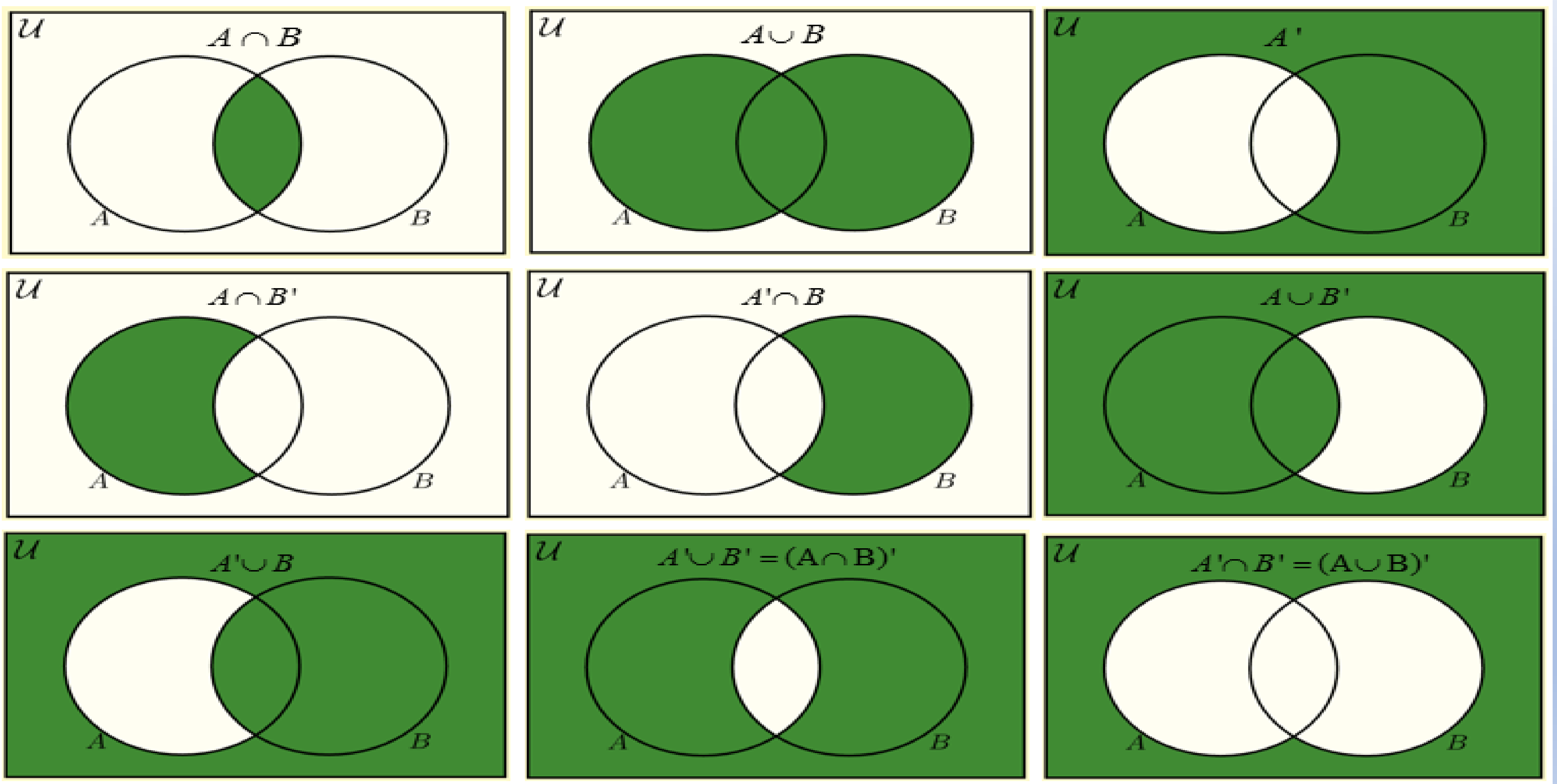
Questions on Set Theory

Venn diagram questions



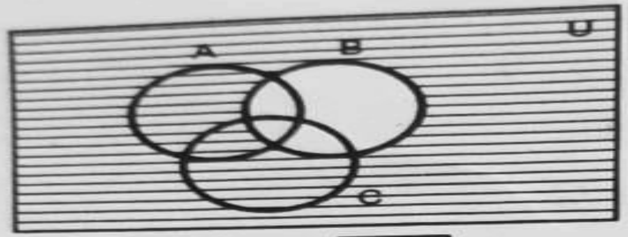
$(U \cap A) \cup (B$ ...

