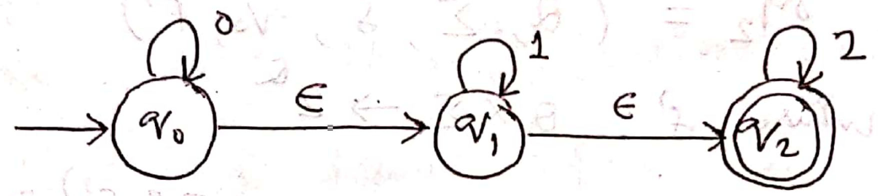
State Transition Table for NFA with ϵ-Transitions

The state transition table for an NFA with ϵ-moves is similar to that of an NFA or DFA. The only difference is that along with the columns labelled by input symbols from Σ, there is an additional column labelled 'ϵ'.

Thus $\delta : Q \times (\Sigma \cup \{\epsilon\}) \rightarrow 2^Q$

e.g.



NFA with ϵ-Move

Q \ $\Sigma \cup \{\epsilon\}$	0	1	2	ϵ
$\rightarrow q_0$	$\{q_0\}$	—	—	$\{q_1\}$
q_1	—	$\{q_1\}$	—	$\{q_2\}$
$*q_2$	—	—	$\{q_2\}$	—

ϵ-Closure of a State

It is the set of all states having distance zero from state 'q' known as ϵ-closure(q)

e.g. $\epsilon\text{-closure}(q_0) = \{q_0, q_1, q_2\}$

Here q_0 is also added to the set because every state is at distance zero from itself.

similarly $\epsilon\text{-closure}(q_1) = \{q_1, q_2\}$

$\epsilon\text{-closure}(q_2) = \{q_2\}$

$$\boxed{\epsilon\text{-closure}(q_0) = \hat{\delta}(q_0, \epsilon) = \{q_0, q_1, q_2\}}$$

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NFA with ε-moves to NFA without ε-moves

Let us consider a NFA with ε-moves

$$M_1 = (Q, \Sigma, \delta, q_0, F)$$

where $\delta : Q \times (\Sigma \cup \{\epsilon\}) \rightarrow 2^Q$

This can be converted to an NFA without ε-moves

$$M_2 = (Q, \Sigma, \delta', q_0, F')$$

where $\delta' : Q \times \Sigma \rightarrow 2^Q$

Here set of final states (F & F') might not be same.

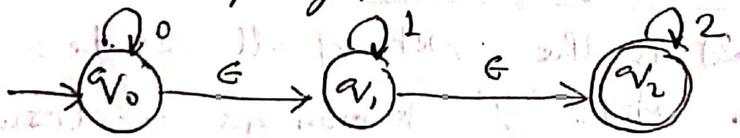
~~Issu~~ For each state that we have you have to check where does this state go on ε-closure and Input set of state you get here

δ' is defined as State $\rightarrow$ $\epsilon^* . a\, \epsilon^*$

have to be check for ~~particular state~~ Input

$$\boxed{\delta'(q, a) = \epsilon\text{-closure}\left(\delta\left(\epsilon\text{-closure}(q), a\right)\right)}$$

Example 1 Convert the following NFA with ε-moves to its equivalent NFA without ε-moves accepting the same language.



Solution: As per Definition of ε-closure

ϵ-closure $(q_0) = \{q_0, q_1, q_2\}$

ϵ-closure $(q_1) = \{q_1, q_2\}$

ϵ-closure $(q_2) = \{q_2\}$

∴ q_2 is the only final state for given NFA with ε-moves

$$F = \{q_2\}$$

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$\because \quad q_2 \in \epsilon\text{-closure}(q_0) = \{q_0, q_1, q_2\}$

i.e. q_2 is at zero distance from q_0 & q_1

Therefore set of final states without ϵ-moves is given by

$$F' = \{q_0, q_1, q_2\}$$

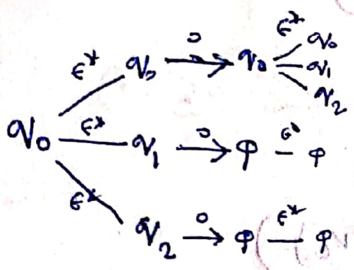
Now we can obtained δ'

$$\delta'(q_0, 0) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_0), 0))$$

$$= \epsilon\text{-closure}(\delta(\{q_0, q_1, q_2\}, 0))$$

$\underline{\epsilon^* \, 0 \, \epsilon^*}$

$$= \epsilon\text{-closure}(\delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_2, 0))$$



$$= \epsilon\text{-closure}(\{q_0\} \cup \phi \cup \phi)$$

$$= \epsilon\text{-closure}(q_0)$$

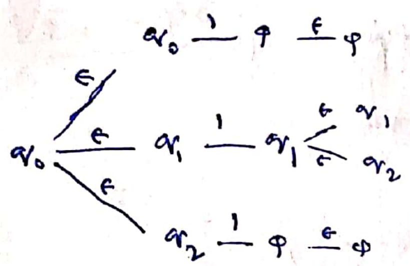
$$= \{q_0, q_1, q_2\}$$

$$\delta'(q_0, 1) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_0), 1))$$

$\underline{\epsilon^* . 1 . \epsilon^*}$

$$= \epsilon\text{-closure}(\delta(\{q_0, q_1, q_2\}, 1))$$

$$= \epsilon\text{-closure}(\delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_2, 1))$$



