Contents

1. Relation

- Relation is a word which is used to indicate a relationship between two objects. There are many kinds of relationships in the world. Such as

- Relation between a student and a teacher,
- Relation between employee and his or her salary,
- Relation between a person and a relative, and so on.

Mathematically

- In this we will discuss the mathematics of relations defined on Set.

- Direct way to express a relationship between elements of two sets is to use Ordered Pair.

- **Ordered Pair,** It is a pair of objects whose components occur in a special order (a,b), where a is called the first component and b is called the second component.

- The relationships between elements of sets are represented using the structure called a relation, which is just a subset of the **Cartesian Product** of the sets.

Cartesian Product:

- Let A and B be two sets. Cartesian product of A and B, denoted by AXB is defined as

$$AXB=\{ (x,y) : x \in A, y \in B\}$$

i.e AXB is the set of all possible ordered pairs whose first component comes from A and second comes from B.

- If we want to describe a relationship between elements of two sets A and B, we can use **ordered pairs** with their first element taken from A and their second element taken from B.
- Since this is a relation between **two sets**, it is called a **binary relation**.

- **Definition:** Let A and B be sets. A binary relation from A to B is a subset of $A \times B$.

- In other words, for a binary relation R we have
$R \subseteq A \times B$. We use the notation aRb to denote that $(a, b) \in R$ and $a\underline{R}b$ to denote that $(a, b) \notin R$.

Relations on a Set

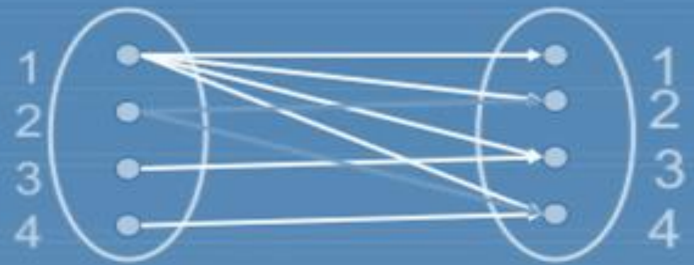
Solution: $R = \{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)\}$

R	1	2	3	4
1		X	X	X
2			X	X
3				X
4				

Example 2

Let A be the set $\{1, 2, 3, 4\}$. Which ordered pairs are in the relation $R = \{ (a, b) |\ a \text{ divides } b \}$?

Sol : Since (a, b) is in R if and only if a and b are positive integers not exceeding 4 such that a divides b



Examples:

Ex 1) Let A and B are two sets as follows:

A= { Calcutta, Patna, lucknow, Chennai}

B={West Bengal, Bihar, Uttar Pradesh, Tamil Nadu}

Find a relation defined by " is capital of" among above.

Sol: R is a relation between the two sets and defined as "is capital of" and is denoted as R= AXB

R= {(Calcutta,West bengal),(Patna, Bihar)...etc}

$$R = \{(x,y): x \in A,\ y \in B,\ xRy\}$$

Relations on a Set

- **How many different relations can we define on a set A with n elements?**

- A relation on a set A is a subset of $A \times A$.
- How many elements are in $A \times A$?

- There are n^2 elements in $A \times A$, so how many subsets (= relations on A) does $A \times A$ have?

- The number of subsets that we can form out of a set with m elements is 2^m. Therefore, 2^{n^2} subsets can be formed out of $A \times A$.

- **Answer:** We can define 2^{n^2} different relation A

Ex 2) Let A and B are the two sets defined as A={1,2,5} and b={2,4}, Find the relation defined by “<”.

Sol: The cartesian product is given as

AXB= {(1,2),(1,4),(2,2),(2,4)(5,2)(5,4)}

and as the relation is defined on “<”, we take all the ordered pairs satisfying (x<y), then relation will be

R={(1,2)(1,4)(2,4)}

Ex3) Let A be the set {1, 2, 3, 4}. Which ordered pairs are in the relation R = {(a, b) | a divides b}?

Sol: The cartesian product is given as AXA

AXA= {(1, 1), (1, 2), (1, 3), (1, 4), (2,2), (2,3), (2,4), (3,4)}

and as the relation is defined as “a divides b”, we will take all the ordered pairs satisfying (a divides b), then relation will be

R = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 4), (3, 3), (4, 4)}.

2. Domain and Range

The set $\{a \in A : (a,b) \in R$ for some $b \in B\}$ is called domain of R and denoted by Dom R.

The set $\{b \in B : (a,b) \in R$ for some $a \in A\}$ is called range of R and denoted by Ran R.

i.e Domain- Set of first element

Range - Set of second element

Operations on Relation

Binary relations are set of ordered pairs, all set operations can be done on relations. Operations:

1) **Union-** If R and S denote two relations then R ∪ S denotes union of R and S,

$$x(R \cup S)y = xRy \ \cup xSy$$

1) **Intersection** : If R and S denote two relations then R ∩ S denotes intersection of R and S,

$$x(R \cap S)y = xRy \ \cap \ xSy$$

Operations on Relation

3) **Difference**: If R and S denote two relations then R - S denotes difference of R and S,

$$x(R - S)y = xRy \cap x(\notin S)y$$

4) **Complement**: If R denote a relation then R′ denotes complement of R and is given as,

$$x(R)y = x(\notin R)y$$

Example

Ex1: Let A = {1, 2, 3} and B = {1, 2, 3, 4}. The relations corresponding to A and B are R1 = {(1, 1), (2, 2), (3, 3)} and R2 = {(1, 1), (1, 2), (1, 3), (1, 4)}. Find?

Sol:

$R1 \cup R2 = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (3, 3)\},$

$R1 \cap R2 = \{(1, 1)\},$

$R1 - R2 = \{(2, 2), (3, 3)\},$

$R2 - R1 = \{(1, 2), (1, 3), (1, 4)\}.$

Example

Ex2: Let A = {1,2,3} and B = {u,v}. The relation corresponding to A and B are R1 = {(1,u), (2,u), (2,v), (3,u)} and R2 = {(1,v),(3,u),(3,v)}. Find?

Sol:

$R1 \cup R2 = \{(1,u),(1,v),(2,u),(2,v),(3,u),(3,v)\}$

$R1 \cap R2 = \{(3,u)\}$

$R1 - R2 = \{(1,u),(2,u),(2,v)\}$

$R2 - R1 = \{(1,v),(3,v)\}$