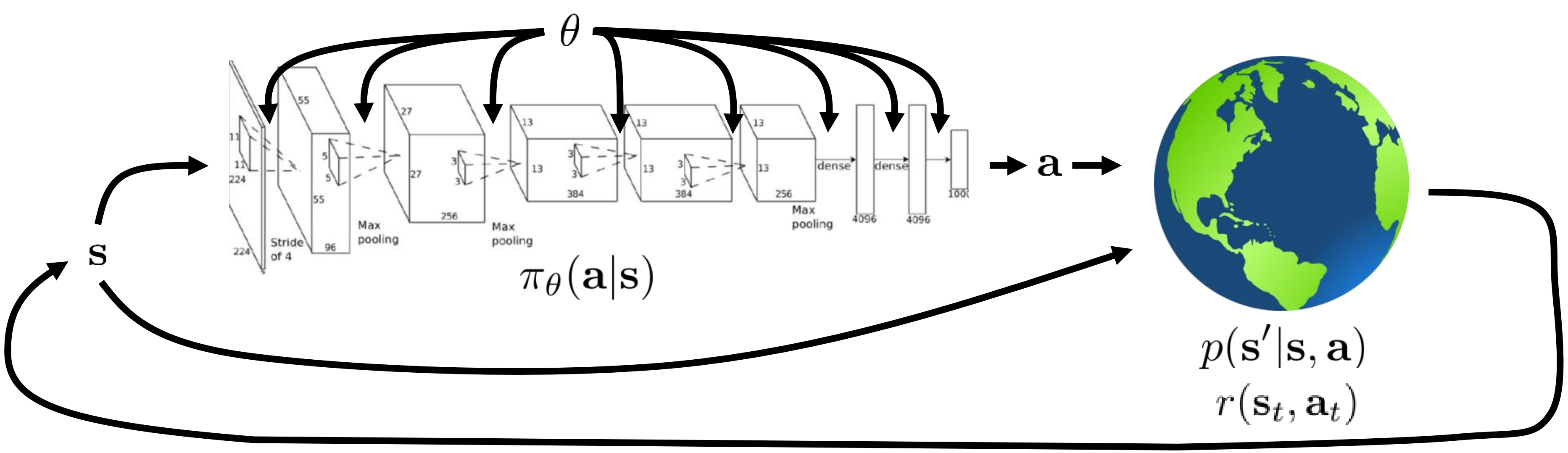

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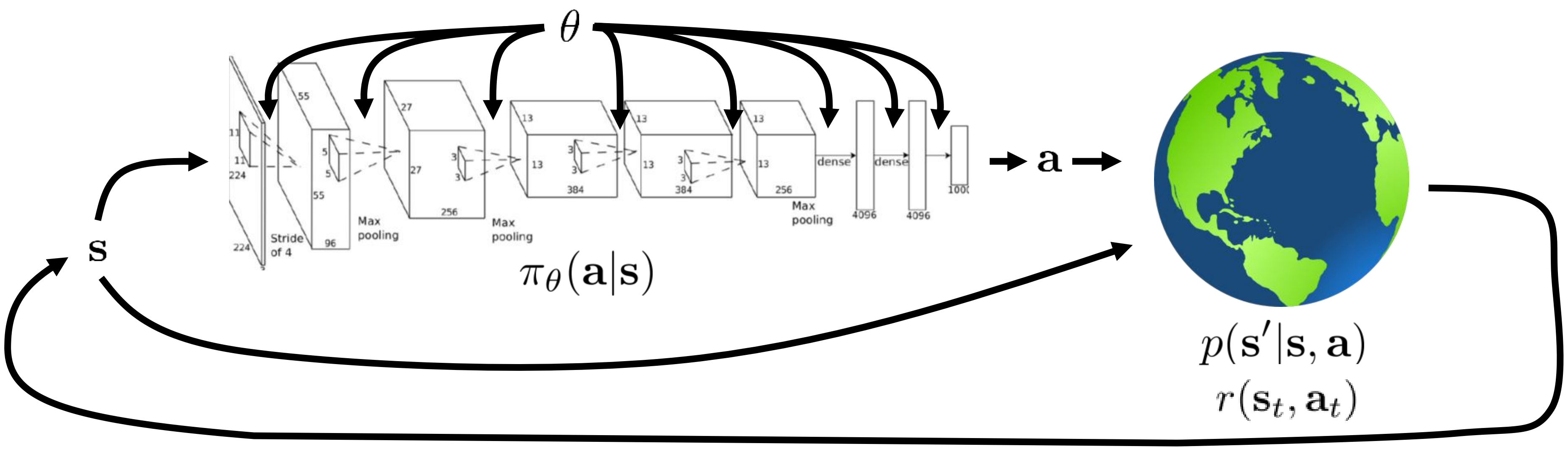

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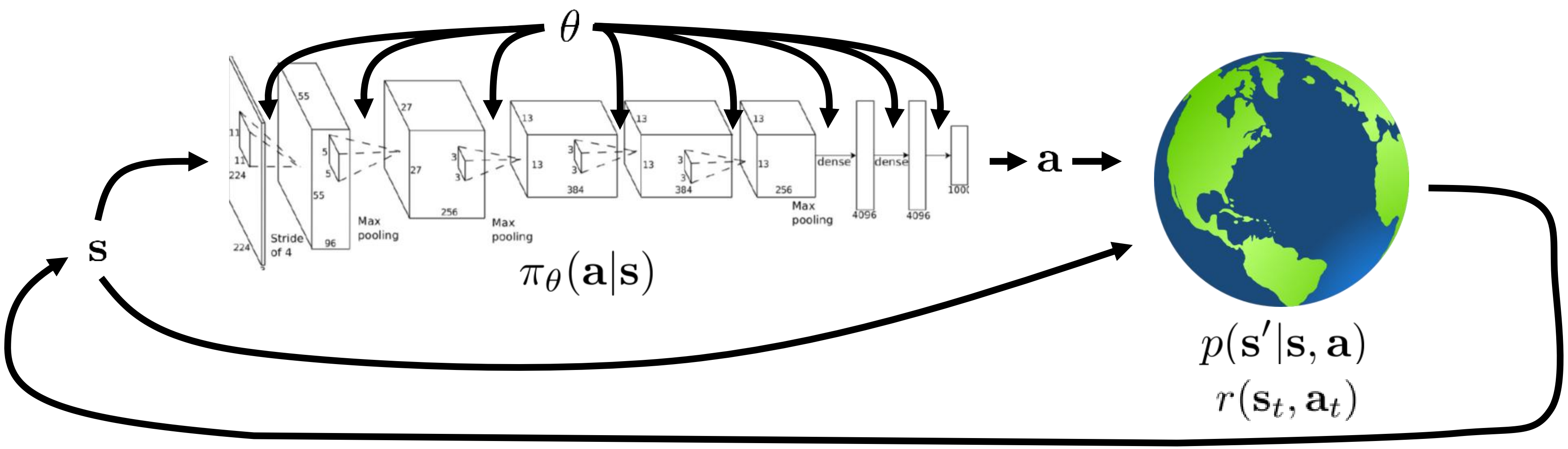

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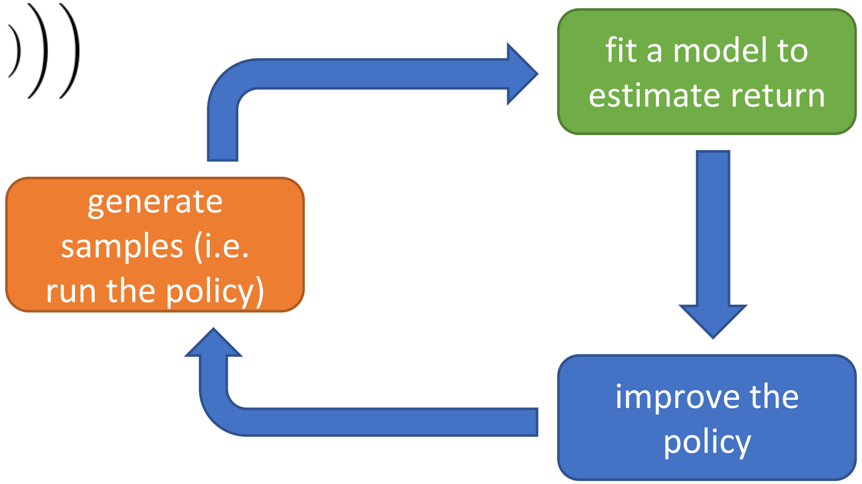

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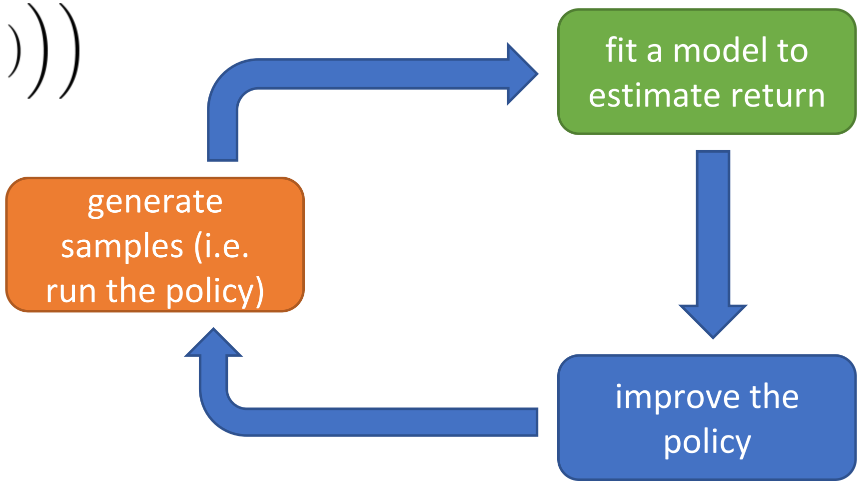

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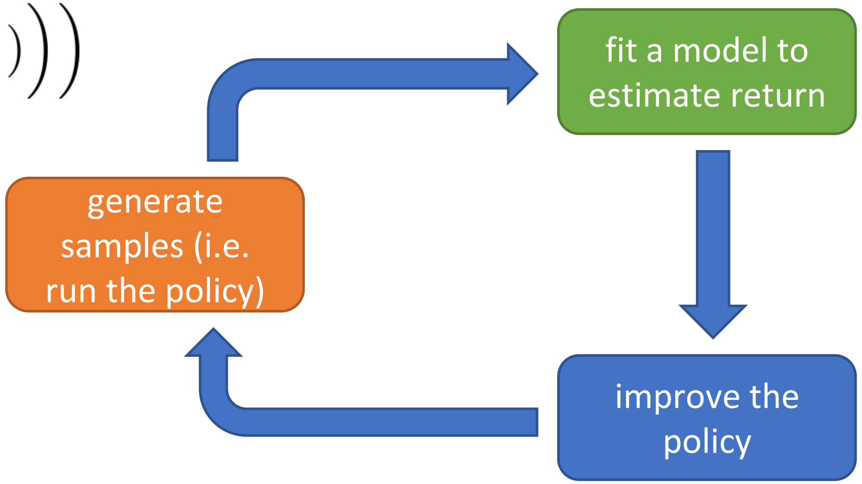

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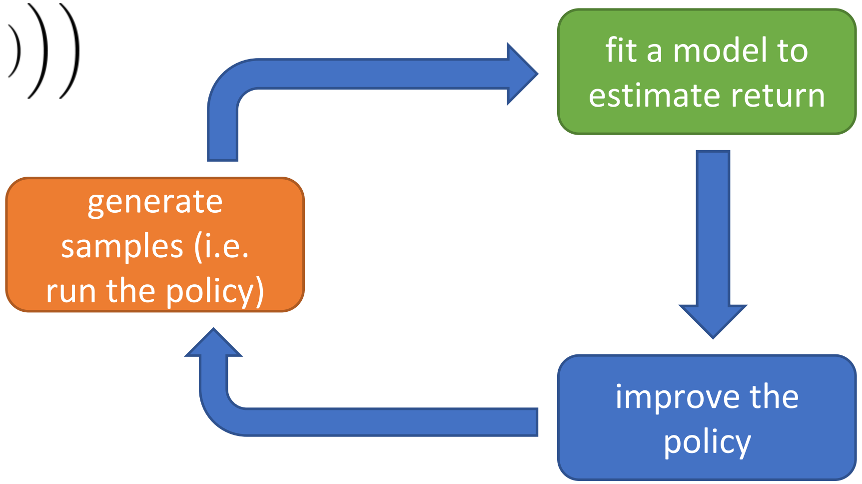

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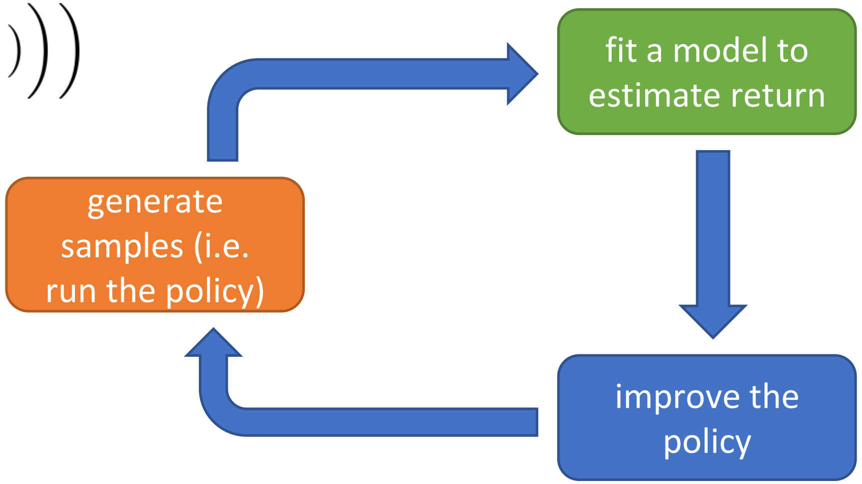

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We also saw advanced PG methods that add a trust region, like TRPO and PPO.

Recap: Value Functions

- We can define the value of a state s_i under a given policy π, $V(s_i)$ as the (discounted) *reward-to-go* from that state:

$$V(s_t) = E_{\pi_\theta}\left[\sum_{t'>t}^{T} \gamma^{t'-t}\, r(s_{t'}, a_{t'}) \mid s_t\right] \Longrightarrow V(s_t) = \sum_{a_t} \overbrace{\pi(a_t|s_t)}^{\text{Probability of taking action } a_i} \big(\underbrace{r(s_t, a_t)}_{\text{Actual reward at current step}} + \gamma \overbrace{\sum_{s_{t+1}} V(s_{t+1})\, p(s_{t+1}|s_t, a_t)}^{\text{Expected total future rewards}}\big)$$

"discount factor" γ

- $0 < \gamma \leq 1$ is typically close to 1.
- $\gamma < 1$ favors short-term rewards, and causes the recurrence to converge on all MDPs.

Recap: Bellman Update

- We can maximize the expected total reward directly in the value recurrence by taking the best (maximum reward) action:

$$V(s) = \max_{a} r(s,a) + \gamma \sum_{s'} V(s')\, p(s' \mid s,a)$$

For today:

Similarly if a Q-function satisfies this equation it corresponds to an optimal policy:

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- The only value function V that satisfies this equation is the one of an OPTIMAL policy!!
- If the state space is small enough to fit in memory, we can solve this recurrence directly using iterative calculation.

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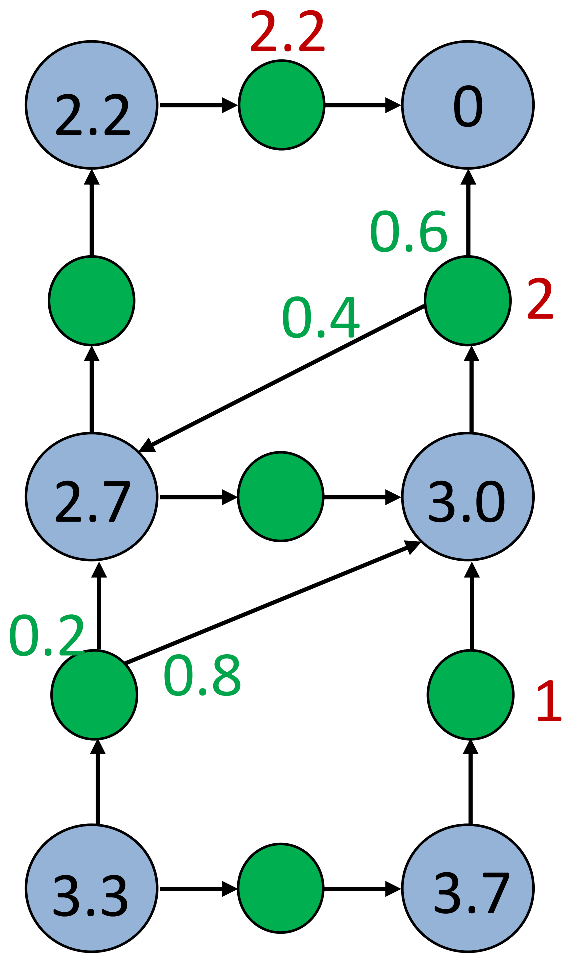
Bellman updates

- If the transition graph is acyclic, then the value function can be computed with dynamic programming. (We would also need dynamics)
- This would require only O(SA) steps, where S is the number of states, and A is the number of actions.
- Since the graph has cycles, repeated updates may be necessary to each node (so this is not a dynamic programming problem).



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  - Understand how actor-critic algorithms work
  - Understand how value-based methods work without a parameterized policy
  - Understand why they can be off-policy and how to stabilize them

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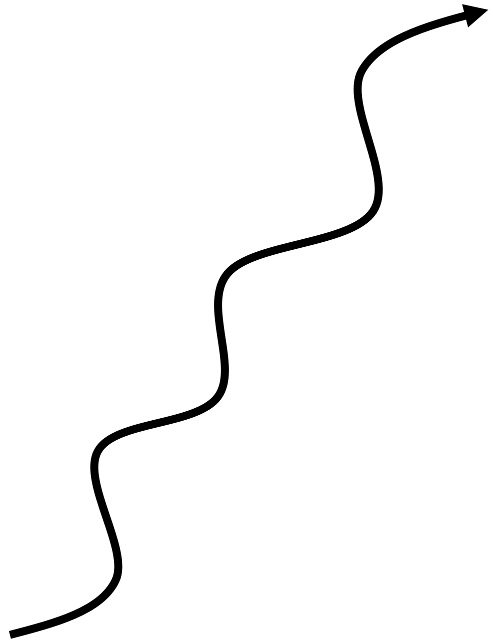

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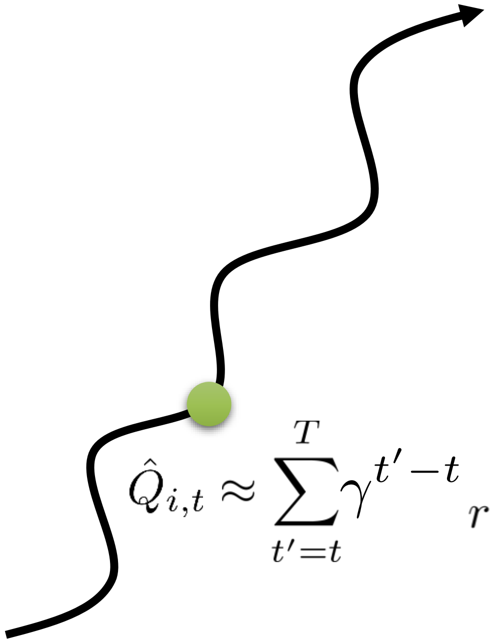

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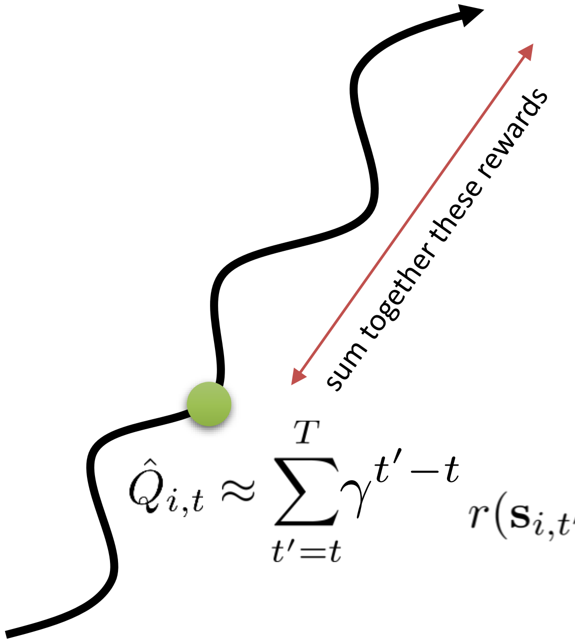

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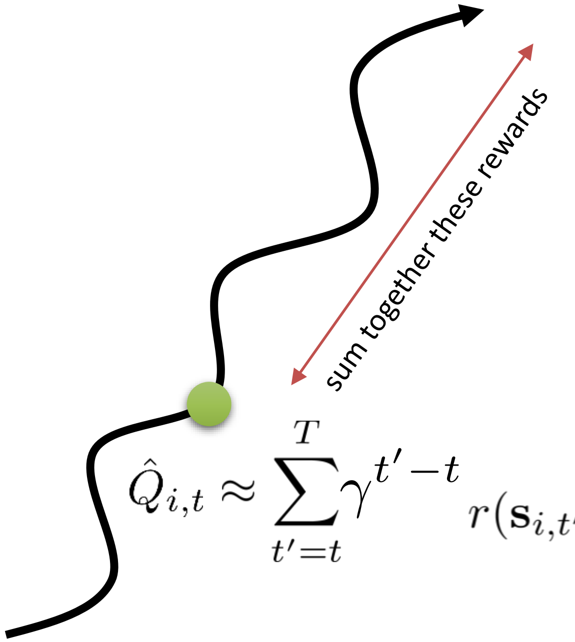

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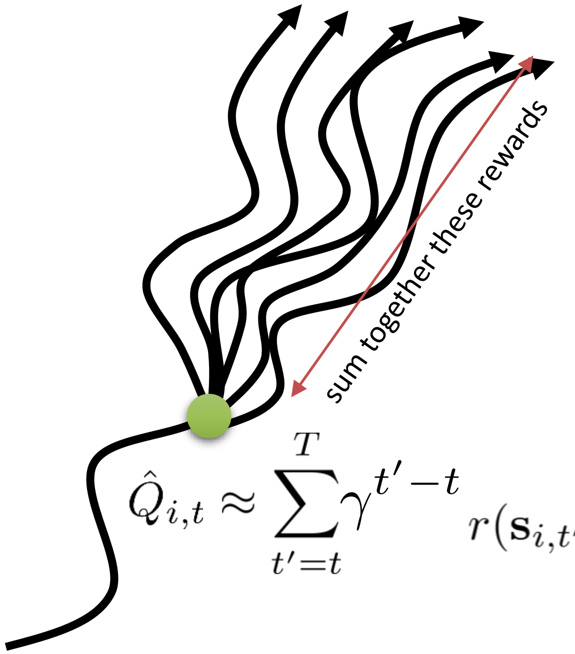

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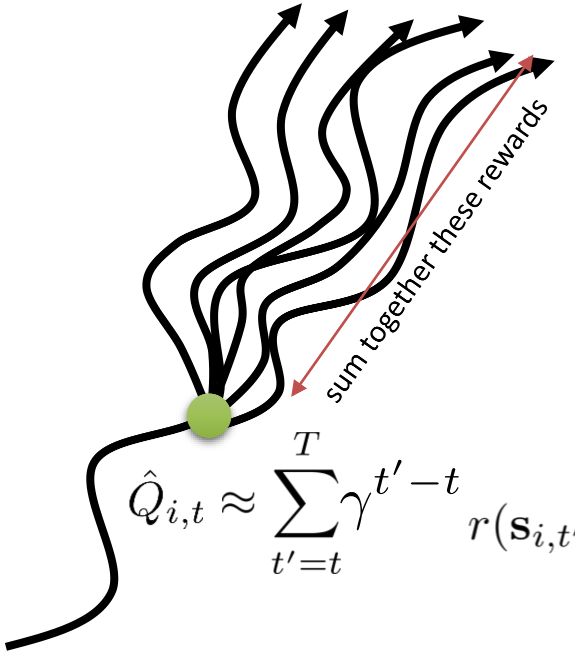

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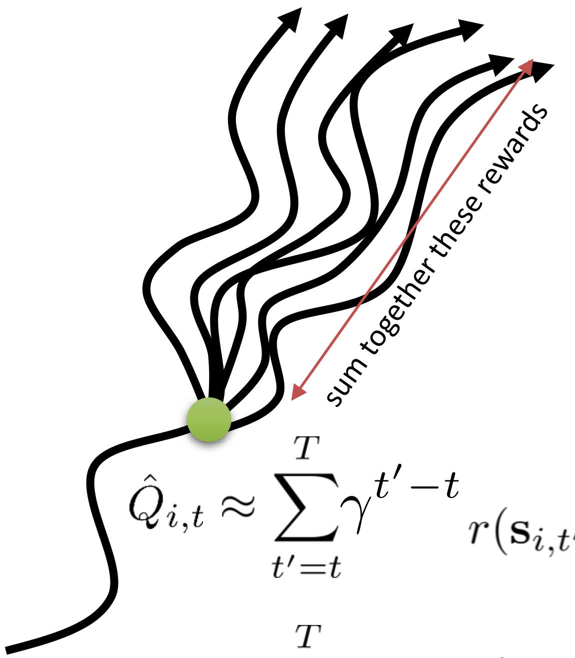
Improving the policy gradient

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \underbrace{\left(\sum_{t'=1}^{T}\gamma^{t'-t} r(\mathbf{s}_{i,t'},\mathbf{a}_{i,t'})\right)}_{\text{“reward to go”}\ \hat{Q}_{i,t}}$$



$$\hat{Q}_{i,t} \approx \sum_{t'=t}^{T}\gamma^{t'-t} r(\mathbf{s}_{i,t'},\mathbf{a}_{i,t'})$$

$$\approx \sum_{t'=t}^{T} E_{\pi_\theta}\left[\gamma^{t'-t} r(\mathbf{s}_{t'},\mathbf{a}_{t'})|\mathbf{s}_t,\mathbf{a}_t\right]$$

$\hat{Q}_{i,t}$: estimate of expected reward if we take action $\mathbf{a}_{i,t}$ in state $\mathbf{s}_{i,t}$

can we get a better estimate? (i.e. lower variance and also unbiased)

$$Q(\mathbf{s}_t,\mathbf{a}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[\gamma^{t'-t} r(\mathbf{s}_{t'},\mathbf{a}_{t'})|\mathbf{s}_t,\mathbf{a}_t\right] \quad \text{: true } \textit{expected} \text{ reward-to-go}$$

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) Q(\mathbf{s}_{i,t},\mathbf{a}_{i,t})$$

What about the baseline?

$Q(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t, \mathbf{a}_t\right]$: true *expected* reward-to-go

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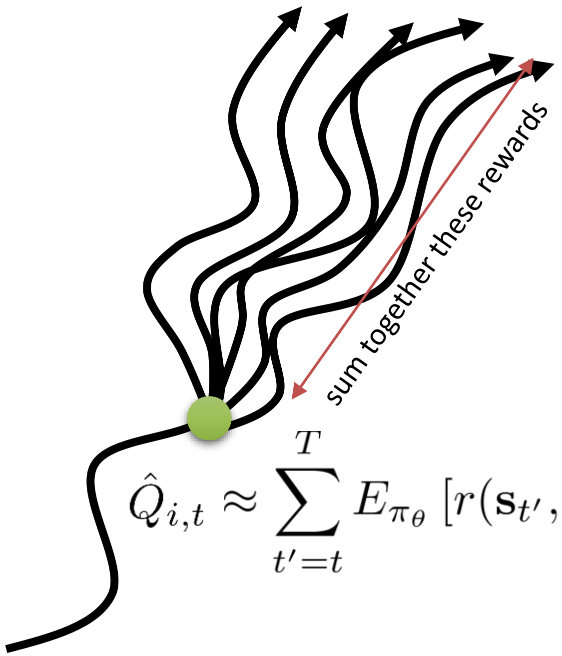

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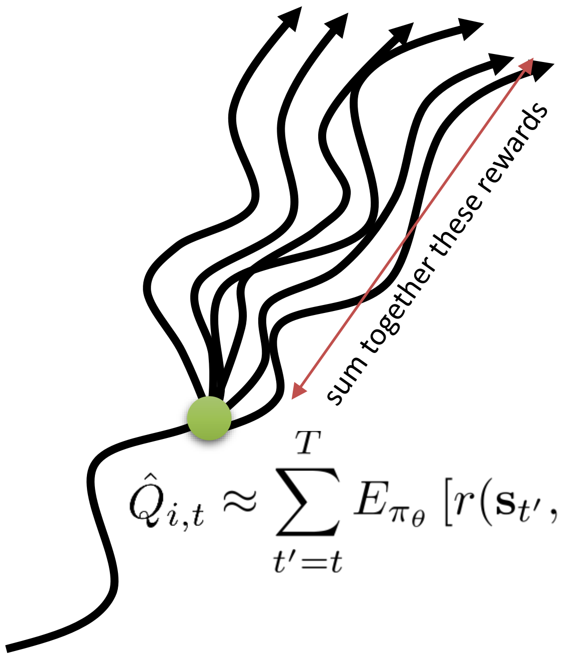

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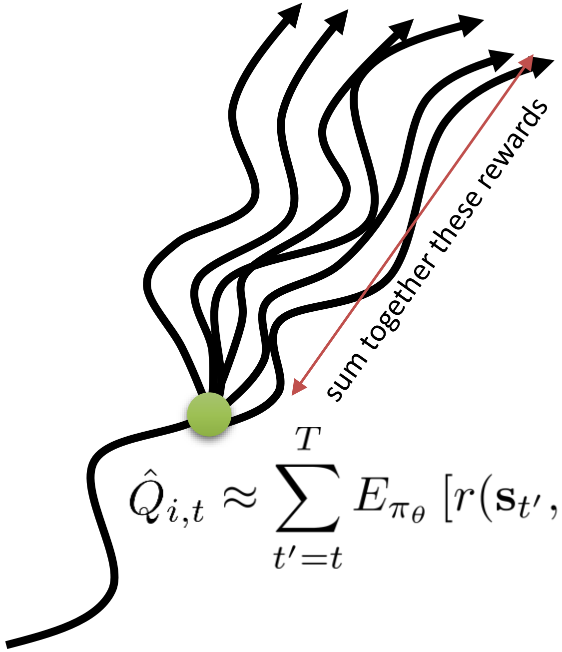
$b = \text{average reward}$



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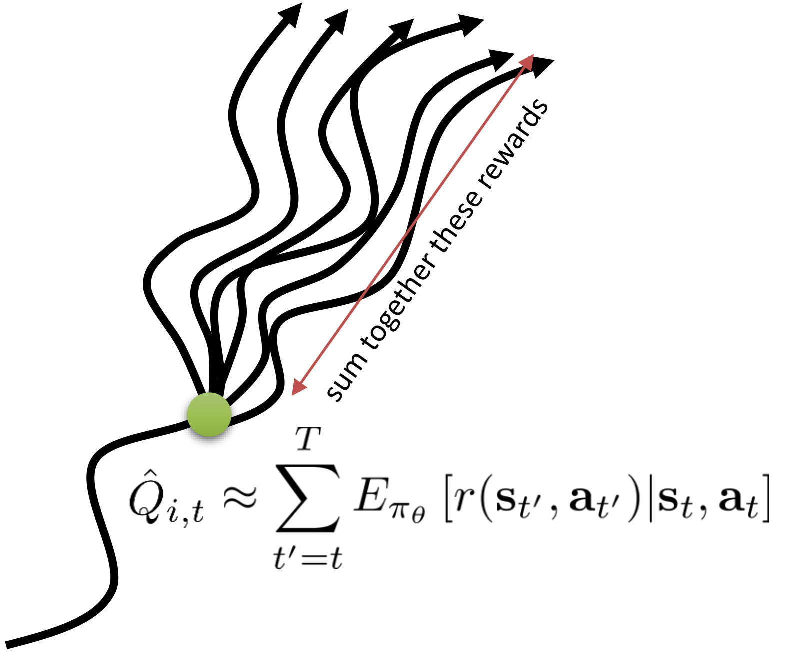
average what?



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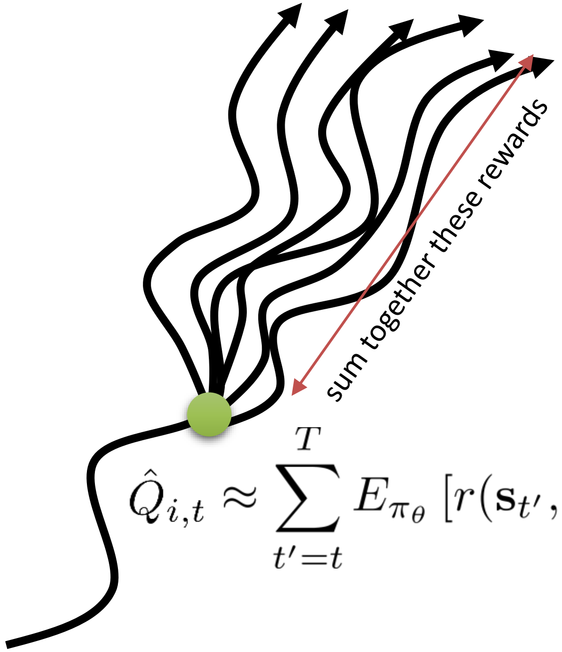

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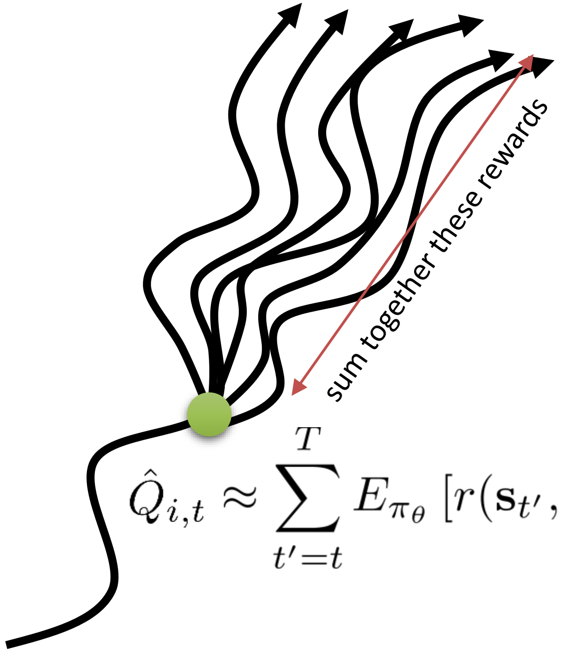
$$b_t = \frac{1}{N}\sum_i Q(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$ average what?