$$= \epsilon\text{-closure}(\phi \cup \{q_1\} \cup \phi)$$

$$= \epsilon\text{-closure}(q_1)$$

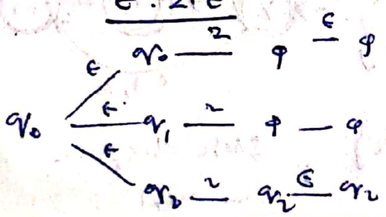
$$= \{q_1, q_2\}$$

$$\delta'(q_0, 2) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_0), 2))$$

$\underline{\epsilon^* . 2 . \epsilon^*}$

$$= \epsilon\text{-closure}(\delta(\{q_0, q_1, q_2\}, 2))$$

$$= \epsilon\text{-closure}(\delta(q_0, 2) \cup \delta(q_1, 2) \cup \delta(q_2, 2))$$



$$= \epsilon\text{-closure}(\phi \cup \phi \cup \{q_2\})$$

$$= \epsilon\text{-closure}(q_2)$$

$$= \{q_2\}$$

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$$\delta'(q_1, 0) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_1), 0))$$

$\underline{\epsilon^* . 0 . \epsilon^*}$

$$= \epsilon\text{-closure}(\delta(\{q_1, q_2\}, 0))$$

$q_1 - q_1 - \phi - \phi$, $q_1 - q_2 - \phi - \phi$

$$= \epsilon\text{-closure}(\delta(q_1, 0) \cup \delta(q_2, 0))$$

$$= \epsilon\text{-closure}(\phi \cup \phi)$$

$$= \epsilon\text{-closure}(\phi)$$

$$= \phi$$

Similarly we can obtain.

$\epsilon^* . 1 \epsilon^*$

$$\delta'(q_1, 1) = \epsilon\text{-closure}(\{q_1\} \cup \phi)$$

$q_1 \xrightarrow{\epsilon} q_1 \xrightarrow{1} q_1 - q_1$, $q_1 - q_2$; $q_1 \xrightarrow{\epsilon} q_2 \xrightarrow{1} \phi - \phi$

$$= \epsilon\text{-closure}(q_1)$$

$$= \{q_1, q_2\}$$

$\epsilon^* . 2 \epsilon^*$

$$\delta'(q_1, 2) = \epsilon\text{-closure}(\phi \cup \{q_2\})$$

$q_1 \xrightarrow{\epsilon} q_1 \xrightarrow{2} \phi - \phi$; $q_1 \xrightarrow{\epsilon} q_2 - q_2 - q_2$

$$= \{q_2\}$$

$$\delta'(q_2, 0) = \epsilon\text{-closure}(\phi)$$

$$= \phi$$

$$\delta'(q_2, 1) = \epsilon\text{-closure}(\phi)$$

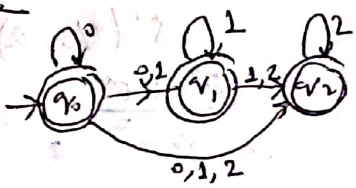
$$= \phi$$

$$\delta'(q_2, 2) = \epsilon\text{-closure}(q_2)$$

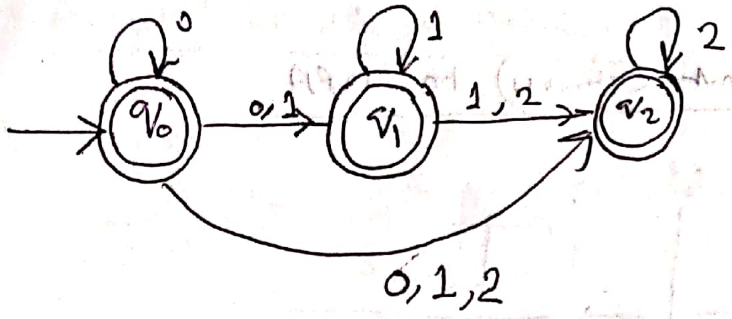
$$= \{q_2\}$$

$\therefore$ State transition table without ϵ-moves

Q \ Σ	0	1	2
→ q_0	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$
q_1	ϕ	$\{q_1, q_2\}$	$\{q_2\}$
q_2	ϕ	ϕ	$\{q_2\}$



State diagram in next page



Equivalence of DFA and NFA with ε-Transitions

There are two approaches

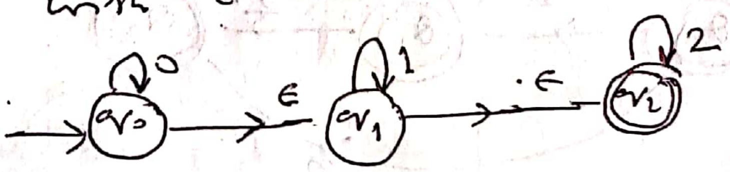
1) Indirect conversion method
2) Direct " "

