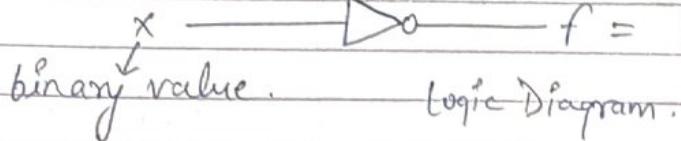


22/07/19

## # LOGIC GATES

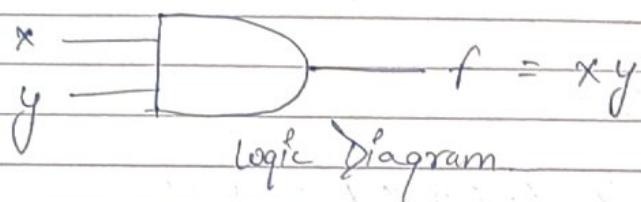
\* NOT -



Truth Table.

$x$	$\bar{x}$
1	0
0	1

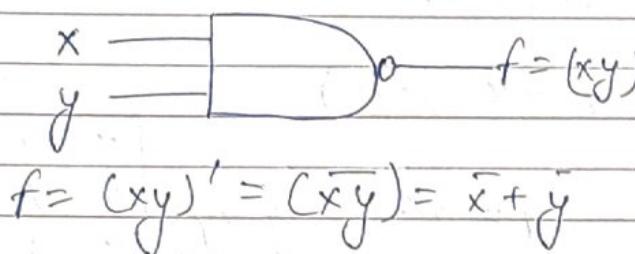
\* AND -



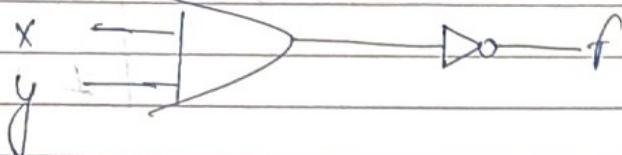
Truth Table.

$x$	$y$	$f = xy$
0	0	0
1	0	0
0	1	0
1	1	1

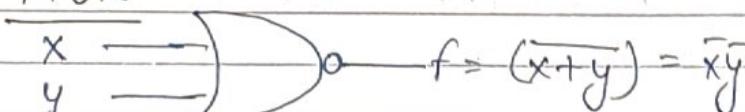
\* NAND -



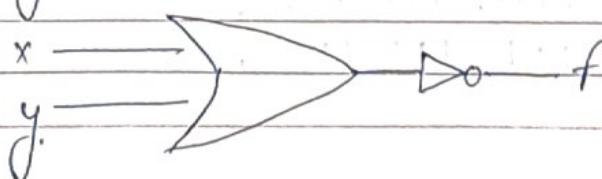
$x$	$y$	$f = (\bar{x}\bar{y})$
0	0	1
0	1	1
1	0	1
1	1	0



\* NOR -



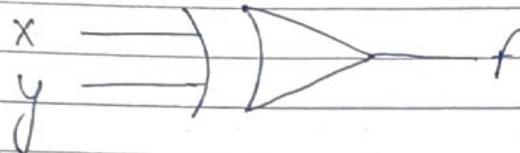
$x$	$y$	$f = (\bar{x} + \bar{y})$
0	0	1
0	1	0
1	0	0
1	1	0



$$② \quad x\bar{x} = 0, \quad x + \bar{x} = 1, \quad \bar{\bar{x}} = x.$$

$n$ - binary variables,  $2^n$  minterms/maxterms,  $2^n$  boolean fns.

### \* XOR (ODD FUNCTION).-



$$f = x \oplus y = \bar{x}y + x\bar{y}$$

x	y	$f = x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0

e.g.  $f = 1 \oplus 1 \oplus 0 \oplus 1 \oplus 1 = 0$

$$f = \text{XOR}(1, 1, 0, 1, 1) = 0.$$

$$f = \text{XOR}(1, 1, 1, 0, 0, 0) = 1.$$

### \* PROPERTIES -

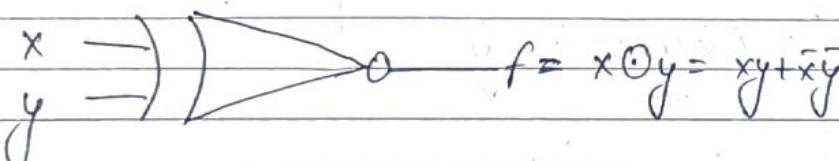
$$x \oplus x = 0 \rightarrow \bar{x} \cdot x + x\bar{x} = 0+0 = 0$$

$$x \oplus \bar{x} = 1 \rightarrow \bar{x}\bar{x} + x\bar{x} = \bar{x} + x = 1$$

$$x \oplus 0 = x \rightarrow \bar{x} \cdot 0 + x \cdot 1 = x$$

$$x \oplus 1 = \bar{x} \rightarrow \bar{x} \cdot 1 + x \cdot 0 = \bar{x}$$

### \* XNOR (EVEN FUNCTION)



$$f = x \odot y = xy + \bar{x}\bar{y}$$

x	y	$f = x \odot y$
0	0	1
0	1	0
1	0	0
1	1	1

### # CONVERSION OF OCTAL INTO BINARY NO.

binary ←  $\begin{smallmatrix} 3 \\ 2^3 = 8 \end{smallmatrix} \rightarrow$  octal units.  
6 bits!

