

MATH 1010 Tutorial Sep. 24th (English III)

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Schedule:

5:30 - 6:05 Tutorial presentation

(5 Questions will be discussed)

6:05 - 6:15 Q & A

(Hosted by: YI, Tianhan)

Remark:

- You can visit yzwang.xyz to download the tutorial notes.
- No tutorial next week. (Oct. 1st)

Q1: Use **interval notation** to indicate the domains of the following functions:

(a) $f(x) = \sqrt{-x^2 + 2x + 3}$

(b) $g(x) = \sqrt[5]{17x^2 - 11x + 1}$

Sol: (a) $-x^2 + 2x + 3 \geq 0$

$$x^2 - 2x - 3 \leq 0$$

$$(x-3)(x+1) \leq 0$$

$$-1 \leq x \leq 3$$

$$x \in [-1, 3]$$



(b) $(-\infty, +\infty)$ or \mathbb{R}

Q2 Given the functions $f(x) = \sqrt{x+6}$ and $g(x) = \frac{x+2}{x+3}$, find the domains of

(a) f, g (b) $f+g$ (c) $\frac{f}{g}, \frac{g}{f}$ (d) $f \circ g, g \circ f$

Sol:

(a) $f: [-6, +\infty)$

$g: (-\infty, -3) \cup (-3, +\infty)$ or $\mathbb{R} - \{-3\}$

(b) $f+g = \sqrt{x+6} + \frac{x+2}{x+3}$
 $x \in [-6, -3) \cup (-3, +\infty)$

(c) $\frac{f}{g} = \frac{\sqrt{x+6}}{\frac{x+2}{x+3}}$ $x \in [-6, -3) \cup (-3, -2) \cup (-2, +\infty)$

$\frac{g}{f} = \frac{\frac{x+2}{x+3}}{\sqrt{x+6}}$ $x \in (-6, -3) \cup (-3, +\infty)$

(d) $f \circ g = \sqrt{\frac{x+2}{x+3} + 6} = \sqrt{\frac{x+2+6x+18}{x+3}} = \sqrt{\frac{7x+20}{x+3}}$

$\frac{7x+20}{x+3} \geq 0$ and $x+3 \neq 0$

$\begin{cases} 7x+20 \geq 0 \\ x+3 > 0 \end{cases}$

or $\begin{cases} 7x+20 \leq 0 \\ x+3 < 0 \end{cases}$

$x \in [-\frac{20}{7}, +\infty)$

$x \in (-\infty, -3)$

$(-\infty, -3) \cup [-\frac{20}{7}, +\infty)$

$$g \circ f = \frac{\sqrt{x+6} + 2}{\sqrt{x+6} + 3}$$

$$x+6 \geq 0 \text{ and } \sqrt{x+6} + 3 \neq 0$$

$$\Rightarrow x \geq -6$$

$$\Rightarrow x \in [-6, +\infty)$$

Q3. Find the inverse function to

$$y = f(x) = \frac{3x+2}{6x-7}$$

Sol: $x = g(y)$ $g?$

$$y(6x-7) = 3x+2$$

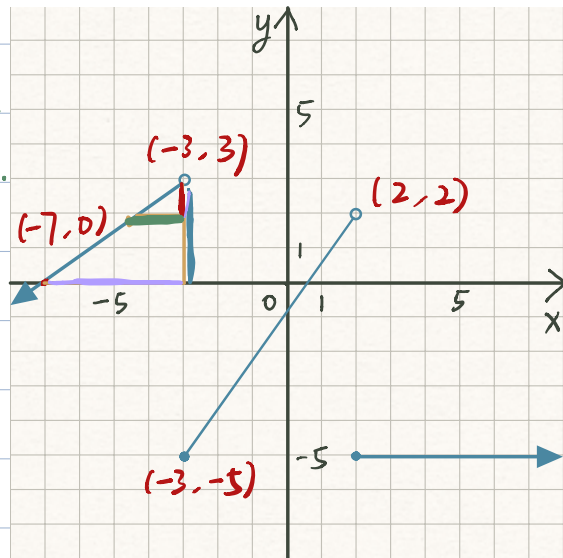
$$6xy - 7y = 3x + 2$$

$$6xy - 3x = 7y + 2$$

$$x(6y-3) = 7y+2$$

$$x = g(y) = \frac{7y+2}{6y-3}$$

Q4. Find the equation of the piecewise function $f(x)$ depicted in the graph.



Sol: ① $x < -3$

$$\frac{y-3}{x-(-3)} = \frac{3-0}{-3-(-7)}$$

$$y-3 = \frac{3}{4}(x+3)$$

$$y = \frac{3}{4}x + \frac{9}{4} + 3$$

$$y = \frac{3}{4}x + \frac{21}{4}$$

② $-3 \leq x < 2$

$$\frac{y-2}{x-2} = \frac{2-(-5)}{2-(-3)}$$

$$y-2 = \frac{7}{5}(x-2)$$

$$\begin{aligned} y &= \frac{7}{5}x - \frac{14}{5} + \frac{10}{5} \\ &= \frac{7}{5}x - \frac{4}{5} \end{aligned}$$

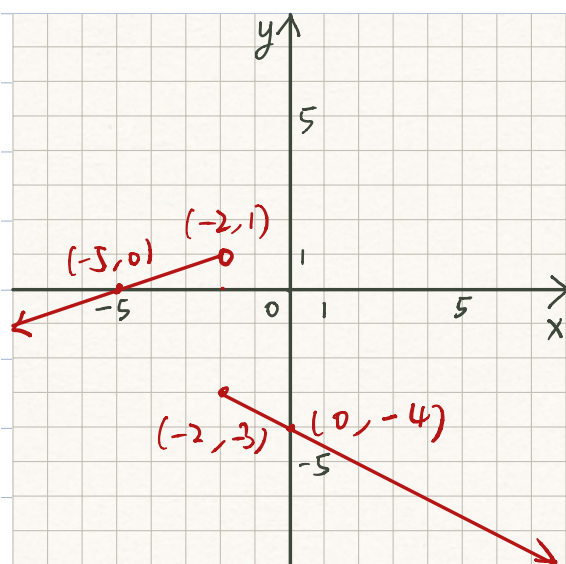
③ $x \geq 2$

$$y = -5$$

$$y = f(x) = \begin{cases} \frac{3}{4}x + \frac{21}{4} & \text{if } x < -3 \\ \frac{7}{5}x - \frac{4}{5} & \text{if } -3 \leq x < 2 \\ -5 & \text{if } x \geq 2 \end{cases}$$

Q5. Sketch the graph of the following piecewise function =

$$f(x) = \begin{cases} \frac{1}{3}x + \frac{5}{3} & x < -2 \\ -\frac{1}{2}x - 4 & x \geq -2 \end{cases}$$



$$y = \frac{1}{3}x + \frac{5}{3}$$

$$\text{Let } x = -2, \quad y = 1$$

$$\text{Let } x = -5, \quad y = 0$$

$$y = -\frac{1}{2}x - 4$$

$$\text{Let } x = -2, \quad y = 1 - 4 = -3$$

$$\text{Let } x = 0, \quad y = -4$$