

Computer Lab 3

Computational Statistics

Linköpings Universitet, IDA, Statistik

2022 XI 23

Kurskod och namn:	732A90 Computational Statistics
Datum:	2022 XI 21—2022 XI 29 (lab session 23 XI 2022)
Delmomentsansvarig:	Krzysztof Bartoszek, Bayu Brahmantio, Jaskirat Marar, Shashi Nagarajan
Instruktioner:	<p>This computer laboratory is part of the examination for the Computational Statistics course</p> <p>Create a group report, (that is directly presentable, if you are a presenting group), on the solutions to the lab as a .PDF file.</p> <p>Be concise and do not include unnecessary printouts and figures produced by the software and not required in the assignments.</p> <p>All R code should be included as an appendix into your report.</p> <p>A typical lab report should 2-4 pages of text plus some amount of figures plus appendix with codes.</p> <p>In the report reference ALL consulted sources and disclose ALL collaborations.</p> <p>The report should be handed in via LISAM</p> <p>(or alternatively in case of problems e-mailed to bayu_dot_brahmantio_at sign_liu_dot_se, jasma356_at sign_student_dot_liu_dot_se, shashi_dot_nagarajan_at sign_@liu_dot_se, or krzysztof_dot_bartoszek_at sign_liu_dot_se),</p> <p>by 23:59 29 November 2022 at latest.</p> <p>Notice there is a deadline for corrections 23:59 22 January 2023 and a final deadline of 23:59 12 February 2023 after which no submissions nor corrections will be considered and you will have to redo the missing labs next year.</p> <p>The seminar for this lab will take place 6 December 2022.</p> <p>The report has to be written in English.</p>

Question 1: Stable distribution

A random variable has a so called *one-sided strictly stable distribution of order 1/2* if for a parameter $c > 0$ it has density equalling

$$f(x) = c(\sqrt{2\pi})^{-1}e^{-c^2/(2x)}x^{-3/2}\mathbf{1}_{(0,\infty)}(x).$$

In particular the support is on $(0, \infty)$. The aim of the assignment is to sample from this distribution by using an acceptance-rejection algorithm with the help of a power-law distribution with density

$$f_p(x) = \frac{\alpha - 1}{T_{\min}} \left(\frac{x}{T_{\min}} \right)^{-\alpha} \mathbf{1}_{(T_{\min}, \infty)}(x)$$

for $T_{\min} > 0$, $\alpha > 1$. In particular the support is on (T_{\min}, ∞) .

1. Plot $f(x)$ and $f_p(x)$ together. Can the power-law distribution be used just by itself or is there a problem at any place of the support Explain what the problem is and how can it be taken care of. Provide values of the power-law distribution's parameters that can be used in the acceptance-rejection algorithm. Derive and implement a majorizing density.
2. Implement an acceptance-rejection algorithm for sampling from the one-sided strictly stable distribution of order 1/2 with the proposal distribution built around the power-law distribution. To sample from a power-law distribution you can use the function `powerLaw::rplcon()`.
3. Generate a large sample using your implemented sampler, for different choices of c . Present the sample graphically. What is the mean and variance and how do they depend on c ? Study the rejection rate.

Question 2: Laplace distribution

The double exponential (Laplace) distribution is given by formula:

$$DE(\mu, \alpha) = \frac{\alpha}{2} \exp(-\alpha|x - \mu|)$$

1. Write a code generating double exponential distribution $DE(0, 1)$ from $\text{Unif}(0, 1)$ by using the inverse CDF method. Explain how you obtained that code step by step. Generate 10000 random numbers from this distribution, plot the histogram and comment whether the result looks reasonable.
2. Use the Acceptance/rejection method with $DE(0, 1)$ as a majorizing density to generate $\mathcal{N}(0, 1)$ variables. Explain step by step how this was done. How did you choose constant c in this method? Generate 2000 random numbers $\mathcal{N}(0, 1)$ using your code and plot the histogram. Compute the average rejection rate R in the acceptance/rejection procedure. What is the expected rejection rate ER and how close is it to R ? Generate 2000 numbers from $\mathcal{N}(0, 1)$ using standard `rnorm()` procedure, plot the histogram and compare the obtained two histograms.