

Unit UNIT-2

Date - 24/10/19

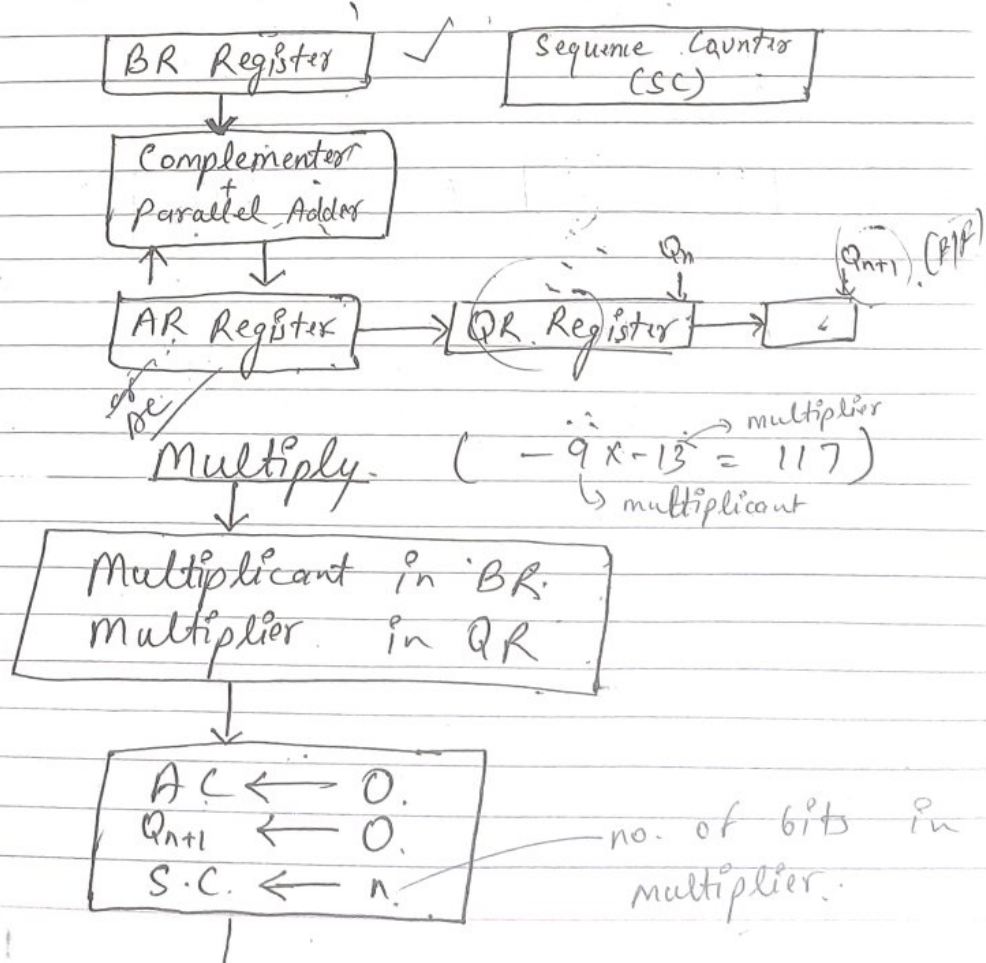
COMPUTER ARITHMETIC -

* MULTIPLICATION HARDWARE ALGO.

(i) BOOTH ALGORITHM -

Booth Algorithm gives a procedure for multiplying binary integers in signed 2's complement representation.

