22920192204097 吴雨娟



一、

解：(1) 由 $\vec{r}=R(\cos\omega t\,\vec{i}+\sin\omega t\,\vec{j})$ 可得

$x=R\cos\omega t$，$y=R\sin\omega t$

消去 $t$ 得

$x^2+y^2=R^2$

∴ 质点的轨道方程为 $x^2+y^2=R^2$

(2) $\vec{v}=\frac{d\vec{r}}{dt}=-\omega R\sin\omega t\,\vec{i}+\omega R\cos\omega t\,\vec{j}$

$v=\sqrt{(-\omega R\sin\omega t)^2+(\omega R\cos\omega t)^2}$

$=\omega R$

∴ 质点的速度为 $\vec{v}=-\omega R\sin\omega t\,\vec{i}+\omega R\cos\omega t\,\vec{j}$

速率为 $\omega R$.

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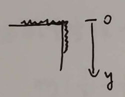
二、解：(1) $\lambda b g = \mu_0 \lambda (l-b) g$

$$b = \frac{\mu_0 l}{1+\mu_0}$$

∴下垂长度b不能超过 $\frac{\mu_0 l}{1+\mu_0}$

(2) $\cancel{F_f = \mu\lambda(l-y)}$ 以桌水平桌面为势能零点.



如图，取竖直向下为y轴正方向，得.

$$\cancel{dF_f = \mu\lambda(l-y)g\,dy} \quad F_f = \mu\lambda(l-y)g$$

摩擦力做功 $W = -\int_b^l \mu\lambda(l-y)g\,dy$

$$= -\left(\frac{\mu\lambda l^2 g}{2} - \mu\lambda l g b + \frac{\mu\lambda g b^2}{2}\right)$$

$$= -\frac{\mu\lambda l^2 g(1+2\mu_0)^2}{2(1+\mu_0)^2}$$

由能量守恒得：

$$\lambda l g \frac{l}{2} - \lambda b g \frac{b}{2} = \frac{1}{2}\lambda l v^2 + \overset{(-W)}{\cancel{\frac{\mu\lambda l^2 g(1+2\mu_0)^2}{2(1+\mu_0)^2}}}$$

$$\frac{1}{2}\lambda l v^2 = \frac{\lambda g l^2 (1+2\mu_0)(1-\mu(1+2\mu_0))}{2(1+\mu_0)^2}$$

$$v = \frac{\sqrt{gl(1+2\mu_0)(1-\mu(1+2\mu_0))}}{1+\mu_0}$$

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三、解：(1) 当它与铅直线成60°角时

$$M = mg \times \frac{1}{2}\cos 30^\circ = \frac{\sqrt{3}}{4} mgl$$

$$M = J\alpha$$

$$J = \frac{1}{3} ml^2$$

$$\therefore \alpha = \frac{3\sqrt{3}g}{4l}$$

由能量守恒得:

$$mg\left(\frac{l}{2} - \frac{l}{2}\cos 60^\circ\right) = \frac{1}{2} J w^2$$

$$J = \frac{1}{3} ml^2$$

$$\therefore w = \sqrt{\frac{3g}{2l}}$$

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四、解：设氦气一边的压强为$p_1$，体积为$V_1$，温度为$T_1$，物质的量为$\frac{m_1}{M_1}$，

氧气一边压强为$p_2$，体积为$V_2$，温度为$T_2$，物质的量为$\frac{m_2}{M_2}$

由已知得：$V_1=V_2$，$p_1=p_2$，$T_1=250K$，$T_2=625K$.

∴ $p_1V_1=\frac{m_1}{M_1}RT_1$

$p_2V_2=\frac{m_2}{M_2}RT_2$

$$\left(\frac{m_1}{M_1}RT_1+\frac{m_2}{M_2}RT_2\right)=\left(\frac{m_1}{M_1}+\frac{m_2}{M_2}\right)RT$$

$$T=\frac{2500}{7}K.$$

∴ 去掉隔板后两种气体混合后的温度为$\frac{2500}{7}K$.