e.g.  $(176)_8 \rightarrow (001\ 111\ 110)_2$   
 $(776)_8 \rightarrow (111\ 111\ 110)_2$

$$\begin{aligned}
 0+0 &= 0 \\
 1+0 &= 1 \\
 0+1 &= 1 \\
 1+1 &= 0
 \end{aligned}$$

(3)

## # CONVERSION OF HEXADECIMAL INTO BINARY.

Base = 16.

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F.

binary  $\xleftarrow{\quad}$   $2^4 = 16$   $\xrightarrow{\quad}$  hexadecimal.

$b_3 \ b_2 \ b_1 \ b_0$   
 8 4 2 1

e.g.  $(1AC)_{16} \rightarrow (000110101100)_2$ .

e.g.  $(1110.111)_2 \rightarrow (E.E)_{16} = E.E(H)$

e.g.  $(1111111.11)_{20} \rightarrow (7F.C)_{16}$ .

e.g.  $(1111111.11)_2 \rightarrow (177.7)_8$ .

## # COMPLEMENTS -

Two types of complements are -

$r=2$   
 r's      (r-1)'s      1's      2's

e.g.  $N = 1011100$ .

$1'(s)(N) = 0100011$

$1^s \text{ method}$        $+ 1$        $2^s \text{ method}$   
 $2^s(N) = \underline{0100100} \quad \underline{0100100}$

e.g.  $N = 111110$

$1^s = 000001$   
 $2^s = 000010$

e.g.  $N = 10101$

$2^s = 10101$

(21)

11001

S.M.R.

## # NUMBER REPRESENTATION -

+ve	+5	$\rightarrow$ 0	$\downarrow^{\text{MSB}}$	0101 ( +5 )
-ve	-5	$\rightarrow$ 1		1101 ( -5 )

Positive no. has only one representation i.e.,  
 Signed Magnitude Representation.  
 e.g. +13  $\rightarrow$  01101.

Negative no. are represented by three ways -

- \* Signed Magnitude Representation.

- \* 1's complement.

- \* 2's complement.

+5  $\rightarrow$  0101 ) .

e.g. \* -5  $\rightarrow$  (1101) . S.M.R.

\* -5  $\rightarrow$  (1010) 1's complement.

\* -5  $\rightarrow$  (1011) 2's complement.

+28  $\rightarrow$  (00011001)

e.g. \* -28  $\rightarrow$  (10011001) . S.M.R.

\* -28  $\rightarrow$  (11100110) 1's

\* -28  $\rightarrow$  (11100111) 2's

## # BINARY ARITHMETIC.

## \* ADDITION -

$$\begin{array}{r}
 & 1 & 1 & 0 & 1 \\
 + & 1 & 1 & 1 & 1 \\
 \hline
 & 1 & 1 & 1 & 0 & 0
 \end{array}
 \quad \frac{2}{2} = \text{rem} = 0 \quad q = 1.$$

(5)

$$\begin{array}{r}
 & 1 & 1 & 1 \\
 + & 1 & 1 & 1 \\
 \hline
 & 1 & 1 & 0 & 0
 \end{array}$$

↑  
Temp

## SUBTRACTION -

↓  
Using 2's complement  
(2's)

e.g.  $(\overset{M}{8} - \overset{N}{3}) = ?$

$M = 5 = 0101$   
 $N = 3 = 0011$

↓  
using  $(\overset{r-1}{7})$ 's complement.  
(1's)

e.g.  $(\overset{M}{3} - \overset{N}{5})$

$M = 3 = 0011$   
 $N = 5 = 0101$

$M = 0101$

$N(2's) = + 1101$

Carry Discard.  $\leftarrow \textcircled{1} 0 0 1 0$ If carry come it means  
 $m \geq n$ .(0010) Answer.

$M = 0011$

$N = + 1011$

$1110 \quad (m < n)$

Here, carry is not found  
which means the result  
is negative. Then we  
convert it into 2's then  
we get  $- (0010)$  Answer

e.g.  $M = 01011$   
 $N = 01001$

$M = 01011$   
 $N = + 10111$   
 $\leftarrow \textcircled{1} 0 0 0 1 0$

(00010) Answer.