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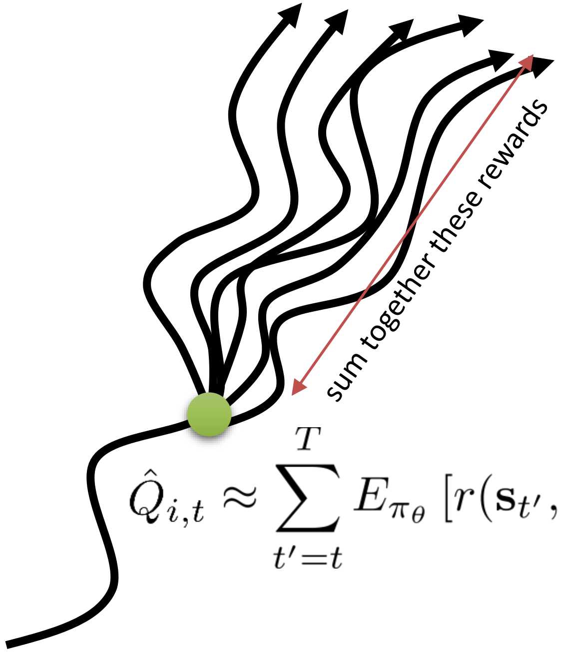
$$V(\mathbf{s}_t) = E_{\mathbf{a}_t \sim \pi_\theta(\mathbf{a}_t | \mathbf{s}_t)}[Q(\mathbf{s}_t, \mathbf{a}_t)]$$



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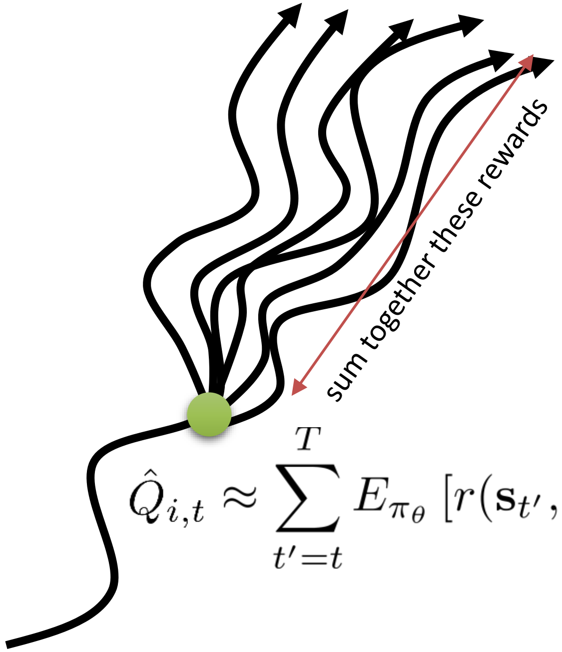

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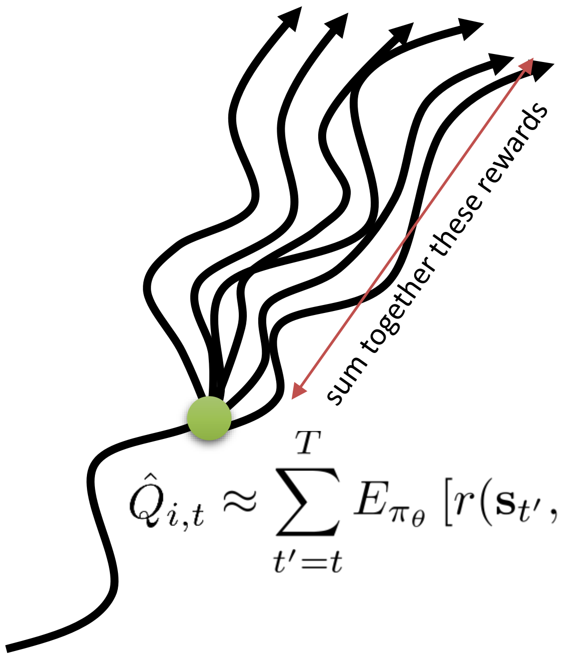

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State & state-action value functions

$$Q(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[\gamma^{t'-t} r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t\right] \quad \text{: true } \textit{expected} \text{ reward-to-go}$$

State & state-action value functions

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: true *expected* reward-to-go

total reward from taking $\mathbf{a}_t$ in $\mathbf{s}_t$

State & state-action value functions

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$V^\pi(\mathbf{s}_t) = E_{\mathbf{a}_t \sim \pi_\theta(\mathbf{a}_t|\mathbf{s}_t)}[Q^\pi(\mathbf{s}_t, \mathbf{a}_t)]$: total reward from $\mathbf{s}_t$

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$A^\pi(\mathbf{s}_t, \mathbf{a}_t) = Q^\pi(\mathbf{s}_t, \mathbf{a}_t) - V^\pi(\mathbf{s}_t)$: how much better $\mathbf{a}_t$ is

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the better this estimate, the lower the variance

State & state-action value functions

$$Q(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[\gamma^{t'-t} r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t\right]$$: true *expected* reward-to-go

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State & state-action value functions

$$Q(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[\gamma^{t'-t} r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t\right]$$

: true *expected* reward-to-go

total reward from taking $\mathbf{a}_t$ in $\mathbf{s}_t$

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State & state-action value functions

$$Q(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[\gamma^{t'-t} r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t\right]$$

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total reward from taking $\mathbf{a}_t$ in $\mathbf{s}_t$

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unbiased, but high variance single-sample estimate

State & state-action value functions

$$Q(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{T} E_{\pi_\theta} \left[\gamma^{t'-t} r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t\right]$$

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total reward from taking $\mathbf{a}_t$ in $\mathbf{s}_t$

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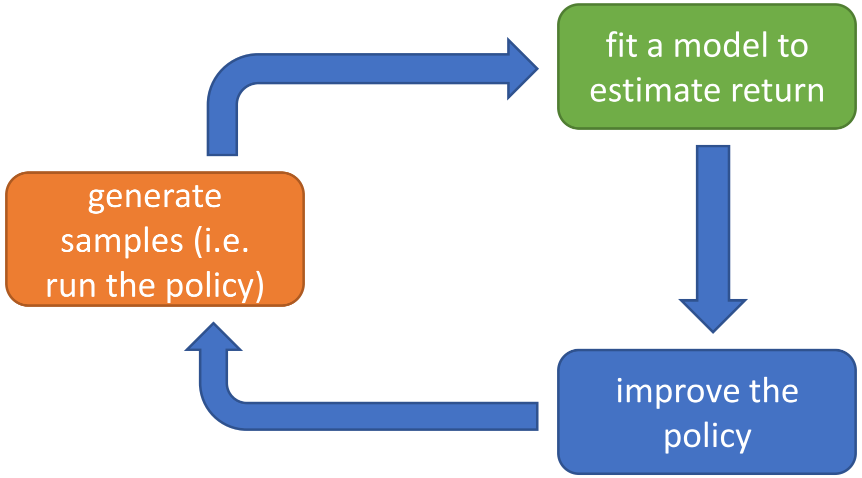

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unbiased, but high variance single-sample estimate

State & state-action value functions

$$Q(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[\gamma^{t'-t} r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t\right]$$: true *expected* reward-to-go
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$V^\pi(\mathbf{s}_t) = E_{\mathbf{a}_t \sim \pi_\theta(\mathbf{a}_t|\mathbf{s}_t)}[Q^\pi(\mathbf{s}_t, \mathbf{a}_t)]$: total reward from $\mathbf{s}_t$

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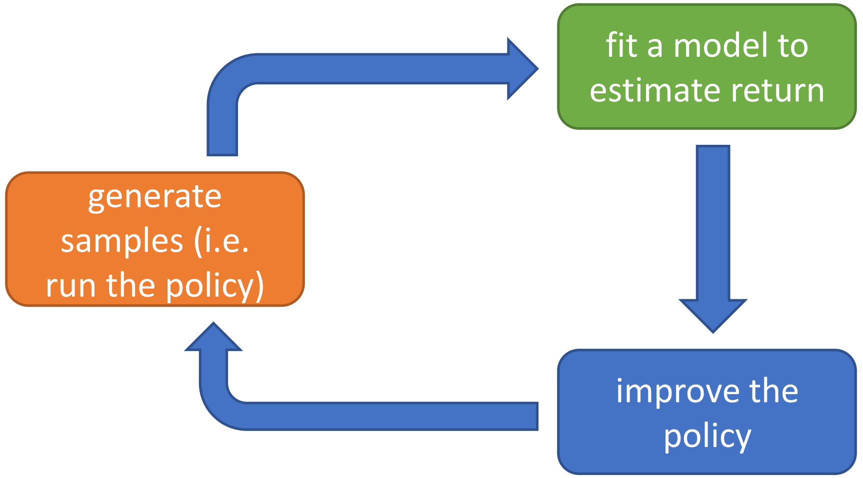

$$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$$

the better this estimate, the lower the variance

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unbiased, but high variance single-sample estimate

State & state-action value functions

$Q(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[\gamma^{t'-t} r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t\right]$: true *expected* reward-to-go
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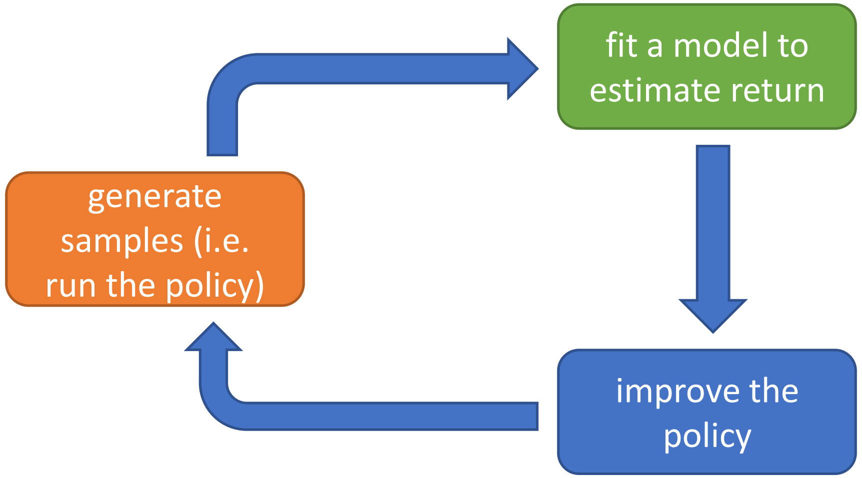
the better this estimate, the lower the variance

fit Q^π, V^π, or A^π



$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \left(\underline{\sum_{t'=1}^{T} \gamma^{t'-t} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'})} - b \right)$$

unbiased, but high variance single-sample estimate

Value function fitting

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t, \mathbf{a}_t\right]$$

$$V^\pi(\mathbf{s}_t) = E_{\mathbf{a}_t \sim \pi_\theta(\mathbf{a}_t|\mathbf{s}_t)}[Q^\pi(\mathbf{s}_t, \mathbf{a}_t)]$$

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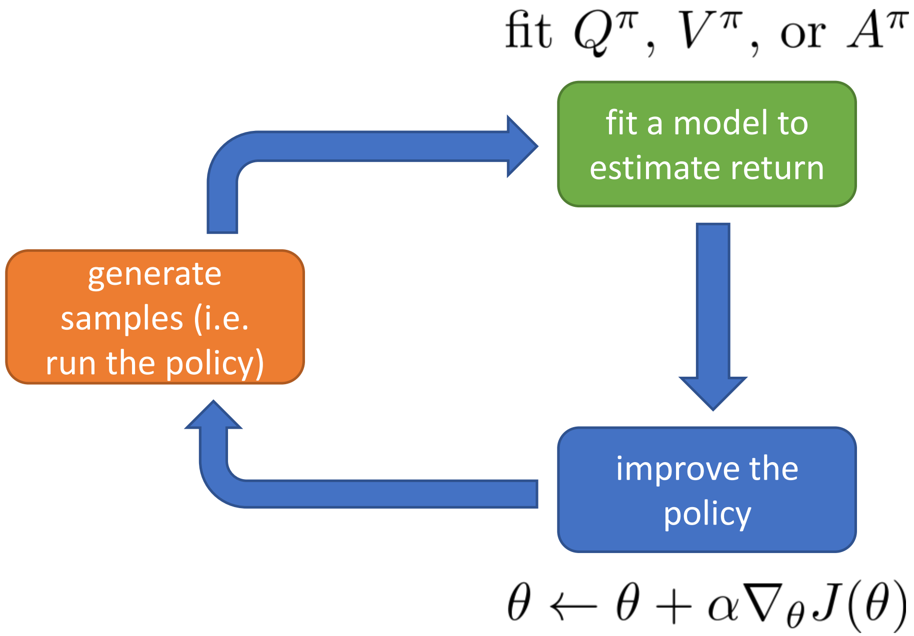
$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) A^\pi(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$



Value function fitting

$$Q^\pi(\mathbf{s}_t,\mathbf{a}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'},\mathbf{a}_{t'})|\mathbf{s}_t,\mathbf{a}_t\right]$$

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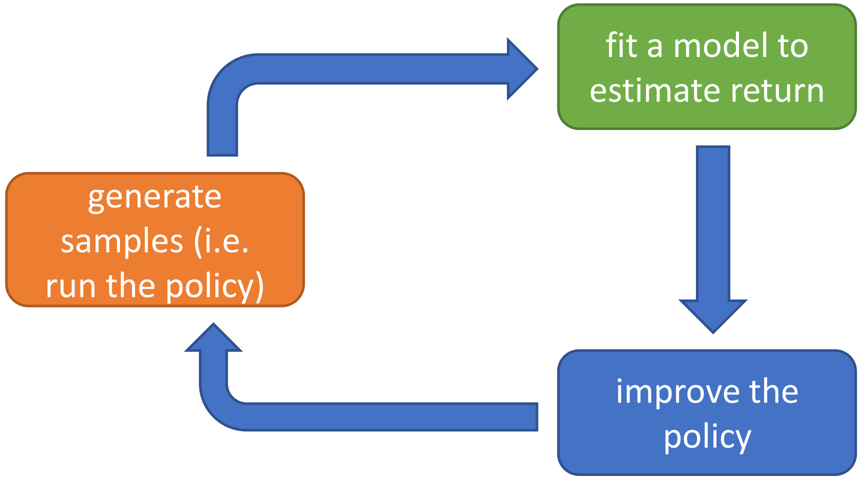
fit *what* to *what*?

fit Q^π, V^π, or A^π



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Value function fitting

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t, \mathbf{a}_t\right]$$

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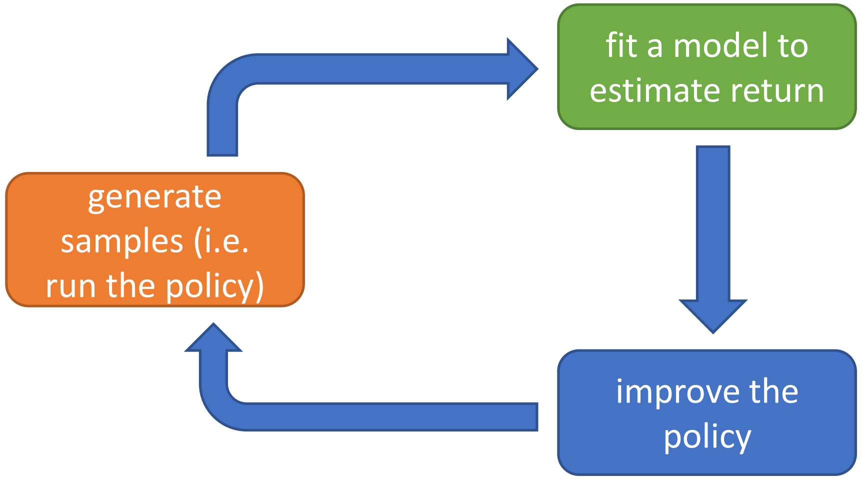
fit *what* to *what*?

Q^π, V^π, A^π?

fit Q^π, V^π, or A^π



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Value function fitting

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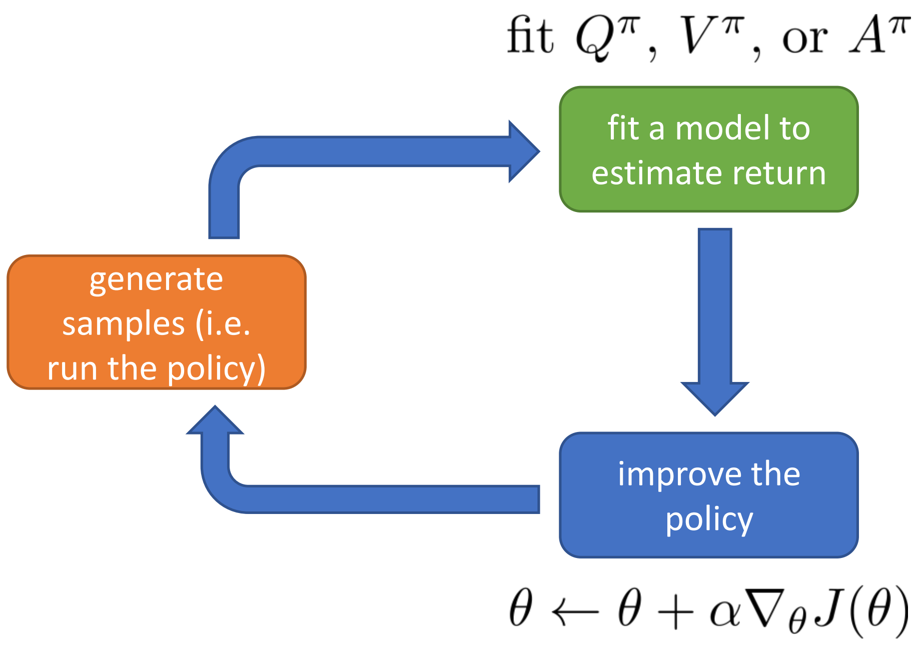
fit *what* to *what?*

$Q^\pi, V^\pi, A^\pi?$

$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t, \mathbf{a}_t\right]$



Value function fitting

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t, \mathbf{a}_t\right]$$

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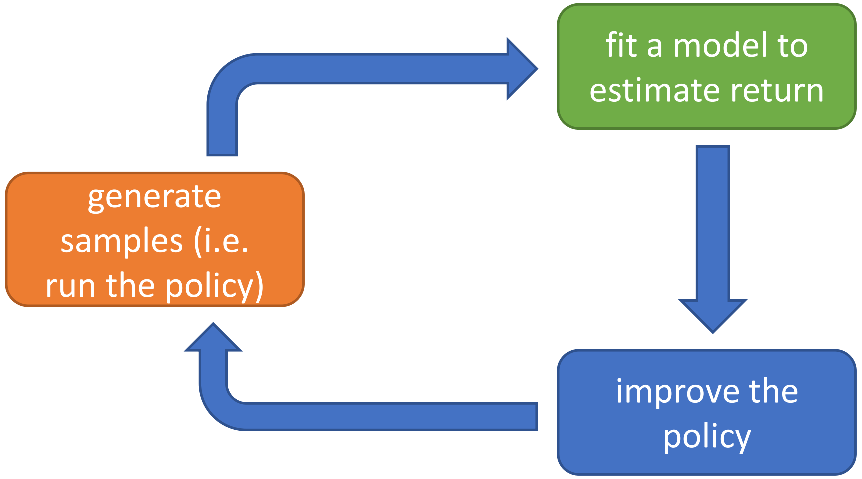
$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) A^\pi(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$

fit *what* to *what?*

Q^π, V^π, A^π?

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = r(\mathbf{s}_t, \mathbf{a}_t) + \sum_{t'=t+1}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t, \mathbf{a}_t\right]$$

fit Q^π, V^π, or A^π



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Value function fitting

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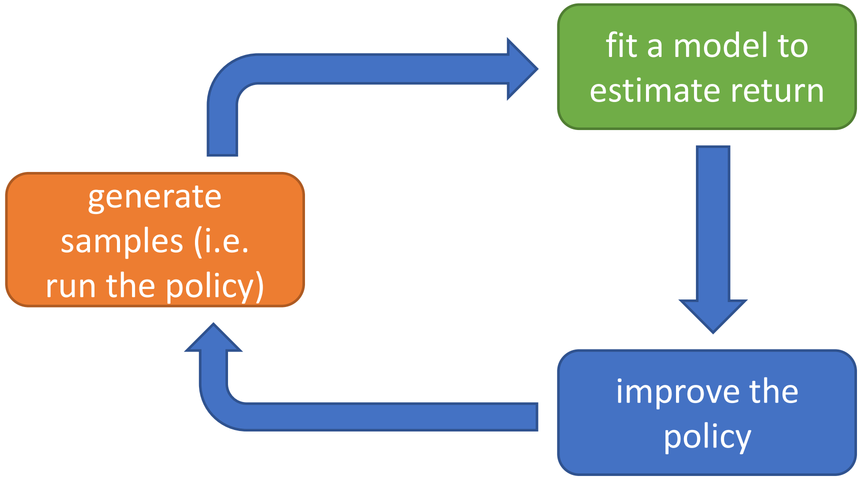
fit *what* to *what?*

$Q^\pi, V^\pi, A^\pi?$

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = r(\mathbf{s}_t, \mathbf{a}_t) + \underbrace{\sum_{t'=t+1}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t, \mathbf{a}_t\right]}_{V^\pi(\mathbf{s}_{t+1})}$$

fit Q^π, V^π, or A^π



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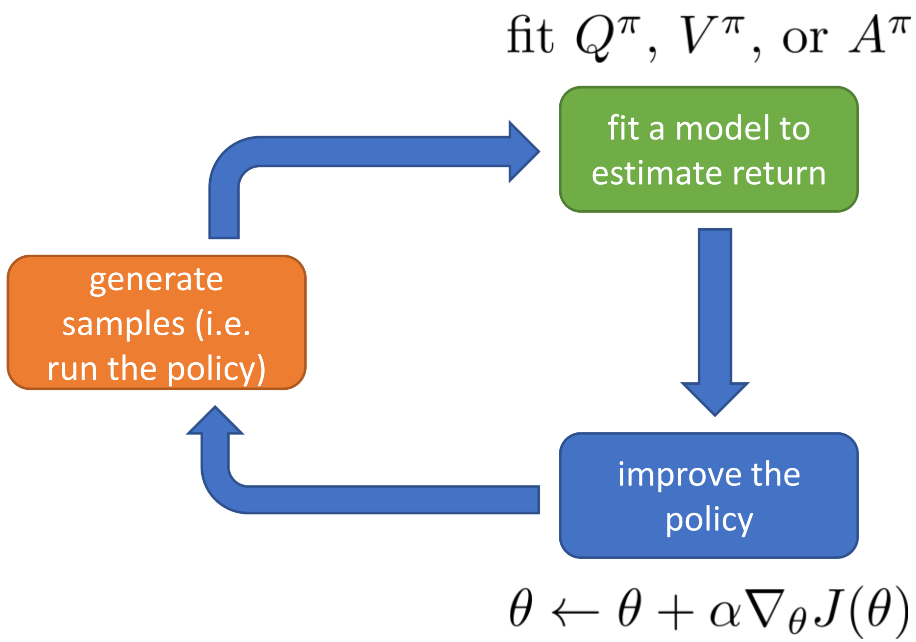
fit *what* to *what?*

$Q^\pi, V^\pi, A^\pi?$

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = r(\mathbf{s}_t, \mathbf{a}_t) + E_{\mathbf{s}_{t+1} \sim p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)}[V^\pi(\mathbf{s}_{t+1})]$$



Value function fitting

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t, \mathbf{a}_t\right]$$

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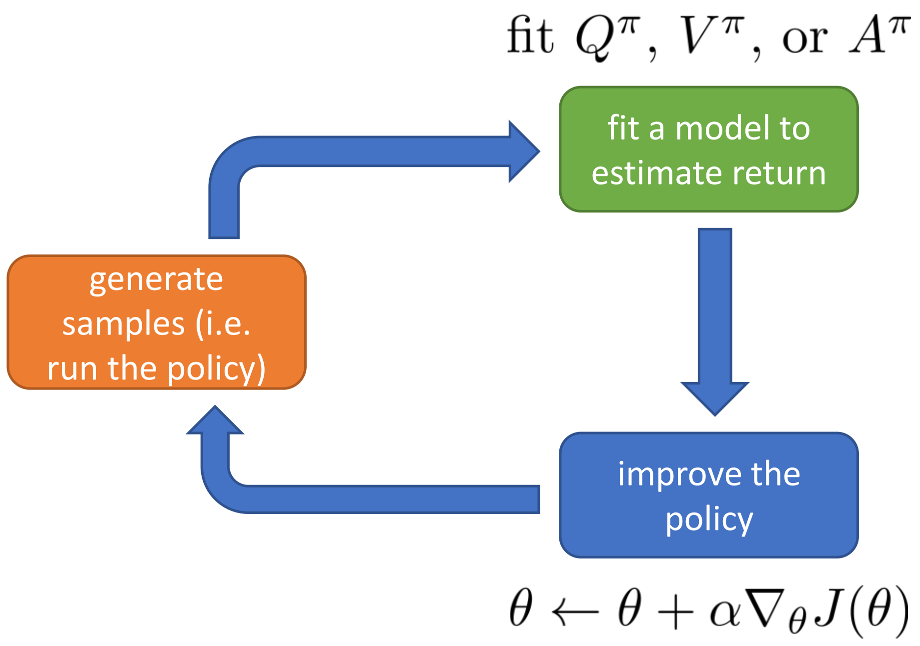
fit *what* to *what*?

Q^π, V^π, A^π?

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) \approx r(\mathbf{s}_t, \mathbf{a}_t) + V^\pi(\mathbf{s}_{t+1})$$



Value function fitting

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t, \mathbf{a}_t\right]$$

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fit *what* to *what?*

Q^π, V^π, A^π?

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) \approx r(\mathbf{s}_t, \mathbf{a}_t) + V^\pi(\mathbf{s}_{t+1})$$

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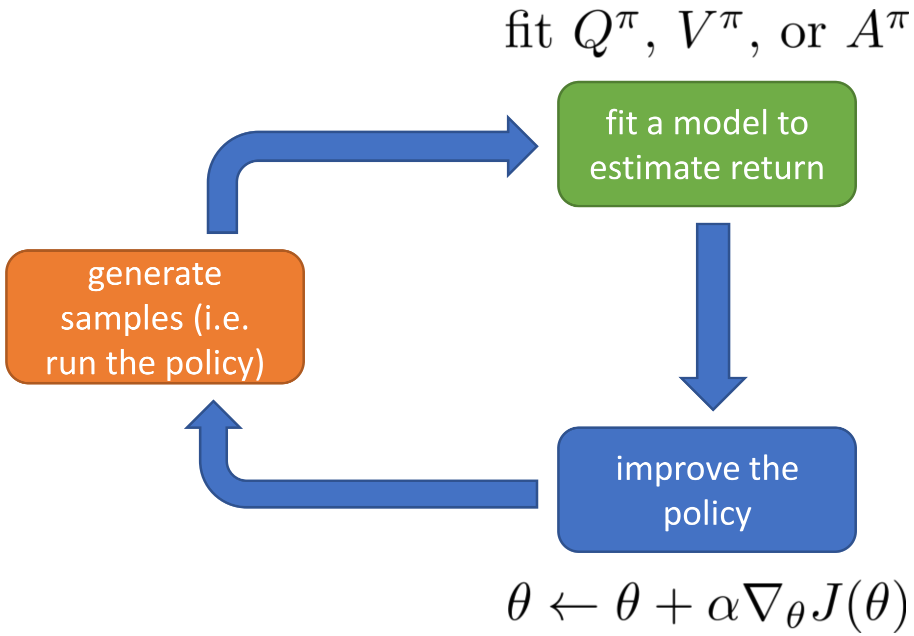

Value function fitting

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t, \mathbf{a}_t\right]$$

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fit *what* to *what*?

Q^π, V^π, A^π?

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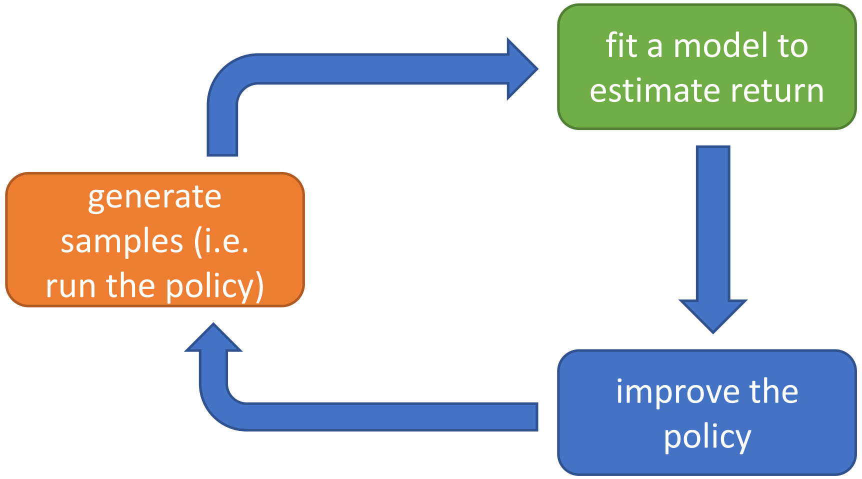
$$A^\pi(\mathbf{s}_t, \mathbf{a}_t) \approx r(\mathbf{s}_t, \mathbf{a}_t) + V^\pi(\mathbf{s}_{t+1}) - V^\pi(\mathbf{s}_t)$$

let's just fit $V^\pi(\mathbf{s})$!

fit Q^π, V^π, or A^π



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Value function fitting

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fit *what* to *what*?

Q^π, V^π, A^π?

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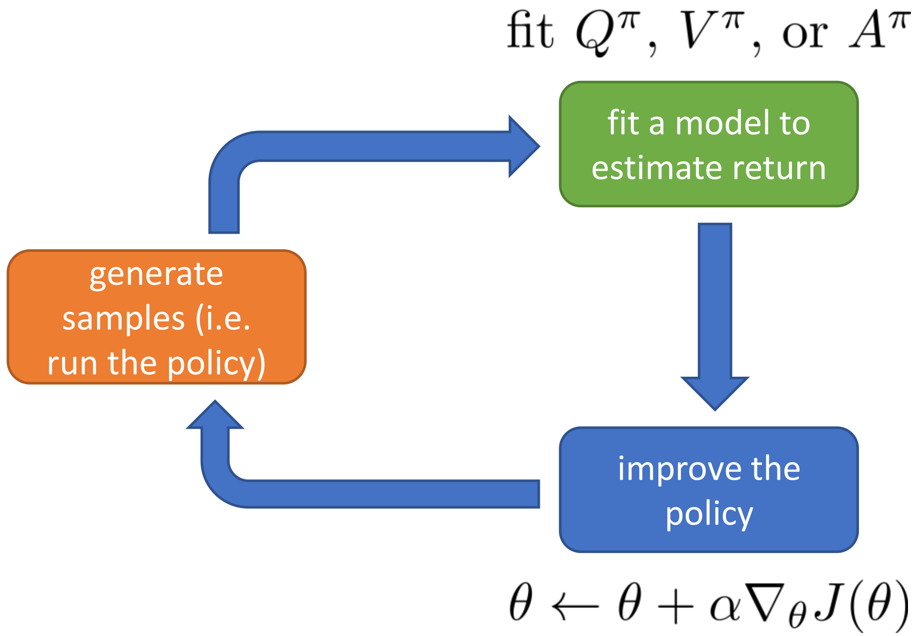

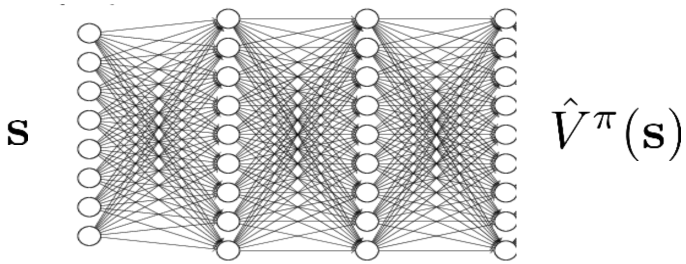

Value function fitting

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fit *what* to *what?*

Q^π, V^π, A^π?