① Indirect conversion method

Rule-1 : construct the NFA without ε-moves from the given NFA with ε-moves

Rule-2 : Construct an equivalent DFA from the newly-constructed NFA without ε-moves

Example: Construct an equivalent DFA for the NFA with ε-moves shown as:



Step 1 :

δ \ Σ	0	1	2	ε
→ q_0	$\{q_0\}$	—	—	$\{q_1\}$
q_1	—	$\{q_1\}$	—	$\{q_2\}$
* q_2	—	—	$\{q_2\}$	[illegible]

NFA to NFA without ε-moves ⟹

δ \ Σ	0	1	2
→ q_0	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$
q_1	—	$\{q_1, q_2\}$	$\{q_2\}$
q_2	—	—	$\{q_2\}$

⑥

Step 2: NFA (without ε-moves) to DFA

Q \ Σ	0	1	2
$\rightarrow * q_0$	$[q_0 q_1 q_2]$	$[q_1 q_2]$	$[q_2]$
$* [q_0 q_1 q_2]$	$[q_0 q_1 q_2]$	$[q_1 q_2]$	$[q_2]$
$* [q_1 q_2]$	φ	$[q_1 q_2]$	$[q_2]$
$* [q_2]$	φ	φ	$[q_2]$

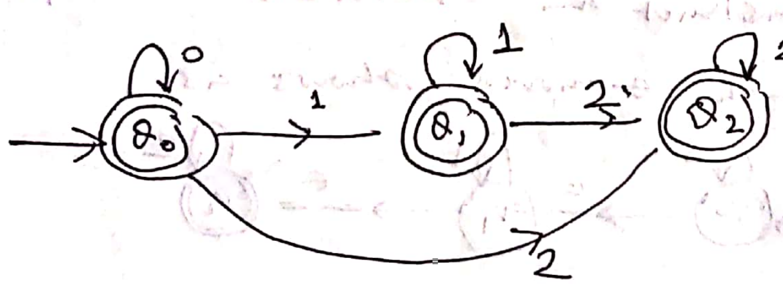
Step 3: Reduce the state transition diagram.

$q_0 = [q_0 q_1 q_2] = Q_0$

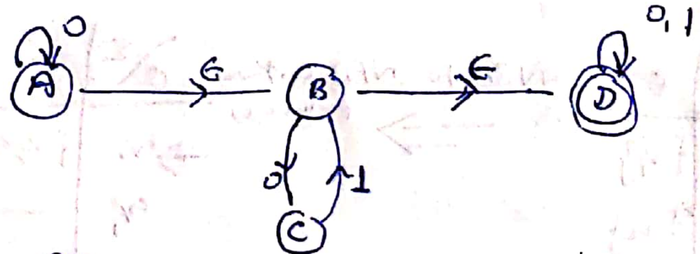
$[q_1 q_2] = Q_1$

$[q_2] = Q_2$

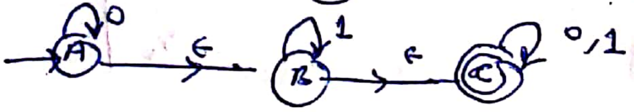
Q \ Σ	0	1	2
$\rightarrow * Q_0$	Q_0	Q_1	Q_2
$* Q_1$	φ	Q_1	Q_2
$* Q_2$	φ	φ	Q_2



Ex-2



Ex-3



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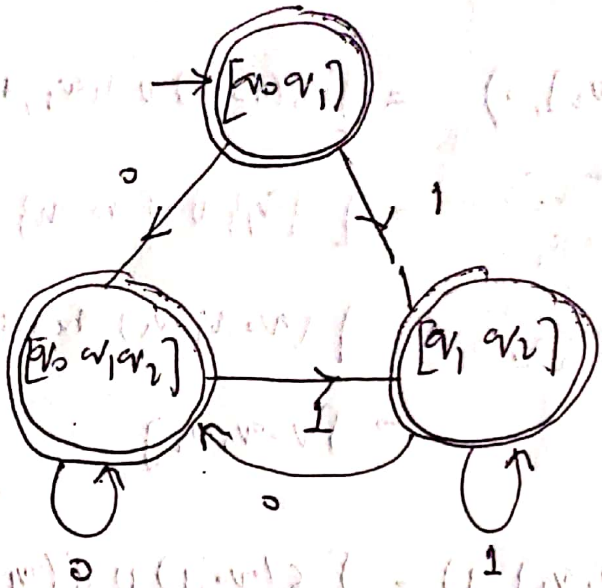

IInd Method : First convert NFA to NFA without ε-closure and convert it into DFA.

Q:- 1 Consider the following ε-NFA, compute the ε-closure of each state.

Q \ Σ	ε	a	b	c
→ p	—	{r}	{q}	—
q	{p}	{q}	{r}	—
* r	{q}	{r}	—	{r}

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ε-closure (p) = {p} ∵ 'p' has 0 distance from 'p'

ε-closure (q) = {q, p}

ε-closure (r) = {r, q, p}

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