$M = 01011$

$N = + 10001$

$11100$

- (00100)

$$\begin{array}{r}
 111\ 2 \\
 -100 \\
 \hline
 10
 \end{array}$$

8 4 2 1

Using 1's Complement -

Q  $M = 0110$  &  $N = 0011$ ,  $(M-N)$ .

$$\begin{array}{r}
 M = 0110 \\
 N(1's) = +1100 \\
 \hline
 \begin{array}{r}
 0010 \\
 \xrightarrow{\rightarrow +1} \\
 \hline
 10011 \quad \text{Ans}
 \end{array}
 \end{array}$$

Q

$$\begin{array}{r}
 0011 \\
 +1001 \\
 \hline
 1100
 \end{array}$$

↓ 1's  
- (0011).

\* OCTAL NUMBER SYSTEM -

7's complement      8's complement.

e.g.  $N = (1235)_8$   $\neq (0001)$

$$7's(N) = (6542)$$

$$8's(N) = (6543)$$

e.g.  $N = (112367)_8$

$$7's(N) = (665410)$$

$$8's(N) = (665411)$$

$\frac{29}{16}$

13

(7)

## \* DECIMAL NUMBER SYSTEM -

$$N = (189954)_{10}.$$

$$9's(N) = (810045)$$

$$10's(N) = (810046)$$

## \* HEXADECIMAL NUMBER SYSTEM.

e.g.  $N = (1EAB6)_{16}$

$$\Rightarrow 15's(N) = (E1549)_{16}$$

$$16's(N) = (E154A)_{16}$$

$$N = (1843)_6$$

$$6's = (4023)$$

$$5's = (4012)$$

e.g. 
$$\begin{array}{r} (A\ 1\ B\ C\ E)_{16} \\ + (5\ F\ 1\ 3\ F)_{16} \\ \hline L\ O\ Q\ D\ A\ D \end{array}$$

$$\begin{array}{r} (F\ F\ F)_{16} \\ + (F\ F\ F)_{16} \\ \hline (1\ F\ F\ E)_{16} \end{array}$$

## \* DECIMAL TO BINARY -

$$(13.3)_{10} \rightarrow (1101.01001)_2$$

for Decimal

$$\begin{array}{r} 0.3 \times 2 = 0.6 \\ 0.6 \times 2 = 1.2 \\ 0.2 \times 2 = 0.4 \\ 0.4 \times 2 = 0.8 \\ 0.8 \times 2 = 1.6. \end{array}$$

$$Q = \left( \begin{array}{r} 28 \\ 8 \end{array} \right)^3 \rightarrow (1) \quad \begin{array}{r} 10 \\ 31 16 8 4 2 1 \\ 110100 \end{array}$$

$$Q = (25.6)_{10}$$

$$(13.46)_8 \quad \begin{array}{l} 0.6 \times 8 = 4.8 \\ 0.8 \times 8 = 6.4 \\ 0.4 \times 6 = 3.2 \end{array} \quad \begin{array}{l} 0.8 \times 2.6 = 2.08 \\ 0.8 \end{array}$$

$$Q = (152.8)_{10} \rightarrow (98.00)_{16}$$

Q Other to Decimal Conversion.

$$Q = (1101.1101)_2 \rightarrow (?)_{10}$$

$$1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2} + 0 \times 2^{-3} + 1 \times 2^{-4} \\ (13.81)_{10} \quad \frac{1}{2} = 0.5, \frac{1}{4} = 0.25, \frac{1}{10} = 0.06$$

## # OTHER BINARY CODES -

\* BCD Code (Binary Coded Decimal)

$$\begin{matrix} b_3 & b_2 & b_1 & b_0 \\ 8 & 4 & 2 & 1 \end{matrix} \quad \begin{matrix} 0 & 1 & 0 & 0 & 0 & 0 \end{matrix}$$

$$(52)_{10} \rightarrow (110100)_2 \rightarrow (01010000)_{BCD}$$

$$(10)_{10} \rightarrow (00010000)_{BCD} \rightarrow (1010)_2$$

$$(15.86)_{10} \rightarrow (00010101.01010110)_{BCD}$$

$\text{XOR} \rightarrow \text{odd } f^n \rightarrow \text{modulo 2} \Rightarrow \text{Symmetric diff.}$

(9)

