Assignment 3.1

H_0 : cost ≥ 120000

H_1 : cost < 120000

cost ~ Normal$(\mu, 12000^2)$

μ ~ normal$(115000, 9000^2)$ Data: $\frac{\sum_{i=1}^{6} cost_i}{6} = 121000$

<a>

prior odds $= (H_0/I) = \frac{P_0}{P_1} = \frac{P(H_0/I)}{P(H_1/I)} = \frac{P(\tilde{\mu}+\tilde{\varepsilon} \geq 120000)}{P(\tilde{\mu}+\tilde{\varepsilon} < 120000)}$

we know that $\tilde{\varepsilon}$ ~ Norm$(0, 12000)$

$\tilde{\mu}$ ~ Norm$(115000, 9000^2)$

So, $\tilde{\mu}+\tilde{\varepsilon}$ ~ $(115000+0, \sqrt{9000^2+12000^2})$

Using R → probability ← pnorm(120000, mean = 115000, sd = $\sqrt{9000^2+12000^2}$)

prior odds $= \frac{1-\text{probability}}{\text{probability}} = 0.58589 \approx 0.59$

<b> Bayes factor

Attachment 2

<c> Loss table

Action	State of nature	
	H_0 true	H_1 true
Accepting H_0	0	6000
Accepting H_1	4000	0

odds$(H_0|Data, I) = B \cdot$ odds$(H_0|I)$

$= 1.63 \cdot 0.59$

$= 0.9617$

$P(H_0|Data, I) = \frac{0.9617}{0.9617+1} \doteq 0.4902$

$P(H_1|Data, I) \doteq 0.5098$

Action	Expected posterior loss
Accepting H_0	$0 \cdot 0.4902 + 6000 \cdot 0.5098 \doteq 3058$
Accepting H_1	$0.4902 \cdot 4000 = 1960.8$

Minimizing the expected posterior loss gives the action "Accepting H_1".