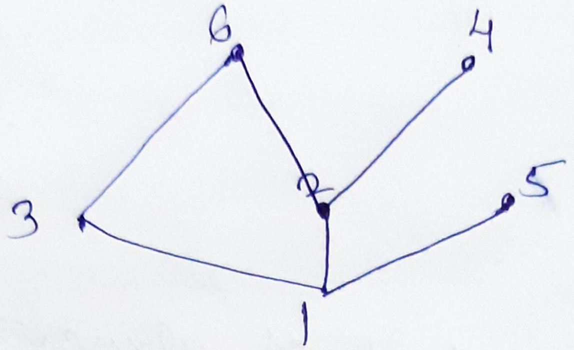
Hasse diagram



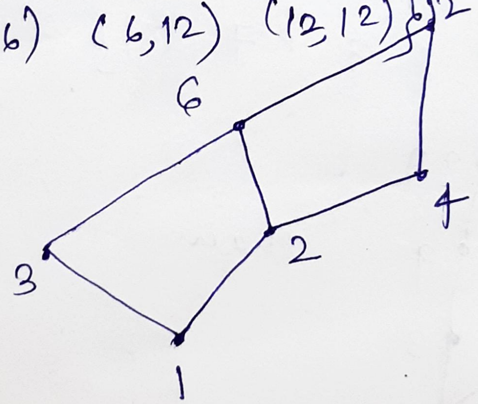
2nd level

1st level

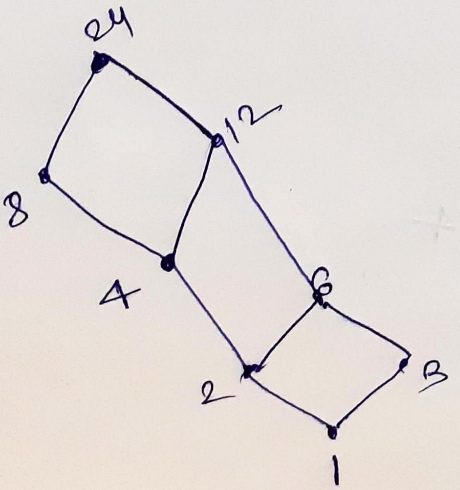
Base

Q) Draw D_{12} i.e x is a divisor of y.
or x divides y or $x \mid y$

$R = \{(1,1)(1,2)(1,3)(1,4)(1,6)(1,12)(2,2)(2,4)$
$(2,6)(2,12)(3,3)(3,6)(3,12)(4,4)(4,12)$
$(6,6)(6,12)(12,12)\}$



Q) $D_{24} = \{1, 2, 3, 4, 6, 8, 12, 24\}$



Q $A = \{2, 3, 5\}$
Relation
$a \leq b$ if
a divides b.

$B = \{2, 3, 4\}$

$C = \{2, 3, 6\}$

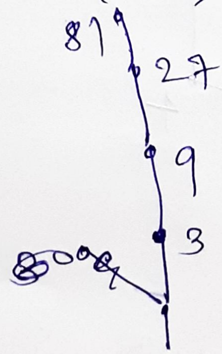
$D = \{2, 4, 6\}$.

$D_{64} = \{1, 2, 4, 8, 16, 32, 64\}$ $(D_{64}, |)$

$|$:- a divides b

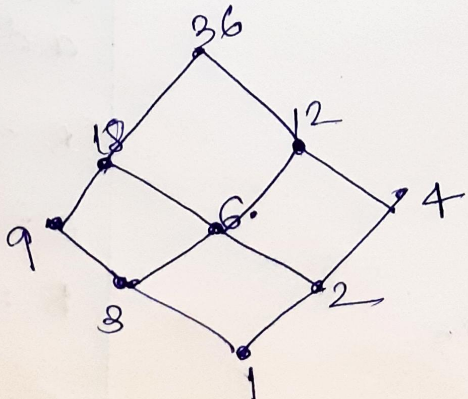
chain of elements

Q) $D_{81} = \{1, 3, 9, 27, 81\}$.