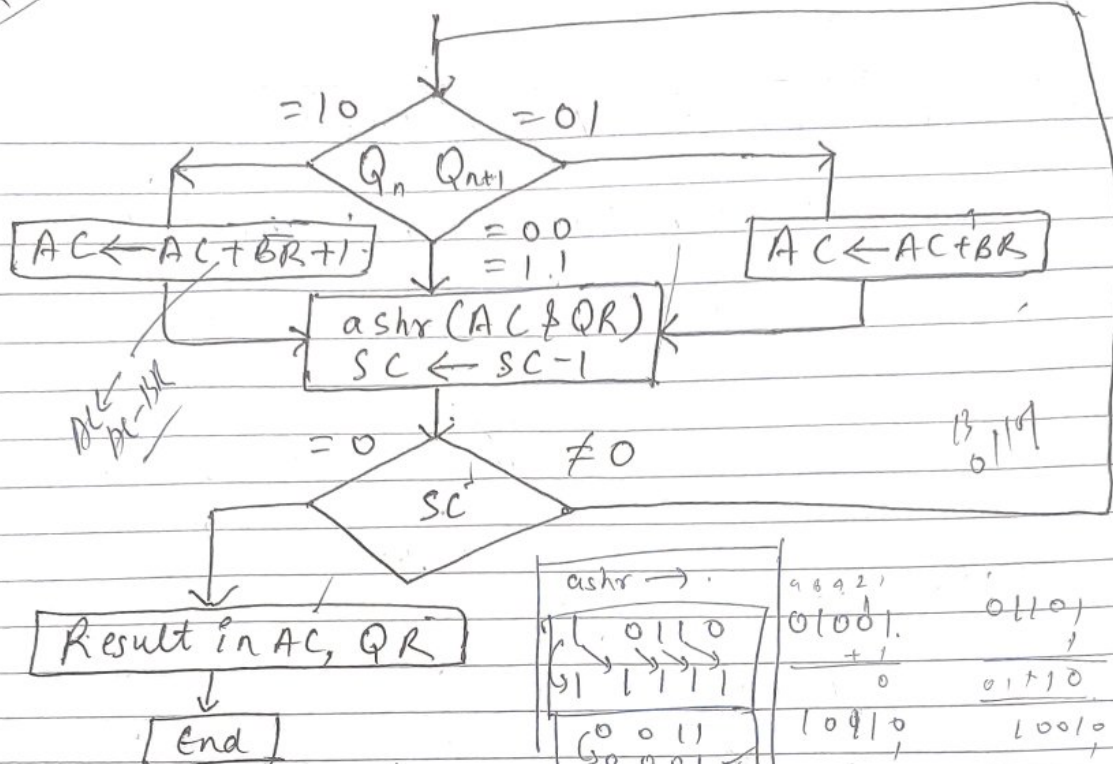
⇒ Hardware Used -



$Q_n \rightarrow$ right most bit

ashr \rightarrow arithmetic shift right

$-x- = +$
 $-x+ = -$
 $+x- = -$
 $+x+ = +$



$Q_n Q_{n+1}$	$BR = 1011$ $BR+1 = 0100$	AC	QR	Q_{n+1}	SC
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1 0	Add $BR+1$	00000	10011	0	10
		01001			
		01001	10011		
	ashr AC & QR	00100	11001	1	100
1 1	ashr AC & QR	00010	01100	1	011
		10111			
0 1	Add BR	11001	01100		
		11100	01100	0	010
0 0	ashr AC & QR	11110	01011	0	001
		01001			
1 0	Add $BR+1$	00111	01011		
	ashr AC & QR	00011	01011	1	000

0101

Q - 5×7 using booth Algo.

$Q_n Q_{n+1}$	$BR+1 = 0101$ $BR-1 = 1011$	AC	QR	Q_{n+1}	SC
		0 0 0 0	0 1 1 1	0	1 0 0
1 0	Add $BR+1$	0 1 0 1			
		0 1 0 1 0 1 1 1			
	ashr AC & QR	0 0 1 0 1 0 1 1		1	0 1 1
1 1	ashr AC & QR	0 0 0 1 0 1 0 1		1	0 1 0
1 1	ashr AC & QR	0 0 0 0 1 0 1 0		1	0 0 1
		1 0 1 1			
0 1	Add BR	1 0 1 1 1 0 1 0			
	ashr AC & QR	1 1 0 1 1 1 0 1		0	0 0 0
		↓ 2's.			
		0 0 1 0 0 0 1 1			
					(-35) ✓

ARRAY MULTIPLIER -
(2-bit Multiplier) -

For j multiplier bits & K multiplicand bits we need $j \times K$ AND gates & $(j-1)$ K -bits Adders to produce a product of $(j+K)$ bits.