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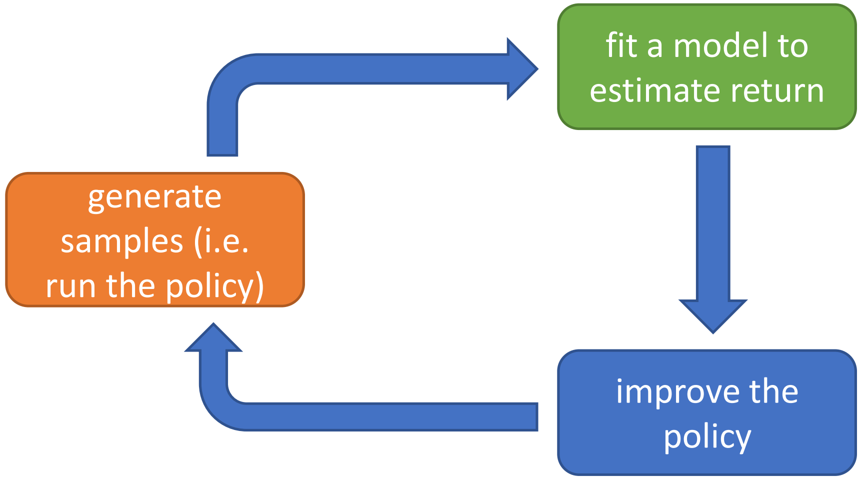
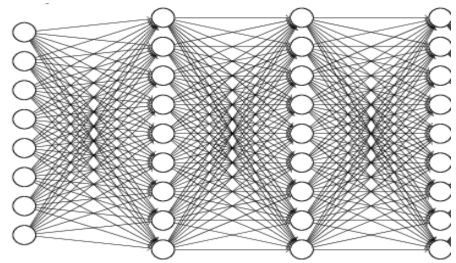


$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

$\mathbf{s}$



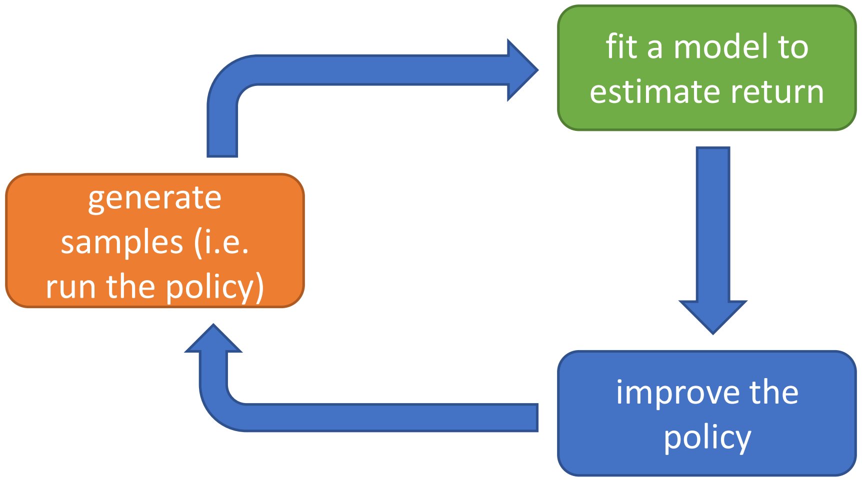
$\hat{V}^\pi(\mathbf{s})$

parameters ϕ

Policy evaluation

$$V^\pi(\mathbf{s}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t\right]\gamma^{t'-t}$$

fit V^π



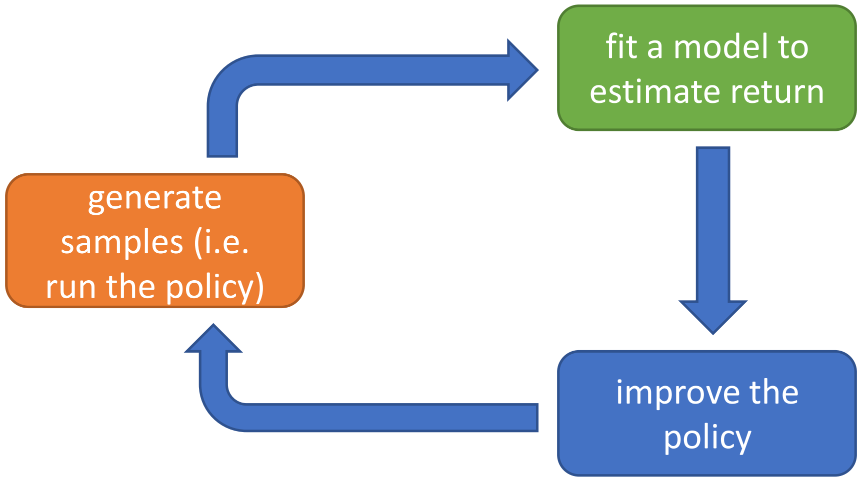
$$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$$

Policy evaluation

$$V^\pi(\mathbf{s}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t\right] \gamma^{t'-t}$$

$$J(\theta) = E_{\mathbf{s}_1 \sim p(\mathbf{s}_1)}[V^\pi(\mathbf{s}_1)]$$

fit V^π



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Policy evaluation

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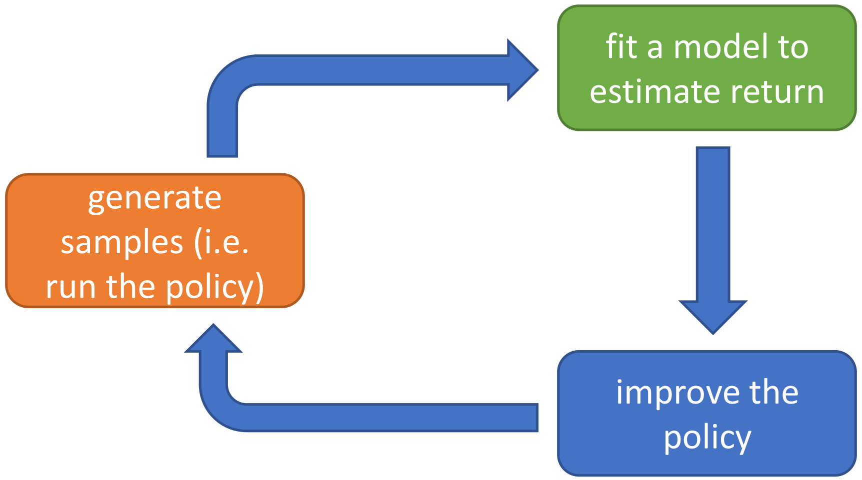
how can we perform policy evaluation?

fit V^π



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Policy evaluation

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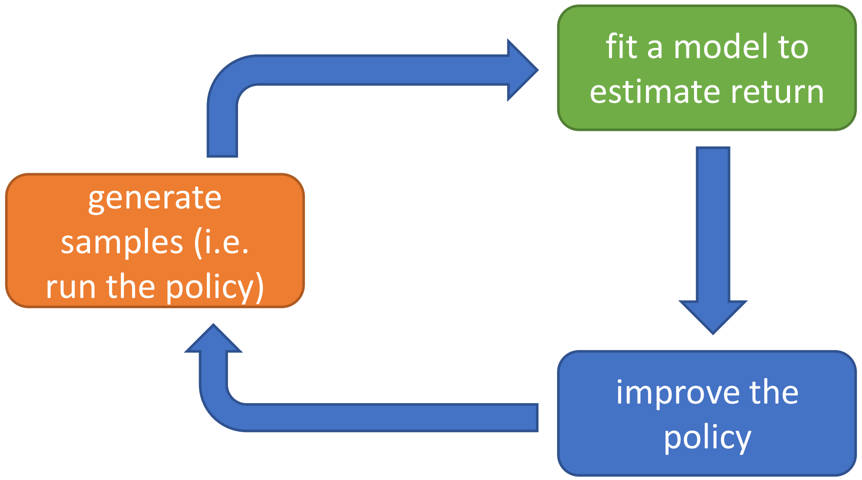
how can we perform policy evaluation?

Monte Carlo policy evaluation (this is what policy gradient does)

fit V^π



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Policy evaluation

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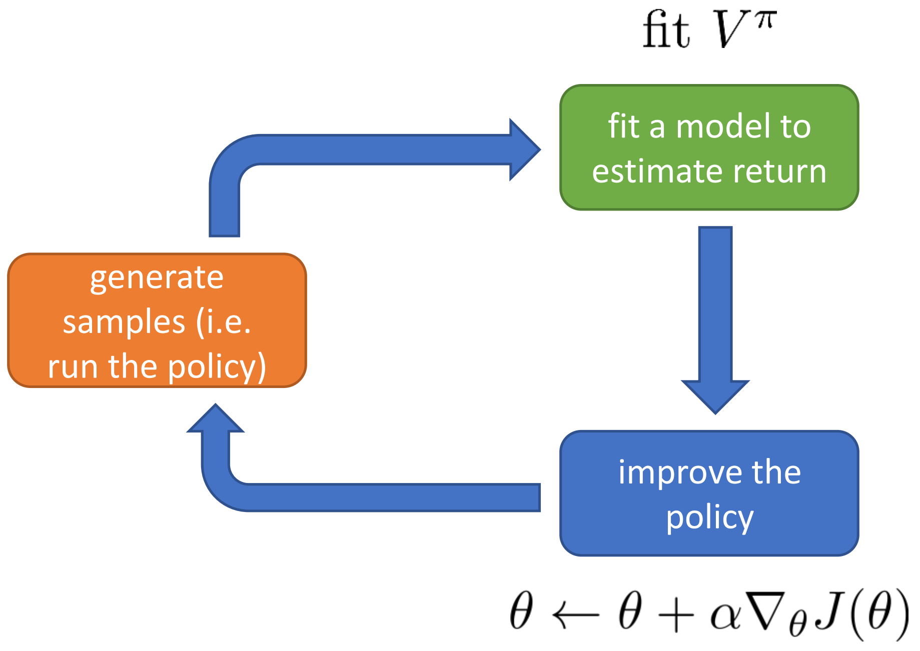
how can we perform policy evaluation?

Monte Carlo policy evaluation (this is what policy gradient does)

$$V^\pi(\mathbf{s}_t) \approx \sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'})\gamma^{t'-t}$$



Policy evaluation

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how can we perform policy evaluation?

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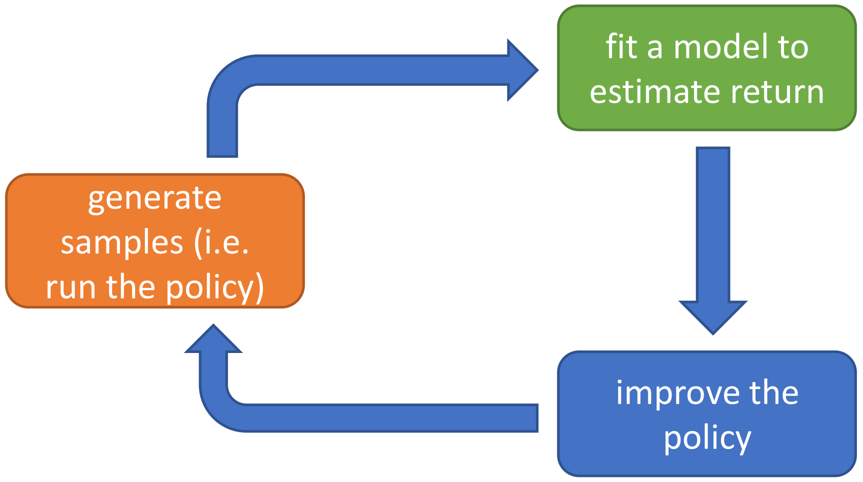
fit V^π



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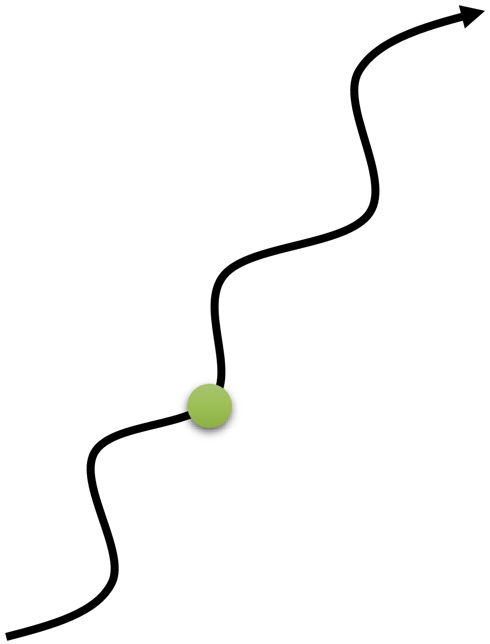

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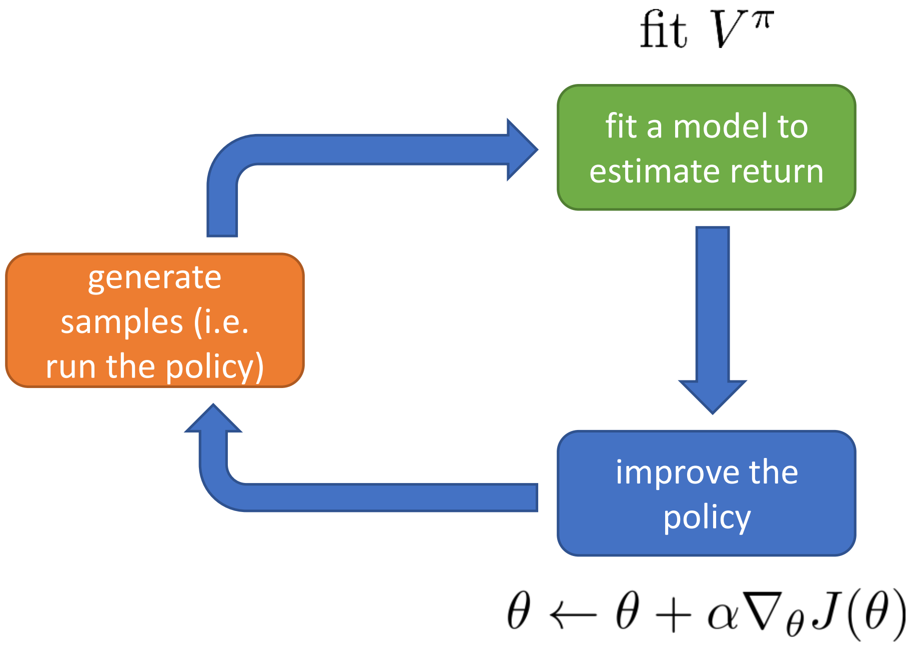
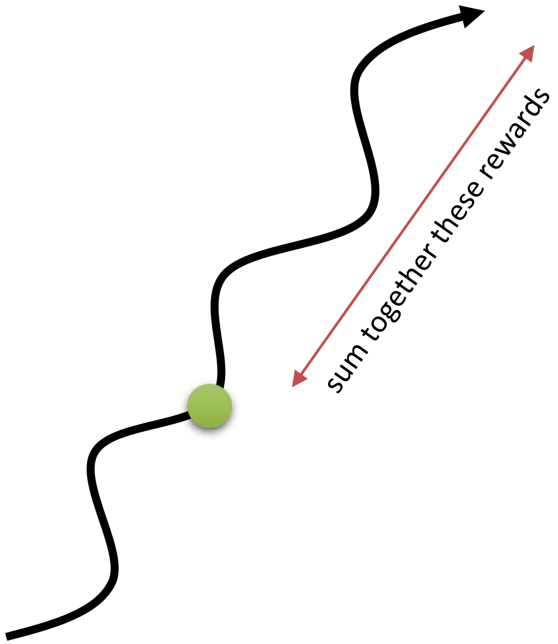
Monte Carlo policy evaluation (this is what policy gradient does)

$$V^\pi(\mathbf{s}_t) \approx \sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'})\gamma^{t'-t}$$





Policy evaluation

$$V^\pi(\mathbf{s}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t\right]\gamma^{t'-t}$$

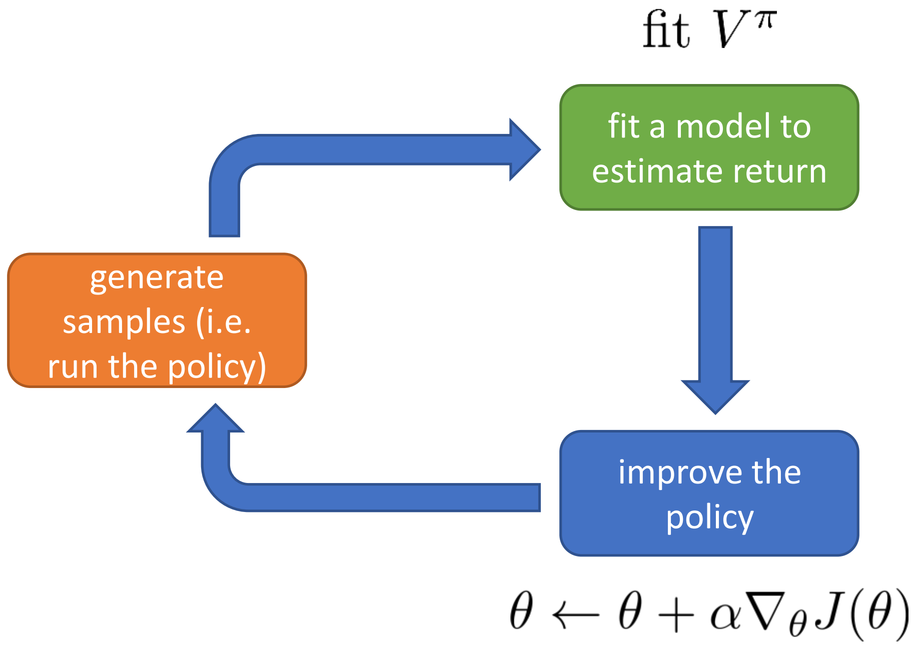
$$J(\theta) = E_{\mathbf{s}_1 \sim p(\mathbf{s}_1)}[V^\pi(\mathbf{s}_1)]$$

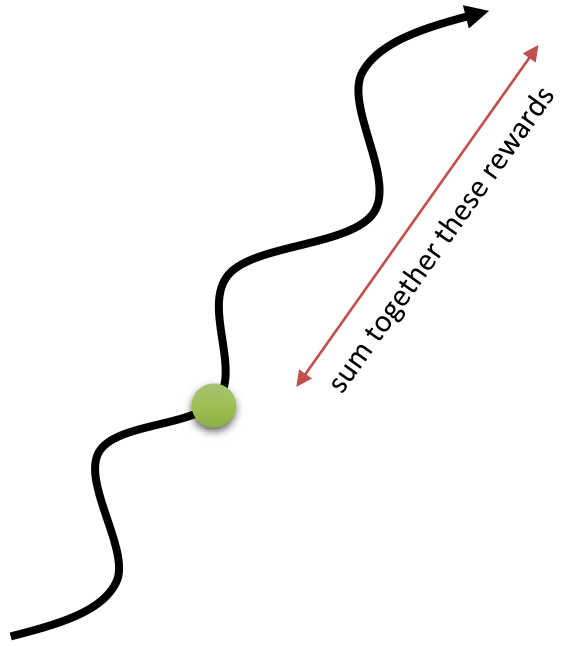
how can we perform policy evaluation?

Monte Carlo policy evaluation (this is what policy gradient does)

$$V^\pi(\mathbf{s}_t) \approx \sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'})\gamma^{t'-t}$$

$$V^\pi(\mathbf{s}_t) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'})\gamma^{t'-t}$$





Policy evaluation

$V^\pi(\mathbf{s}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t\right]\gamma^{t'-t}$

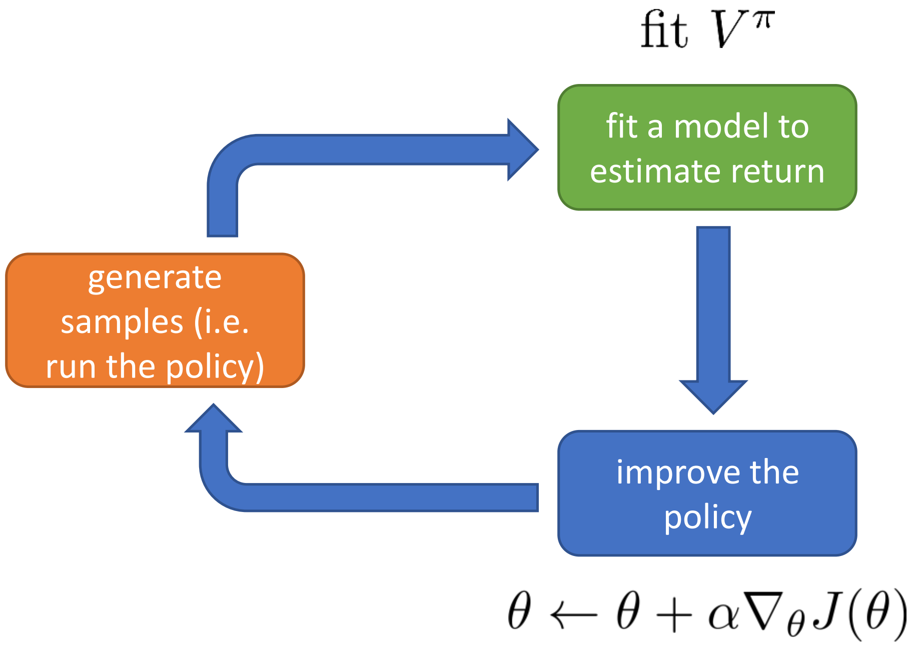
$J(\theta) = E_{\mathbf{s}_1 \sim p(\mathbf{s}_1)}[V^\pi(\mathbf{s}_1)]$

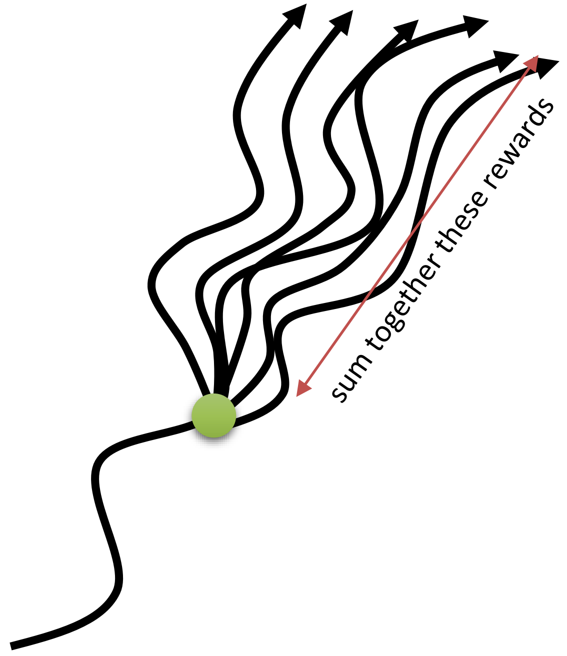
how can we perform policy evaluation?

Monte Carlo policy evaluation (this is what policy gradient does)

$$V^\pi(\mathbf{s}_t) \approx \sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'})\gamma^{t'-t}$$

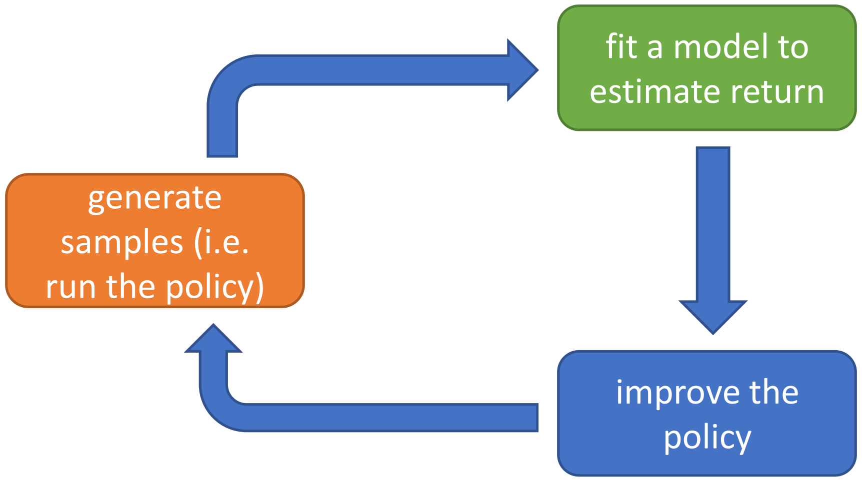
$$V^\pi(\mathbf{s}_t) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'})\gamma^{t'-t}$$





Policy evaluation

fit V^π



$$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$$

$$V^\pi(\mathbf{s}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t\right]\gamma^{t'-t}$$

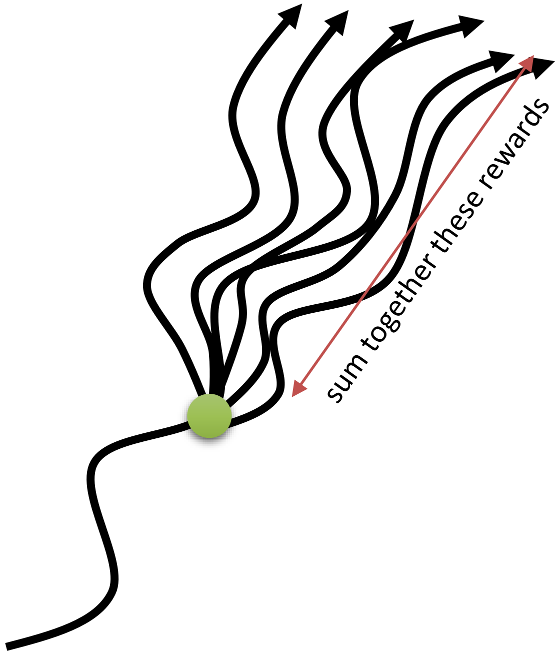
$$J(\theta) = E_{\mathbf{s}_1 \sim p(\mathbf{s}_1)}[V^\pi(\mathbf{s}_1)]$$

how can we perform policy evaluation?

Monte Carlo policy evaluation (this is what policy gradient does)

$$V^\pi(\mathbf{s}_t) \approx \sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'})\gamma^{t'-t}$$

$$V^\pi(\mathbf{s}_t) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'})\gamma^{t'-t}$$ (requires us to reset the simulator)



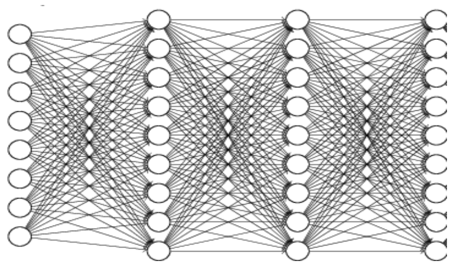
Monte Carlo evaluation with function approximation

$$V^\pi(\mathbf{s}_t) \approx \sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'})\gamma^{t'-t}$$

Monte Carlo evaluation with function approximation

$$V^{\pi}(\mathbf{s}_t) \approx \sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) \gamma^{t'-t}$$

$\mathbf{s}$

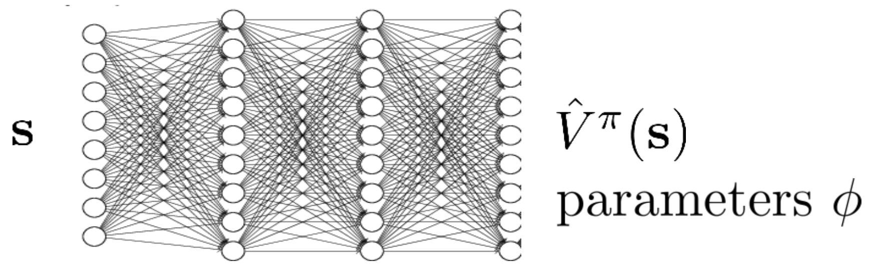


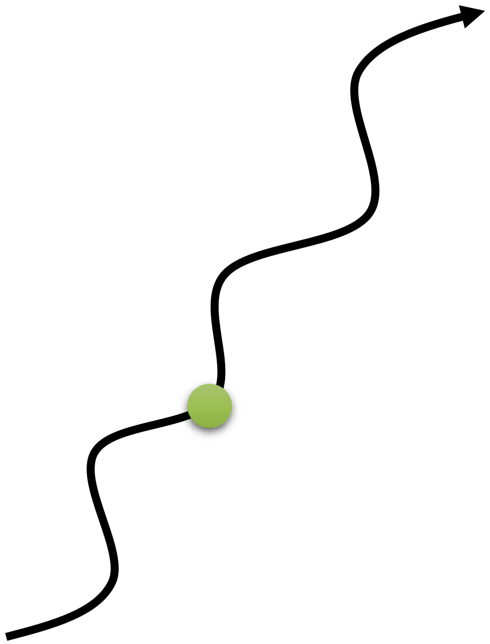
$\hat{V}^{\pi}(\mathbf{s})$

parameters ϕ

Monte Carlo evaluation with function approximation

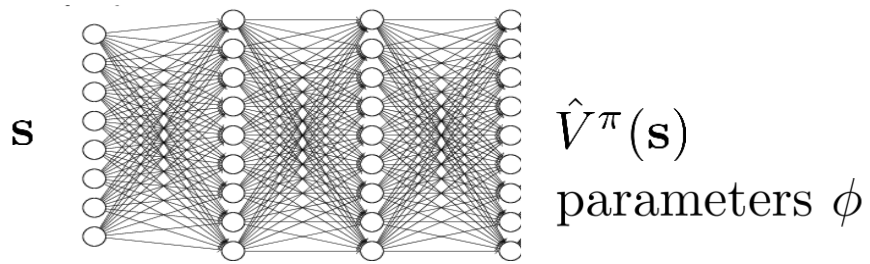
$$V^\pi(\mathbf{s}_t) \approx \sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) \gamma^{t'-t}$$

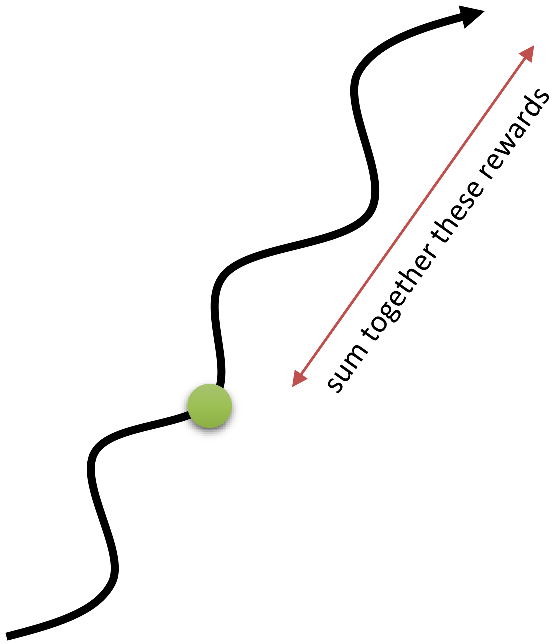




Monte Carlo evaluation with function approximation

$$V^\pi(\mathbf{s}_t) \approx \sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'})\gamma^{t'-t}$$





Monte Carlo evaluation with function approximation

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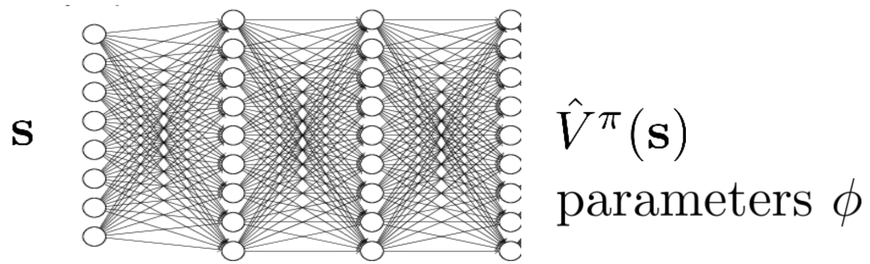
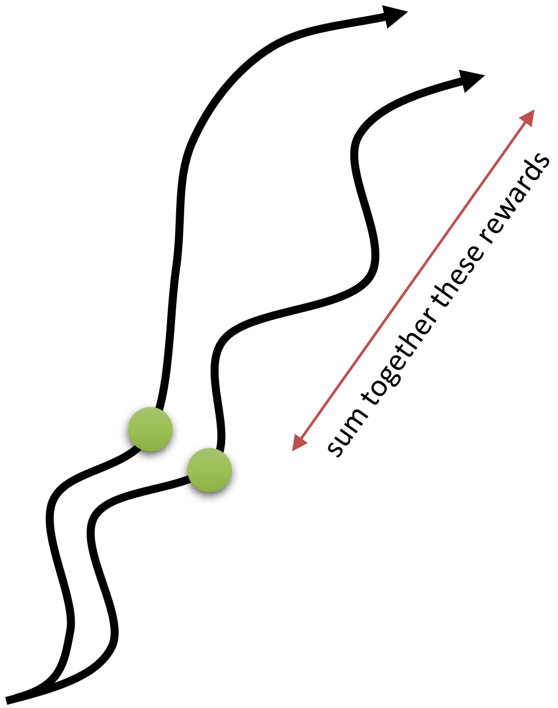
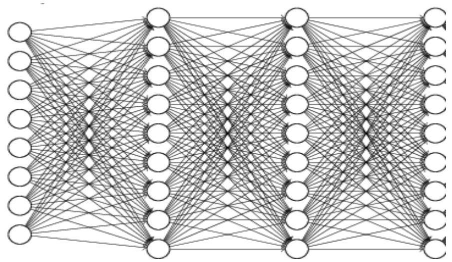




Monte Carlo evaluation with function approximation

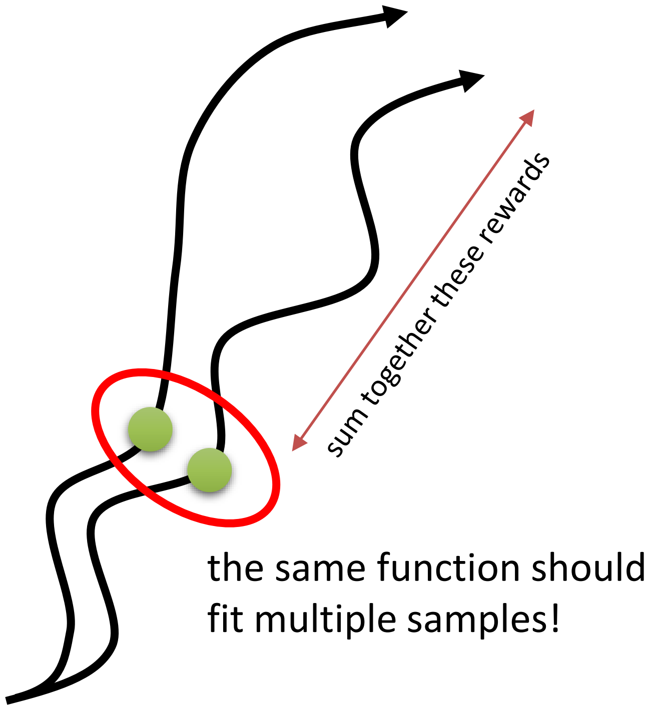
$$V^\pi(\mathbf{s}_t) \approx \sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'})\gamma^{t'-t}$$

$\mathbf{s}$



$\hat{V}^\pi(\mathbf{s})$

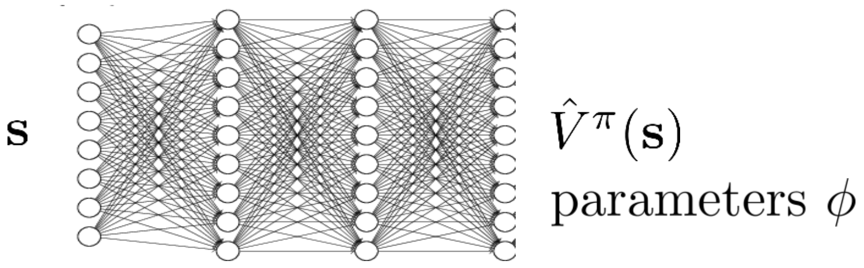
parameters ϕ

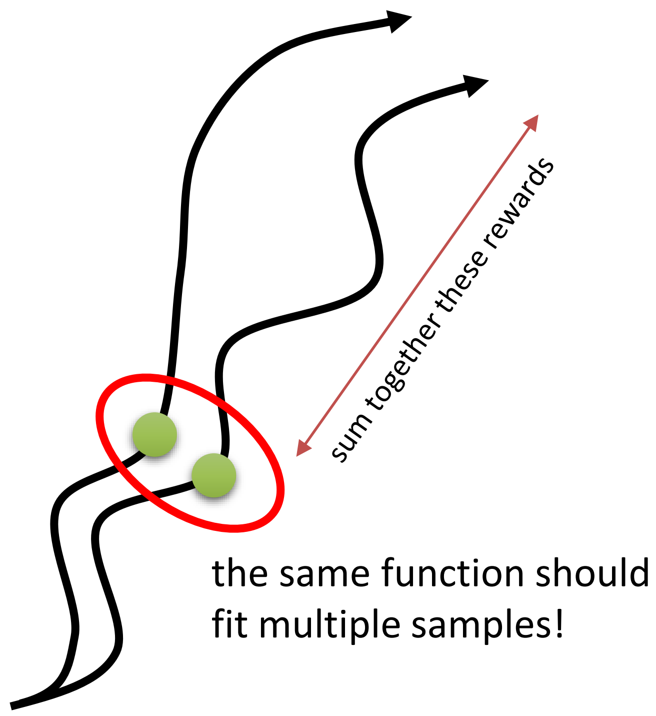


Monte Carlo evaluation with function approximation

$$V^\pi(\mathbf{s}_t) \approx \sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) \gamma^{t'-t}$$

not as good as this: $V^\pi(\mathbf{s}_t) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) \gamma^{t'-t}$



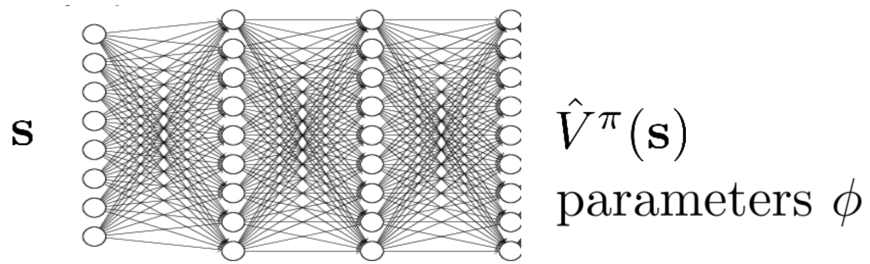


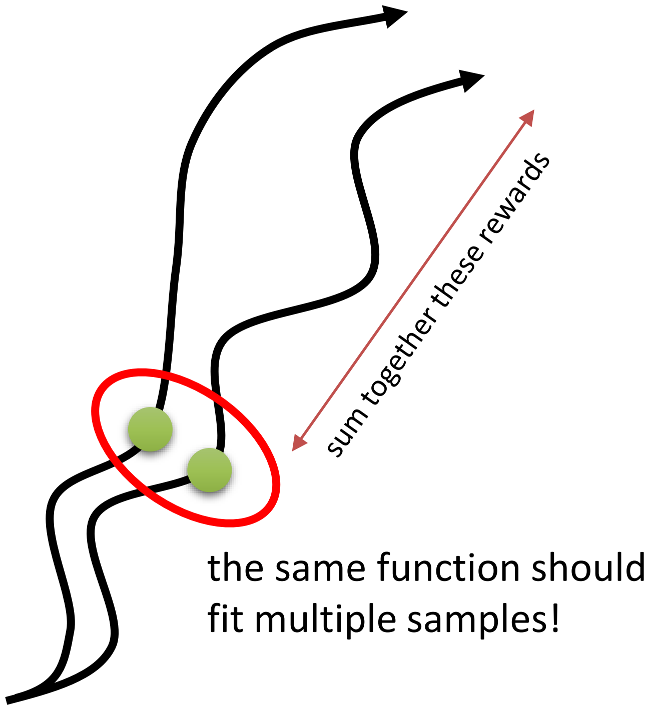
Monte Carlo evaluation with function approximation

$$V^\pi(\mathbf{s}_t) \approx \sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'})\gamma^{t'-t}$$

not as good as this: $V^\pi(\mathbf{s}_t) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'})\,\gamma^{t'-t}$

but still pretty good!





