Example 51: *Construct a truth table for the following statement:*

(*i*) $(p \vee q \Rightarrow s) \Leftrightarrow (p \Leftrightarrow s) \vee (q \Rightarrow s)$

(*ii*) $[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow [p \Rightarrow r]$

[U.P.T.U. (B.Tech.) 2005]

Solution: (i) Truth table for $(p \vee q \Rightarrow s) \Leftrightarrow (p \Rightarrow s) \vee (q \Rightarrow s)$:

p	q	s	$p \vee q$	$p \vee q \Rightarrow s$	$p \Rightarrow s$	$q \Rightarrow s$	$(p \Rightarrow s) \vee (q \Rightarrow s)$	$(p \vee q \Rightarrow s) \Leftrightarrow (p \Rightarrow s) \vee (q \Rightarrow s)$
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	T	T	T	T	T	T
T	F	F	T	F	F	T	T	F
F	T	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T	F
F	F	T	F	T	T	T	T	T
F	F	F	F	T	T	T	T	T

(ii) Truth table for $[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow [p \Rightarrow r]$:

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$p \Rightarrow r$	$(p \Rightarrow q) \wedge (q \Rightarrow r)$	$[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow [p \Rightarrow r]$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Example 52: Prove that the following statement is a tautology.

$$(\sim B) \wedge (A \Rightarrow B) \Rightarrow (\sim A)$$

[U.P.T.U. (B.Tech.) 2005]

Solution: Truth table for given statement:

A	B	$\sim A$	$\sim B$	$A \Rightarrow B$	$\sim B \wedge (A \Rightarrow B)$	$(\sim B) \wedge (A \Rightarrow B) \Rightarrow (\sim A)$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

Hence, $(\sim B) \wedge (A \Rightarrow B) \Rightarrow (\sim A)$ is a tautology.

Example 53: *If $p \equiv$ Ram is beautiful, $q \equiv$ Ram is mixable, $r \equiv$ His friends like Ram, then write the following statement in language:*

(a) $(p \Rightarrow q) \vee (p \Rightarrow r)$ (b) $p \Rightarrow (q \vee r)$

Examine, are the above statement equivalent?

Solution:

(a) If Ram is beautiful then either Ram is mixable or his friends like Ram.

(b) If Ram is beautiful then he is mixable or his friends like him.

Combined truth table for (a) and (b):

p	q	r	$p \Rightarrow q$	$p \Rightarrow r$	$(p \Rightarrow q) \vee (p \Rightarrow r)$	$q \vee r$	$p \Rightarrow (q \vee r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	F	T

From above truth table, we see that the entries in sixth and eighth column are identical.

Hence, the statements given by (a) and (b) above are logically equivalent.

***Example* 54:** *Prepare the tables of the following statement:*

(*i*) $(p \Leftrightarrow q) \wedge (r \vee q)$ [U.P.T.U. (B.Tech.) 2007]

(*ii*) $\{(p \vee q) \wedge r\} \Rightarrow q$

Solution: (i) Truth table of $(p \Leftrightarrow q) \wedge (r \vee q)$:

p	q	r	$p \Leftrightarrow q$	$r \vee q$	$(p \Leftrightarrow q) \wedge (r \vee q)$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	T	F
T	F	F	F	F	F
F	T	T	F	T	F
F	T	F	F	T	F
F	F	T	T	T	T
F	F	F	T	F	F

(ii) Truth table of $\{(p \vee q) \wedge r\} \Rightarrow q$:

p	q	r	$p \vee q$	$(p \vee q) \wedge r$	$\{(p \vee q) \wedge r\} \Rightarrow q$
T	T	T	T	T	T
T	T	F	T	F	T
T	F	T	T	T	F
T	F	F	T	F	T
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	F	F	T
F	F	F	F	F	T