ex:-

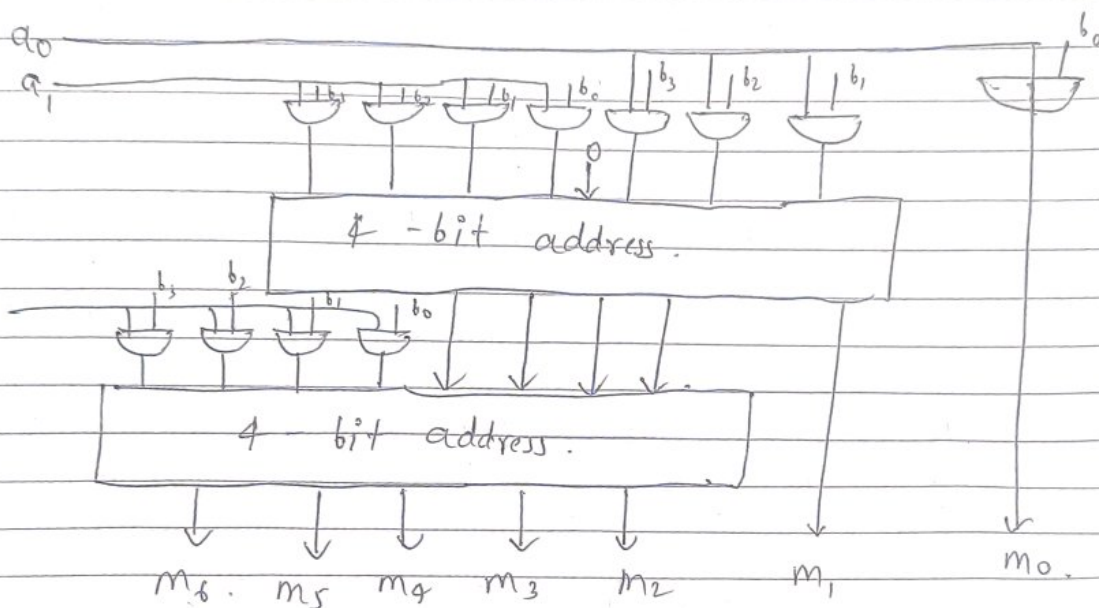
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      1 1 1 1 (4 bits)
      x 1 1 1 (3 bits)
      -----
      1 1 1 1
      1 1 1 1
      1 1 1 1
      -----
    1 0 1 0 0 0 1 (7 bits)
  
```

* 4 bit by 3-bit array multiplier.

$$B = b_3, b_2, b_1, b_0.$$

$$A = a_2, a_1, a_0.$$



$$b_3, b_2, b_1, b_0$$

$$a_2, a_1, a_0$$

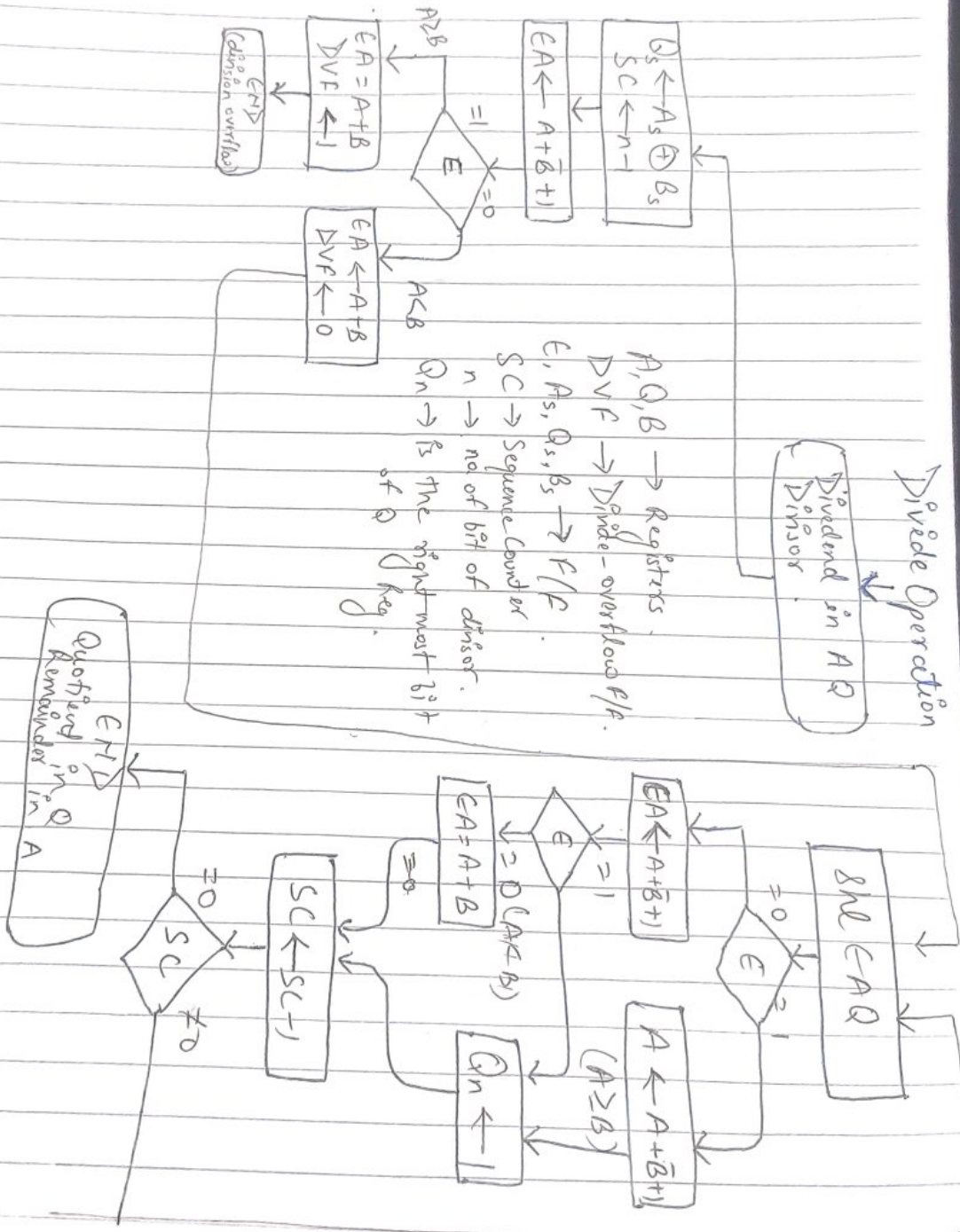
$$a_0 b_3, a_0 b_2, a_0 b_1, a_0 b_0$$

$$a_1 b_3, a_1 b_2, a_1 b_1, a_1 b_0$$

$$a_2 b_3, a_2 b_2, a_2 b_1, a_2 b_0$$

$$m_6, m_5, m_4, m_3, m_2, m_1, m_0$$

DIVISION ALGORITHM (HARDWARE) -



when $A = \begin{Bmatrix} 100 \\ 101 \\ 110 \end{Bmatrix}$ then division overflow error come.

Q 15/4 using divisor algorithm

$$Q = 3, A = 3 \mid B = 100, \bar{B} + 1 = 100 \mid \begin{array}{r} 15. \\ \overline{) 00111} \\ 4 \quad 3 \\ \hline \end{array}$$

E A Q SC.

0 001 111 011

SHLEAQ 0 011 110

Add $\bar{B} + 1$ 100

0 111 110

100