EX.1.10.21 Using Venn diagram show that :

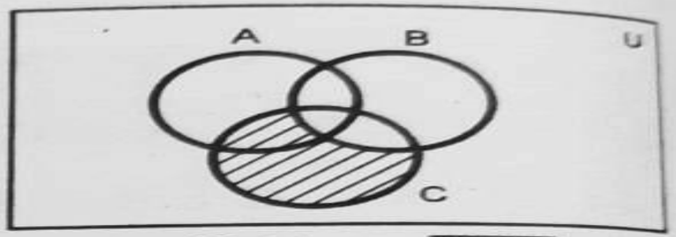
$$A \cup (\overline{B} \cap C) = (A \cup \overline{B}) \cap (A \cup C)$$

Sol. : Consider the following venn diagrams

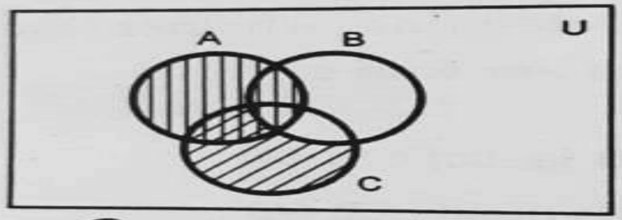
$$\overline{B} = \{x \,/\, x \notin B\}$$



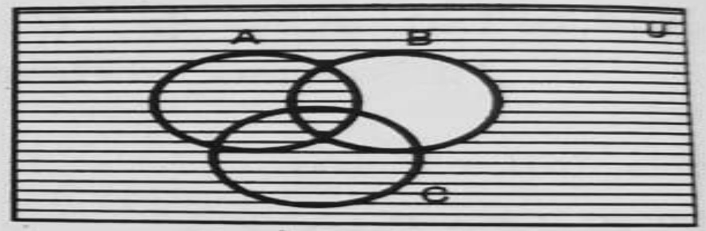
① $\overline{B}$ =



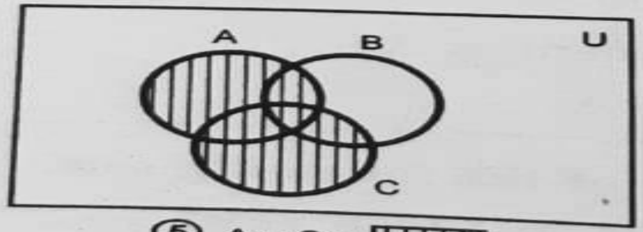
② $\overline{B} \cap C$ =



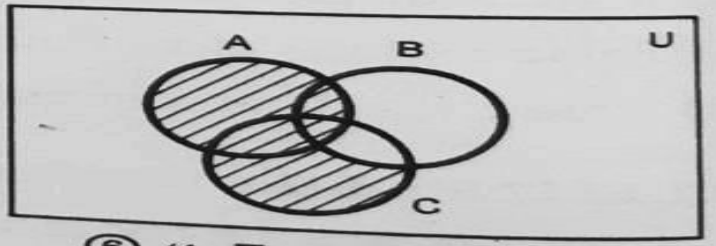
③ $A \cup (\overline{B} \cap C)$ = and



④ $A \cup \overline{B}$ =



⑤ $A \cup C$ =



⑥ $(A \cup \overline{B}) \cap (A \cup C)$ =

From ③ and ⑥,

$$A \cup (\overline{B} \cap C) = (A \cup \overline{B}) \cap (A \cup C)$$

Question 1: If U = {1, 3, 5, 7, 9, 11, 13}, then which of the following are subsets of U.

- B = {2, 4}
- A = {0}
- C = {1, 9, 5, 13}
- D = {5, 11, 1}
- E = {13, 7, 9, 11, 5, 3, 1}
- F = {2, 3, 4, 5}

Answer 1: Here, we can see that C, D and E have the terms which are there in ∪. Therefore, C, D and E are the subsets of ∪.

Some Important formulae: For any three sets A, B, C.

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(ii) If $A \cap B = \phi$, then $n(A \cup B) = n(A) + n(B)$

(iii) $n(A - B) + n(A \cap B) = n(A)$

(iv) $n(B - A) + n(A \cap B) = n(B)$

(v) $n(A \cup B) = n(A - B) + n(A \cap B) + n(B - A)$

(vi) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n($

$n(A \cap B \cap C)$.

