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2.Recursive Definition of Relation

➢ It is a function in which the definition of function refers to itself. This must have following properties:

a) There must be certain arguments called base values for which does't refers to itself.

a) Each time the function refer to itself, the argument of the function must be closer to a base value.

Examples:

Ex 1) The factorial function $f(n) = n!$ is defined recursively as follows: Initial Condition: $f(0) = 1$, Recursion: $f(n + 1) = (n + 1).f(n)$

Discussion:

Starting with the initial condition $f(0) = 1$ and the recurrence relation $f(n + 1) = (n + 1) . f(n)$, For example,

$$1! = f(1) = 1.f(0) = 1,$$

$$2! = f(2) = 2.f(1) = 2,$$

$$3! = f(3) = 3.f(2) = 6, \text{ etc}$$

When a function $f(n)$, is defined recursively then the equation giving $f(n+1)$ in terms of previous values of f is called as **Recurrence Relation**.

Ackermann Function(Example of Recursive Function)

It is a function defined as by, A(m,n) where m and n are non-negative integers.

Conditions:

1) if m=0 then $\boxed{A(m,n)=n+1}$
2) if m $\neq$ 0 then $\boxed{A(m,n)=A(m-1,1)}$
3) if m,n $\neq$ 0 then

$$\boxed{A(m,n)=A(m-1,A(m,n-1))}$$

Ackermann Function(Example of Recursive Function)

Ex1 : Compute A(1,2)

Sol- A(1,2)= A(1-1,A(1,1))

= A(0,A(1,1))

A(m,n)=n+1, m=0

A(1,1)=A(0,A(1,0))

A(m,n)=A(m-1,A(m,n-1)), m,n≠0

A(1,0)=A(0,1)

A(0,1)=2

Therefore, A(1,0)=2

A(1,1)=A(0,2)

A(0,2)=3

A(1,1)=3

A(1,2)=A(0,3)

A(m,n)=A(m-1,1), m≠0

= 3+1=4

A(1,2)=4

3. ORDER RELATIONS

Partial Orderings (POSET) :A binary relation R is a partial order over a set A iff it is

- *Reflexive,*
- *Antisymmetric, and*
- *Transitive.*

- A pair (A, R), where R is a partial order over A, is called a partially ordered set or poset.

Properties of Order Relations

A binary relation R over a set A is called **antisymmetric** iff

For any $x \in A$ and $y \in A$,

if xRy and yRx, then $x = y$.

Properties of Order Relations

$x \leq y$

$42 \leq 137$

$137 \leq 42$?

Not Antisymmetric

$x \leq y$

$137 \leq 137$

$137 \leq 137$?

Antisymmetric

Properties of Order Relations

$x \leq y$

$x \leq x$

Reflexivity

$x \leq y$

$137 \leq 137$

137 ≤ 137?

Antisymmetric

$x \leq y$ and $y \leq z$

$x \leq z$

Transitivity

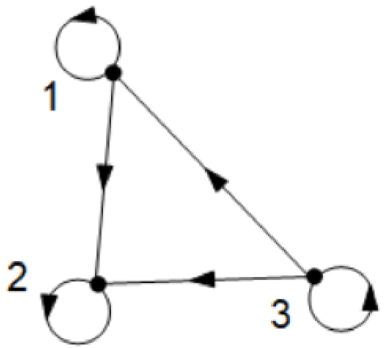
Example 1: The less-than-or-equal-to relation on the set of integers ***I*** is a partial order, and the set ***I*** with this relation is a poset.

Example 2: The subset relation on the power set of a set, say **{1, 2}**, is also a partial order, and the set **{1, 2}** with the subset relation is a poset.

3. Examples:

Let S = {1, 2, 3} and

let R = {(1,1), (2,2), (3,3), (1, 2), (3,1), (3,2)}



P={(1,1), (2,2), (3,3), (1,3), (2,3), (1,2)} ?

- Let's consider set A as follows:

A = {p,q,r}

- Let's analyze if this subset of A*A {(p,p),(q,q),(r,r),(p,r),(q,r)} is partially ordered or not

Solution:

1. Check if it is reflexive
2. Check if it is anti-symmetric
3. Check if it is transitive

- Step 1: The subset is reflexive as it contains the pairs, (p,p), (q,q) and (r,r).
- Step 2: It is anti-symmetric as (p,r) and (q,r) do not have their symmetric pairs (r,p) or (r,q) in it. In addition, it also contains the pairs (p,p), (q,q), (r,r) in which the elements are equal to each other.
- Step 3: The subset contains (p,p) and (p,r). Therefore according to the definition of a transitive relation, it must contain (p,r), which, you can see, is already present in it.
- Hence it is transitive.
- Since all the three conditions are satisfied, we could now call the subset as a **partially ordered set**

Total Order Relation

Consider the relation R on the set A. If it is also called the case that for all, a, b ∈ A,

we have either (a, b) ∈ R or (b, a) ∈ R or a = b, then the relation R is known total order relation on set A.

Example: Show that the relation '<' (less than) defined on N, the set of +ve integers is neither an equivalence relation nor partially ordered relation but is a total order relation.

Solution:

- **Reflexive:** Let a ∊ N, then a < a
  ⟹ '<' is not reflexive.
- As, the relation '<' (less than) is not reflexive, it is neither an equivalence relation nor the partial order relation.
- But, as ∀ a, b ∊ N, we have either a < b or b < a or a = b. So, the relation is a total order relation.