E=0, Add B \leftarrow 1011 110 010.

SHLEAQ 0 111 100

100

E=0, Add $\bar{B} + 1$ 1011 100

E=1, $Q_n \leftarrow 1$ 011 101 001

SHLEAQ 0 111 010

100

E=0, Add $\bar{B} + 1$ 1011 010.

E=1, $Q_n \leftarrow 1$ 1011 011 000.

↓
Remainder
(3)

↓
Quotient
(3).

$$\left(\frac{25}{7} \right) \rightarrow 3 \frac{4}{7}$$

$$A = 4, Q = 3.$$

$$\begin{array}{r} 16 \ 0 \ 4 \ 2 \ 1 \\ 1 \ 1 \ 0 \ 0 \ 1 \end{array}$$

$$B = 111, \bar{B} + 1 = 001 \quad \& \quad A \bar{Q} = \underline{011001}$$

Q 25/7 using divisor algorithm.

	E	A	Q	SC
	0	011	001	011
shl EAQ	0	110	010	
		001		
Add $\bar{B} + 1$	0	111	010	
		111		
E=0, Add B	1	010	010	010
E=1, Q=1	1	010	011	
shl EAQ	1	100	100	
		001		
Add $\bar{B} + 1$	1	101	100	
		101		
Q=1	1	101	101	001
shl EAQ	1	011	010	
Add $\bar{B} + 1$	1	100	010	
Qn ← 1	1	100	011	000
		<u>100</u>	<u>011</u>	
		Remainder	Quotient	
		(=4)	(=3)	

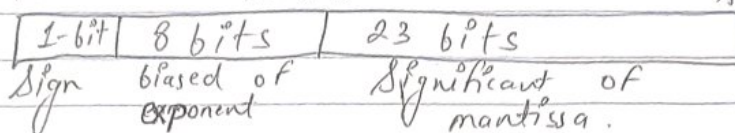
Institute of Electrical & Electronic Engineers
(It is a society which recommend)