Theorem 10: *For any sets A and B, prove that*

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Proof: We have by venn diagram

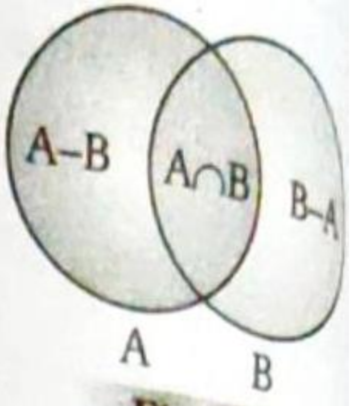


Fig. 1.8

The set $A - B, B - A, (A \cap B)$ are disjoint and their union is $(A \cup B)$.

$$\therefore \quad n(A \cup B) = n(A - B) + n(A \cap B) + n(B - A)$$

$$= n(A - B) + n(A \cap B) + n(B - A) + n(A \cap B) - n(A \cap B)$$

$$= n(A) + n(B) - n(A \cap B)$$

$[\because \ n(A - B) + n(A \cap B) = n(A)$ and $n(B - A) + n(A \cap B) = n(B)]$

Hence

$$\therefore \quad n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Theorem 11: *For any sets A,B,C prove that*

$$n(A \cup B \cup C) = [n(A) + n(B) + n(C) + n(A \cap B \cap C)] - [n(A \cap B) + n(B \cap C) + n(A \cap C)]$$

Proof: We have $n(A \cup B \cup C) = n[(A \cup B) \cup C]$

Question 2: Let A and B be two finite sets such that $n(A) = 20$, $n(B) = 28$ and $n(A \cup B) = 36$, find $n(A \cap B)$.

Solution: Using the formula $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

then $n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$= 20 + 28 - 36$

$= 48 - 36$

$= 12$

Question 3: In a group of 60 people, 27 like cold drinks and 42 like hot drinks and each person likes at least one of the two drinks. How many like both coffee and tea?

Solution: Let A = Set of people who like cold drinks B = Set of people who like hot drinks Given,

$(A \cup B) = 60 \quad n(A) = 27 \quad n(B) = 42$ then;

$n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$= 27 + 42 - 60$

$= 69 - 60 = 9$

$= 9$

Therefore, 9 people like both tea and coffee

Question 4: In a competition, a school awarded medals in different categories. 36 medals in dance, 12 medals in dramatics and 18 medals in music. If these medals went to a total of 45 persons and only 4 persons got medals in all the three categories, how many received medals in exactly two of these categories?

Solution: Let A = set of persons who got medals in dance.

B = set of persons who got medals in dramatics.

C = set of persons who got medals in music.

Given,

$n(A) = 36$

$n(B) = 12$

$n(C) = 18$

$n(A \cup B \cup C) = 45$

$n(A \cap B \cap C) = 4$

We know that number of elements belonging to exactly two of the three sets A, B, C

$= n(A \cap B) + n(B \cap C) + n(A \cap C) - 3n(A \cap B \cap C)$

$= n(A \cap B) + n(B \cap C) + n(A \cap C) - 3 \times 4$ ……..(i)

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

Therefore, $n(A \cap B) + n(B \cap C) + n(A \cap C) = n(A) + n(B) + n(C) + n(A \cap B \cap C) - n(A \cup B \cup C)$

From (i) required number

$= n(A) + n(B) + n(C) + n(A \cap B \cap C) - n(A \cup B \cup C) - 12$

$= 36 + 12 + 18 + 4 - 45 - 1$

$= 70 - 67$

$= 2$

Question 5: In a group of 100 persons, 72 people can speak English and 43 can speak French. How many can speak English only? How many can speak French only and how many can speak both English and French?

Solution: Let A be the set of people who speak English.

B be the set of people who speak French.

A - B be the set of people who speak English and not French.

B - A be the set of people who speak French and not English.

$A \cap B$ be the set of people who speak both French and English.

Given,

$n(A) = 72$

$n(B) = 43$

$n(A \cup B) = 100$

Now, $n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$= 72 + 43 - 100$

$= 115 - 100$

$= 15$

Therefore, Number of persons who speak both French and English = 15

$n(A) = n(A - B) + n(A \cap B) \Rightarrow$

$n(A - B) = n(A) - n(A \cap B)$

$= 72 - 15$

$= 57$

and $n(B - A) = n(B) - n(A \cap B)$

$= 43 - 15$

$= 28$

Therefore, Number of people speaking English only = 57

Number of people speaking French only = 28