Assignment 3.2

(a) H_0 : 70% red 30% blue

H_1 : 70% blue 30% red

✓ +5 ✗ −3

prior probability $P(H_0)=0.4$. $p(H_1)=0.6$

The rest of details are in Attachment 1

	H_0	H_1
pick H_0	5	−3
pick H_1	−3	5

$E(\text{pick } H_0) = 5\cdot0.4 - 0.6\cdot3 = 2-1.8 = 0.2$

$E(\text{pick } H_1) = -3\cdot0.4 + 5\cdot0.6 = 1.8$ ✓

a' is accept H_1

Assignment 3.3

(a)

Zero-one loss : $\hat{\theta}_B(x)$ is the posterior mode of θ^2 given x.

we need to get the posterior probability

$$f^{y}(\sigma^2) \propto L(\sigma^2 | \text{Data}) \cdot f'(\sigma^2)$$

$$\propto \frac{1}{(\sigma\sqrt{2\pi})^3} \cdot e^{-\frac{1}{2}\left(\frac{x_1-\mu}{\sigma}\right)^2 - \frac{1}{2}\left(\frac{x_2-\mu}{\sigma}\right)^2 - \frac{1}{2}\left(\frac{x_3-\mu}{\sigma}\right)^2} \cdot f'(\sigma^2)$$

$$\propto \frac{1}{(\sigma\sqrt{2\pi})^3} \cdot e^{-\frac{1}{2}\left(\frac{(x_1-\mu)^2+(x_2-\mu)^2+(x_3-\mu)^2}{\sigma^2}\right)} \cdot f'(\sigma^2)$$

$$\propto \frac{1}{\sigma^3} \cdot e^{-\frac{1}{2}\left(\frac{6842\times10^{-5}}{\sigma^2}\right)} \cdot f'(\sigma^2)$$

$$\propto \frac{1}{\sigma^3} \cdot e^{\frac{-3.421\cdot10^{-5}}{\sigma^2}} \cdot f'(\sigma^2)$$

$$\propto \sigma^{-3} \cdot e^{-3.421\cdot10^{-5}\cdot\sigma^{-2}} \cdot (\sigma^2)^{-(\alpha+1)} \cdot e^{-\beta/\sigma^2}$$

(with $\alpha = 2$, $\beta = 10^{-5}$)

$$\propto \sigma^{-3}\cdot(\sigma^2)^{-3} \cdot e^{\sigma^{-2}\cdot -3.421\cdot10^{-5} + \sigma^{-2}\cdot -10^{-5}}$$

$$\propto (\sigma^{-3})^3 \cdot e^{\sigma^{-2}(-4.421\cdot10^{-5})}$$

$$\propto (\sigma^2)^{-(3.5+1)} \cdot e^{-(4.421\cdot10^{-5})/\sigma^2}$$

$\alpha^* = 3.5$ $\beta^* = 4.421\cdot10^{-5}$

$$\text{Mode} = \frac{\beta^*}{\alpha^*+1} = \frac{4.421\cdot10^{-5}}{3.5+1} = 9.82\cdot10^{-6}$$

(b) Attachment 2