FLOATING POINT REPRESENTATION.

$$\begin{array}{l} \textcircled{25.67} \times \textcircled{10^5} \rightarrow \text{exponent} \quad 25.67 \times 10^5 \\ \quad \downarrow \quad \quad \quad \downarrow \quad \quad \quad \downarrow \\ \text{mantissa} \quad \quad \text{Base/Radix.} \quad 2.567 \times 10^6 \\ \quad \quad \quad \quad \quad \quad \quad \quad \quad 256.7 \times 10^4 \end{array}$$

$$\textcircled{1.1101} \times 2^{\textcircled{5}} \rightarrow \text{exponent}$$

* IEEE 754 FORMAT (32-bits format)



$$\boxed{\text{Bias value} = 2^{k-1} - 1} \quad \text{where } k \rightarrow \text{no. of bits in exponent.}$$

$$\text{Bias value} = 2^{8-1} - 1 = 127. \quad (\text{for 32-bits format})$$

Mantissa = normalized.
(Significant) = (1. —)

ex- (i) -1.5 & (ii) 384 represent these no.'s into 32-bits IEEE format.

Solution: (i) -1.5 $\Rightarrow (-1.1)_2 \rightarrow$ normalized. $.5 \times 2 = 1$
 \downarrow mantissa. $\text{exp} = 0 + 127 = 127.$

IEEE format