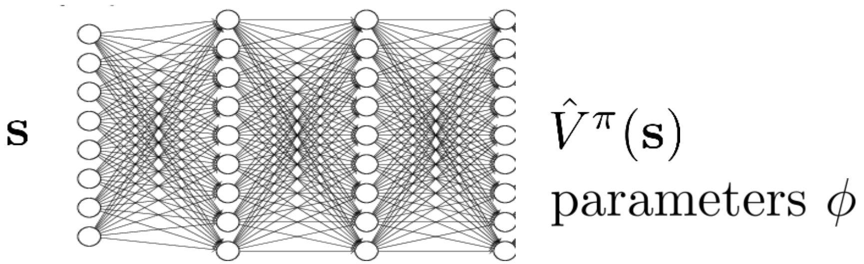
Monte Carlo evaluation with function approximation

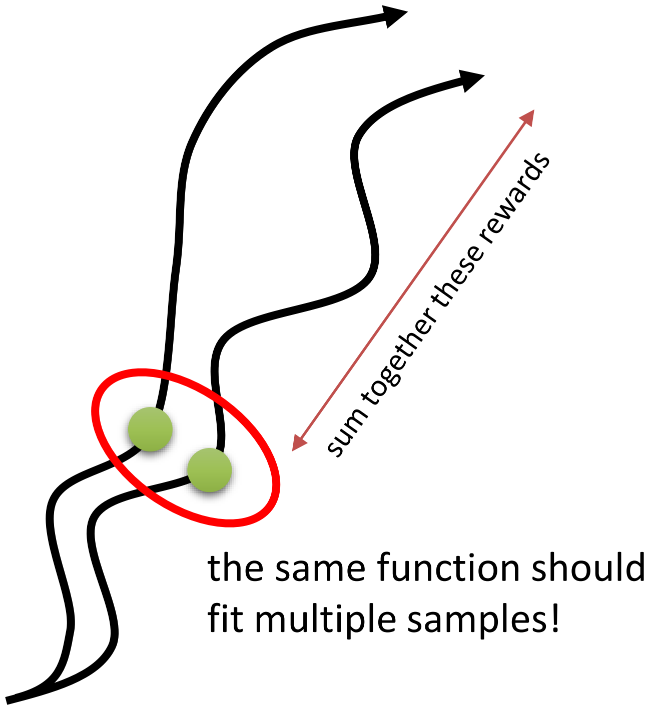
$$V^\pi(\mathbf{s}_t) \approx \sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'})\gamma^{t'-t}$$

not as good as this: $V^\pi(\mathbf{s}_t) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'})\,\gamma^{t'-t}$

but still pretty good!

Training data $\{(s_{i,t}, \sum_{t'=t}^{T} r(s_{i,t'}, a_{i,t'})\gamma^{t'-t})\}$



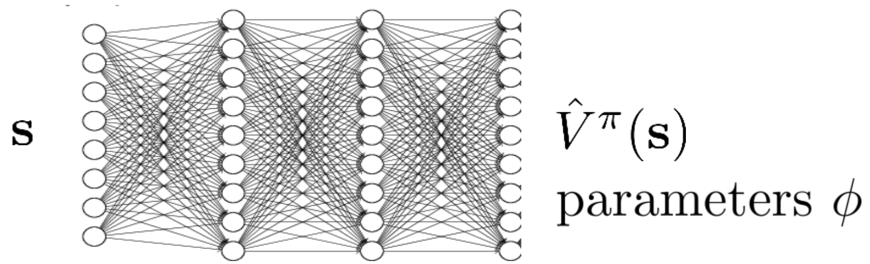


Monte Carlo evaluation with function approximation

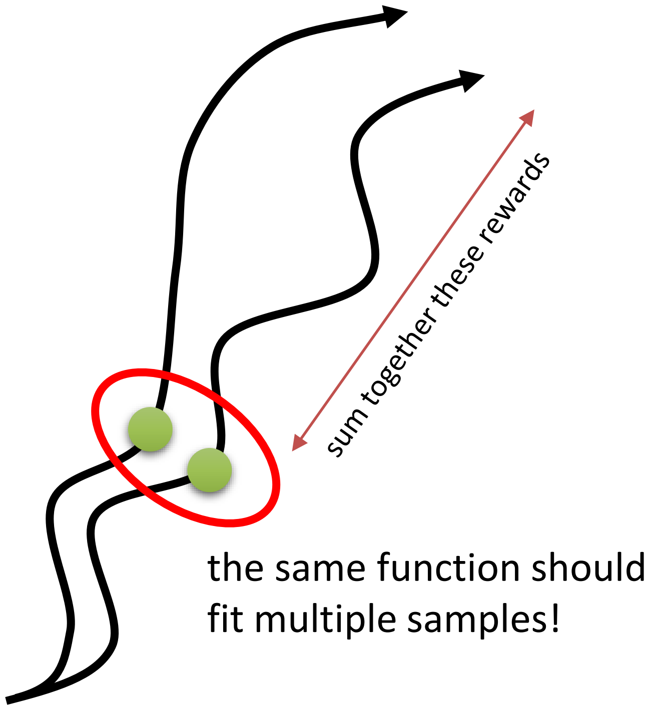
$$V^\pi(\mathbf{s}_t) \approx \sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'})\gamma^{t'-t}$$

not as good as this: $V^\pi(\mathbf{s}_t) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'})\,\gamma^{t'-t}$

but still pretty good!



Training data $\{(s_{i,t}, \underbrace{\sum_{t'=t}^{T} r(s_{i,t'}, a_{i,t'})\gamma^{t'-t}}_{y_{i,t}})\}$



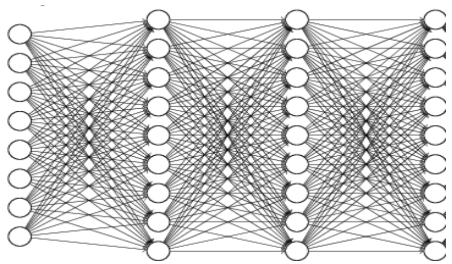
Monte Carlo evaluation with function approximation

$$V^\pi(\mathbf{s}_t) \approx \sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'})\gamma^{t'-t}$$

not as good as this: $V^\pi(\mathbf{s}_t) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'})\,\gamma^{t'-t}$

but still pretty good!

$\mathbf{s}$

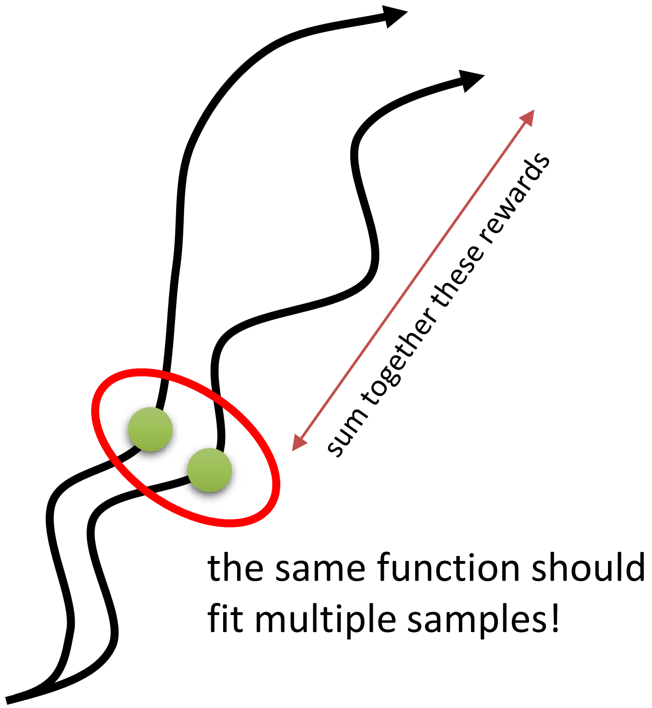


$\hat{V}^\pi(\mathbf{s})$

parameters ϕ

Training data $\{(s_{i,t}, \underbrace{\sum_{t'=t}^{T} r(s_{i,t'}, a_{i,t'})\gamma^{t'-t}}_{y_{i,t}})\}$

supervised regression: $\mathcal{L}(\phi) = \frac{1}{2}\sum_i \left\|\hat{V}_\phi^\pi(\mathbf{s}_i) - y_i\right\|^2$



Can we do better?

ideal target: $y_{i,t} = \sum_{t'=t}^{T} E_{\pi_\theta} \left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_{i,t} \right]$

Can we do better?

ideal target: $y_{i,t} = \sum_{t'=t}^{T} E_{\pi_\theta} \left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_{i,t} \right]$

Monte Carlo target: $y_{i,t} = \sum_{t'=t}^{T} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'})$

Can we do better?

ideal target: $y_{i,t} = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_{i,t}\right] \approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \sum_{t'=t+1}^{T} E_{\pi_\theta}[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_{i,t+1}]$

Monte Carlo target: $y_{i,t} = \sum_{t'=t}^{T} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'})$

Can we do better?

ideal target: $y_{i,t} = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_{i,t}\right]$ $\gamma^{t'-t}$

$$\approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + V^{\pi}(\mathbf{s}_{i,t+1})\gamma$$

Monte Carlo target: $y_{i,t} = \sum_{t'=t}^{T} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'})$

Can we do better?

ideal target: $y_{i,t} = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_{i,t}\right]$ $\gamma^{t'-t}$

$$\approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + V^\pi(\mathbf{s}_{i,t+1})\gamma \approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \hat{V}_\phi^\pi(\mathbf{s}_{i,t+1})\gamma$$

Monte Carlo target: $y_{i,t} = \sum_{t'=t}^{T} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'})$

Can we do better?

$\gamma^{t'-t}$

ideal target: $y_{i,t} = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_{i,t}\right]$

$$\approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + V^{\pi}(\mathbf{s}_{i,t+1})\gamma \approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \underbrace{\hat{V}_{\phi}^{\pi}(\mathbf{s}_{i,t+1})}_{\text{directly use previous fitted value function!}}\gamma$$

Monte Carlo target: $y_{i,t} = \sum_{t'=t}^{T} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'})$

Can we do better?

$\gamma^{t'-t}$

ideal target: $y_{i,t} = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_{i,t}\right]$

$$\approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \gamma V^{\pi}(\mathbf{s}_{i,t+1}) \approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \gamma \underbrace{\hat{V}^{\pi}_{\phi}(\mathbf{s}_{i,t+1})}_{\text{directly use previous fitted value function!}}$$

Monte Carlo target: $y_{i,t} = \sum_{t'=t}^{T} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'})$

training data: $\left\{\left(\mathbf{s}_{i,t}, r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \gamma \hat{V}^{\pi}_{\phi}(\mathbf{s}_{i,t+1})\right)\right\}$

Can we do better?

$\gamma^{t'-t}$

ideal target: $y_{i,t} = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_{i,t}\right]$

$$\approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \gamma V^\pi(\mathbf{s}_{i,t+1}) \approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \gamma \underbrace{\hat{V}_\phi^\pi(\mathbf{s}_{i,t+1})}_{\text{directly use previous fitted value function!}}$$

Monte Carlo target: $y_{i,t} = \sum_{t'=t}^{T} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'})$

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Can we do better?

ideal target: $y_{i,t} = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_{i,t}\right]$

$\gamma^{t'-t}$

$$\approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + V^\pi(\mathbf{s}_{i,t+1}) \gamma \approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \underbrace{\hat{V}_\phi^\pi(\mathbf{s}_{i,t+1})}_{\text{directly use previous fitted value function!}} \gamma$$

Monte Carlo target: $y_{i,t} = \sum_{t'=t}^{T} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'})$

training data: $\left\{\left(\mathbf{s}_{i,t}, \underbrace{r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \gamma \hat{V}_\phi^\pi(\mathbf{s}_{i,t+1})}_{y_{i,t}}\right)\right\}$

supervised regression: $\mathcal{L}(\phi) = \frac{1}{2} \sum_i \left\| \hat{V}_\phi^\pi(\mathbf{s}_i) - y_i \right\|^2$

sometimes referred to as a "bootstrapped" estimate

Policy evaluation examples

Policy evaluation examples

TD-Gammon, Gerald Tesauro 1992

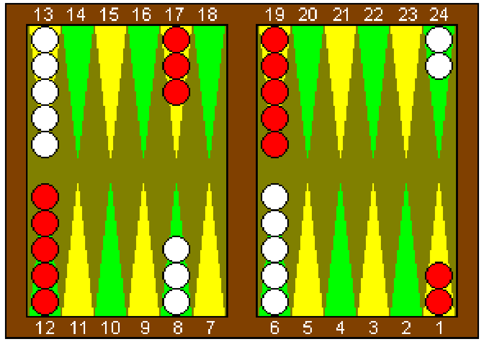


Figure 2. An illustration of the normal opening position in backgammon. TD-Gammon has sparked a near-universal conversion in the way experts play certain opening rolls. For example, with an opening roll of 4-1, most players have now switched from the traditional move of 13-9, 6-5, to TD-Gammon's preference, 13-9, 24-23. TD-Gammon's analysis is given in Table 2.

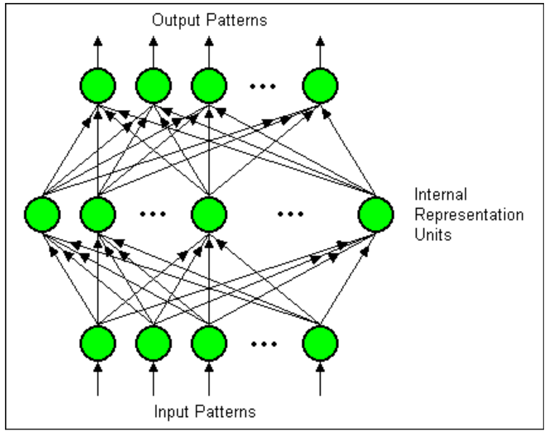
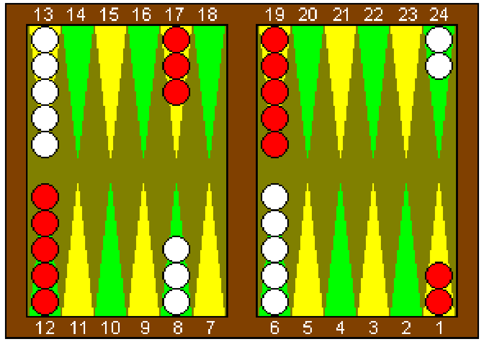


Figure 1. An illustration of the multilayer perception architecture used in TD-Gammon's neural network. This architecture is also used in the popular backpropagation learning procedure. Figure reproduced from [9].

Policy evaluation examples

TD-Gammon, Gerald Tesauro 1992



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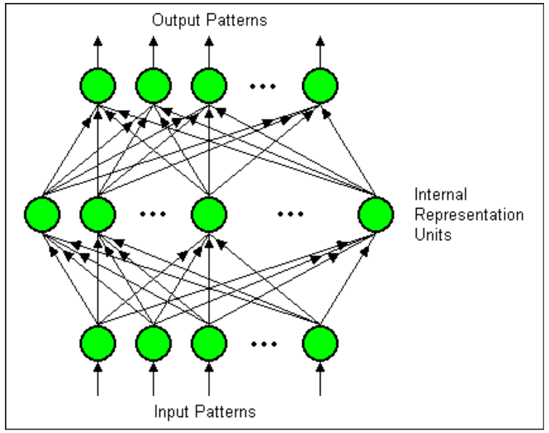

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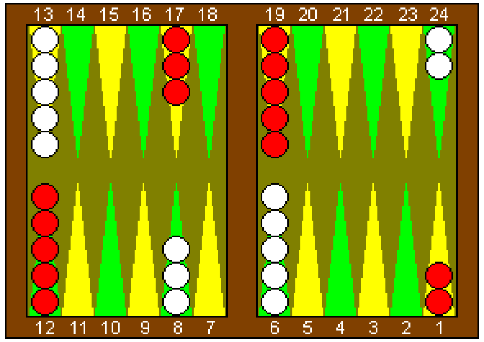
$\text{reward: game outcome}$

Policy evaluation examples

TD-Gammon, Gerald Tesauro 1992



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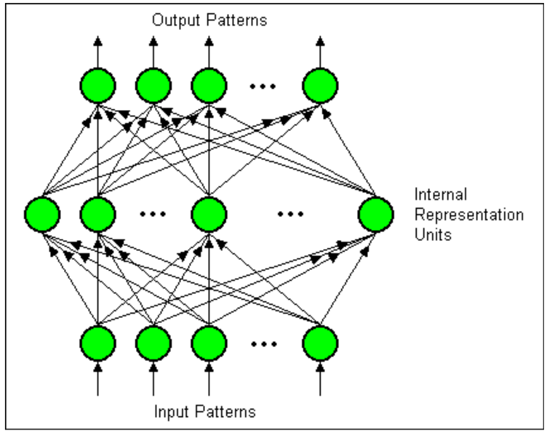

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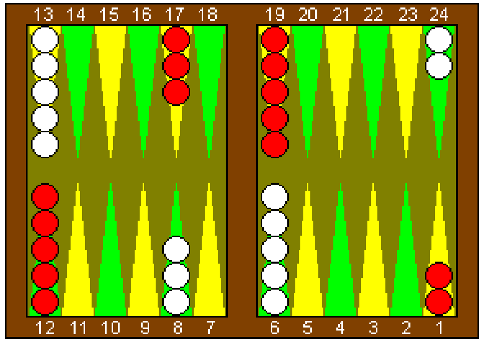
reward: game outcome

value function $\hat{V}^{\pi}_{\phi}(\mathbf{s}_t)$:
expected outcome given board state

Policy evaluation examples

TD-Gammon, Gerald Tesauro 1992



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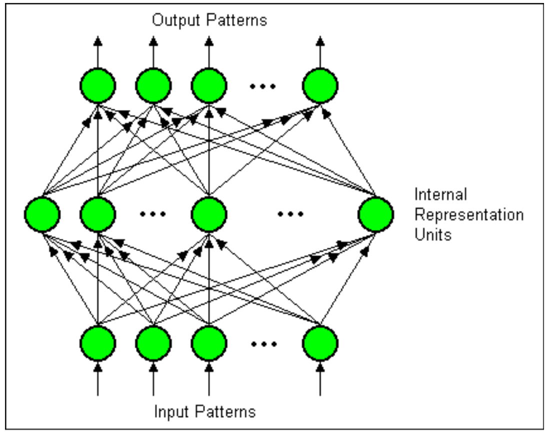
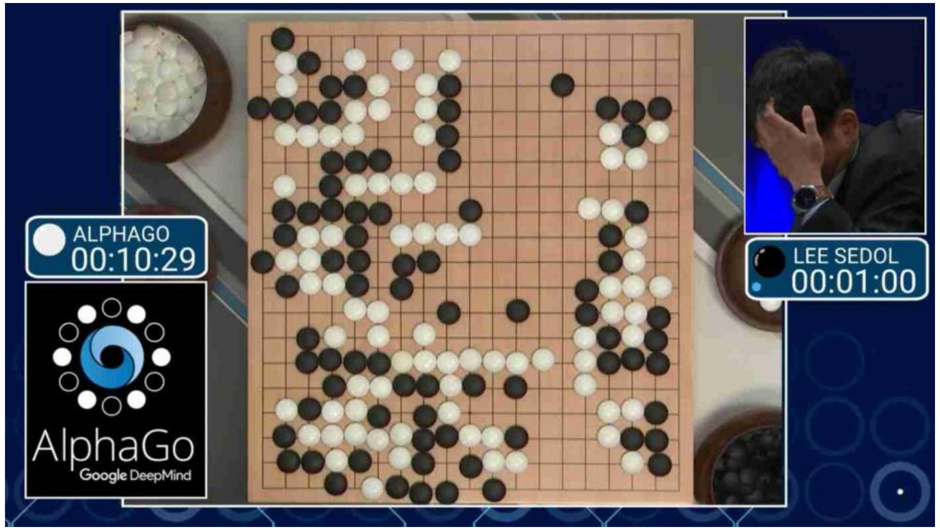


Figure 1. An illustration of the multilayer perception architecture used in TD-Gammon's neural network. This architecture is also used in the popular backpropagation learning procedure. Figure reproduced from [9].

AlphaGo, Silver et al. 2016



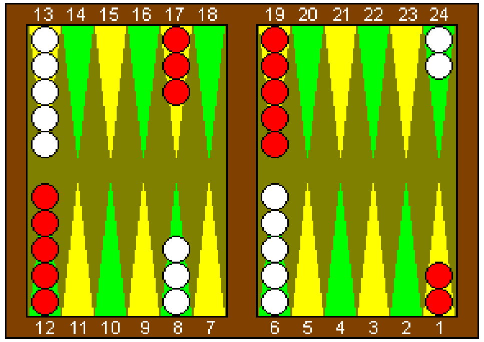
reward: game outcome

value function $\hat{V}_\phi^\pi(\mathbf{s}_t)$:

expected outcome given board state

Policy evaluation examples

TD-Gammon, Gerald Tesauro 1992



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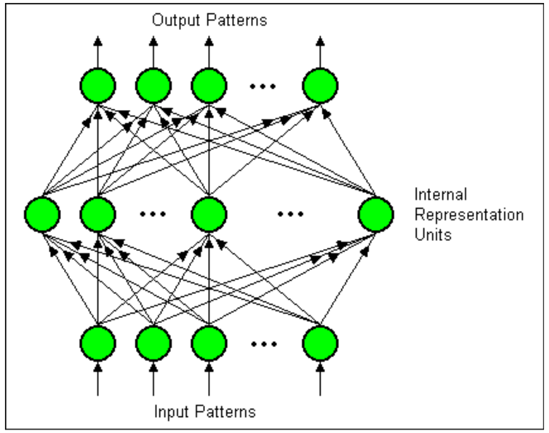

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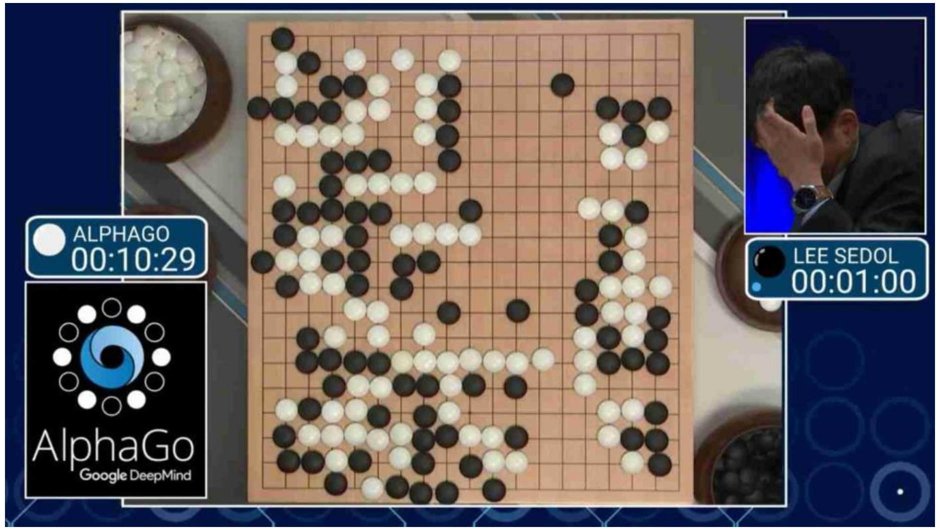

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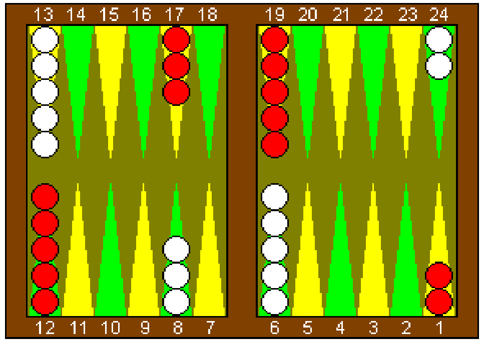

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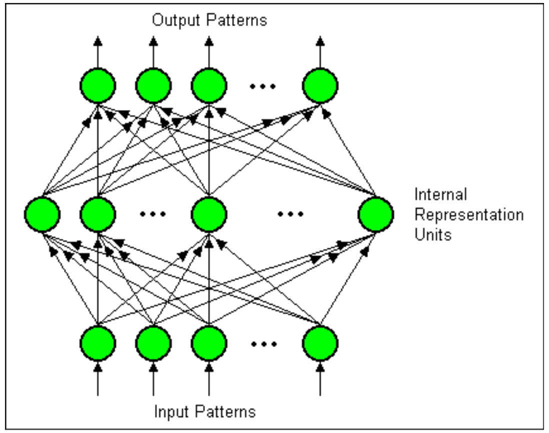

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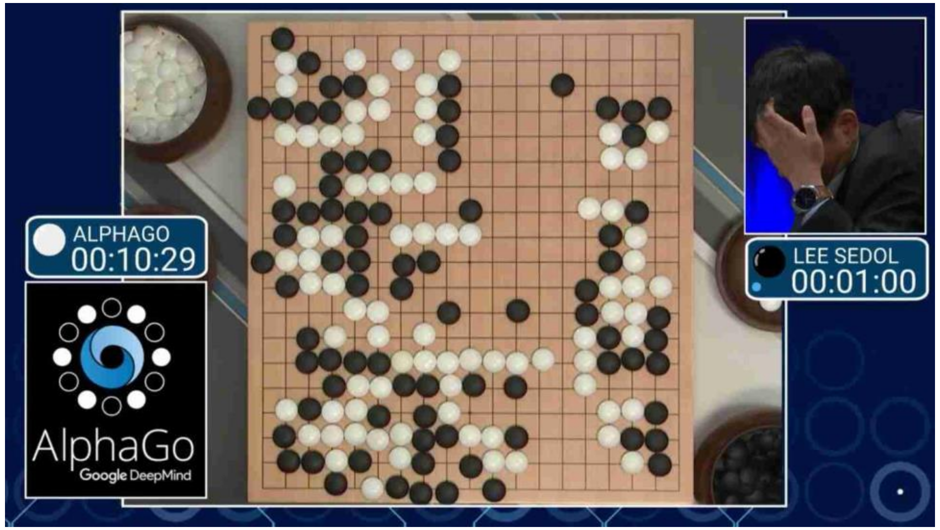
AlphaGo, Silver et al. 2016



reward: game outcome

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Actor-critic algorithms

batch actor-critic algorithm:

1. sample $\{\mathbf{s}_i, \mathbf{a}_i\}$ from $\pi_\theta(\mathbf{a}|\mathbf{s})$ (run it on the robot)
2. fit $\hat{V}^\pi_\phi(\mathbf{s})$ to sampled reward sums
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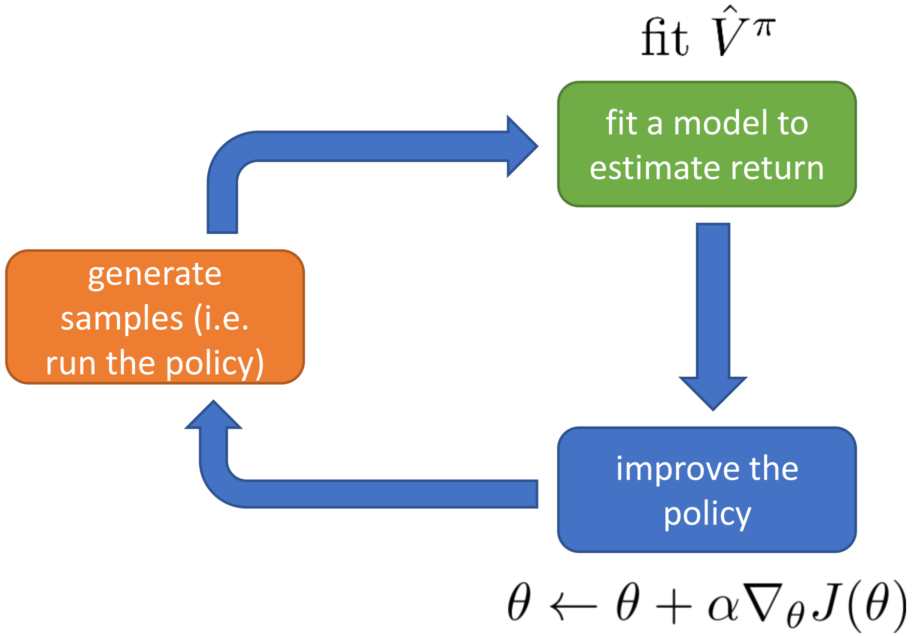

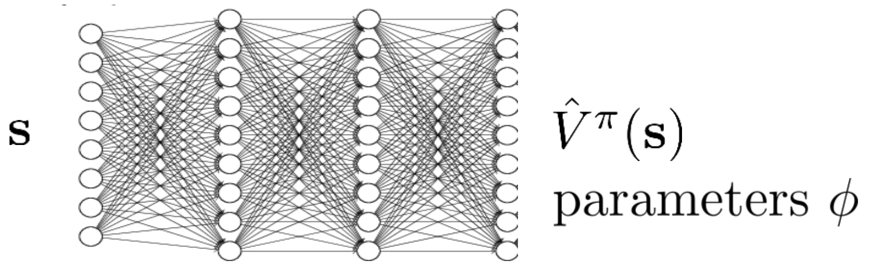

Actor-critic algorithms

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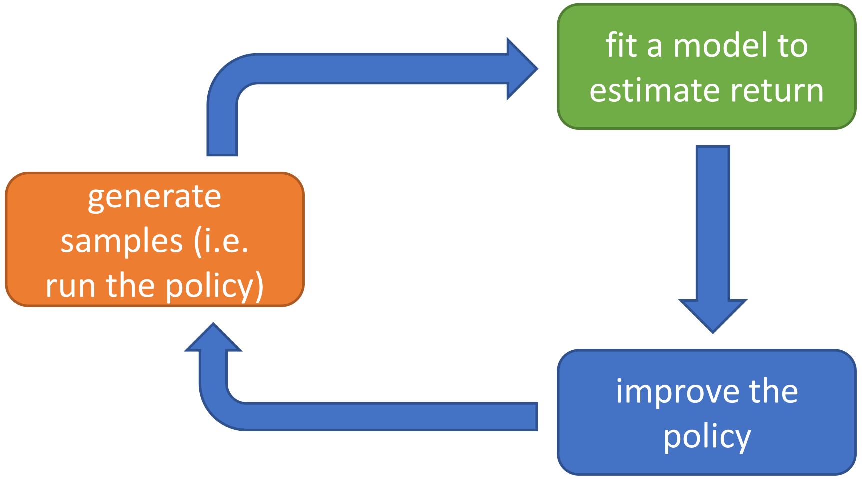
online actor-critic algorithm:

fit $\hat{V}^\pi$



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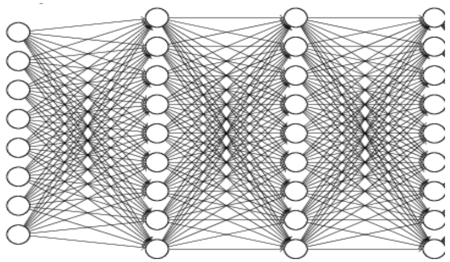
$\mathbf{s}$