\* Excess - 3 ( $X_3$ ) -

$$(0)_{10} \rightarrow (0011)_{\text{excess 3}}$$

$$(1)_{10} \rightarrow (0100)_{\text{excess 3}}$$

$$(2)_{10} \rightarrow (0101)_{\text{excess 3}}$$

$$(9)_{10} \rightarrow (1100)_{\text{excess 3}}$$

$$(167)_{10} \rightarrow (010010011010)_{\text{excess 3}}$$

$$\begin{array}{r} (000101100111)_{\text{BCD}} \\ + 001100110011 \\ \hline \end{array}$$

$$(25.3)_{10} \rightarrow (01011000.0110)_{X_3}$$

~~$\times$  Gray code (Reflected/Cyclic)~~

• Binary to Gray -

$$N = (10001000101)_2 \rightarrow (111110)_{\text{gray code}}$$

$$N = (100000001)_{\text{binary}} \rightarrow (1010010)_{\text{gray code}}$$

• Gray to Binary -

$$N = (10111)_{\text{gray code}} \rightarrow (11010)_2$$

$$N = (0111110)_{\text{gray code}}$$

$$(01010100)_2$$

$$Q \quad (211)_x = (152)_8. \quad (x = ? \text{ find base } x)$$

$$(2x^2 + 1x + 1x^0) = (1 \times 8^2 + 5 \times 8 + 5 \times 8^0)$$

$$(2x^2 + x + 1) = 105$$

$$2x^2 + x - 105 = 0$$

$$x = \frac{-1 \pm \sqrt{1 + 864}}{4}$$

$$\begin{array}{r} 2 | 210 \\ 3 | 105 \\ 7 | 35 \\ 5 | 5 \\ \hline & 1 \end{array}$$

$$2x^2 + 18x + 18x - 105 = 0$$

$$2x(x+9) - 15(x+9) = 0.$$

$$(2x+15)(x+7) = 0.$$

$$x = -15/2. \quad x = +7$$

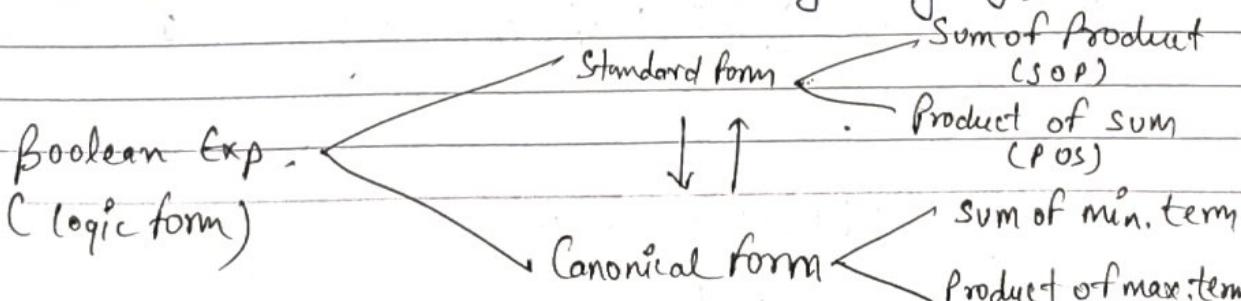
## # BOOLEAN EXPRESSION OR FUNCTION (LOGIC)

$$f(x,y) = xy + \bar{x}y + x\bar{y} \quad x, y, \bar{x}, \bar{y}$$

$$f. = x + \bar{x}y + \bar{y}$$

↓  
term

term      Product term ( $xy, x\bar{y}, \bar{x}\bar{y}$ )  
 sum term ( $x+y, x+\bar{y}$ )



$n \rightarrow 2^n \rightarrow \text{min/max term}$   $(\bar{x}\bar{y})' = x+y$ . (1)

↓  
Binary no.

x	y	min. term	maxterm	f1	f2	f3
		$\bar{x}\bar{y} (m_0)$	$x+y (m_0)$	0	0	1
		$\bar{x}y (m_1)$	$(x+\bar{y}) (m_1)$	0	0	0
		$x\bar{y} (m_2)$	$(\bar{x}+y) (m_2)$	1	1	1
		$xy (m_3)$	$(\bar{x}+y) (m_3)$	0	1	0

$$\begin{aligned} f_1 &= \bar{x}\bar{y} + xy \\ f_2 &= \bar{x}y + \bar{y} \\ f_3 &= (\bar{x}\bar{y} + \bar{y}) \end{aligned} \quad \left. \begin{array}{l} \text{sum of minterms.} \\ \hline \end{array} \right.$$

$$f_1 = \sum (m_2) = \Sigma(2)$$

$$f_2 = \sum (m_2, m_3) = \Sigma(2, 3)$$

$$f_3 = \sum (m_0, m_2) = \Sigma(0, 2)$$

$$f(x, y) = \begin{matrix} m_0 & 00 & \bar{x}\bar{y} \\ m_1 & 01 & \bar{x}y \\ m_3 & 11 & xy \end{matrix}$$

$$f(x, y, z) = \begin{matrix} m_7 & 100 & \bar{x}\bar{y}z \\ m_6 & 110 & x\bar{y}z \end{matrix}$$

$$f(x, y, z, w) = \begin{matrix} m_{11} & 1011 & \bar{x}\bar{y}z\bar{w} \\ m_{13} & 1101 & xy\bar{z}\bar{w} \end{matrix}$$

$$f = xy + \bar{x}\bar{y} + x\bar{y} \quad (\text{Canonical Form})$$

$$f = (\bar{x} + xy + \bar{y}) \quad (\text{Standard Form})$$

(sum of product).

product.

ex-Q  $f = x + \bar{y}$  (sum of product).