1	01111111	100000000000000000000000
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(ii) 384 $\Rightarrow (110000000)_2 \Rightarrow$ normalize (1.1×2^8)
 $\text{exp} = 127 + 8 = 135.$
 $= 10000111$

512 256 128 64 32 16 8 4 2 1
1 1

IEEE
Format

[0 | 10000111 | 100000000000000000000000]

ex-

111.1010110 $\times 2^6$

$\Rightarrow 1.111010110 \times 2^6$ | $\text{exp} = 8 + 27 = 135$
10000111

IEEE
Format

[0 | 10000111 | 111010110000000000000000]

* 64-Bits FORMAT

1-bit	11-bits	52-bits
Sign	biased of exponent	significant or mantissa

Bias Value = $2^{k-1} - 1$ where $k \rightarrow$ no. of bits of exponent

Bias Value = $2^{11-1} - 1 = 2^{10} - 1 = 1023$

ex- 1110.0101 $\times 2^{-20}$

$\Rightarrow 1.1100101 \times 2^{-17}$ | $\text{exp} = 127 - 17 = 110$
 $\text{exp} = 11 + 1023 = 1036$
 $= 0111000110$

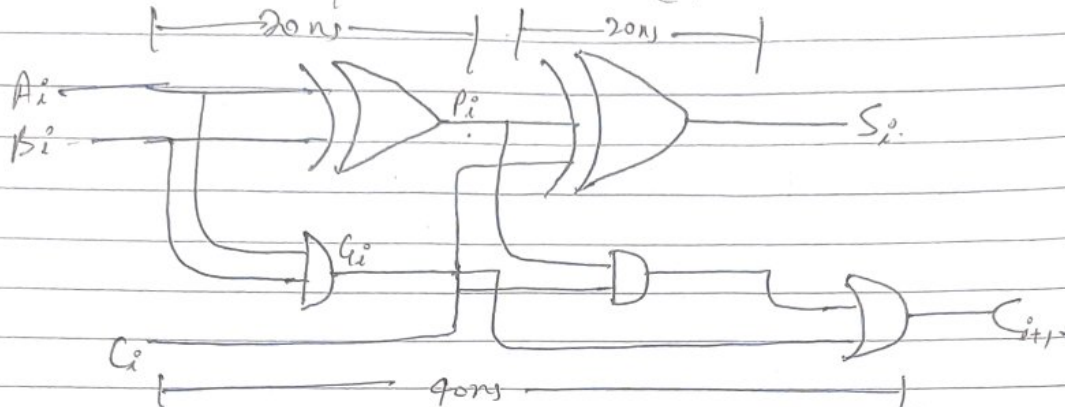
IEEE
64-bit
Format

[0 | 0111000110 | 110010100...0]

$$S_i = P_i \oplus C_i$$

$$C_{i+1} = G_i + P_i C_i$$