$\hat{V}^\pi(\mathbf{s})$

parameters ϕ

Actor-critic algorithms

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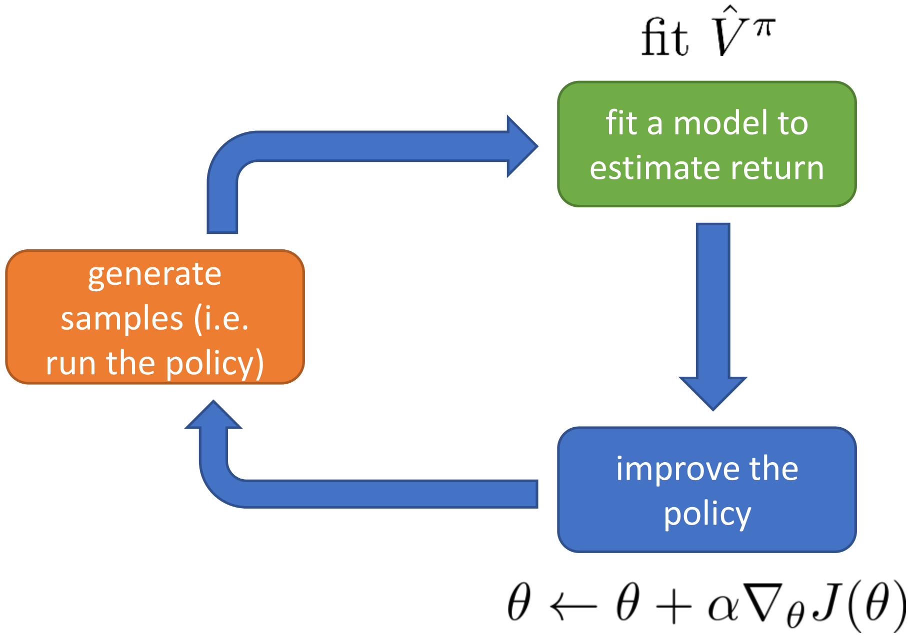
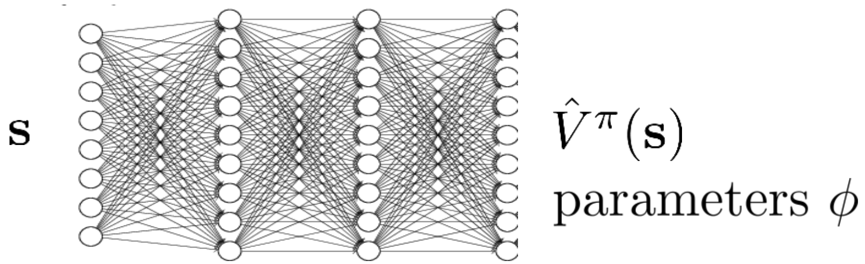


online actor-critic algorithm:

1. take action $\mathbf{a} \sim \pi_\theta(\mathbf{a}|\mathbf{s})$, get $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$



Actor-critic algorithms

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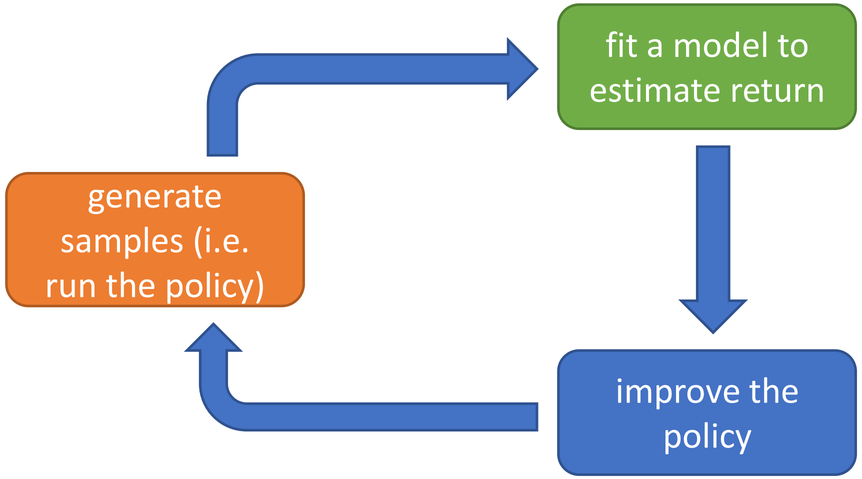
fit $\hat{V}^\pi$



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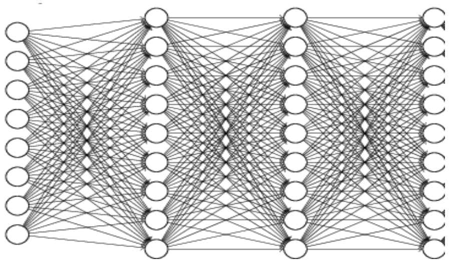
$\mathbf{s}$



$\hat{V}^\pi(\mathbf{s})$

parameters ϕ

Actor-critic algorithms

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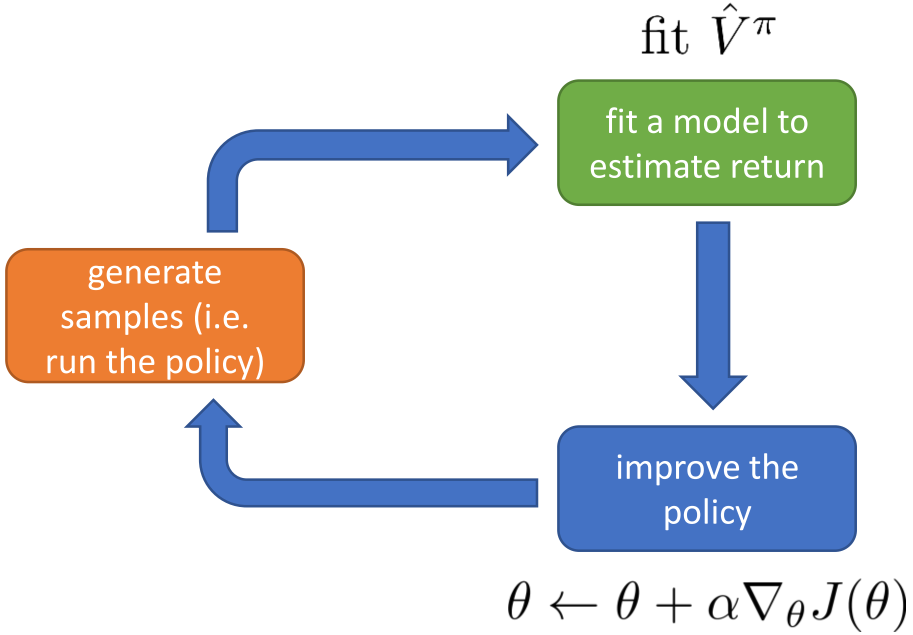

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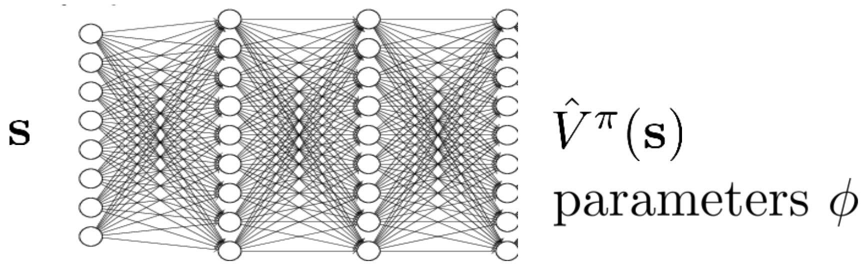

Actor-critic algorithms

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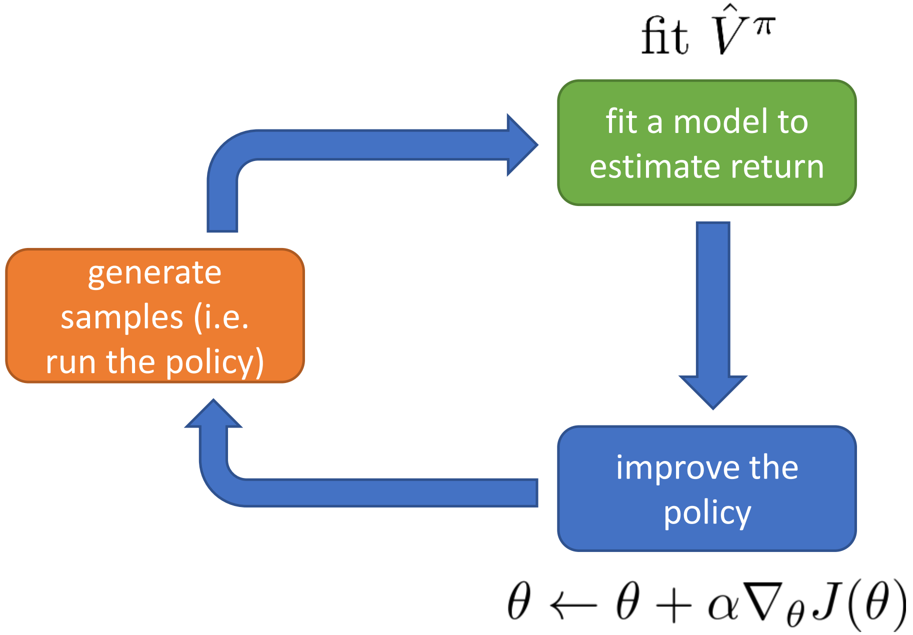

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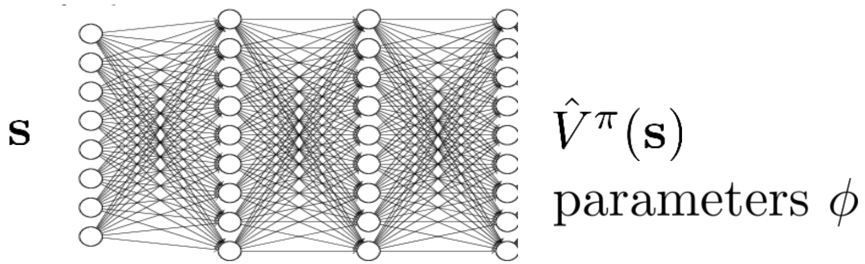

Actor-critic algorithms

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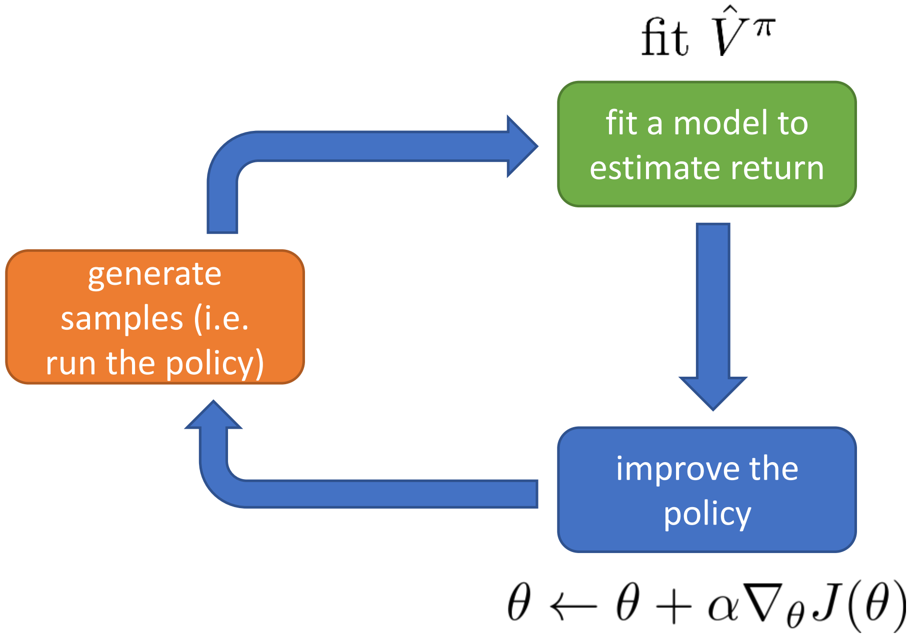

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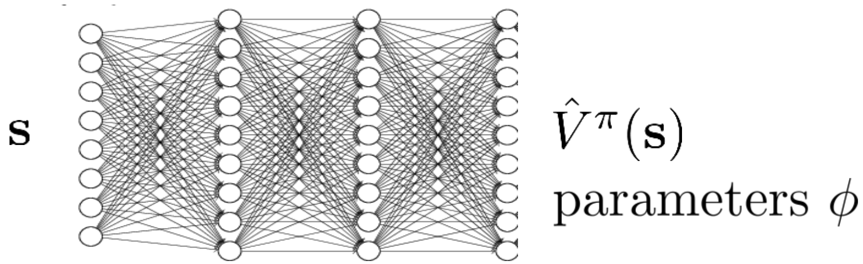

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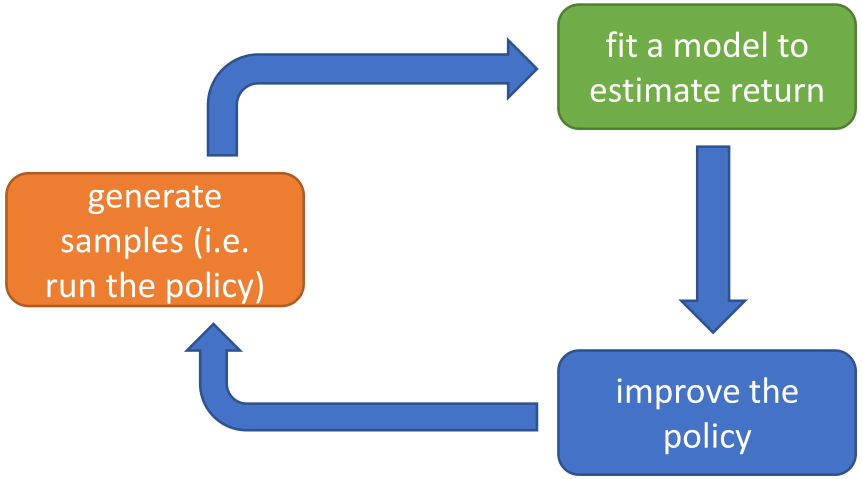
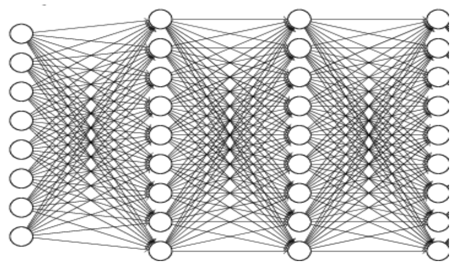
fit $\hat{V}^\pi$



$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

$\mathbf{s}$



$\hat{V}^\pi(\mathbf{s})$

parameters ϕ

Actor-critic algorithms

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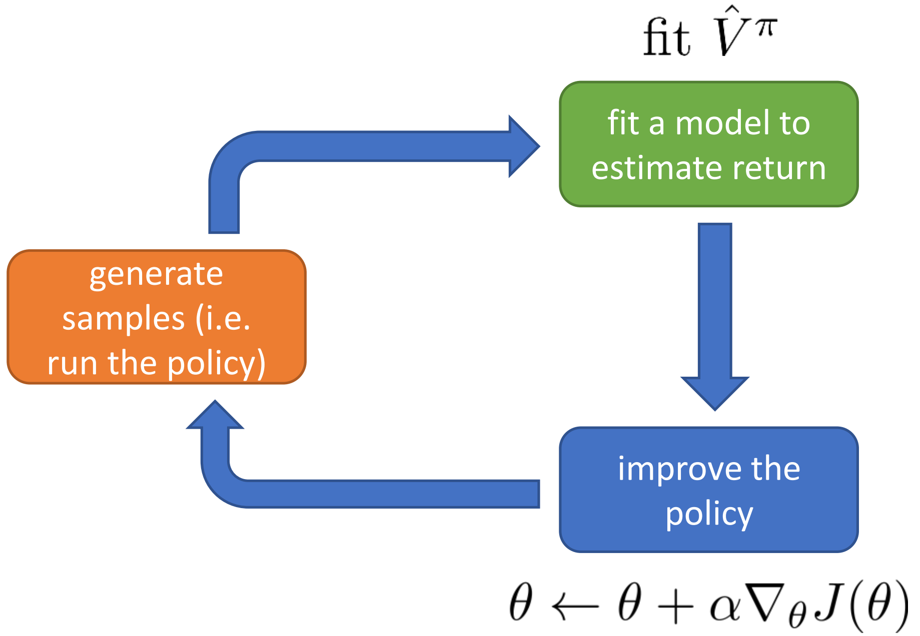
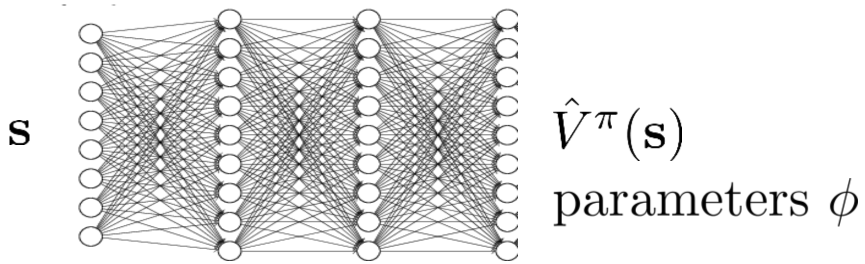
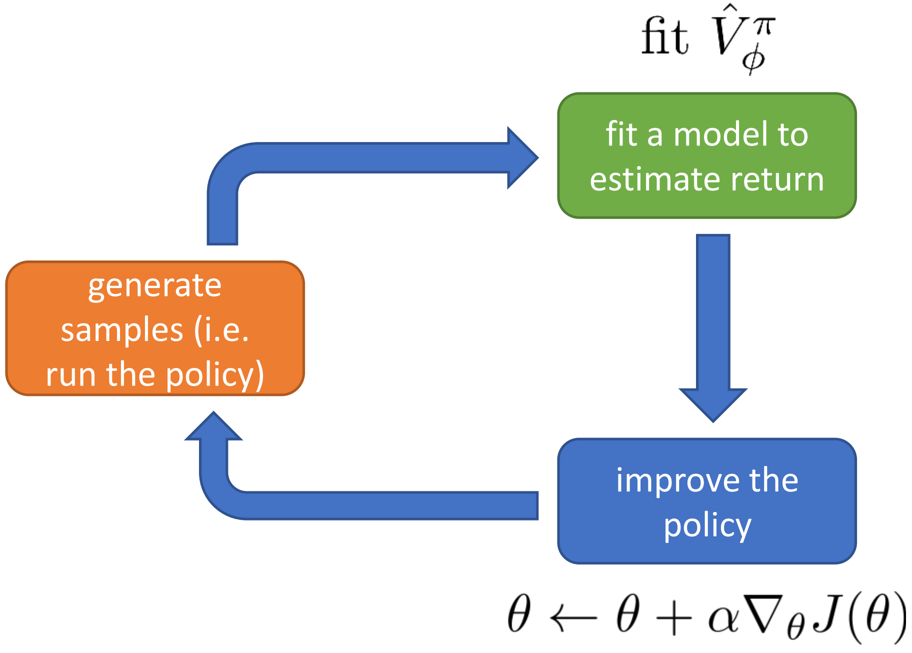




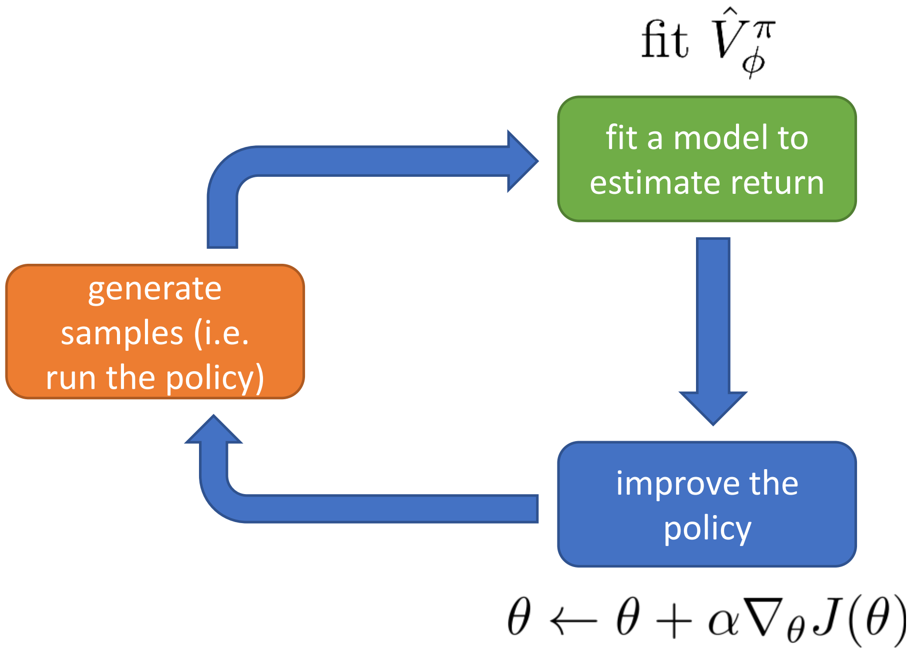
Review



fit $\hat{V}_\phi^\pi$
fit a model to estimate return
generate samples (i.e. run the policy)
improve the policy
$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

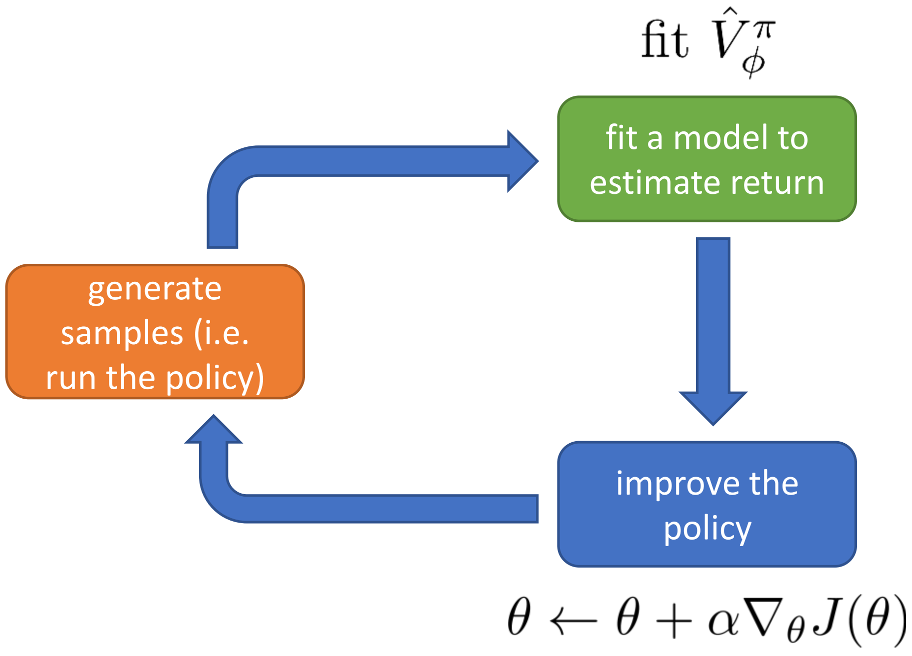
Review

- Actor-critic algorithms:



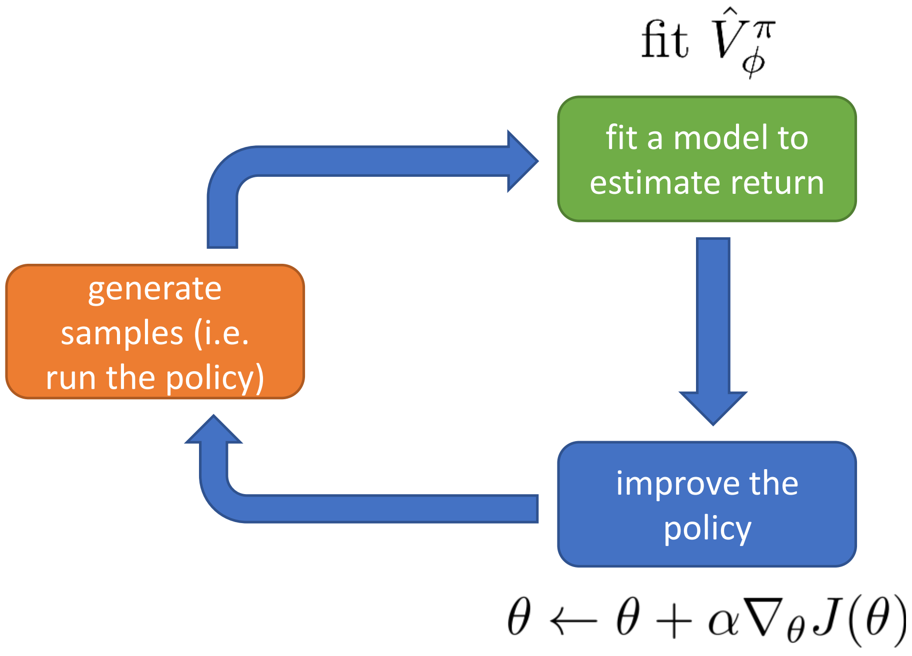
Review

- Actor-critic algorithms:
  - Actor: the policy



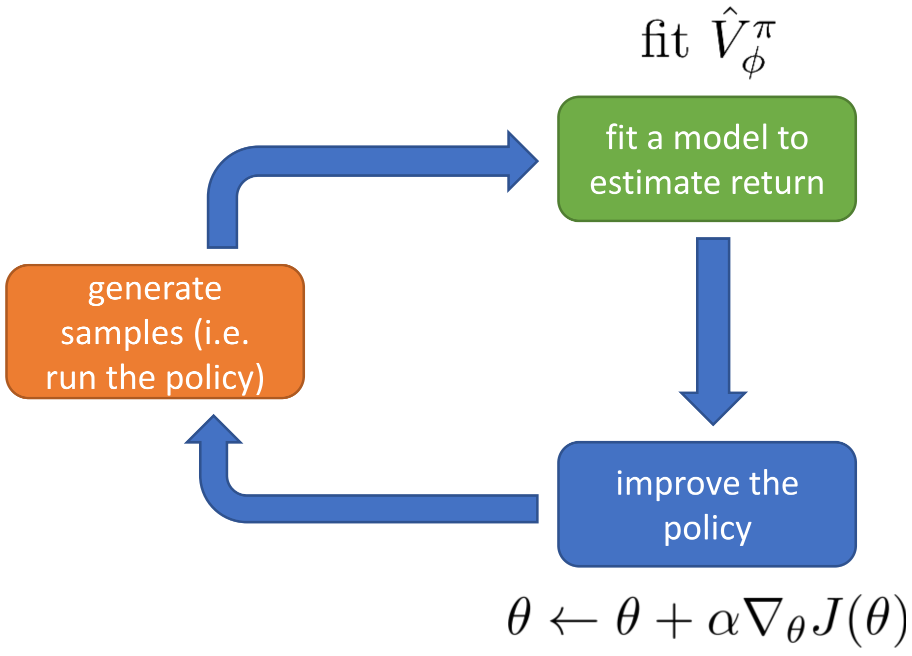
Review

- Actor-critic algorithms:
  - Actor: the policy
  - Critic: value function



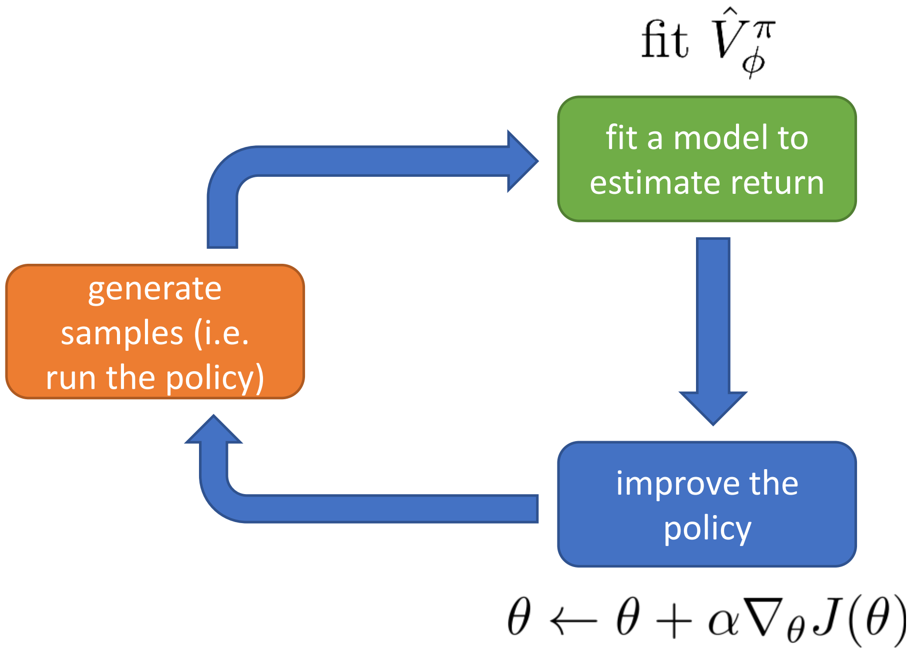
Review

- Actor-critic algorithms:
  - Actor: the policy
  - Critic: value function
  - Reduce variance of policy gradient



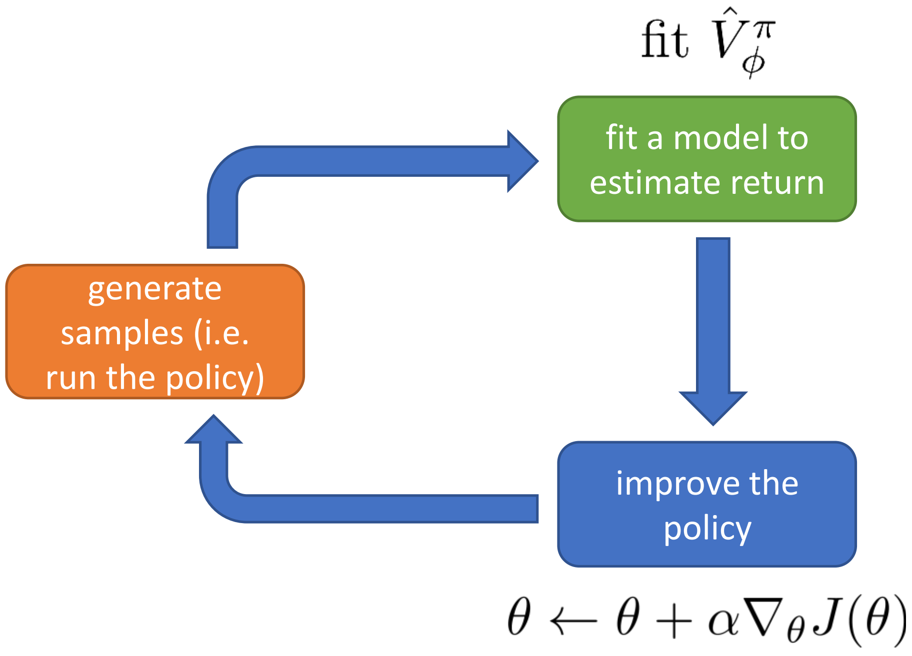
Review

- Actor-critic algorithms:
  - Actor: the policy
  - Critic: value function
  - Reduce variance of policy gradient
- Policy evaluation



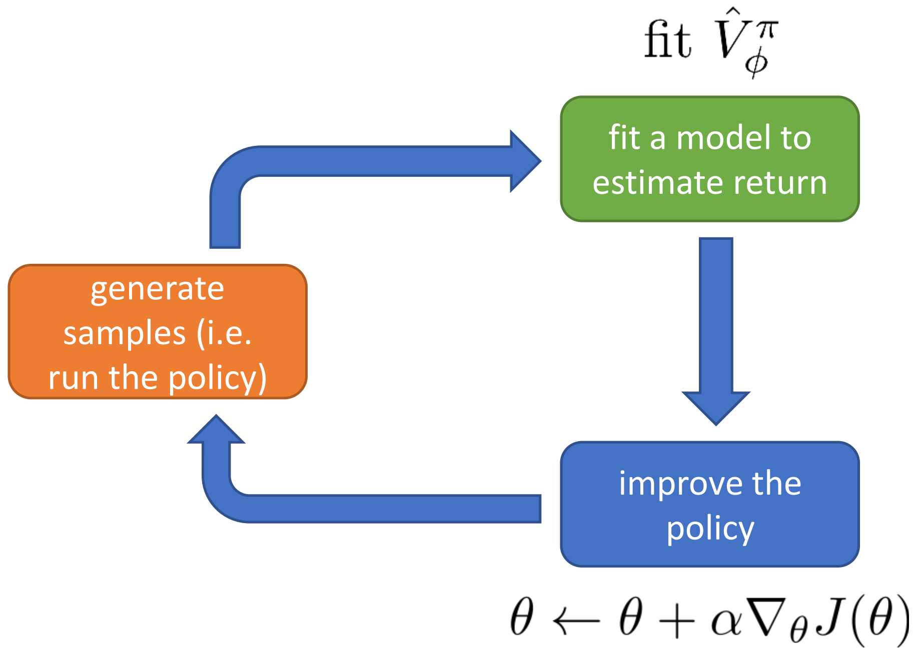
Review

- Actor-critic algorithms:
  - Actor: the policy
  - Critic: value function
  - Reduce variance of policy gradient
- Policy evaluation
  - Fitting value function to policy



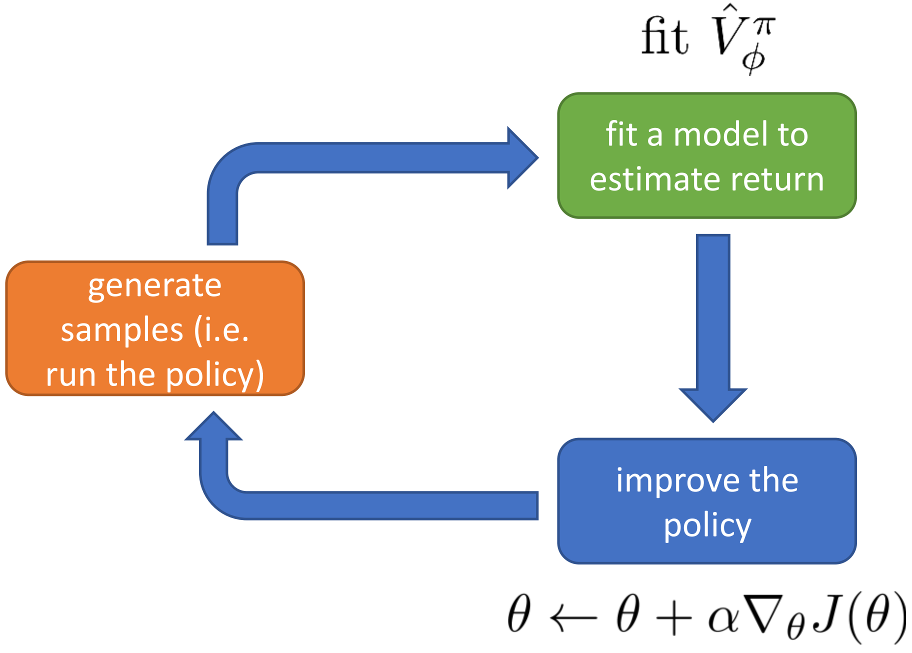
Review

- Actor-critic algorithms:
  - Actor: the policy
  - Critic: value function
  - Reduce variance of policy gradient
- Policy evaluation
  - Fitting value function to policy
- Actor-critic algorithm design