chain

Q) $(D_{36}, |)$

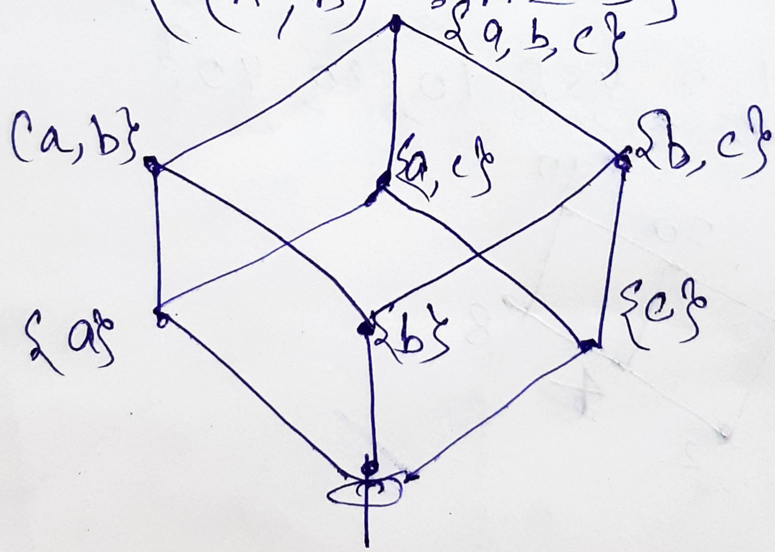


Two elements a and b in a poset (S, R) are said to be comparable if either aRb or bRa

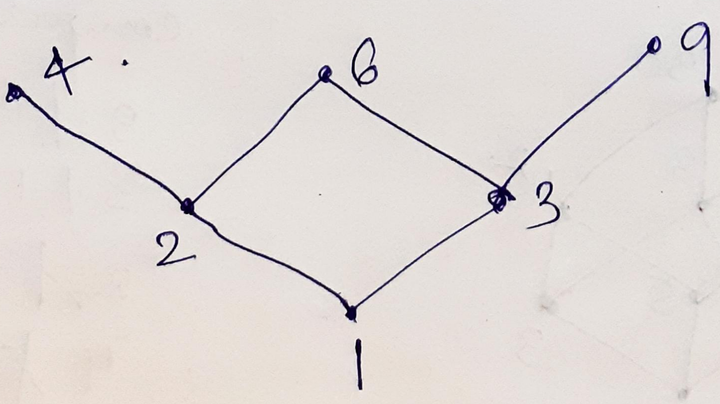
Incomparable : If neither $a \le b$ nor $b \le a$

Q) P(S) Power of S. S = {a, b, c}

{(A, B) : A ⊂ B} Relation.

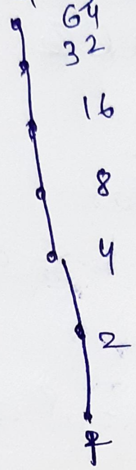


Q) A = {1, 2, 3, 4, 6, 9} '|' (divisor)

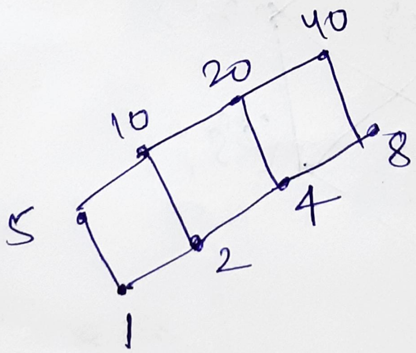


(D_{64}, D_{40}, D_{60}), $|$ x divides y

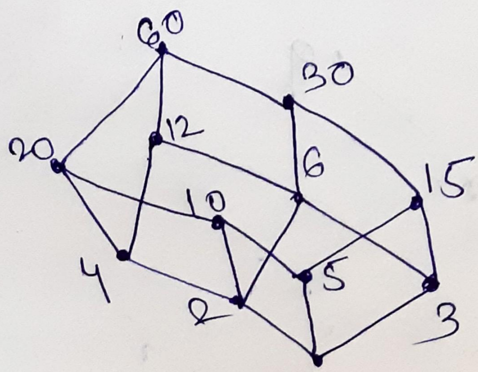
$(D_{64}, |) = \{1, 2, 4, 8, 16, 32, 64\}$



$(D_{40}, |) = \{1, 2, 4, 5, 8, 10, 20, 40\}$



$(D_{60}, |)$ $\{1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60\}$



comparable

$3 \mid 15$

$3 \mid 12$

$2 \nmid 15$

$4 \nmid 30$.

