SEMINAR EXERCISES IN PROBABILITY THEORY 732A63

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2 Conditioning

Exercise 2.0.1 (self-study)

Prove Theorem 2.1 in page 34 in Gut's book, in the discrete distribution case.

Exercise 2.0.2 (self-study)

Prove Theorem 2.2(a,b) in page 36 in Gut's book, in the continous distribution case.

Exercise 2.1 (2.8 in Gut's book)

Let X and Y have the joint density function

$$f_{X,Y}(x,y) = \begin{cases} \frac{x^2}{2y^3} \cdot e^{-x/y} & ,0 < x < \infty, 0 < y < 1 \\ 0 & \text{, otherwise.} \end{cases}$$

(a) Determine the distribution of Y.

(b) Find the conditional distribution of X given that $Y = y$.

(c) Using the results from (a) and (b), compute $E[X]$ and $Var[X]$.

Exercise 2.2 (2.11 in Gut's book)

Suppose that X and Y have a joint density function given by

$$f_{X,Y}(x,y) = \begin{cases} c \cdot x^2 y & ,0 < y < x < 1 \\ 0 & \text{, otherwise.} \end{cases}$$

Compute c, the marginal densities, $E[X]$, $E[Y]$, and the conditional expectations $E[Y|X = x]$ and $E[X|Y = y]$.

Exercise 2.3 (2.33 in Gut's book)

Suppose that a random variable X is uniformly distributed symmetrically around zero, but in such a way that the parameter is uniform on (0,1); that is, suppose that

$$X|A = a \sim U(-a, a) \text{ with } A \sim U(0,1).$$

Find the distribution of X, and also $E[X]$ and $Var[X]$.

Exercise 2.4 (2.35 in Gut's book)

Let X and Y be jointly distributed random variables such that

$$Y|X = x \sim Bin(n, x) \text{ with } X \sim U(0,1).$$

Compute $E[Y]$, $Var[Y]$ and $Cov(X,Y)$, without using what is known from Section 4 about the distribution of Y.

Exercise 2.5* (2.23 in Gut's book)

The joint density function of X and Y is given by

$$f_{X,Y}(x,y) = \begin{cases} c \cdot xy & , x, y > 0,\ 4x^2 + y^2 \leq 1 \\ 0 & , \text{otherwise.} \end{cases}$$

Compute c, the marginal densities, and the conditional expectations $E[Y|X = x]$ and $E[X|Y = y]$.

Exercise 2.6* (2.30 in Gut's book)

Show that a suitable power of a Weibull-distributed random variable whose parameter is gamma-distributed is Pareto-distributed. More precisely, show that if

$$X|A = a \sim W\left(\frac{1}{a}, \frac{1}{b}\right) \text{ with } A \sim \Gamma(p, \theta),$$

then X^b has a (translated) Pareto distribution.

Self-study exercises are excluded from the bonus-point deal. Exercises marked with * are a bit more challenging.

May Gauss be with you!