Review

- Actor-critic algorithms:
  - Actor: the policy
  - Critic: value function
  - Reduce variance of policy gradient
- Policy evaluation
  - Fitting value function to policy
- Actor-critic algorithm design
  - Batch-mode, or online (+ parallel)



Review

- Actor-critic algorithms:
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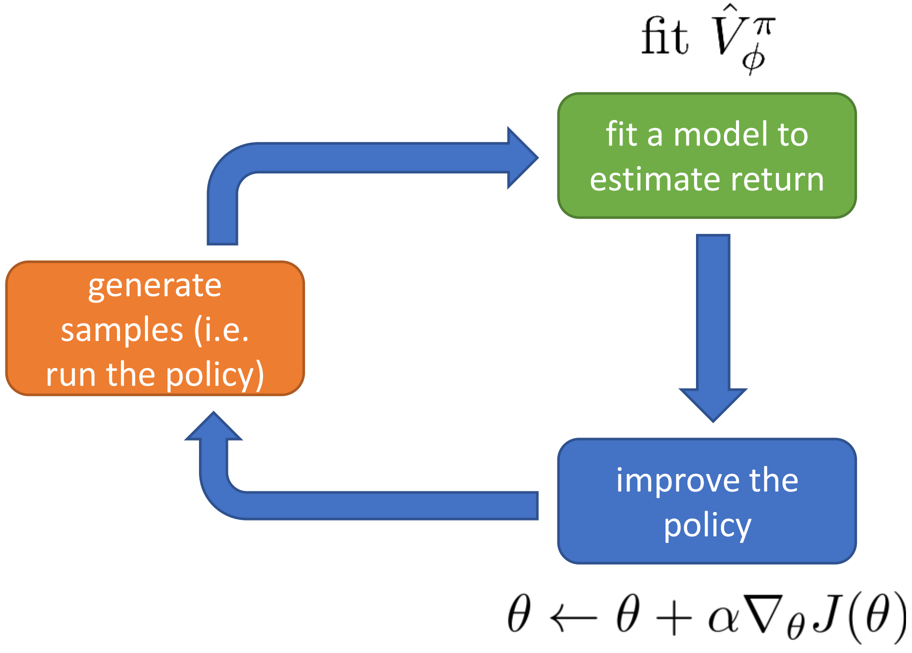

Review

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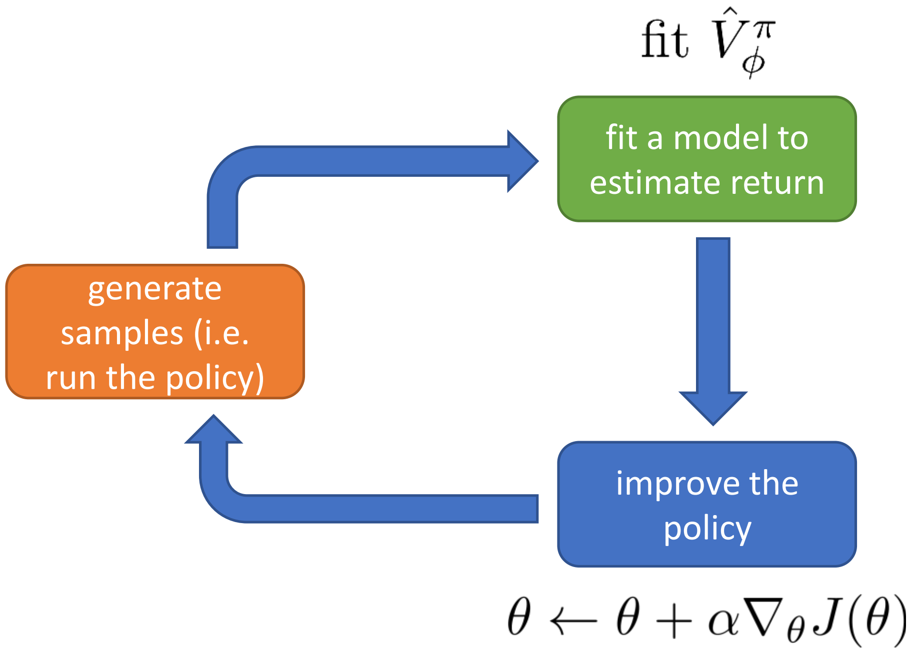

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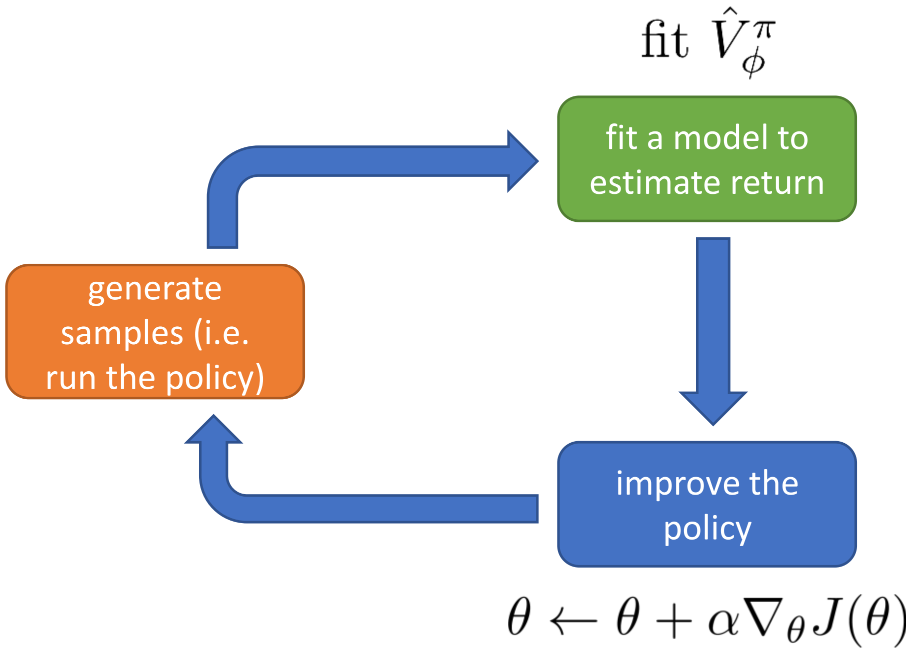

Actor-critic examples

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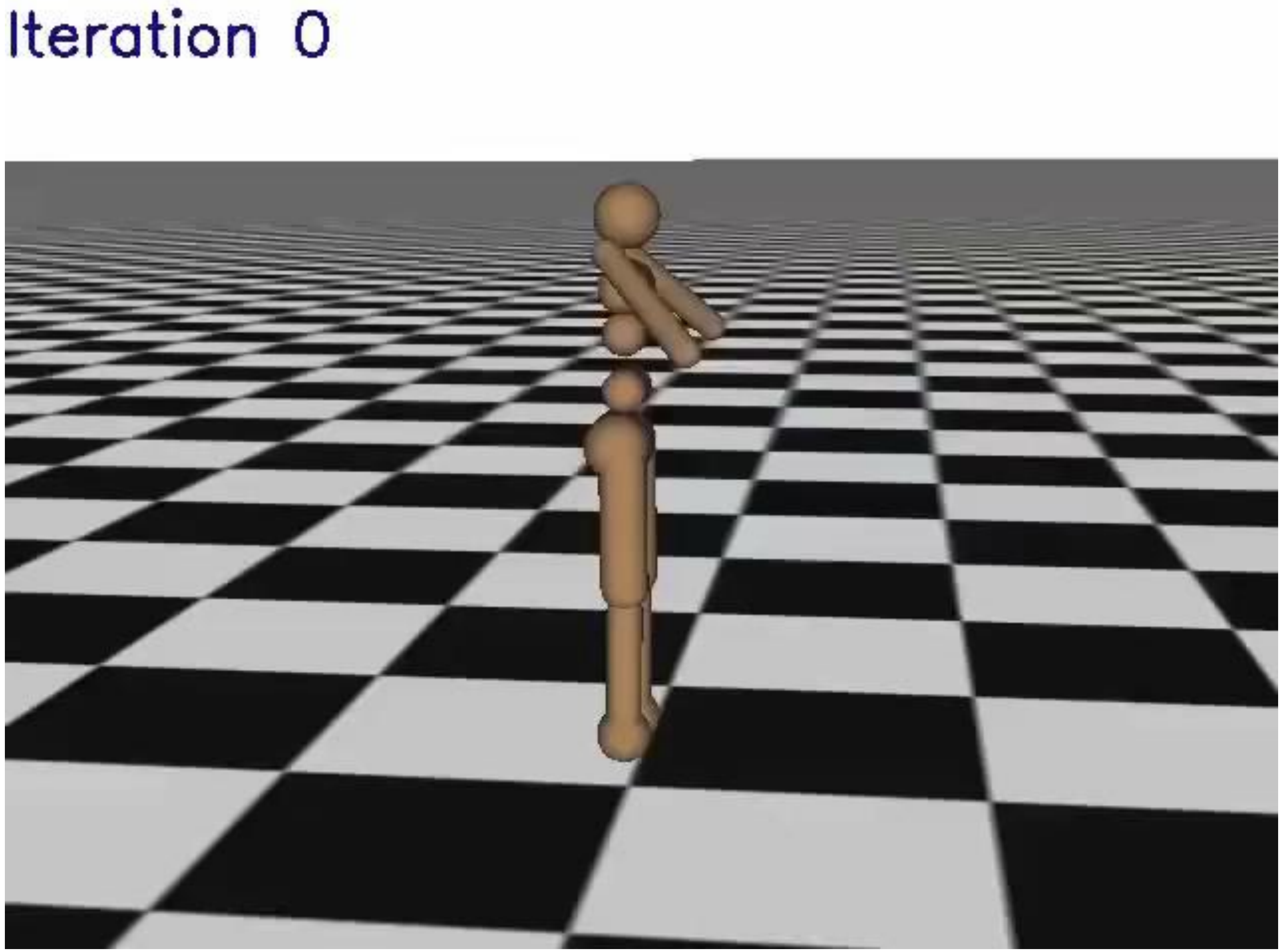

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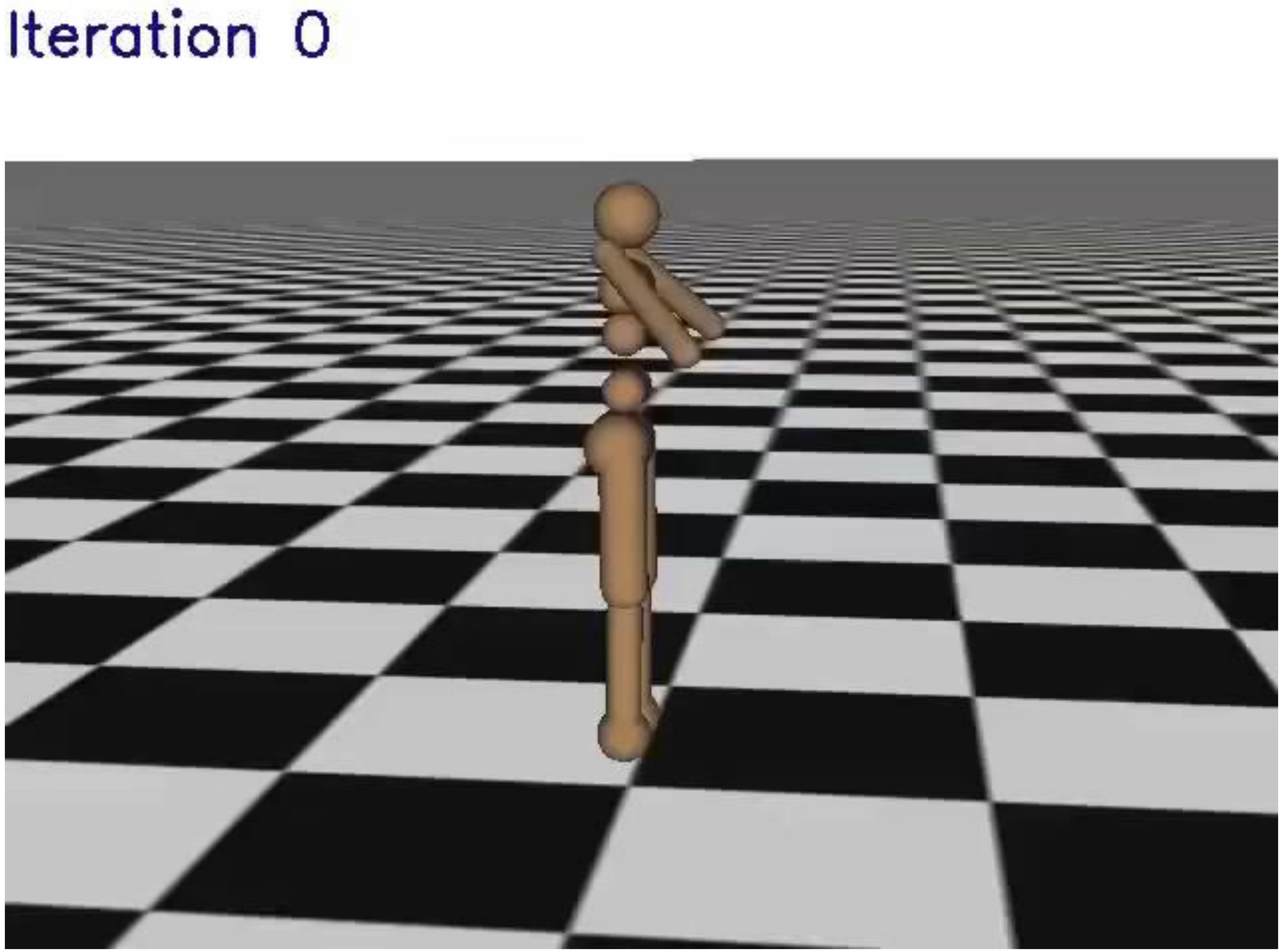

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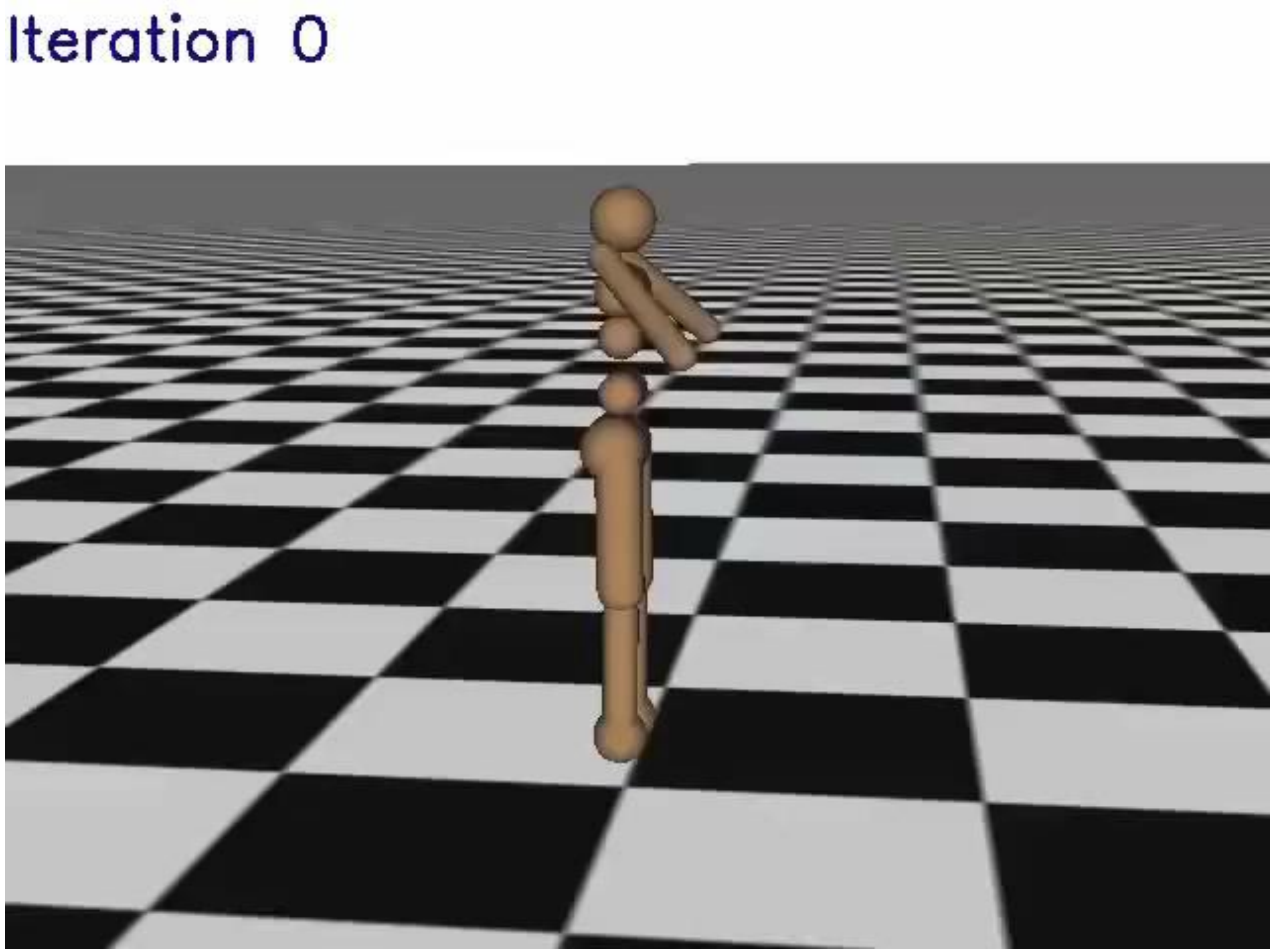

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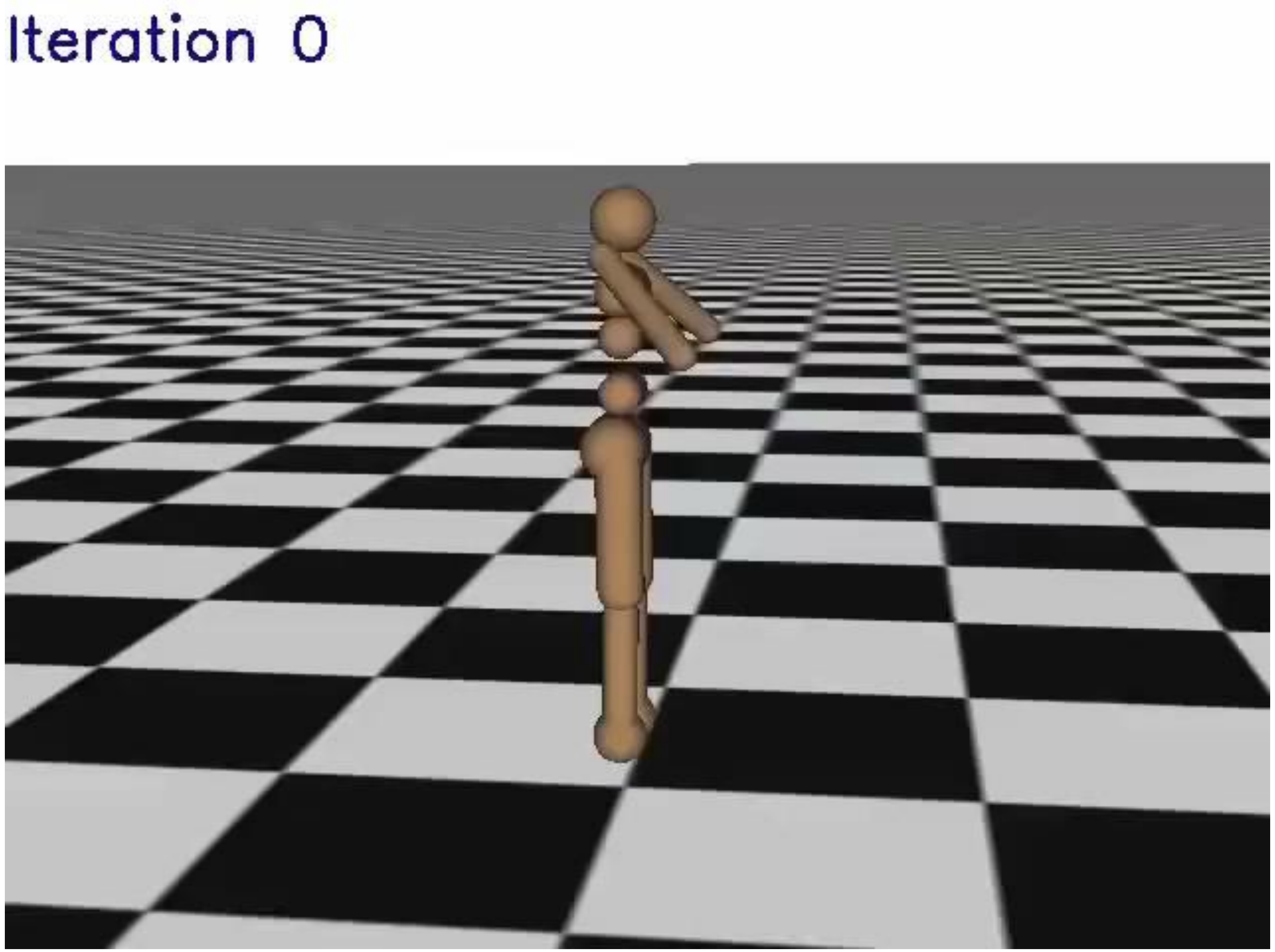

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- High dimensional continuous control with generalized advantage estimation (Schulman, Moritz, L., Jordan, Abbeel ‘16)
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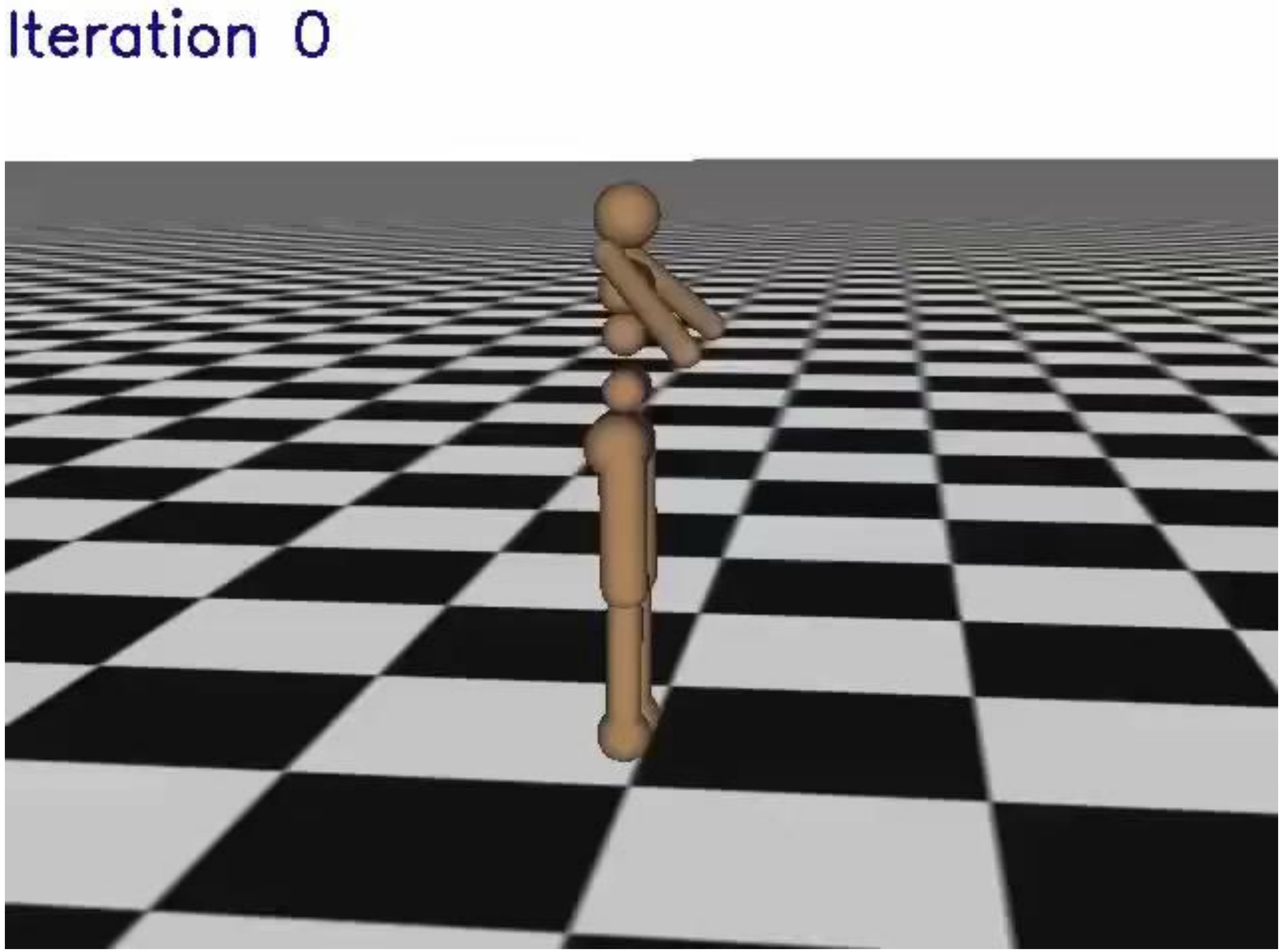
- Asynchronous methods for DRL (Mnih et al. ‘16) → online



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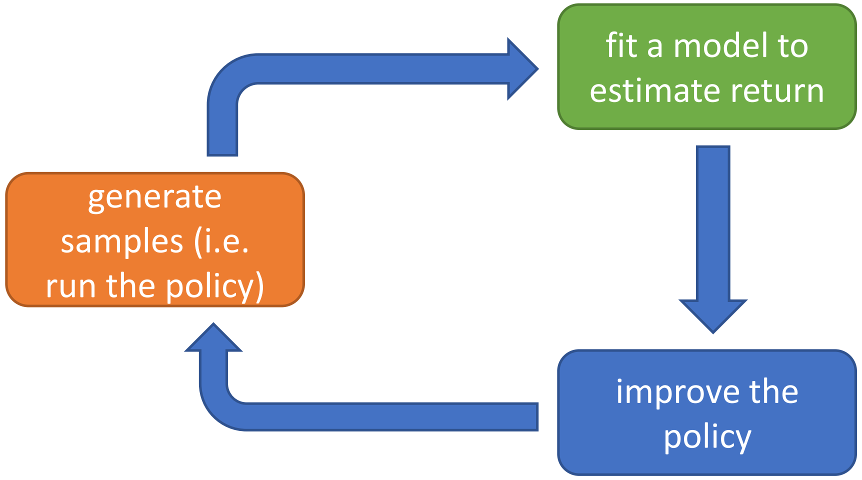

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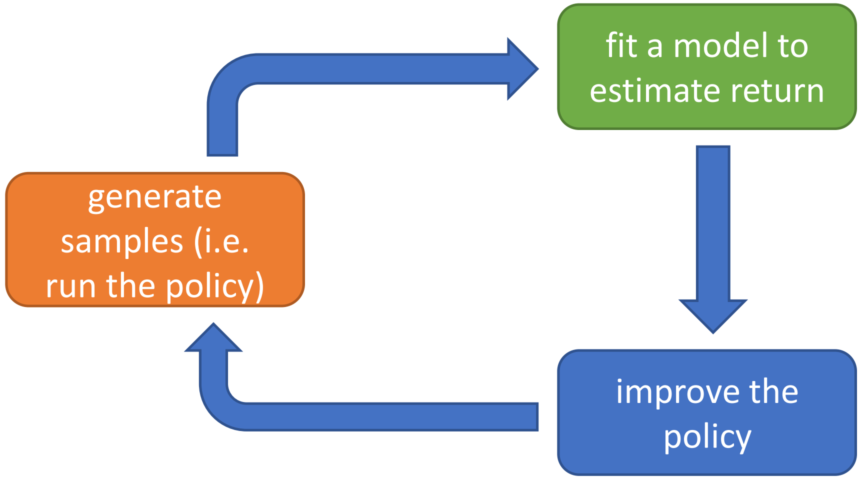

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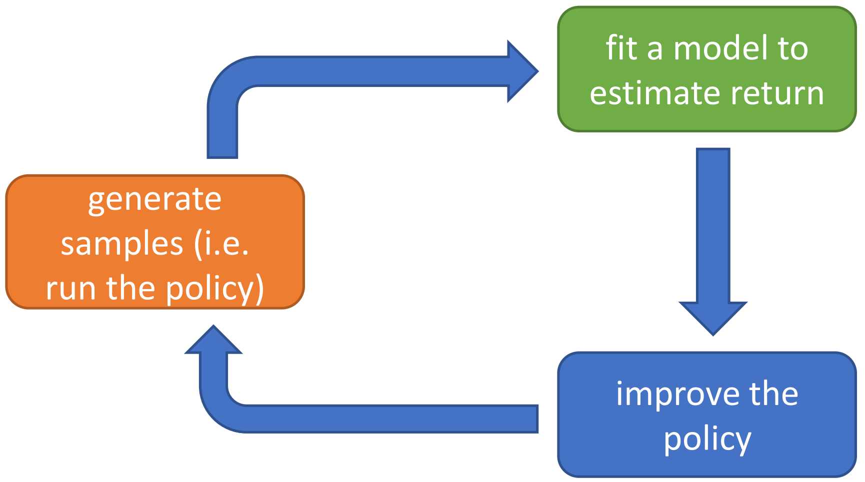
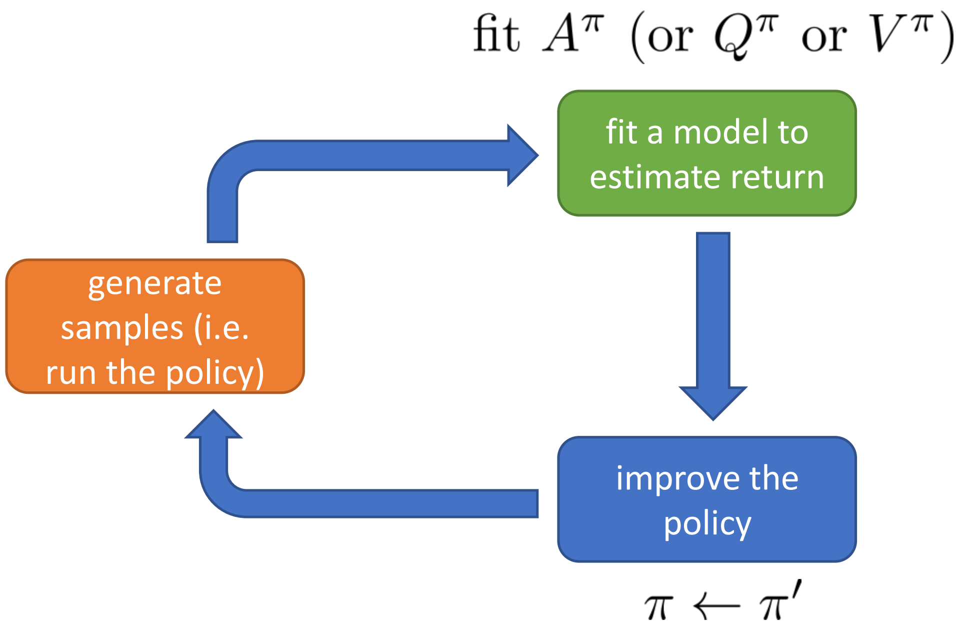


$\pi \leftarrow \pi'$

Policy iteration

High level idea:

policy iteration algorithm:



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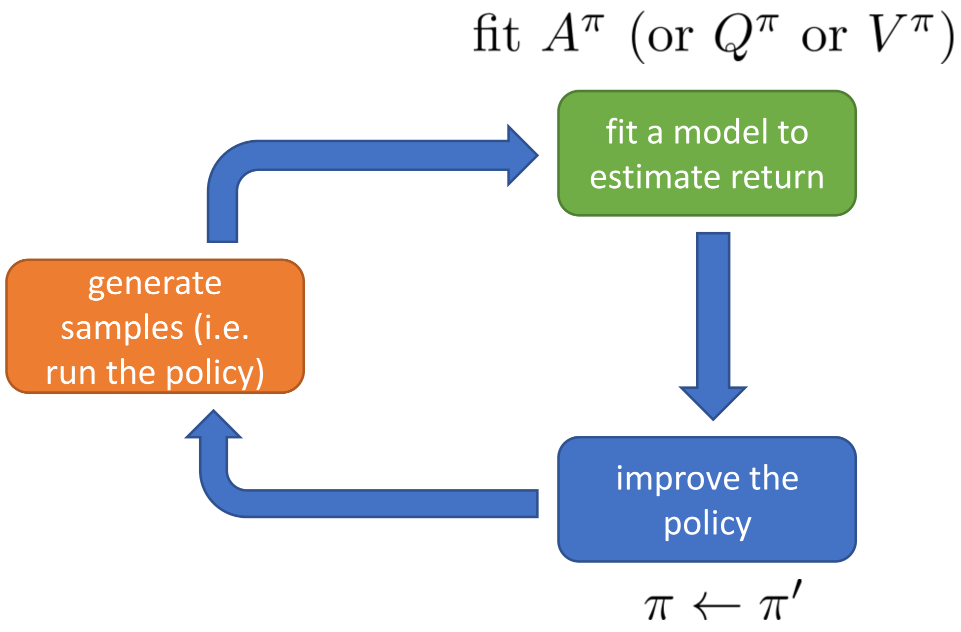