Convert it into sum of minterms.

Solution  $x \cdot (y + \bar{y}) = xy + x\bar{y} \rightarrow \textcircled{i}$ .

$$\bar{y} \cdot (x + \bar{x}) = x\bar{y} + \bar{x}\bar{y} \rightarrow \textcircled{ii}$$

$$f = xy + x\bar{y} + x\bar{y} + \bar{x}\bar{y}$$

$$f = xy + x\bar{y} + \bar{x}\bar{y}$$

$$f = \sum (m_3, m_2, m_0) \text{ or } \sum (3, 2, 0).$$

Q  $f = \bar{x} + x\bar{y} + y$

$$\bar{x}(y + \bar{y}) = \bar{x}y + \bar{x}\bar{y}$$

$$y(x + \bar{x}) = xy + x\bar{y}$$

$$f = \bar{x}y + xy + x\bar{y} + \bar{x}\bar{y}$$

$$f = \sum (m_0, m_1, m_2, m_3) = \sum (0, 1, 2, 3).$$

Q  $f = xy + \bar{x} + yz$

$$\bar{x}(y + \bar{y})(z + \bar{z}) = (\bar{x}y + \bar{x}\bar{y})(z + \bar{z}) = \bar{x}yz + \bar{x}y\bar{z} \\ + \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z}$$

$$xy(z + \bar{z}) = xyz + xy\bar{z}$$

$$yz(x + \bar{x}) = xyz + \bar{x}yz$$

1	1
1	1
1	1

→ 1 (constant).

421

K-map

(13)

$$f = \bar{x}\bar{y}\bar{z} + x\bar{y}\bar{z} + xy\bar{z} + \bar{x}\bar{y}z + \bar{x}y\bar{z}$$

$$+ x\bar{y}z -$$

$$f = \sum(0, 6, 7, 3, 2, 1) = \sum(0, 1, 2, 3, 6, 7)$$

$$f = (x+y+z) \cdot (\bar{x}+\bar{y}+\bar{z}) \cdot (\bar{x}+\bar{y}+z)$$

(product of max terms.)  
(Canonical form)

$$f = (x) \cdot (x+y) \cdot (\bar{x}+\bar{y}+z)$$

(product of sum) (Standard form).

## # BOOLEAN FUNCTION SIMPLIFICATION -

\* USING K-MAP

• 2-variable K-map.

x 0	y 0	0	1	→ $\bar{x}y$
1	0	$m_0$	$m_1$	
	1	$m_2$	$m_3$	→ $xy$

Q  $f = \sum(m_0, m_1)$ . minimize it using K-map.

Solution-

x 0	y 0	0	1	→ $\bar{y} + y = 1$
1	0	1	1	
	1	1	1	

=  $\bar{x}$

Q  $f = \sum(m_0, m_2, m_3)$ .

$$\Rightarrow \bar{y} + x$$

x 0	y 0	0	1	1
0	0	1	1	1
1	0	1	1	1

Q  $f = \sum (m_0, m_1)$

x	y	0	1
0	0	1	1
1	1	1	1

$$f = \bar{x}\bar{y} + xy.$$

• 3-variable K-map

x	y	z	00	01	11	10
0	$m_0$	$m_1$	$m_3$	$m_2$		
1	$m_4$	$m_5$	$m_7$	$m_6$		

$$\rightarrow x\bar{y}z$$

x	y	z	00	01	11	10
0	1	1	1	1		
1	1	1	1	1		

$$= \bar{z} \text{ Answer}$$

$$\rightarrow f = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + A\bar{B}\bar{C} + A\bar{B}\bar{C}$$

Q

x	y	z	00	01	11	10
0	1	1	1	1	1	1
1	1	1	1	1	1	1

$$= yz + \bar{x}$$

Q

x	y	z	00	01	11	10
0	1	1	1	1	1	1
1	1	1	1	1	1	1

$$= \bar{x}\bar{z} + \bar{x}y$$

Q

x	y	z	00	01	11	10
0	1	1	1	1	1	1
1	1	1	1	1	1	1

$$= \bar{x}z + z + xy$$

$$8 \text{ Pair} = 1.$$

$$4 \text{ Pair} = \text{term (1 literal)}$$

$$2 \text{ Pair} = \text{term (2 literals)}$$

$$No \text{ Pair} = \text{term (3 literals)}$$

8 4 2 1

(15)

$$Q \quad f = \sum (0, 1, 2, 5, 6, 7) \\ \Rightarrow \begin{array}{c} xy^2 \\ \times \quad 00 \quad 01 \quad 11 \quad 10 \\ 0 \quad | \quad 0 \quad 1 \quad 1 \quad 1 \\ 1 \quad | \quad 1 \quad 1 \quad 1 \end{array}$$

$$\bar{x}y + \bar{y}z + xz + xy + y\bar{z}$$

• 4-value K-map.

