SEMINAR EXERCISES IN PROBABILITY THEORY 732A63

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1 Introduction and Multivariate Random Variables

Exercise 1.0.1 (self-study)

Use the univariate transformation formula from page 13 in the slides to solve yourself the Example 2.2 in page 20 in Gut's book.

Exercise 1.0.2 (self-study)

Use the multivariate transformation theorem from page 20 in the slides to solve yourself the Example 2.4 in page 21 in Gut's book.

Exercise 1.1

Let X and Y have the joint density

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{\sqrt{2\pi}} \exp\left[-\left(\frac{x^2}{8} + 2y\right)\right] & , -\infty < x < \infty, 0 < y < \infty \\ 0 & , \text{otherwise.} \end{cases}$$

(a) Determine the marginal distributions $f_X(x)$ and $f_Y(y)$. Do X and Y belong to some known distributions?

(b) Compute $E[(X+1)Y^2]$.

Exercise 1.2

Suppose that X and Y are random variables with joint density

$$f_{X,Y}(x,y) = \begin{cases} \frac{2}{3}(2x+y) & , 0 < x, y < 1 \\ 0 & , \text{otherwise.} \end{cases}$$

(a) Compute the probability density function of and cumulative distribution function of X.

(b) Compute $P(2X + Y < 1)$.

Exercise 1.3 (1.1 in Gut's book)

Show that if $X \sim C(0,1)$, then so is $1/X$.

Exercise 1.4 (1.8 in Gut's book)

Show that if X and Y are independent, $N(0,1)$-distributed random variables, then $X/Y \sim C(0,1)$.

Exercise 1.5* (1.16 in Gut's book)

A certain chemistry problem involves the numeric study of a lognormal random variable X. Suppose that the software package used requires the input of $E(Y)$ and $Var(Y)$ into the computer, where Y is normal and such that $X = e^Y$, but that one knows only the values of $E(X)$ and $Var(X)$. Find expressions for the former mean and variance in terms of the latter.

Exercise 1.6* (1.43a in Gut's book)

Let X and Y be independent random variables. Determine the distribution of $(X-Y)/(X+Y)$, if $X \sim Exp(1)$ and $Y \sim Exp(1)$.

Self-study exercises are excluded from the bonus-point deal. Exercises marked with * are a bit more challenging.

May Gauss be with you!