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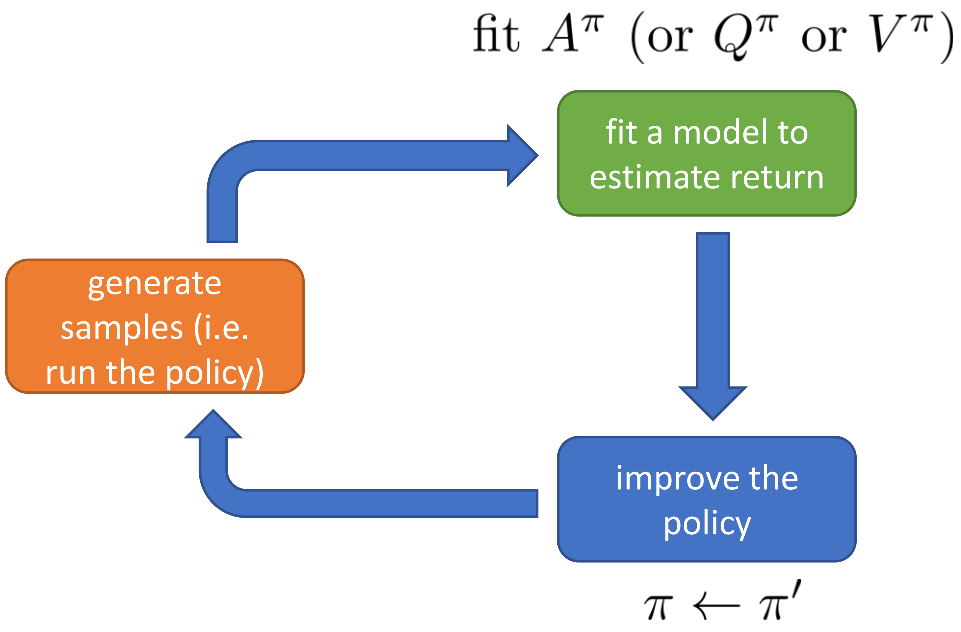

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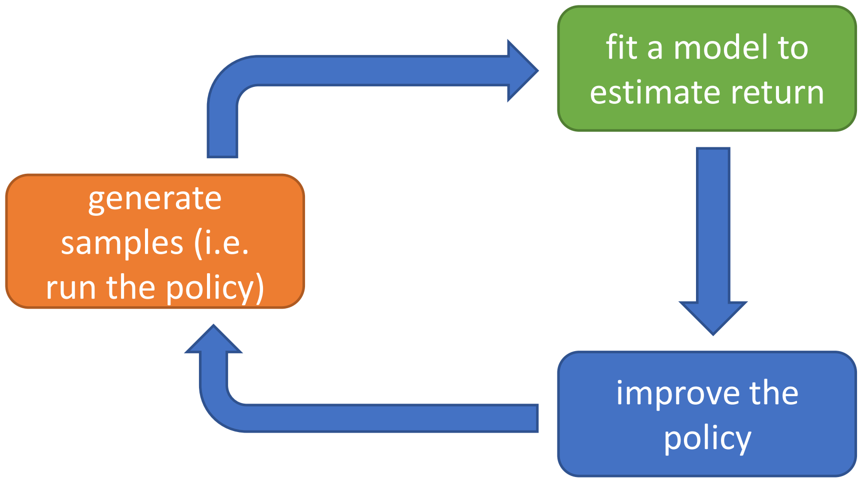

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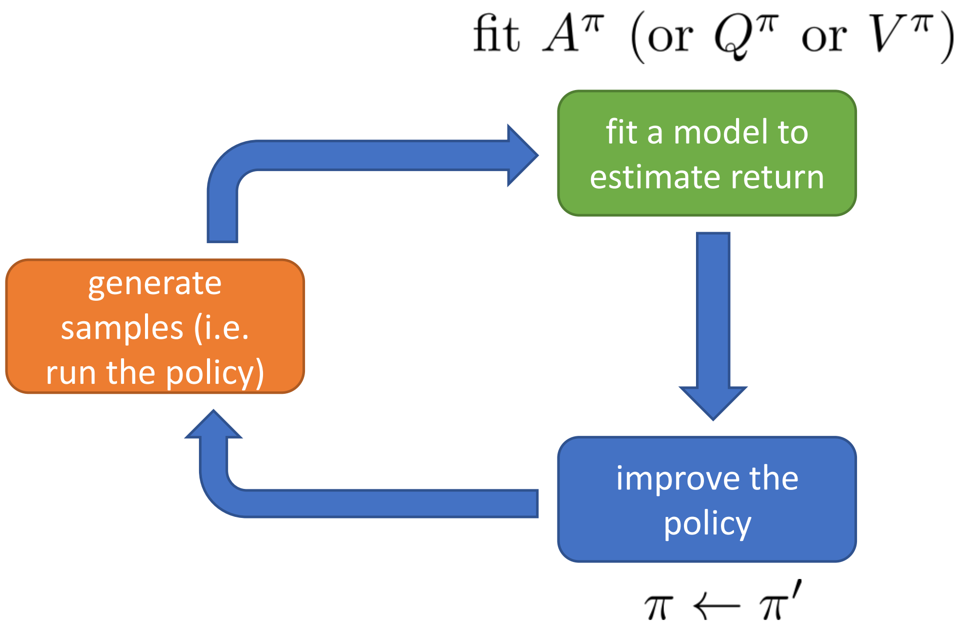

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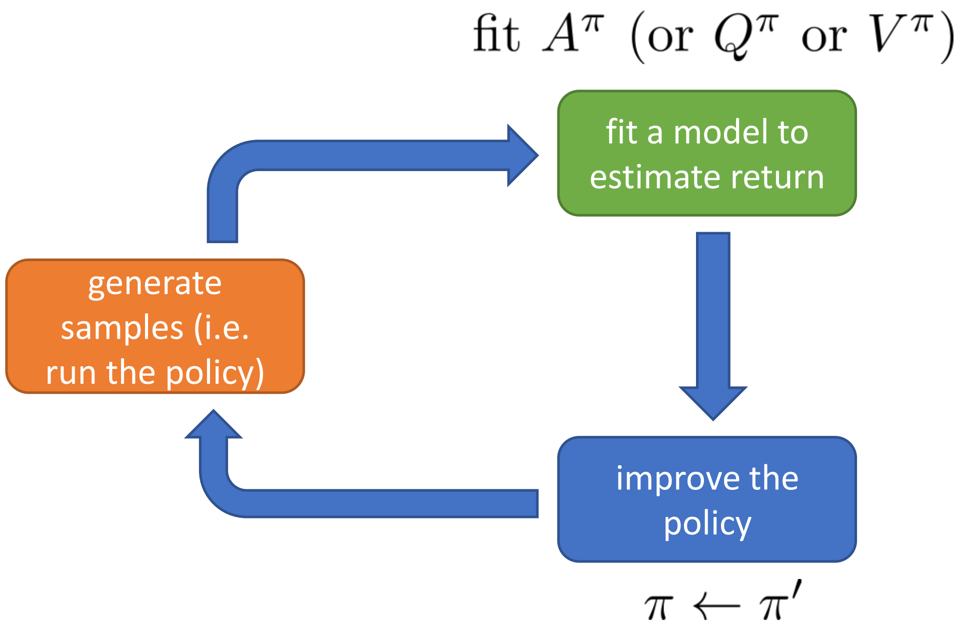

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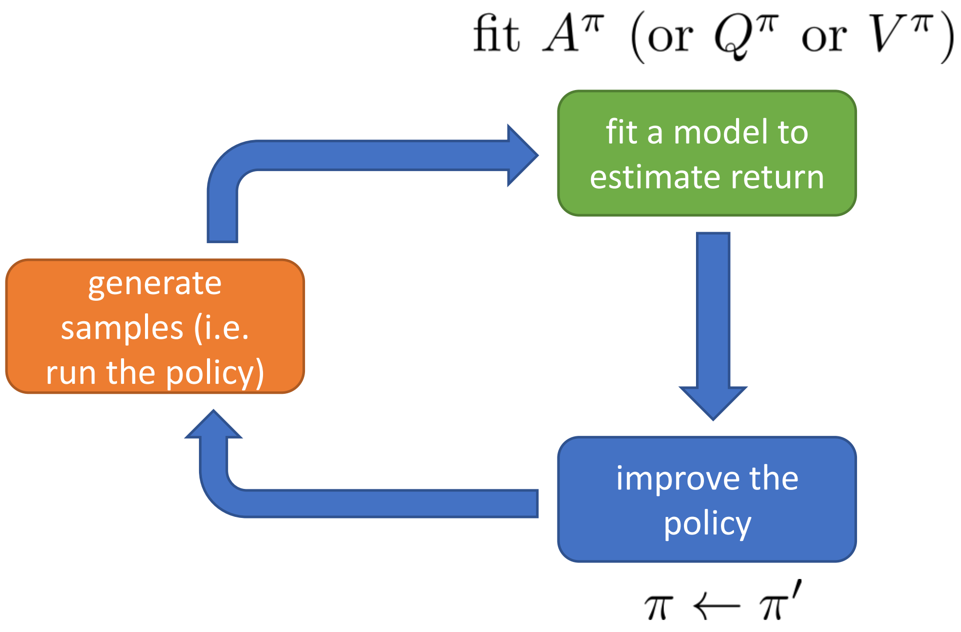

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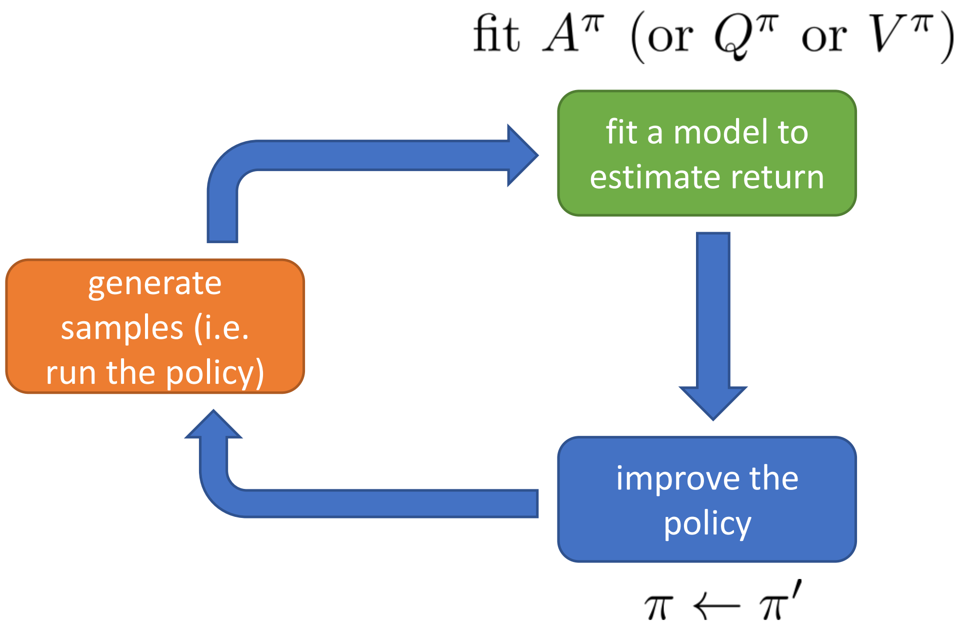

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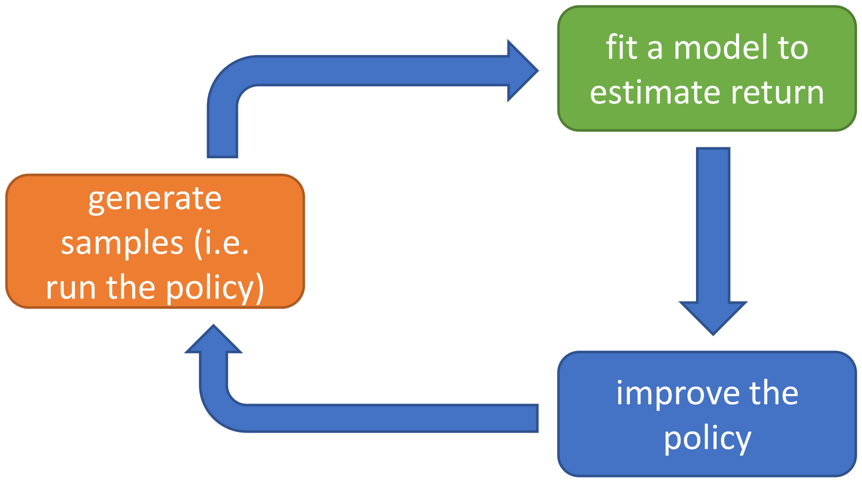

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Fitted Q-iteration

full fitted Q-iteration algorithm:

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parameters

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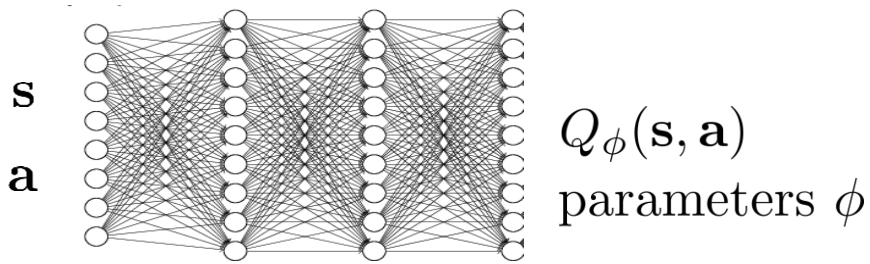

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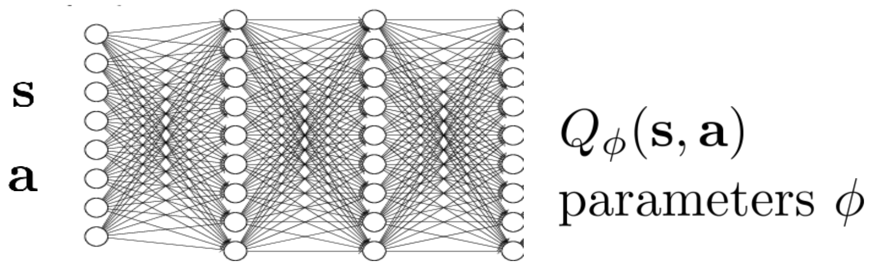

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$K\times$

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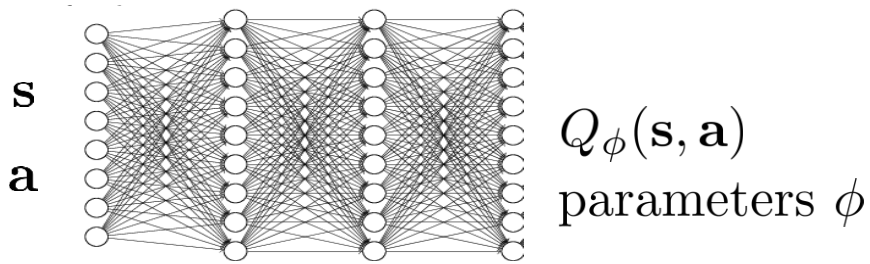

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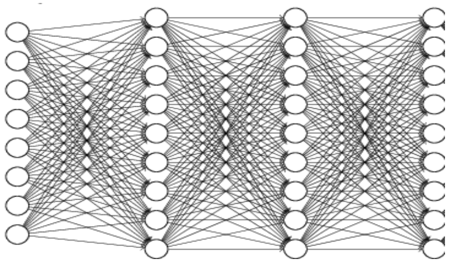
iterations K

gradient steps S

$\mathbf{s}$

$\mathbf{a}$



$Q_\phi(\mathbf{s}, \mathbf{a})$

parameters ϕ

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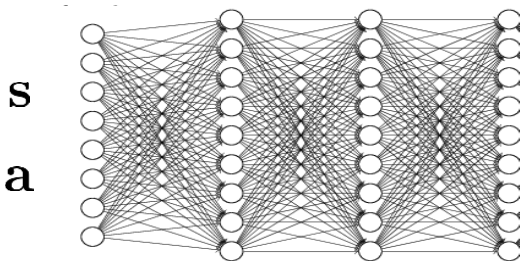

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Why is this algorithm off-policy?

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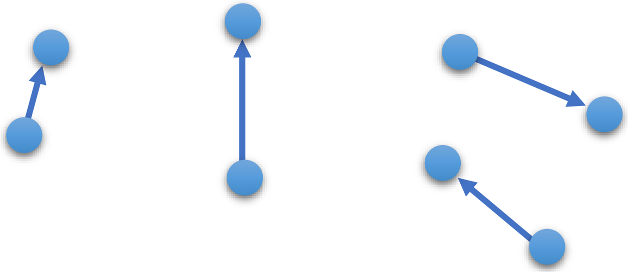

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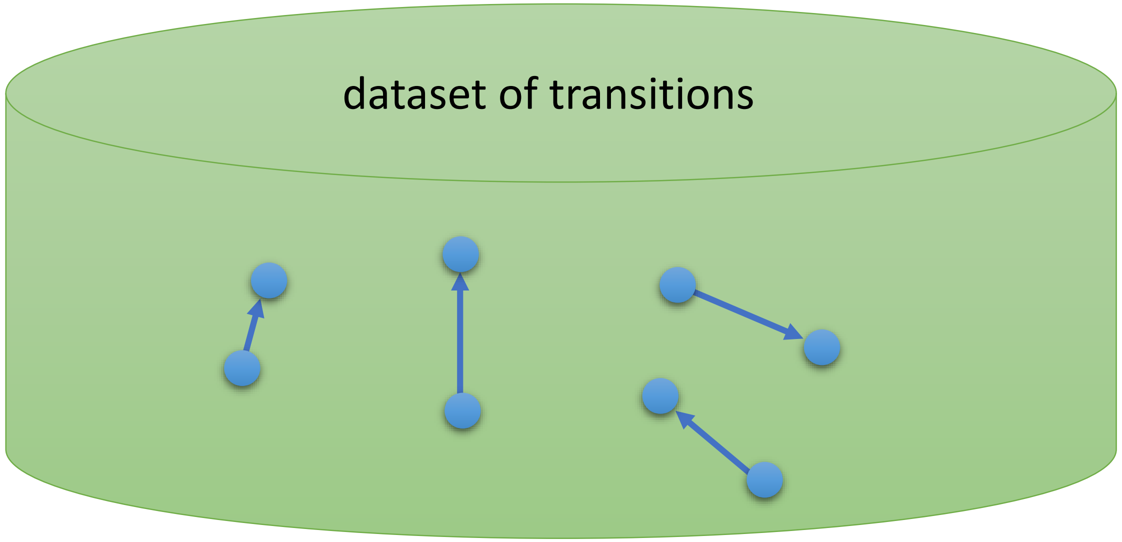

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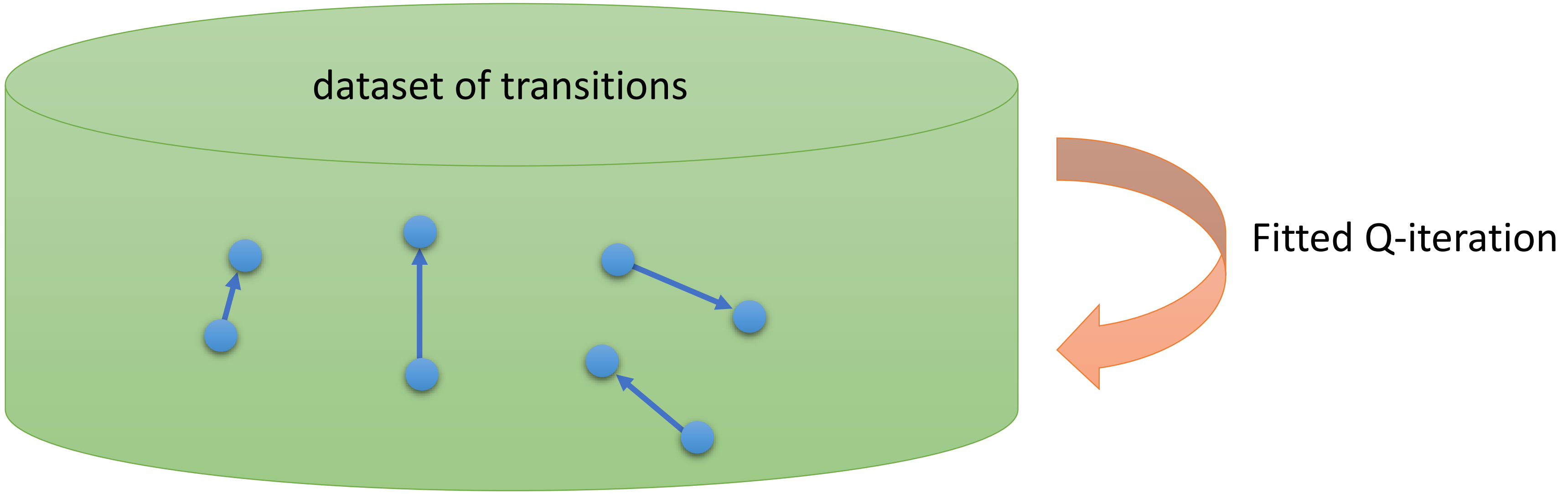

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most guarantees are lost when we leave the tabular case (e.g., when we use neural network function approximation)

Exploration with Q-learning

online Q iteration algorithm:

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“epsilon-greedy”

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"epsilon-greedy"

$$\pi(\mathbf{a}_t|\mathbf{s}_t) \propto \exp(Q_\phi(\mathbf{s}_t, \mathbf{a}_t))$$

"Boltzmann exploration" (usually with a temperature parameter to control the spread independently of the reward scale)

Correlated samples in online Q-learning

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- sequential states are strongly correlated

Correlated samples in online Q-learning

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- sequential states are strongly correlated
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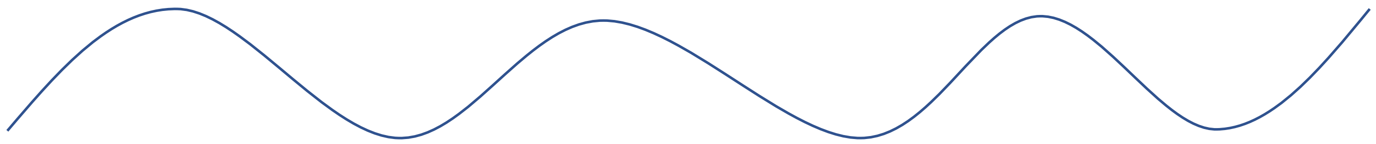

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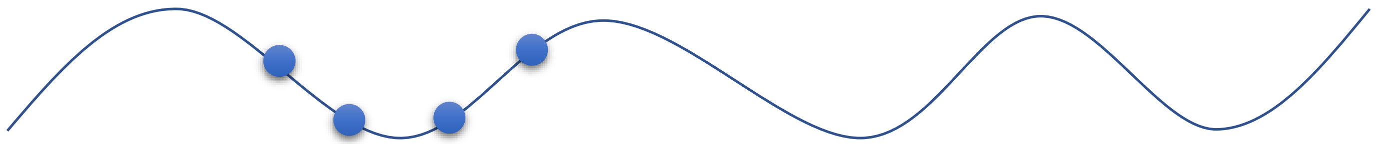

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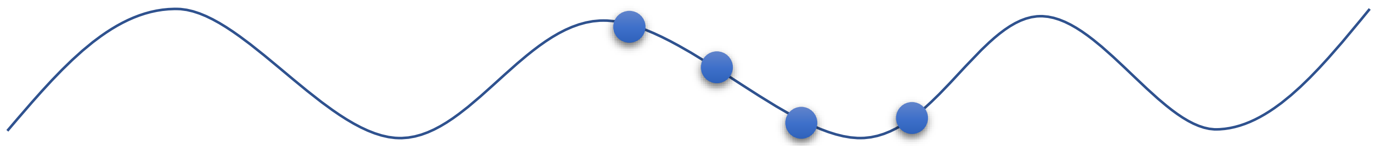

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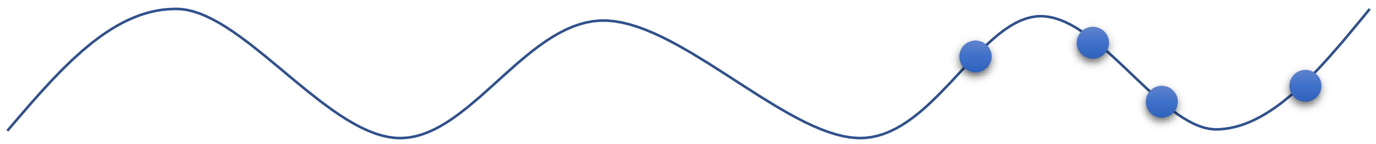

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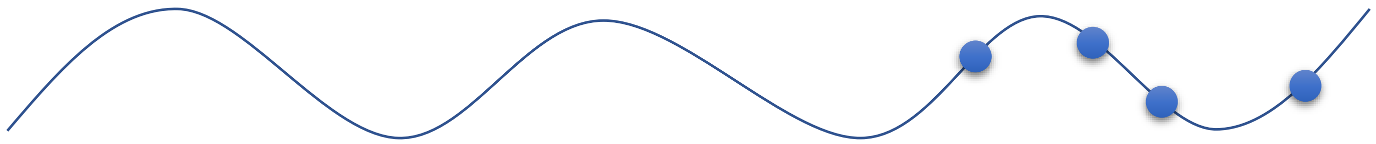

synchronized parallel Q-learning

Correlated samples in online Q-learning

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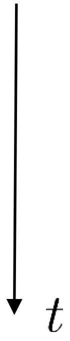

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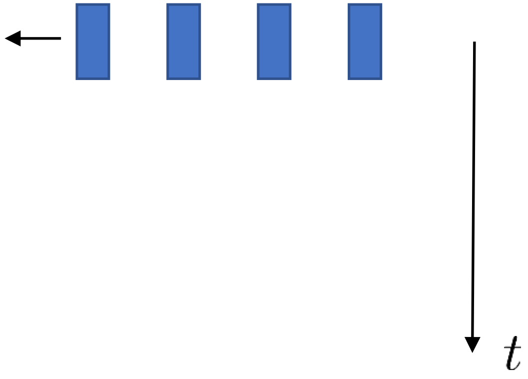
get $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r) \leftarrow$



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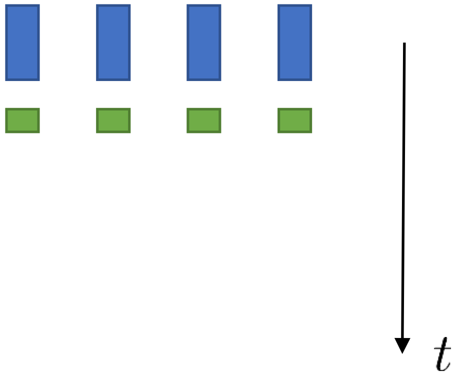

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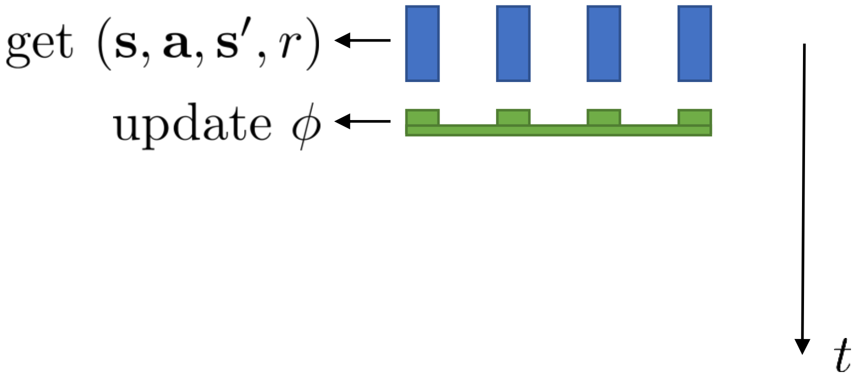

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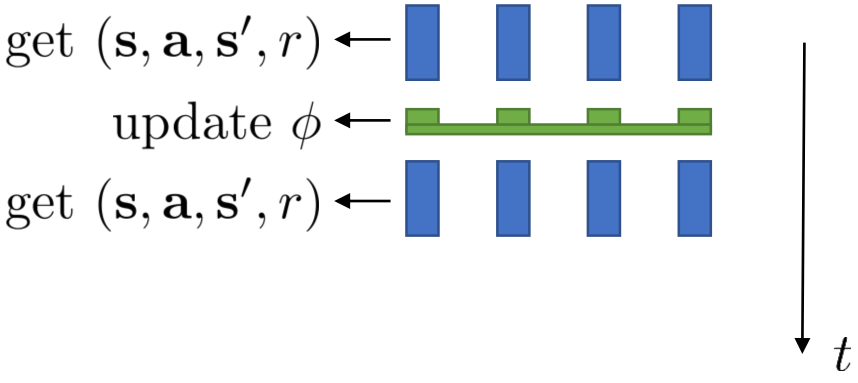

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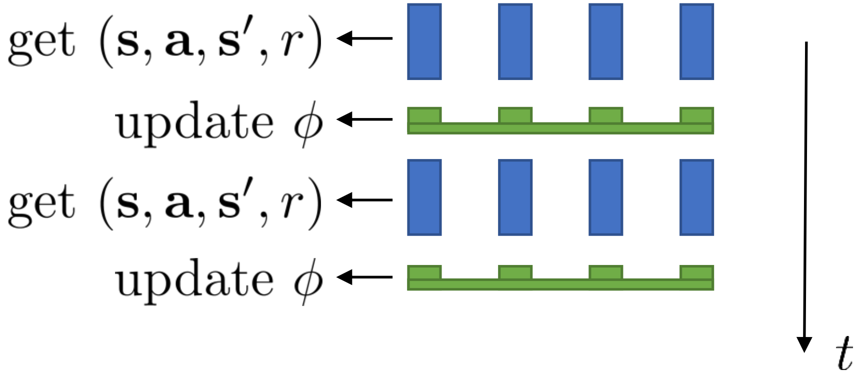

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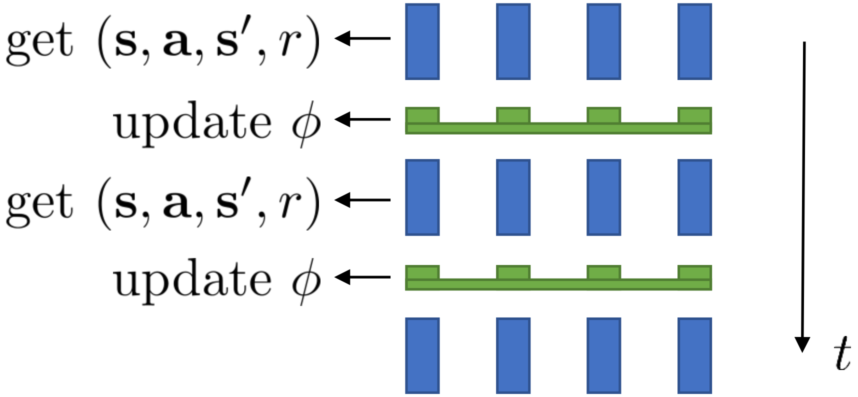

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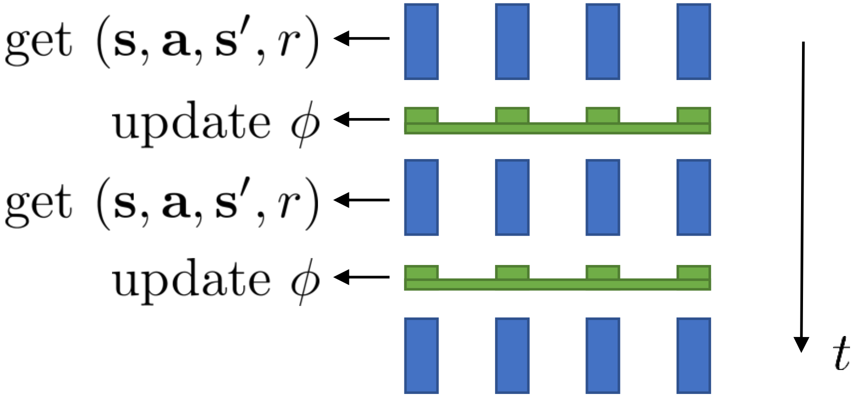
synchronized parallel Q-learning

asynchronous parallel Q-learning



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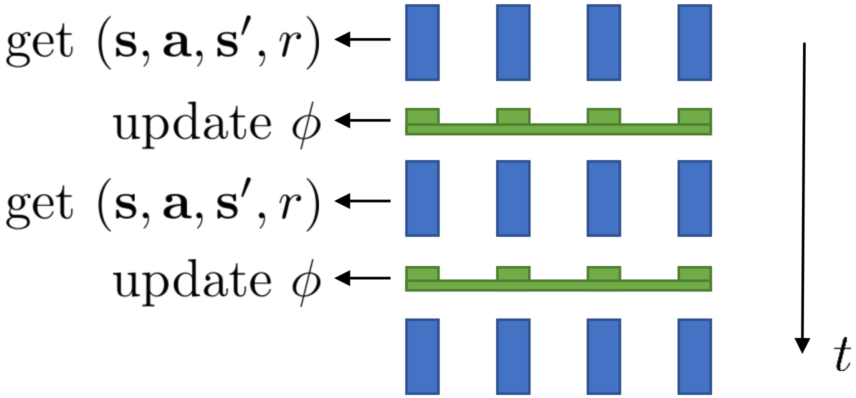

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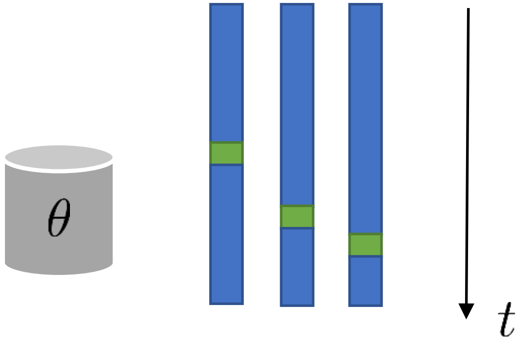

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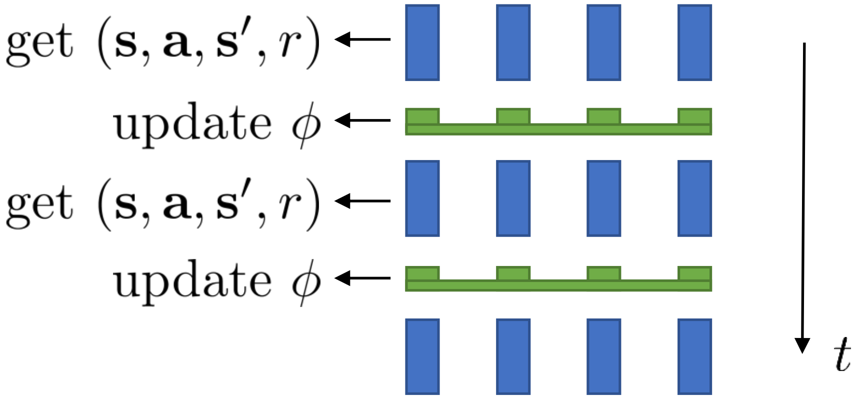

