逐次加倍的FFT算法:

令 $W_N = e^{-j2\pi/N}$，为N次单位根 $(W_N^N = e^{-j2\pi} = 1)$

设 $N = 2^\nu$，ν 为正整数

由(3')式定义可得:

DFT: $F_n = \frac{1}{N}\sum_{i=0}^{N-1} f(i)\, W_N^{ni}$ (7) $n = 0, 1, 2, \cdots, N-1$

记 $N = 2M$，M为正整数

$$F_n = \frac{1}{2M}\sum_{i=0}^{2M-1} f_i \cdot W_{2M}^{ni}$$

← 偶下标 ← 奇下标

$$= \frac{1}{2}\left\{\frac{1}{M}\sum_{i=0}^{M-1} f_{2i}\, W_{2M}^{n2i} + \frac{1}{M}\sum_{i=0}^{M-1} f_{2i+1}\, W_{2M}^{n(2i+1)}\right\}$$

易证 $W_{2M}^{n2i} = W_M^{ni}$，故有

前一半

$$F_n = \frac{1}{2}\left\{\frac{1}{M}\sum_{i=0}^{M-1} f_{2i}\, W_M^{ni} + \frac{1}{M}\sum_{i=0}^{M-1} f_{2i+1}\, W_M^{ni}\cdot W_{2M}^{n}\right\} = \frac{1}{2}\left\{F_n^{\text{偶}} + F_n^{\text{奇}}\cdot W_{2M}^{n}\right\}$$

定义: $F_n^{\text{偶}} = \frac{1}{M}\sum_{i=0}^{M-1} f_{2i}\, W_M^{ni}$，$F_n^{\text{奇}} = \frac{1}{M}\sum_{i=0}^{M-1} f_{2i+1}\, W_M^{ni}$ $n = 0, 1, 2, \cdots, M-1$

$n = M + m$

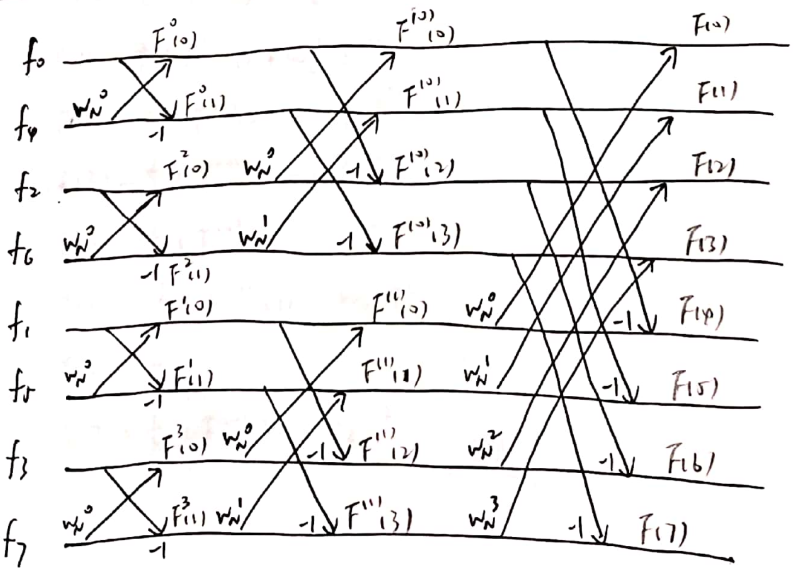
又∵ $W_M^{n+M} = [e^{-j2\pi/M}]^{n+M} = W_M^n\, e^{-j2\pi} = W_M^n$

$$W_{2M}^{n+M} = [e^{-j2\pi/2M}]^{n+M} = -W_{2M}^n$$

后一半

∴ $F_{n+M} = \frac{1}{2}\left\{F_n^{\text{偶}} - F_n^{\text{奇}}\cdot W_{2M}^{n}\right\}$ $n = 0, 1, \cdots, M-1$

$$= \frac{1}{2}\left\{F_{n+M}^{\text{偶}} + F_{n+M}^{\text{奇}}\cdot W_{2M}^{n+M}\right\}$$



8点FFT信号计算流图.

第一步: $N=2$

$$F^0(0)=\frac{1}{2}(f_0+W_2^0f_4)$$
$$F^0(1)=\frac{1}{2}(f_0-W_2^0f_4)$$
$$F^2(0)=\frac{1}{2}(f_2+W_2^0f_6)$$
$$F^2(1)=\frac{1}{2}(f_2-W_2^0f_6)$$
$$F^1(0)=\frac{1}{2}(f_1+W_2^0f_5)$$
$$F^1(1)=\frac{1}{2}(f_1-W_2^0f_5)$$
$$F^2(0)=\frac{1}{2}(f_3+W_2^0f_7)$$
$$F^2(1)=\frac{1}{2}(f_3-W_2^0f_7)$$

第二步: $N=4$

$$F^{(0)}(0)=\frac{1}{2}(F^0(0)+W_4^0F^{(2)}(0))$$
$$F^{(0)}(1)=\frac{1}{2}(F^{(0)}(1)+W_4^1F^{(2)}(1))$$
$$F^{(0)}(2)=\frac{1}{2}(F^{(0)}(0)-W_4F^{(2)}(0))$$
$$F^{(0)}(3)=\frac{1}{2}(F^{(0)}(1)-W_4^1F^{(2)}(1))$$

$$F^{(1)}(0)=\frac{1}{2}(F^{(1)}(0)+W_4^0F^{(3)}(0))$$
$$F^{(1)}(1)=\frac{1}{2}(F^{(1)}(1)+W_4^1F^{(3)}(1))$$
$$F^{(1)}(2)=\frac{1}{2}(F^{(1)}(0)-W_4^0F^{(3)}(0))$$
$$F^{(1)}(3)=\frac{1}{2}(F^{(1)}(1)-W_4^1F^{(3)}(1))$$

第三步: $N=8$

$$F(0)=\frac{1}{2}(F^{(0)}(0)+W_8^0F^{(1)}(0))$$
$$F(1)=\frac{1}{2}(F^{(0)}(1)+W_8^1F^{(1)}(1))$$
$$F(2)=\frac{1}{2}(F^{(0)}(2)+W_8^2F^{(1)}(2))$$
$$F(3)=\frac{1}{2}(F^{(0)}(3)+W_8^3F^{(1)}(3))$$
$$F(4)=\frac{1}{2}(F^{(0)}(0)-W_8^0F^{(1)}(0))$$
$$F(5)=\frac{1}{2}(F^{(0)}(1)-W_8^1F^{(1)}(1))$$
$$F(6)=\frac{1}{2}(F^{(0)}(2)-W_8^2F^{(1)}(2))$$
$$F(7)=\frac{1}{2}(F^{(0)}(3)-W_8^3F^{(1)}(3))$$