	AB	CD	00	01	10	11
Q	00	1	1			
	01					
	11					
	10	1	1			

	AB	CD	00	01	11	10
	00		$m_0$	$m_1$	$m_3$	$m_2$
	01		$m_4$	$m_5$	$m_7$	$m_6$
	11		$m_{12}$	$m_{13}$	$m_{15}$	$m_{14}$
	10		$m_8$	$m_9$	$m_{11}$	$m_{16}$

$$= \bar{B}\bar{D}$$

$$Q \quad f = \sum (0, 1, 4, 5, 8, 9, 13, 15)$$

	AB	CD	00	01	11	10
Q	00	1	1			
	01	1	1	1		
	11		1	1		
	10	1	1			

$$= \bar{A}\bar{C} + \bar{B}\bar{C} + ABD -$$

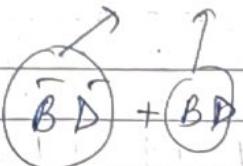
	AB	CD	00	01	11	10
Q	00	1	1	1	1	1
	01	1	1	1	1	1
	11		1	1	1	1
	10	1	1	1	1	1

$$= D + C + \bar{C}D + \bar{A}\bar{B}C - \bar{A}B.$$

Q

	AB\CD	00	01	11	10
00		1	1	1	1
01		1	1	1	1
11		1	1	1	1
10		1	1	1	1

essential prime implicant.



$$\bar{B}\bar{D} + B\bar{D} + \bar{A}\bar{C}\bar{D}$$

prime implicant.

- 5-variable K-maps

BC\DE	00	01	10	11	10	00	01	11	10
00	0	1	3	2	4	16	17	19	18
01	4	5	7	6	12	20	21	23	22
11	12	13	15	14	1	28	29	31	30
10	8	9	11	10	10	24	25	27	26

$$2^5 = 32 \text{ minterms } (0-31)$$

in

It uses BCD adder or Decimal adder.

BC\DE	00	01	11	10	BC\DE	00	01	11	10
00	0	1	1	1	00	1	1	1	1
01	1	1	1	1	01	1	1	1	1
11	1	1	1	1	11	1	1	1	1
10	1	1	1	1	10	1	1	1	1

$$f = \sum (5, 7, 13, 15, 21, 23, 29, 31) \text{ minimize it.}$$

$$= CE \text{ Answer} + \bar{A}\bar{B}\bar{C}\bar{D}\bar{E} + A\bar{B}\bar{C}\bar{D}$$

Q 2-bit binary input + provide its square as an output.



(17)

A	B	x	y	z	w	$x = xy$
0	0	0	0	0	0	$y = x \bar{y}$
0	1	0	0	0	1	$z = 0$
1	0	0	1	0	0	$w = \bar{x}y + xy$
1	1	1	0	0	1	$= y (\bar{x}+x) = y.$

## # DIGITAL CIRCUIT

↓  
Combinational  
(Adding, Subtraction, Mux,  
Decoding, Encoding)  
multiplexer.

↓  
Sequential:  
(Flip Flops, Registers,  
Counters.)

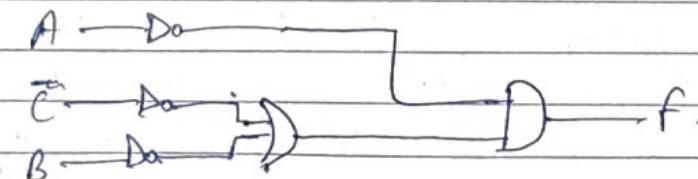
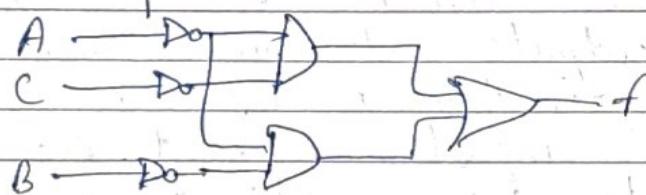
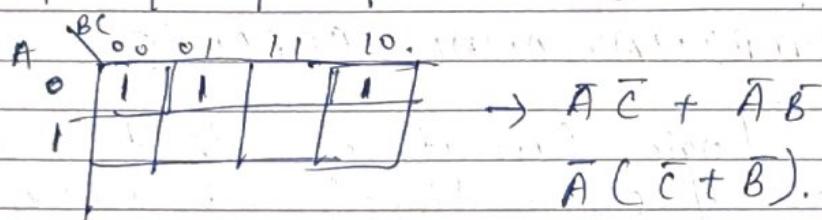
### \* COMBINATIONAL CIRCUIT -

Logic Circuit for digital systems may be combinational or sequential. A combinational circuit consists of logic gates whose outputs at any time are determined directly from the present combination of inputs without regard to previous inputs. A combinational circuit performs a specific information processing operation fully specified logically by a set of boolean functions.