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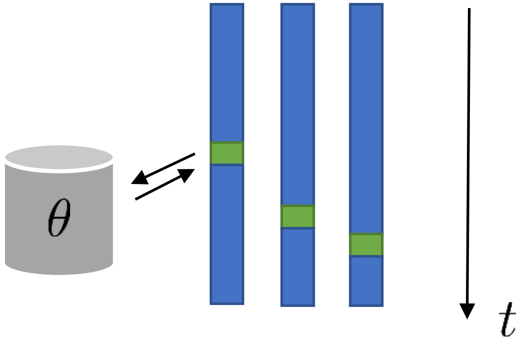

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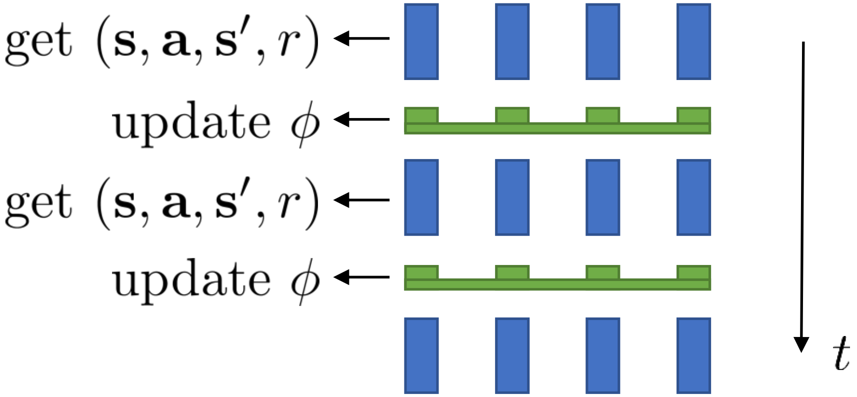

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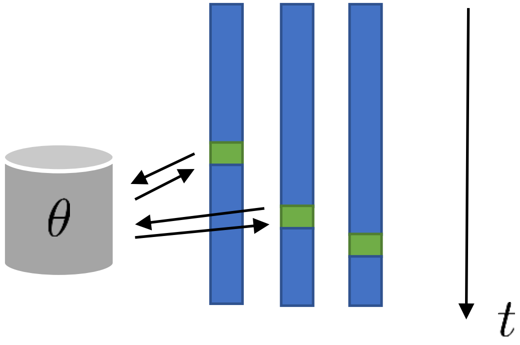

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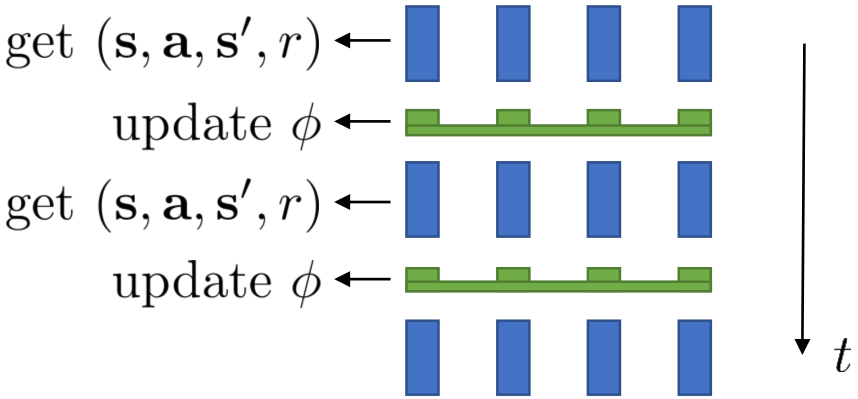

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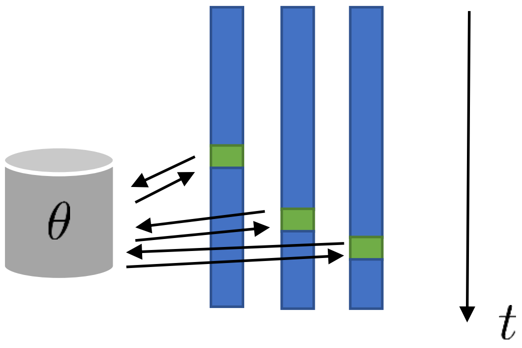

Another solution: replay buffers

online Q iteration algorithm:

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full fitted Q-iteration algorithm:

1. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy
2. set $\mathbf{y}_i \leftarrow r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'_i} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$

$K\times$

3. set $\phi \leftarrow \arg\min_\phi \frac{1}{2} \sum_i \|Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i\|^2$

Another solution: replay buffers

online Q iteration algorithm:

special case with K = 1, and one gradient step

1. take some action $\mathbf{a}_i$ and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$
2. $\phi \leftarrow \phi - \alpha \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)])$

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full fitted Q-iteration algorithm:

1. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy

any policy will work! (with broad support)

$K\times$

2. set $\mathbf{y}_i \leftarrow r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'_i} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$
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full fitted Q-iteration algorithm:

1. ~~collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy~~ **any policy will work! (with broad support)**
2. set $\mathbf{y}_i \leftarrow r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'_i} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$ **just load data from a buffer here**

$K\times$

3. set $\phi \leftarrow \arg\min_\phi \frac{1}{2} \sum_i \|Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i\|^2$

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special case with K = 1, and one gradient step

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$K\times$

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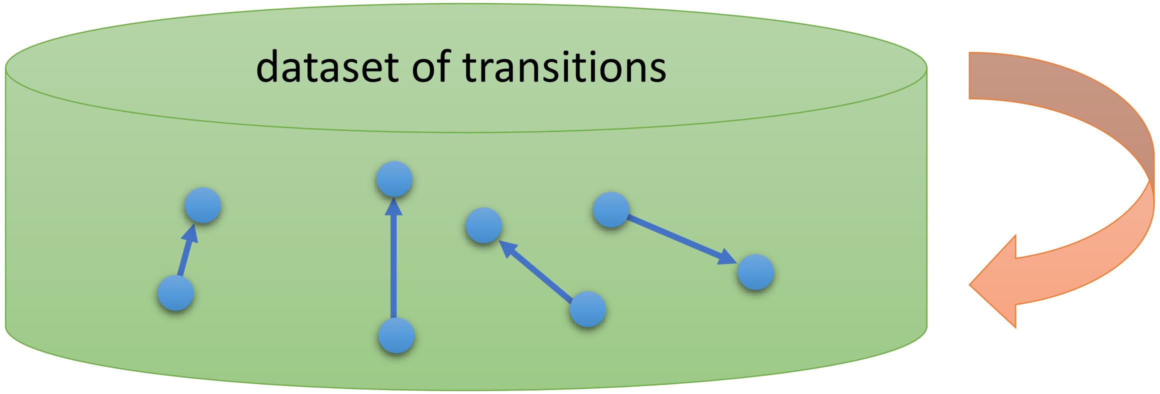


Fitted Q-iteration

Another solution: replay buffers

online Q iteration algorithm:

special case with K = 1, and one gradient step

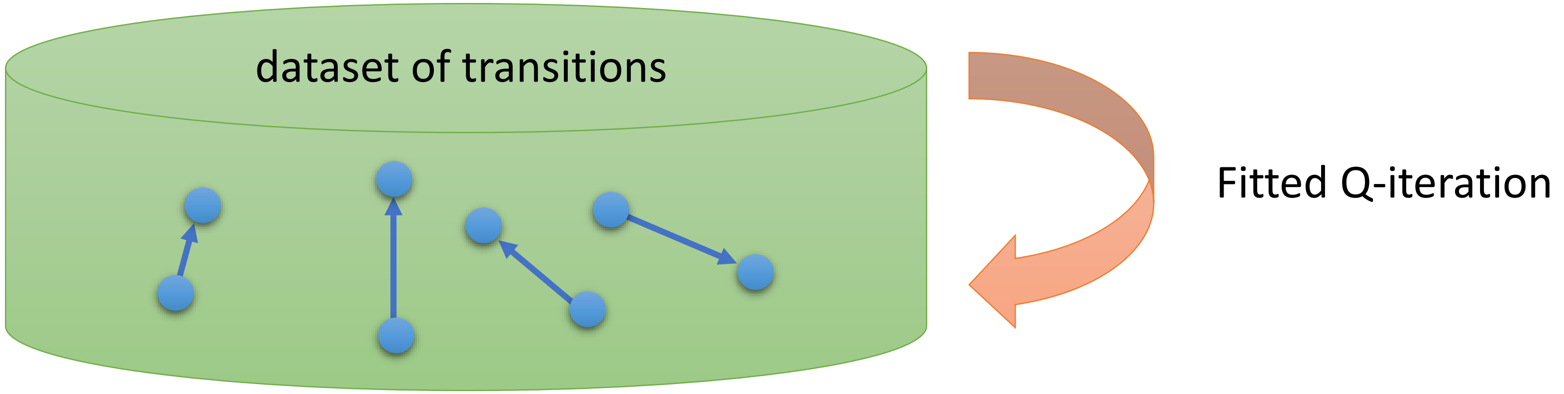
1. take some action $\mathbf{a}_i$ and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$
2. $\phi \leftarrow \phi - \alpha \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)])$

full fitted Q-iteration algorithm:

1. ~~collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy~~ — **any policy will work! (with broad support)**

$K \times$

2. set $\mathbf{y}_i \leftarrow r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'_i} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$ — **just load data from a buffer here**
3. set $\phi \leftarrow \arg\min_\phi \frac{1}{2} \sum_i \|Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i\|^2$ — **still use one gradient step**



Another solution: replay buffers

Q-learning with a replay buffer:

1. sample a batch $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$ from $\mathcal{B}$
2. $\phi \leftarrow \phi - \alpha \sum_i \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)])$

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+ samples are no longer correlated

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+ multiple samples in the batch (low-variance gradient)

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Q-learning with a replay buffer:

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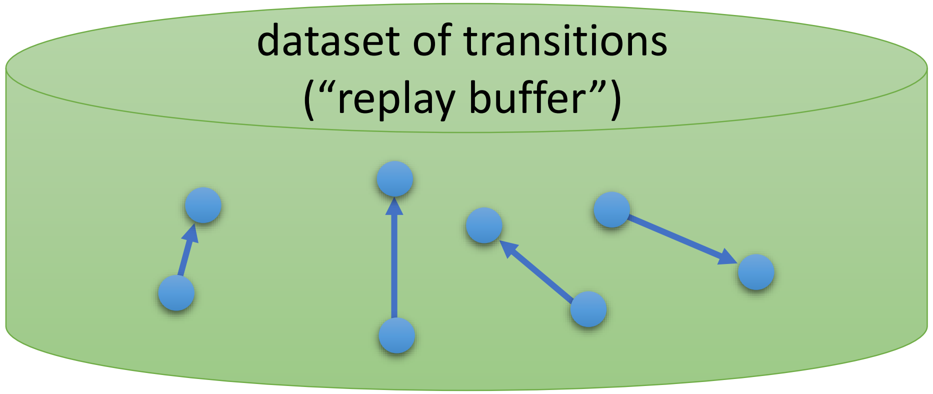

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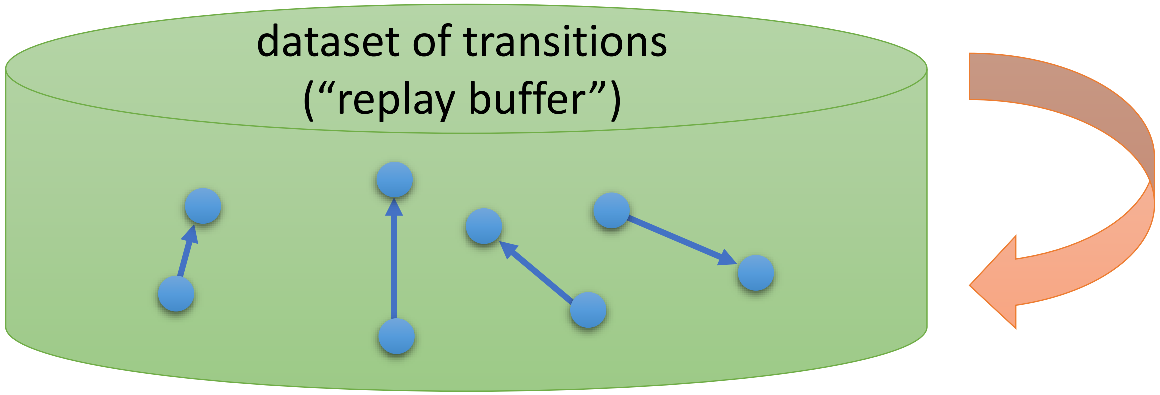

off-policy
Q-learning

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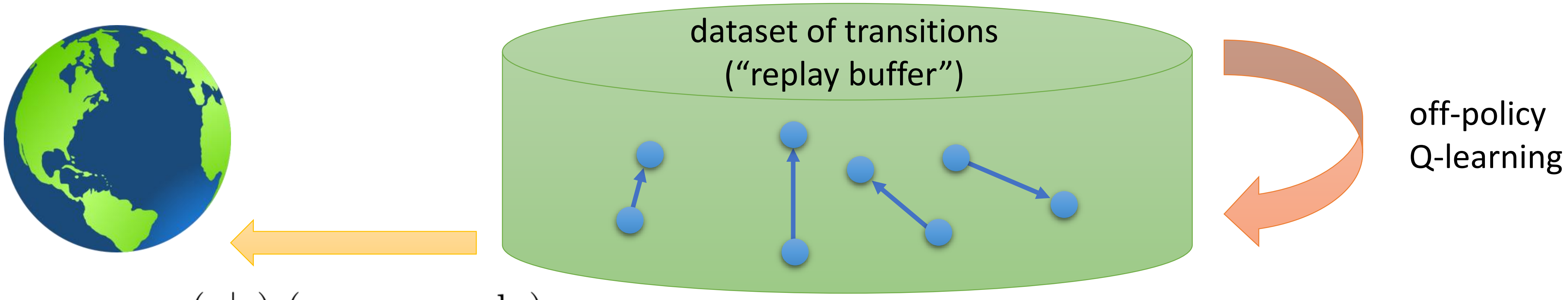

$\pi(\mathbf{a}|\mathbf{s})$ (e.g., ϵ-greedy)

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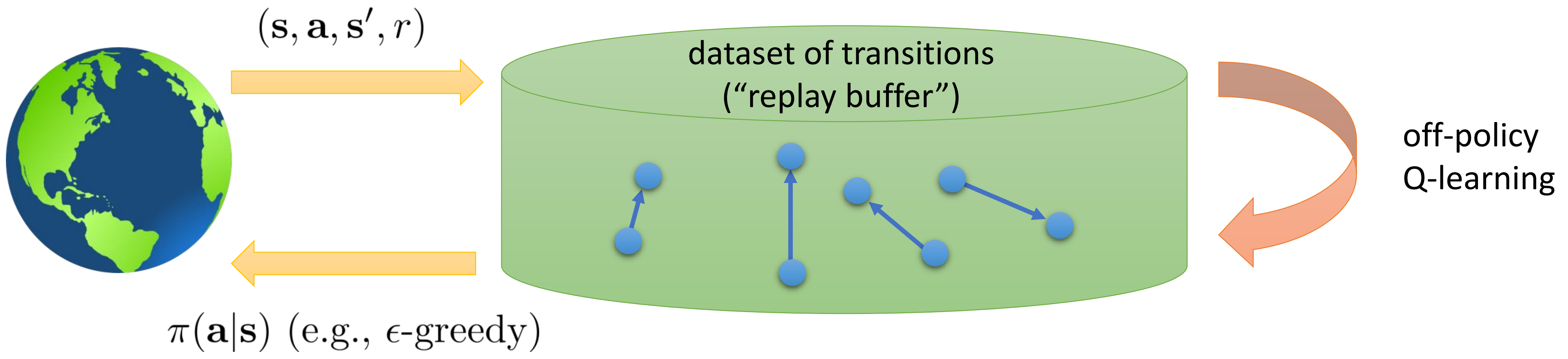
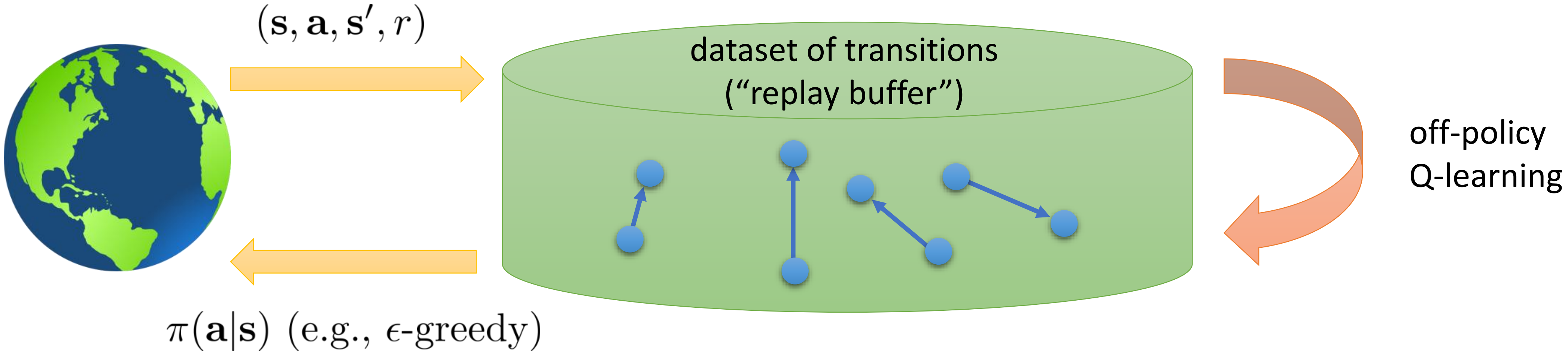


Putting it together

full Q-learning with replay buffer:



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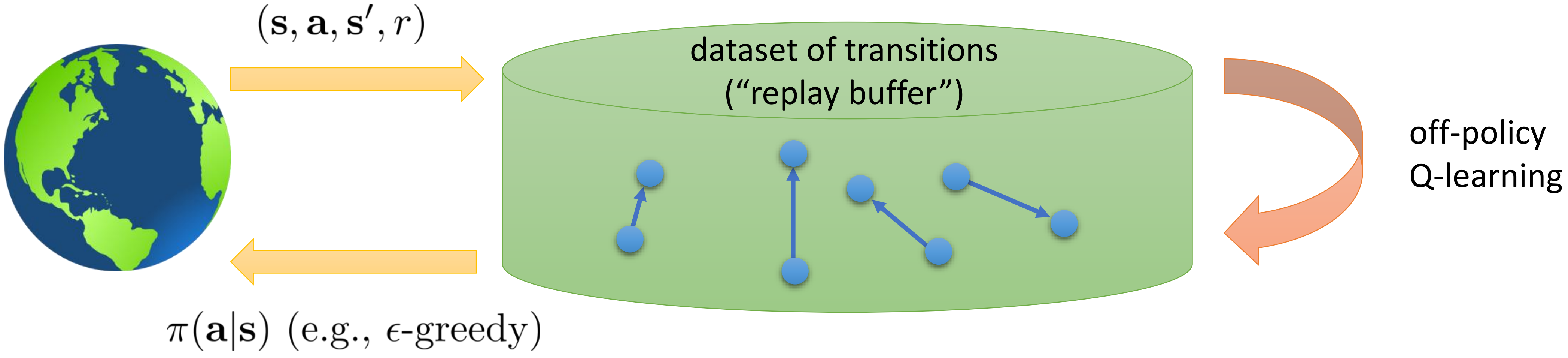
full Q-learning with replay buffer:

1. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy, add it to $\mathcal{B}$



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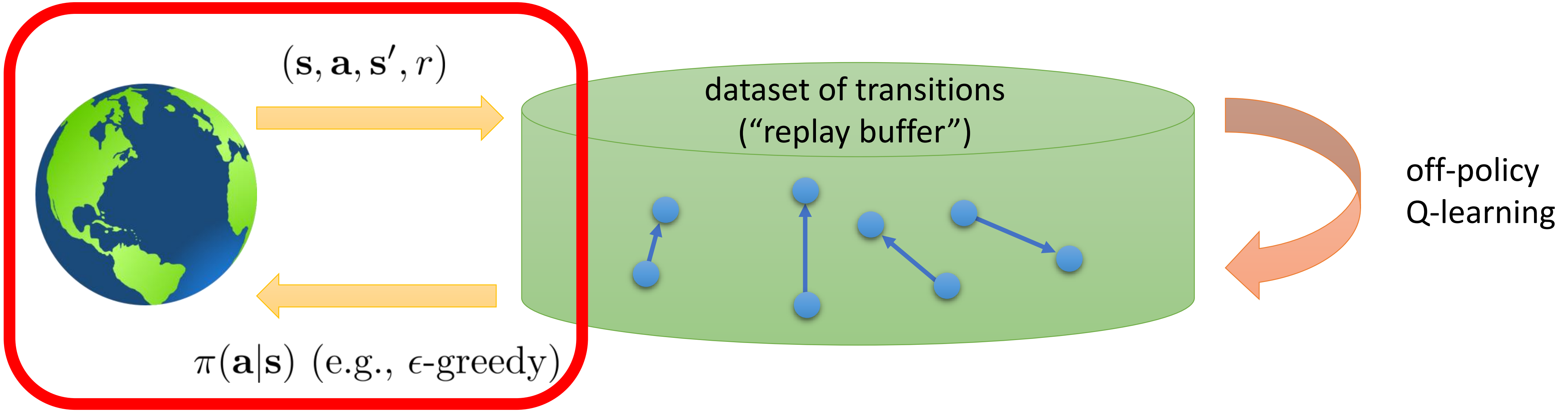

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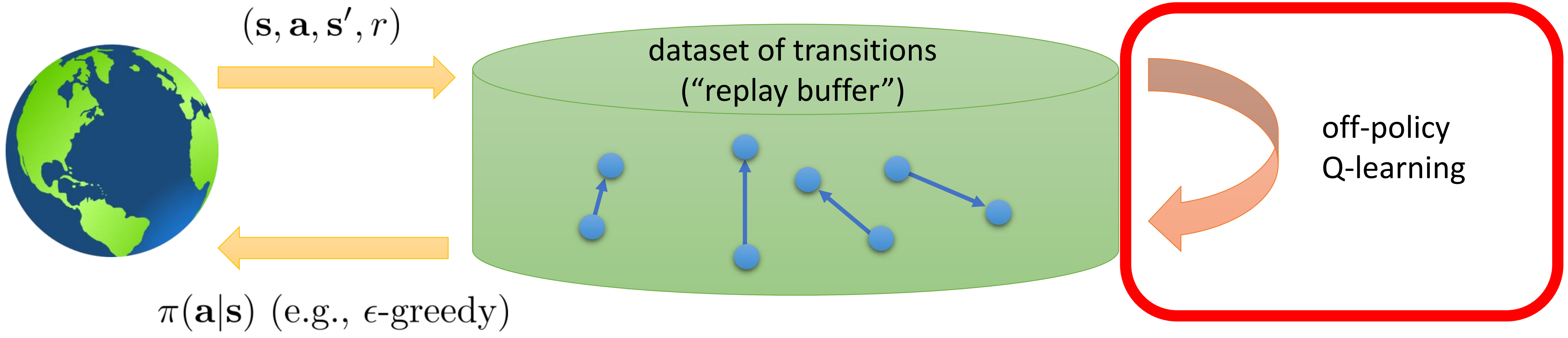

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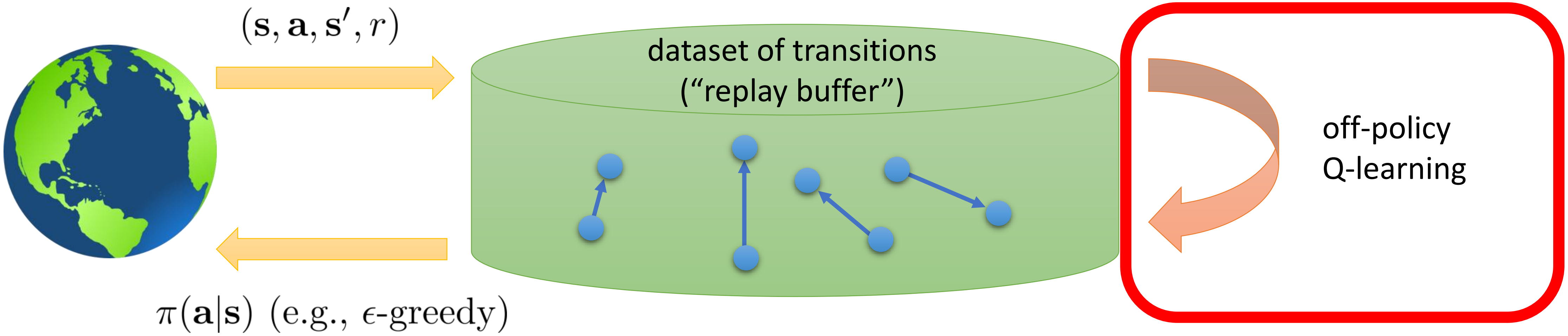

Putting it together

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$K\times$

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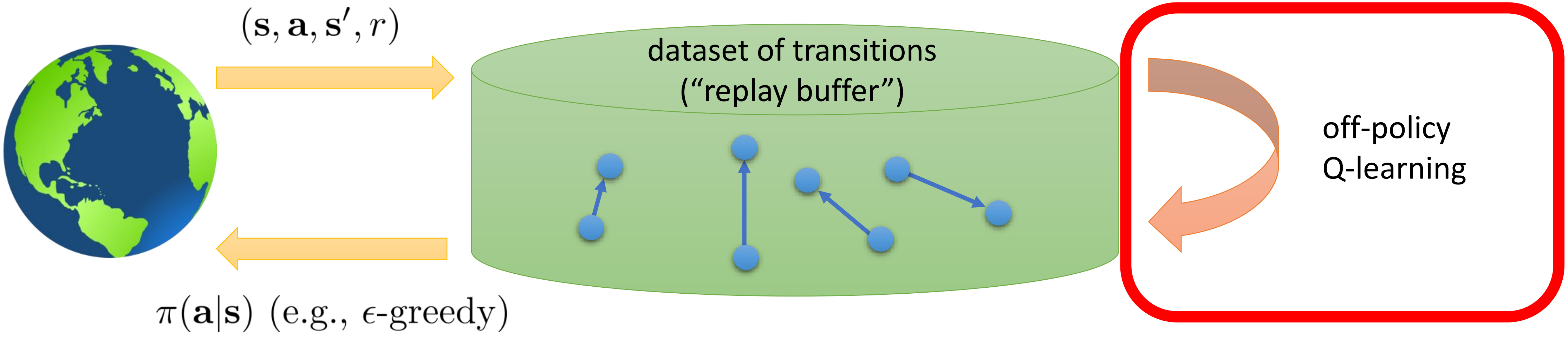

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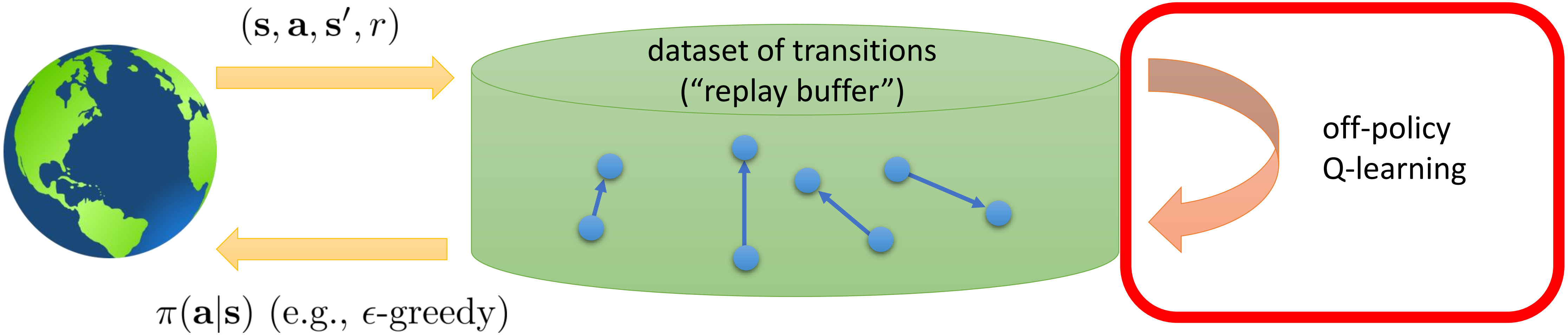
K = 1 is common, though larger K more efficient



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$K\times$

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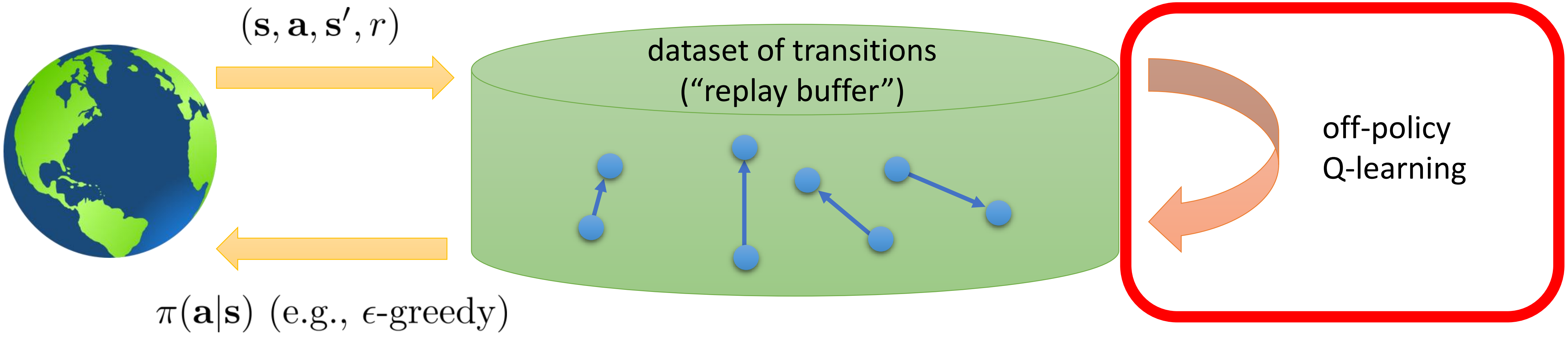

What's wrong?

online Q iteration algorithm:

1. take some action $\mathbf{a}_i$ and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$
2. $\mathbf{y}_i = r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$
3. $\phi \leftarrow \phi - \alpha \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i)$

these are correlated!

Q-learning is *not* gradient descent!

$$\phi \leftarrow \phi - \alpha \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)])$$

no gradient through target value

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use replay buffer

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no gradient through target value

This is a problem!

Q-Learning and Regression

full Q-learning with replay buffer:

1. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy, add it to $\mathcal{B}$

$K \times$

2. sample a batch $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$ from $\mathcal{B}$
3. $\phi \leftarrow \phi - \alpha \sum_i \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)])$

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full fitted Q-iteration algorithm:

1. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy

$K\times$

2. set $\mathbf{y}_i \leftarrow r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'_i} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$
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one gradient step, moving target

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perfectly well-defined, stable regression

Q-Learning with target networks

Q-learning with replay buffer and target network:

Q-Learning with target networks

Q-learning with replay buffer and target network:

2. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy, add it to $\mathcal{B}$

Q-Learning with target networks

Q-learning with replay buffer and target network:

2. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy, add it to $\mathcal{B}$
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Q-Learning with target networks

Q-learning with replay buffer and target network:

1. save target network parameters: $\phi' \leftarrow \phi$

   2. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy, add it to $\mathcal{B}$

$N\times$

      3. sample a batch $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$ from $\mathcal{B}$

   $K\times$

      4. $\phi \leftarrow \phi - \alpha \sum_i \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}'_i, \mathbf{a}'_i)])$

Q-Learning with target networks

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3. sample a batch $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$ from $\mathcal{B}$
4. $\phi \leftarrow \phi - \alpha \sum_i \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \underline{[r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}'_i, \mathbf{a}'_i)]})$

Q-Learning with target networks

Q-learning with replay buffer and target network:

1. save target network parameters: $\phi' \leftarrow \phi$

$N\times$

2. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy, add it to $\mathcal{B}$

$K\times$

3. sample a batch $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$ from $\mathcal{B}$
4. $\phi \leftarrow \phi - \alpha \sum_i \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \underline{[r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}'_i, \mathbf{a}'_i)]})$

targets don't change in inner loop!

Q-Learning with target networks

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targets don't change in inner loop!

supervised regression

“Classic” deep Q-learning algorithm (DQN)

Q-learning with replay buffer and target network:

$N\times$ $K\times$

1. save target network parameters: $\phi' \leftarrow \phi$
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“Classic” deep Q-learning algorithm (DQN)

Q-learning with replay buffer and target network:

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“classic” deep Q-learning algorithm:

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"Classic" deep Q-learning algorithm (DQN)

Q-learning with replay buffer and target network:

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$N\times$ $K\times$

"classic" deep Q-learning algorithm:

1. take some action $\mathbf{a}_i$ and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$, add it to $\mathcal{B}$

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"Classic" deep Q-learning algorithm (DQN)

Q-learning with replay buffer and target network:

$N\times$

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$K\times$

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"classic" deep Q-learning algorithm:

1. take some action $\mathbf{a}_i$ and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$, add it to $\mathcal{B}$
2. sample mini-batch $\{\mathbf{s}_j, \mathbf{a}_j, \mathbf{s}'_j, r_j\}$ from $\mathcal{B}$ uniformly

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“Classic” deep Q-learning algorithm (DQN)

Q-learning with replay buffer and target network:

$N\times$ $K\times$

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3. compute $y_j = r_j + \gamma \max_{\mathbf{a}'_j} Q_{\phi'}(\mathbf{s}'_j, \mathbf{a}'_j)$ using *target* network $Q_{\phi'}$

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“Classic” deep Q-learning algorithm (DQN)

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$N\times$ $K\times$

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4. $\phi \leftarrow \phi - \alpha \sum_j \frac{dQ_\phi}{d\phi}(\mathbf{s}_j, \mathbf{a}_j)(Q_\phi(\mathbf{s}_j, \mathbf{a}_j) - y_j)$

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“Classic” deep Q-learning algorithm (DQN)

Q-learning with replay buffer and target network:

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$N\times$ $K\times$

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5. update ϕ': copy ϕ every N steps

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“Classic” deep Q-learning algorithm (DQN)

Q-learning with replay buffer and target network:

$N\times$

$K\times$

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“Classic” deep Q-learning algorithm (DQN)

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$N\times$ $K\times$

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$K = 1$

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