PARALLEL ADDER WITH LOOK-AHEAD CARRY GENERATOR (FAST ADDER).

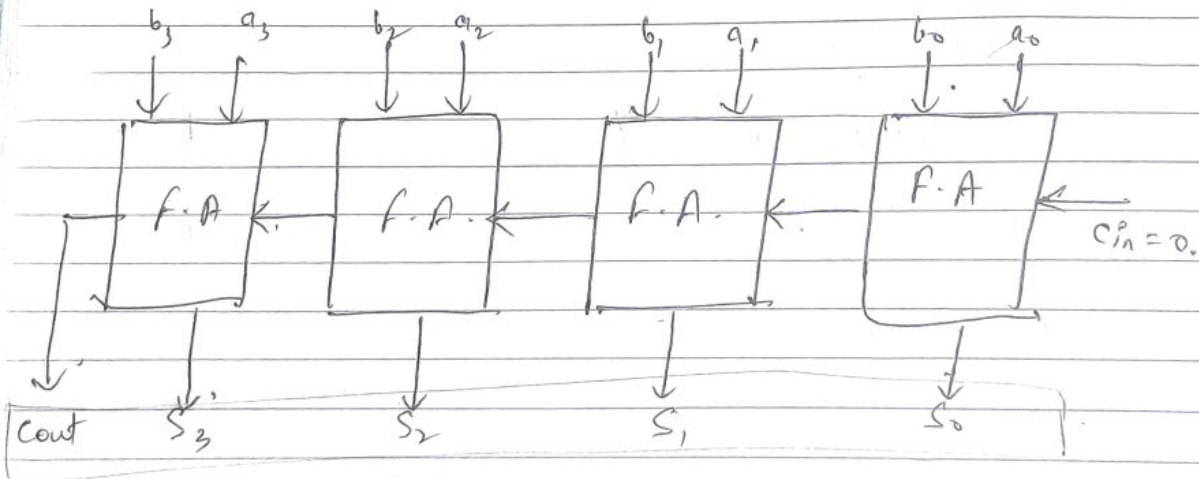


Propagation Delay - $P_i = A_i \oplus B_i$ (Carry Propagator).
 $G_i = A_i B_i$ (Carry Generator).

XOR - 20ns, OR & AND - 10ns.

FULL ADDER

* 4-bit Parallel Adder



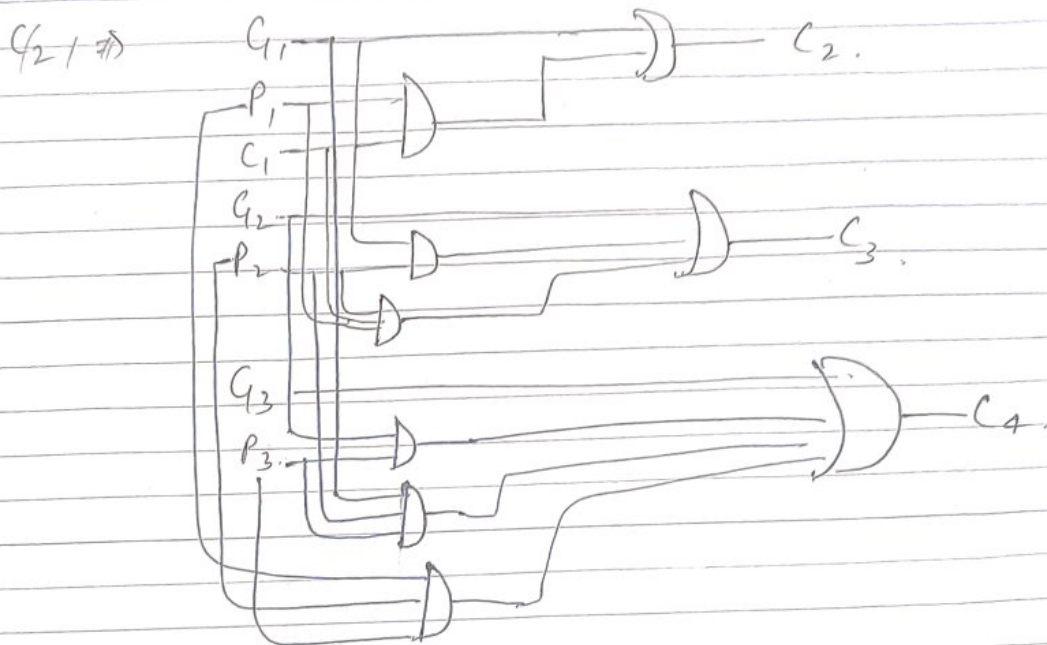
Propagation delay = $4 \times 40\text{ns} = 160\text{ns}$.

In General = n (no. of bit of operands) $\times 40\text{ns}$.

If $i=1$; $C_2 = G_1 + P_1 C_1$ — (1)

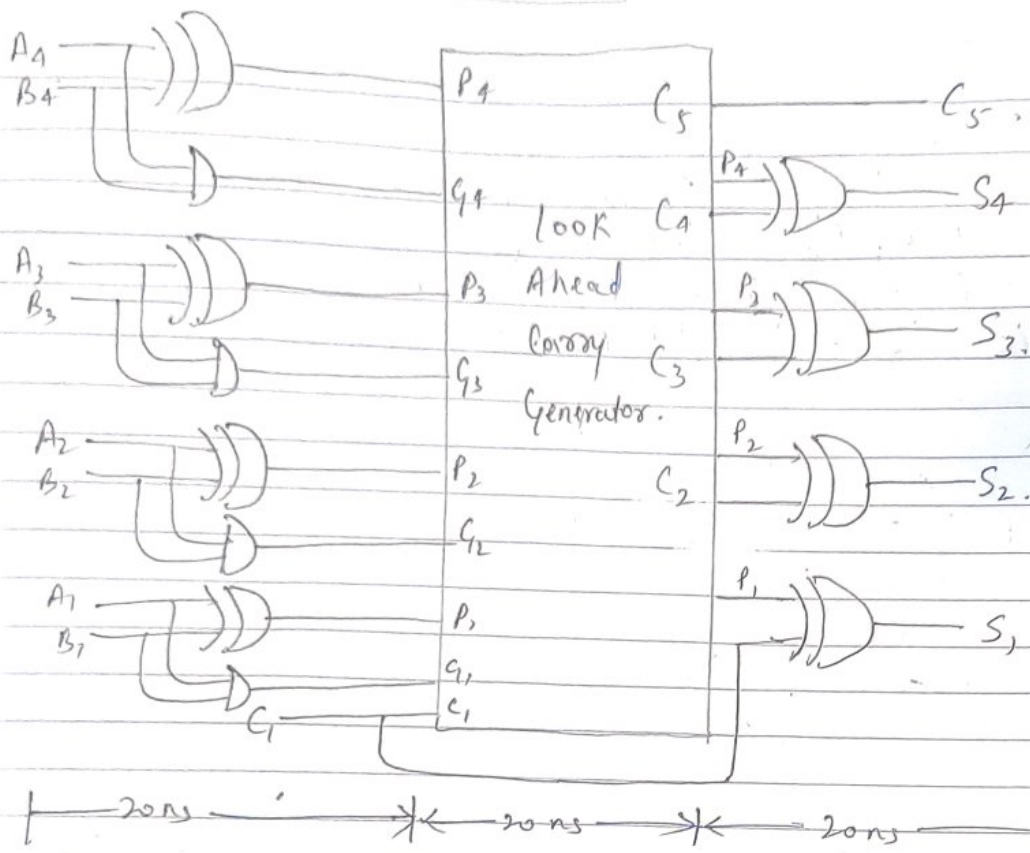
If $i=2$; $C_3 = G_2 + P_2 C_2 = G_2 + P_2 (G_1 + P_1 C_1)$
 $C_3 = G_2 + G_1 P_2 + P_1 P_2 C_1$ — (2)

If $i=3$; $C_4 = G_3 + P_3 C_3$
 $C_4 = G_3 + P_3 (G_2 + G_1 P_2 + P_1 P_2 C_1)$
 $C_4 = G_3 + G_2 P_3 + G_1 P_2 P_3 + P_1 P_2 P_3 C_1$ — (3)



Propagation delay of look ahead carry generator is constant & it is 20ns.

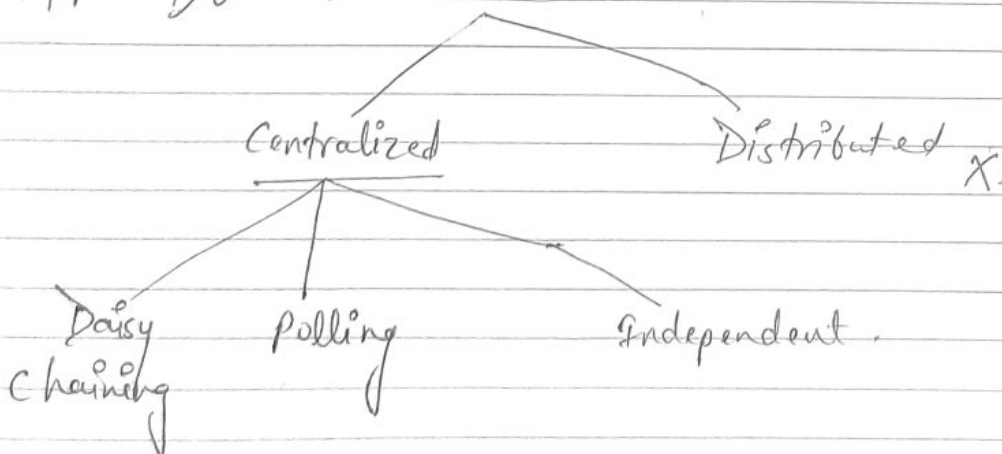
*** FAST ADDER



Propagation delay of Fast Adder is 60ns.

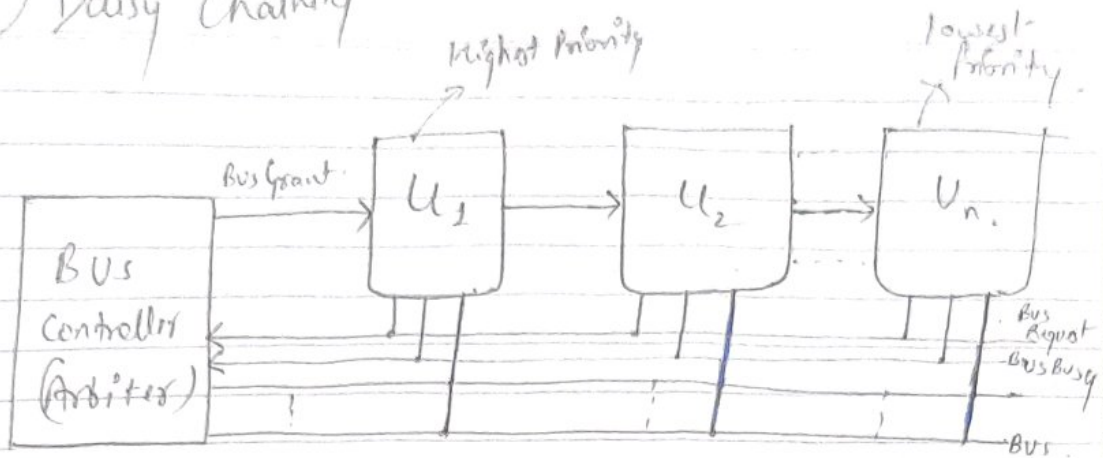