Examples - adder, subtractor, multiplexer, encoder & decoder.

Ex- Design a combinational circuit with three inputs & one output. The output is equal to logic 1 when the binary value of the input is less than 3. The output is logic 0 otherwise.

$\Rightarrow$	A	B	C	X
	0	0	0	<del>ABC</del>
	0	0	1	<del>ABC</del>
	0	1	0	<del>ABC</del>
	1	0	0	0
	1	0	1	0
	1	1	0	0
	1	1	1	0



(19)

Q Design a combinational circuit with three inputs of 3 outputs. When the binary input is 0, 1, 2 or 3, the binary output is one greater than the input. When the binary input is 4, 5, 6 or 7 the binary output is one less than one.

Truth Table:

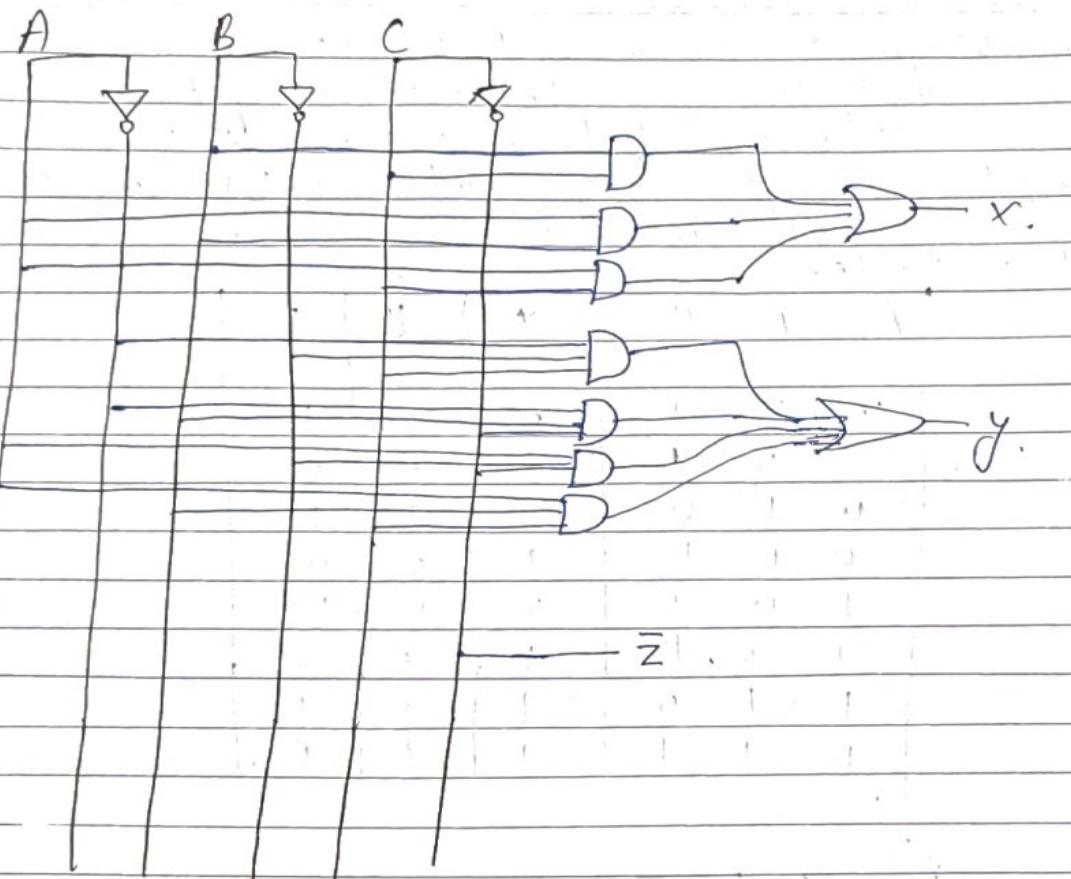
A	B	C	X	Y	Z
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0	0	1	1
1	0	1	1	0	0
1	1	0	1	0	1
1	1	1	1	1	0

$$X = \begin{array}{|c|c|c|c|} \hline & A & B & C \\ \hline \end{array} \quad \begin{array}{|c|c|c|c|} \hline & 0 & 0 & 0 \\ \hline & 0 & 0 & 1 \\ \hline & 0 & 1 & 0 \\ \hline & 0 & 1 & 1 \\ \hline & 1 & 0 & 0 \\ \hline & 1 & 0 & 1 \\ \hline & 1 & 1 & 0 \\ \hline & 1 & 1 & 1 \\ \hline \end{array} = BC + AB + AC$$