~~Ma~~ximal & Minimal Elements:

An element of POSET is called Maximal element if it is not less than any element of POSET.
i.e a is maximal element in a POSET.
($a \preceq c$) (a precedes c)

An element $b \in A$ is minimal element of A if there is no element c in A such that $c \preceq b$.

UPPER BOND (UB)

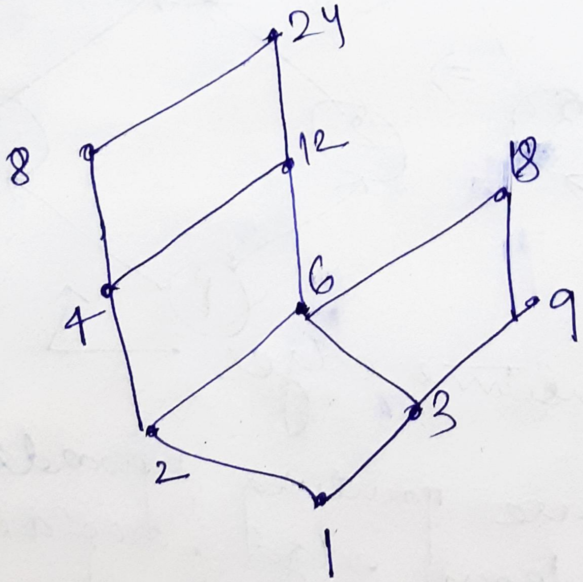
LOWER BOND (LB)

($\vee$) LUB(Supremum)

($\wedge$) GLB(infinum)

(LB)
- Greatest (last) element
- Smallest (first) element.

Q) $\{1, 2, 3, 4, 6, 8, 9, 12, 18, 24\}$



LB $= \{1\}$

UB $= \{24, 12, 6, 18\}$

Maximal $= \{24, 18\}$

First - 1

Last $= \phi$

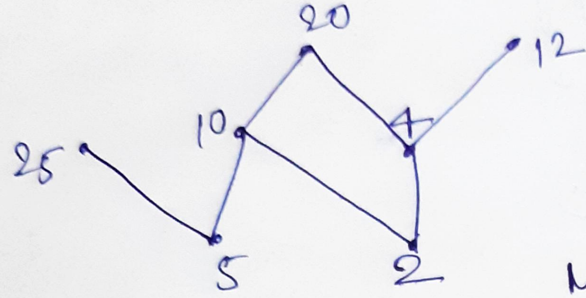
Minimal $= \{1\}$

$(2, 6)$ - (UB) $= \{6, 18, 12, 24\}$

LUB = 6

$(8, 12)$ $\begin{bmatrix} LUB = 24 \\ GLB = 4. \end{bmatrix}$ (LB $= \{4, 2, 1\}$

$X = \{2, 4, 5, 10, 12, 20, 25\}$



$U.B = \{25, 10, 20, 4, 12\}$

Maximal $= \{25, 20, 12\}$

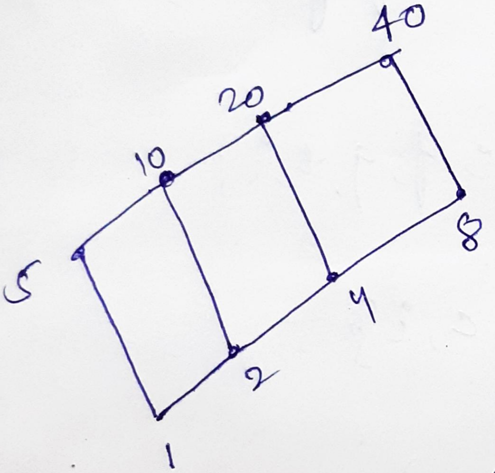
$LB = \{5, 2\}$

Minimal $\{2, 5\}$.

Last $= \phi$

First $= \phi$

Q)



Find Upper Bounds
Lower Bounds
LUB GLB of the following.

