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Q) Prove the following by M.I method:-

$$P(n) = 7+77+777+ \cdots 7 \cdots n \text{ times}$$
$$= \frac{7}{81}\left(10^{n+1} - 9n - 10\right)$$

$\forall n \in N$.

Solⁿ Step Basic step:-

For $n=1$ LHS $P(1) = 1$

$$RHS = \frac{7}{81}(100-9-10) = \frac{7}{81} \times 81 = 7$$

LHS = RHS.

Step II. Let statement $P(n)$ be true for $n=k$ i.e

$$\boxed{7+77+ \cdots 77 \cdots K \text{ times} = \frac{7}{81}\left(100^{k+1} - 9k - 10\right)}$$

(1).

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Step 3: To Prove $P(K+1)$ is true we have to show

$7 + 77 + 777 + \ldots 7 \ldots (m+1)$ times

$$= \frac{7}{81}\left(10^{m+2} - 9(m+1) - 10\right)$$

We can write,

$$7 = 10^0 \times 7$$

$$77 = 10^1 \times 7 + 7$$

$\vdots$

$$\underset{(K+1)\text{ times}}{77\ldots7} = 10^K \times 7 + 10^{K-1} \times 7 + 10 \times 7 + 7$$

$$\Rightarrow 7(\underbrace{1 + 10 + 10^2 + \ldots 10^K}_{\text{GP with } r = 10\ (r > 0)})$$

$$\boxed{S_n = \frac{a(r^n - 1)}{r + 1}}$$

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$$= 7\left(\frac{10^{k+1} - 1}{10 - 1}\right)$$

$$= \frac{7}{9} \times \left(10^{k+1} - 1\right)$$

LHS :-

$$\overbrace{\frac{7}{81}\left(10^{k+1} - 9k - 10\right)}^{\text{from eq}^n\ (1)} + \frac{7}{9}\left(10^{k+1} - 1\right)$$

$$= \frac{7\left(10^{k+1} - 9k - 10\right) + 7 \times 9\left(10^{k+1} - 1\right)}{81}$$

$$= \frac{7}{81} \times \left(10^{k+1} + 9 \times 10^{k+1} - 9k - 9 - 10\right)$$

$$= \frac{7}{81}\left\{10^{k+1} - 9(k+1) - 10\right\}$$

$$= \text{RHS}.$$

$\therefore$ $P(k+1)$ is also true.