3(b)

Attachment 2

$$f(\sigma^2) \propto L(\sigma^2|Data) \cdot f'(\sigma^2)$$

$$\propto \frac{1}{\sigma^3} \cdot e^{\frac{-3.421 \cdot 10^{-5}}{\sigma^2}} \cdot f'(\sigma^2)$$

$$\propto \frac{1}{\sigma^3} \cdot e^{\frac{-3.421 \cdot 10^{-5}}{\sigma^2}} \cdot 0.114 \cdot \sigma^{-2}$$

$$\propto \sigma^{-5} \cdot e^{\frac{-3.421 \cdot 10^{-5}}{\sigma^2}}$$

$$\propto (\sigma^2)^{-2.5} \cdot e^{-3.421 \cdot 10^{-5}/\sigma^2}$$

$$\alpha^* + 1 = 2.5 \Rightarrow \alpha^* = 1.5$$

$$\beta^* = 3.421 \cdot 10^{-5}$$

$$\text{Mode} = \frac{3.421 \cdot 10^{-5}}{1.5+1} = 1.3684 \times 10^{-5}$$

1(b) $$\mu_{posterior} = \frac{\sigma^2}{n\sigma_0^2+\sigma^2}\mu_0 + \frac{n\sigma_0^2}{n\sigma_0^2+\sigma^2}\bar{X}$$

$$\frac{1}{\sigma^2_{posterior}} = \frac{n}{\sigma^2} + \frac{1}{\sigma_0^2}$$

$\sigma = 12000$, $\sigma_0 = 9000$, $n = 6$, miu_0 = 115000, x_bar = 121000

$\Rightarrow$ $\mu_{posterior} = 119628.571$ $\sigma_{posterior} = 4302.823$

So, cost $\sim N(119628.571, \sqrt{4302.823^2 + 12000^2})$

propability <- pnorm(120000, 119628.571, $\sqrt{4302.823^2+12000^2}$)

$$\text{posterior odds} = \frac{1-\text{probability}}{\text{probability}} = \frac{1-0.5116}{0.5116} \approx 0.9545682$$

$$B = \frac{\text{posterior odds}}{\text{prior odds}} = 1.631791 \approx 1.63$$