Sol^n

$U.B(4) = \{4, 8, 20, 40\}$

$U.B(10) = \{10, 20, 40\}$

$U.B(4, 10) = \{20, 40\}$

$LUB(4, 10) = 20$

$GLB(4, 10) = 2$

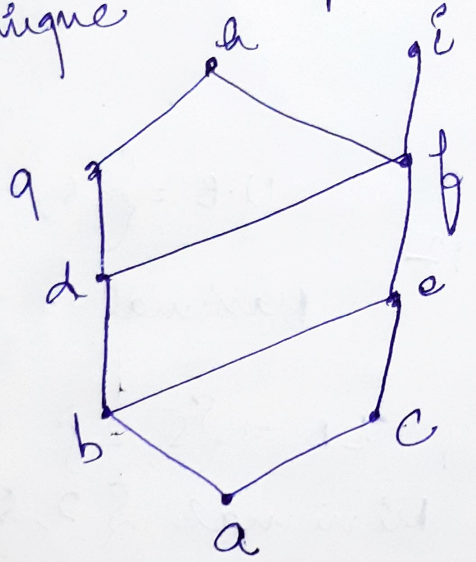
$LB(4) = \{1, 2, 4\}$

$LB(10) = \{1, 2, 5, 10\}$

$LB(4, 20) = \{1, 2, 5)$

$LB(4, 10) = \{1, 2\}$.

Supremum & Infimum if exists is alwa[ys]
unique



LB (a) = {a}

UB(a) = {a b c d e f g h i}

LB(b) = {a, b}

UB(b) = { b, d, e, f, g, h, i}

LB(c} = {a, c}

UB(c) = {e, f, h, c, i}

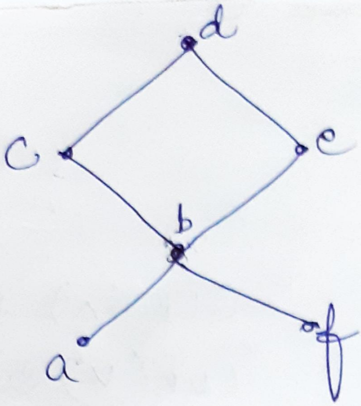
LB(d) = {b, d, a}

UB(d} = {f, g, h, i}

LB(g) = {d, b, a, g}

UB(g) = {h}

UB {a, c, a, f} = {f, i, h}



Sol^n ① $UB(c,e) = \{d\}$

$LUB = \{d\}$

② $LB(c,e) = \{b, a, f\}$

$GLB = \{b\}$

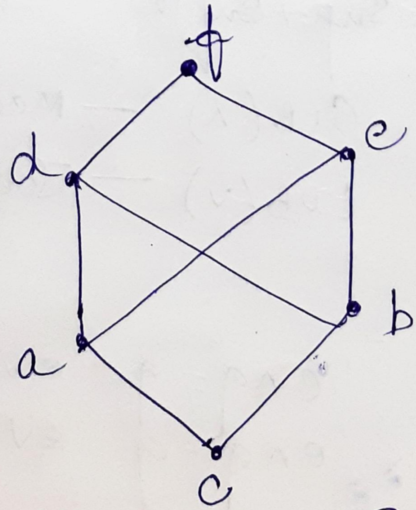
③ $UB(a,f) = \{b, c, e, d\}$

$LUB = \{b\}$

④ $LB(a,f) = \phi$.

GLB does not exist.

Q)



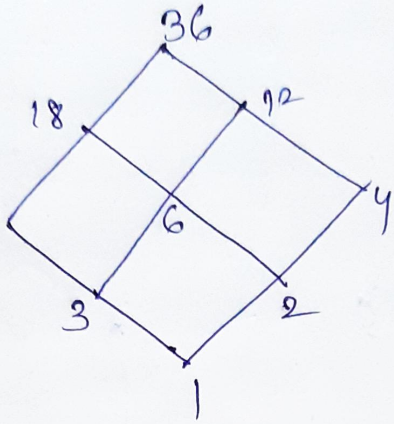
① $UB(a,b) = \{d, e, f\}$

$LUB = \phi$

$LB\{a,b\} = c$

$GLB = c.$ (unique)

9)



$GLB(\wedge)$ Infi

$LUB(\vee)$ Sup.

