LINKÖPINGS UNIVERSITET
Department of Computer and
Information Science
Division of Statistics and Machine learning/ANd

732A66 Decision theory
Fall semester 2023
Assignment 1

# Assignment 1

Below are two tasks that you shall try to solve. All questions put should be answered. Prepare your solutions in a nice format that can be easily read. You are allowed to help each other but there must be individual submissions (that are not just copies of one submission).

Your solutions should be submitted at latest on **Friday 13 October 2023**.

1. A bank official is concerned about the rate at which the bank's tellers provide service for their customers. He feels that all of the tellers work at about the same speed, which is either 30, 40 or 50 customers per hour. Furthermore, 40 customers per hour is twice as likely as each of the two other values, which are assumed to be equally likely. In order to obtain more information, the official observes all five tellers for a two-hour period, noting that 380 customers are served during that period.

   Show that the posterior probabilities for the three possible speeds are approximately 0.000045, 0.99996 and 0.00000012 respectively.

   (*Hint:* The total amount of 380 customers cannot be equalized with an average amount of customers per teller and hour. Using formal calculations instead of simulating data, it is easier to confirm the posterior probabilities. The formal calculations may very well be made using R.)

2. Assume that data follows a beta distribution with parameters *a* and *b*, i.e. the probability density function is $f(y|a,b) = y^{a-1}(1-y)^{b-1}/B(a,b)$ where $B(a,b)$ is the beta function also equal to $\Gamma(a)\Gamma(b)/\Gamma(a+b)$. This could be the case when data consists of the analysed purity (in percent) of a narcotic substance in a number of seized packages.

   a) Show – *by using the properties of distributions belonging to the exponential class* – that the probability density function of a conjugate family to the family of beta distributions can be expressed as

   $$f'(a,b|\gamma,\delta,\theta) \propto \frac{e^{\gamma a+\delta b}}{\left(B(a,b)\right)^{\theta}}$$

   b) Find the corresponding expression for the probability density function of the posterior distribution when a sample of size *n* has been obtained.