五、解：(1) $\int_0^{\infty} f(v)\,dv$

$= \frac{1}{2}\times v_0 \times a \times 2 + v_0 \times a = 1$

$\therefore a = \frac{1}{2v_0}$

(2) ~~$N_1 = N\frac{1}{4}$~~

$$f(v) = \begin{cases} \frac{1}{2v_0^2}v & 0 < v < v_0 \\ \frac{1}{2v_0} & v_0 \le v < 2v_0 \\ -\frac{1}{2v_0^2}v + \frac{3}{2v_0} & 2v_0 \le v < 3v_0 \\ 0 & v \geqslant 3v_0 \end{cases}$$

$\therefore N_1 = N\int_{1.5v_0}^{2v_0} f(v)\,dv$

$= N\int_{1.5v_0}^{2v_0} \frac{1}{2v_0}\,dv$

$= \frac{1}{4}N$

$\therefore$ 速率在 $1.5v_0$ 和 $2v_0$ 之间的粒子数为 $\frac{1}{4}N$.

(3) $\bar{v} = \int_0^{\infty} v f(v)\,dv$

$= \int_0^{v_0} \frac{1}{2v_0^2} v^2\,dv + \int_{v_0}^{2v_0} \frac{v}{2v_0}\,dv$

$+ \int_{2v_0}^{3v_0} \left(-\frac{v^2}{2v_0^2} + \frac{3v}{2v_0}\right) dv$

$= \frac{v_0}{6} + \frac{3v_0}{4} + \frac{7v_0}{12}$

$= \frac{3v_0}{2}$

$\therefore$ 粒子的平均速率为 $\frac{3v_0}{2}$

(4) $\bar{v}_1 = \dfrac{\int_0^{1.5v_0} v\,dN}{N\int_0^{1.5v_0} f(v)\,dv}$

$= \dfrac{N\int_0^{1.5v_0} v f(v)\,dv}{N\int_0^{1.5v_0} f(v)\,dv}$

$\int_0^{1.5v_0} f(v)\,dv$

$= \int_0^{v_0} f(v)\,dv + \int_{v_0}^{1.5v_0} f(v)\,dv$

$= \frac{1}{2}$

$\int_0^{1.5v_0} v f(v)\,dv$

$= \int_0^{v_0} f(v) v\,dv + \int_{v_0}^{1.5v_0} f(v) v\,dv$

$= \frac{v_0}{6} + \frac{5v_0}{16}$

$= \frac{23v_0}{48}$

$\therefore \bar{v}_1 = \dfrac{\frac{23v_0}{48}}{\frac{1}{2}} = \frac{23}{24}v_0$

$\therefore$ 0到 $1.5v_0$ 之间的分子的平均速率为 $\frac{23}{24}v_0$.

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六、解:(1) $l_0 = 4.3 \times 10^{16}\,m$，$v = 0.8c = 0.8 \times 3 \times 10^8\,m/s$

$$l = l_0\sqrt{1 - v^2/c^2} = \frac{3}{5}l_0 = 2.580 \times 10^{16}\,m.$$

∴飞船中的观察者测得地球和该星之间的距离为 $2.580 \times 10^{16}\,m$.

(2) $t_1 = \dfrac{2l_0}{v} \approx 3.583 \times 10^8\,s$

∴按地球上的钟计算，飞船往返一次需 $3.583 \times 10^8\,s$

(3) $t_2 = \dfrac{2l}{v} = 2.150 \times 10^8\,s$

∴按飞船上的钟计算，飞船往返一次需 $2.150 \times 10^8\,s$.

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七. 解: (1) $a_y = -k v_y^2$

$$\frac{dv_y}{dt} = -k v_y^2$$

$$-\frac{1}{v_y^2} dv_y = k\,dt$$

$$\cancel{\therefore \int_0^v}$$

$$\therefore \int_{v_0}^{v} -\frac{1}{v_y^2} dv_y = \int_0^t k\,dt$$

$$\frac{1}{v} - \frac{1}{v_0} = kt$$

$$v = \frac{v_0}{1+kv_0 t}$$

(2)

$$\cancel{a_y = -k}$$

$$a_y = -kv_y^2$$

$$\frac{dv_y}{dt} = -kv_y^2$$

$$\frac{dv_y}{dt} = \frac{dv_y}{dy}\cdot\frac{dy}{dt} = v_y\frac{dv_y}{dy}$$

$$\therefore v_y\frac{dv_y}{dy} = -kv_y^2$$

$$\frac{dv_y}{dy} = -kv_y$$

$$\frac{1}{v_y} dv_y = -k\,dy$$

$$\int_{v_0}^{\frac{v_0}{10}} \frac{1}{v_y} dv_y = \int_0^y -k\,dy$$

$$\ln\frac{v_0}{10} - \ln v_0 = -ky$$

$$y = \frac{\ln 10}{k}$$

$\therefore$ 运动员入水的深度为 $\frac{\ln 10}{k}$ m.