Soln

$6 \wedge 4 = 2$

$6 \vee 4 = 12$

$9 \vee 4 = 36$

$9 \wedge 4 = 1$

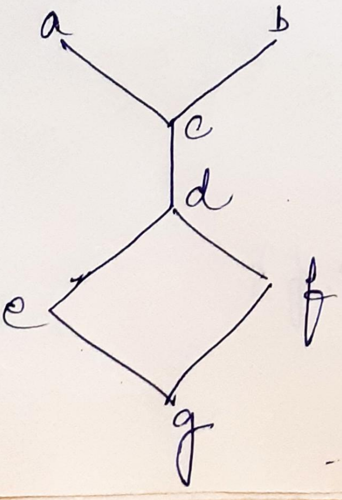
LATTICE :-

A Poset in which every pair of elements has both (GLB + LUB) or (Sup + Inf)

$(L, (\wedge, \vee))$

$GLB(\wedge)$ — Meet (And

$LUB(\vee)$ — Joint (or



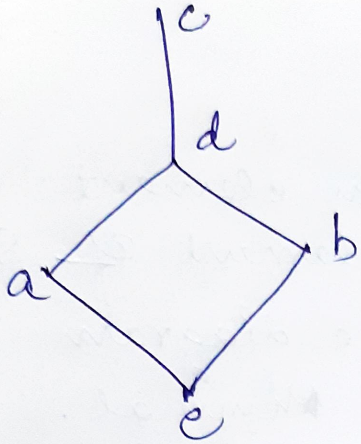
$c \wedge g = g \quad e \vee g = e$

$e \wedge g = g \quad e \vee f = d$

$c \wedge d = d \quad c \vee d = c$

$a \wedge b = c \quad \underline{a \vee b = \phi}$

$\therefore$ NOT A LATTICE.



This is a lattice

WELL ORDERED POSET (WPOSET)

(TOSET) - Total ordered set (CHAIN) OR (LINEAR ORDER)

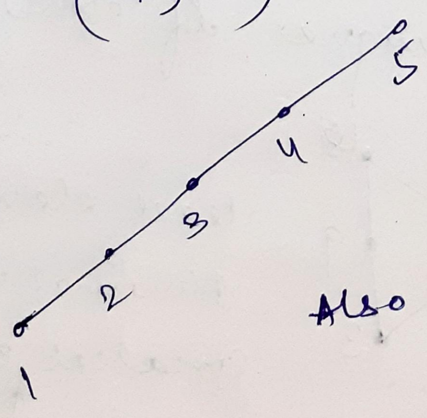
A partial ordered relation in which every pairs are related i.e aRb or bRa. eg $R(\le)$ $(a \le b)$ or $(b \le a)$

(Means any two elements are comparable.

Any set is WOSET if it is a POSET such that $\le$ is a (TOSET) and every non-empty subset of A has a least element (first).

Eg $(N, \le)$

$x \le y$ means x precedes y.



WOSET.

Also it is TOSET.

Maximal Elements!

⇒ Let $(S, \leq)$ is a poset. An element $a \in S$ is minimal if no other element of S strictly procedes 'a'. In Hasse diagram if no edge enters 'a' then a is minimal.

⇒ An element b in S is maximal if no other element of S strictly succeeds b or if no edge leaves 'b'

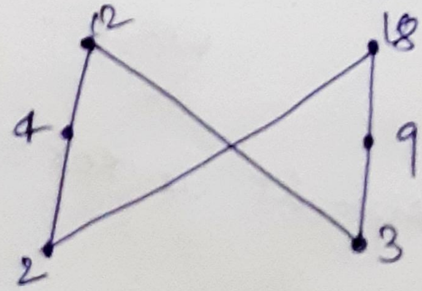
NOTE! S can have more than one minimal & more than one maximal elements

"FIRST/LAST ELEMENTS"

⇒ An element $a \in S$ is the greatest (last) element of S if $x \leq a$ for all $x \in S$
i.e every element in S preceeds 'a'.

⇒ An element 'b' $\in S$ is called a first or least element if $b \leq x$ for all $x \in S$
i.e, every element in S succeeds 'b'.

Both should be unique if exists

Q) 

least element = None

Minimal = 2, 3.

Greatest element = None

Maximal - 12, 18