If XOR = 20ns & AND OR = 10ns.

Imp *** # BUS ARBITRATION:



Arbitration \rightarrow mediator.

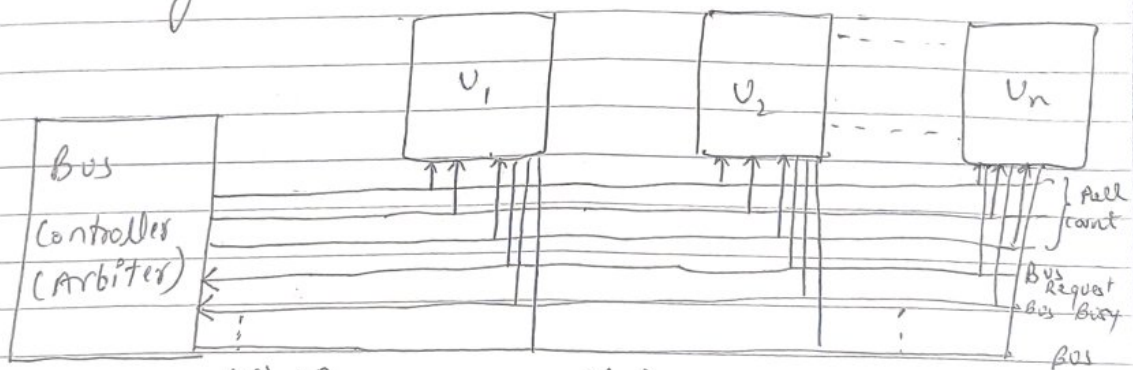
(i) Daisy Chaining



The drawback of daisy chaining is when any device is break the whole chain will break.

If n -devices then 2 control line $\begin{cases} Bg \\ BR \end{cases}$

(ii) Polling



If 8 devices then 5 poll count lines are there

The drawback of polling is that here the no. of wires increases so much.

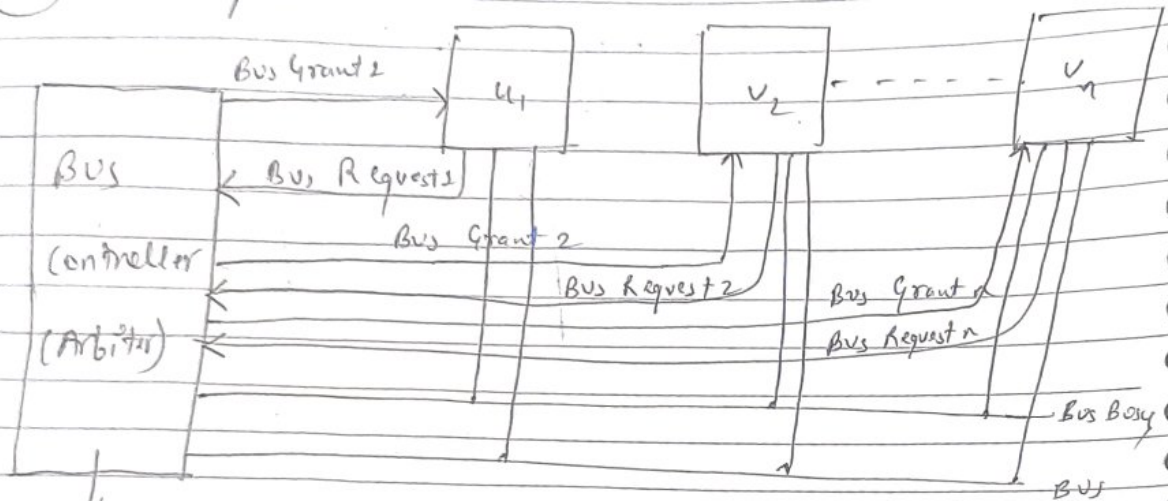
n -devices then lines $= \log_2 n$.

Both Daisy chaining & polling are slow method.

Short Notes \rightarrow CISC / RISC.

Generations of Computers. (2nd & 3rd Generation write arithmetic operation of floating point Advantages)

(iii) Independent -



priority encoder. (If more than one bus request issues by devices then Bus Controller uses P.E.)

The drawback of Independent is the no. of wire increases.

• If n devices then $2n$ lines are required.

HARD