$$Y = \begin{array}{|c|c|c|c|} \hline & A & B & C \\ \hline \end{array} \quad \begin{array}{|c|c|c|c|} \hline & 0 & 0 & 0 \\ \hline & 0 & 0 & 1 \\ \hline & 0 & 1 & 0 \\ \hline & 0 & 1 & 1 \\ \hline & 1 & 0 & 0 \\ \hline & 1 & 0 & 1 \\ \hline & 1 & 1 & 0 \\ \hline & 1 & 1 & 1 \\ \hline \end{array} = \bar{A}BC + \bar{A}B\bar{C} + A\bar{B}\bar{C} + A\bar{B}C \Rightarrow X \oplus B \oplus C$$

$$Z = \begin{array}{|c|c|c|c|} \hline & A & B & C \\ \hline \end{array} \quad \begin{array}{|c|c|c|c|} \hline & 0 & 0 & 0 \\ \hline & 0 & 0 & 1 \\ \hline & 0 & 1 & 0 \\ \hline & 0 & 1 & 1 \\ \hline & 1 & 0 & 0 \\ \hline & 1 & 0 & 1 \\ \hline & 1 & 1 & 0 \\ \hline & 1 & 1 & 1 \\ \hline \end{array} = \bar{C}$$

(29)



## # ADDER -

- \* Half adder
- \* full adder
- \* Parallel adder
- \* BCD / Decimal adder
- \* Add 'eq' with carry look ahead generator.

Inputs. $\rightarrow$ XY		C	S	→ carry	→ sum,
0	0	0	0		
0	1	0	1		
1	0	0	1		
1	1	1	0		

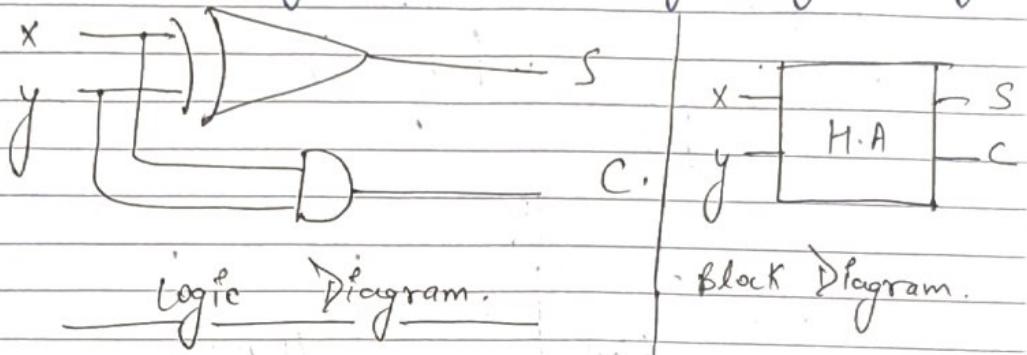
Truth Table

Boolean expression.

logic expression.

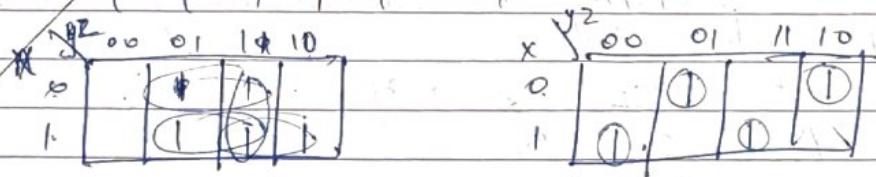
(21)

$$C = xy \quad S = \bar{x}y + x\bar{y} = x \oplus y$$



\* FULL ADDER - (Add 3-binary bits).

x	y	z	c	s
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1



$$\bar{x}z + xy + x'y'z + x'y'z + xyz$$

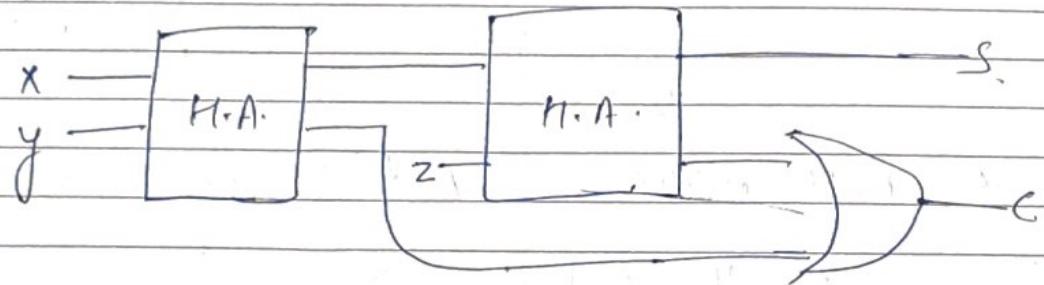
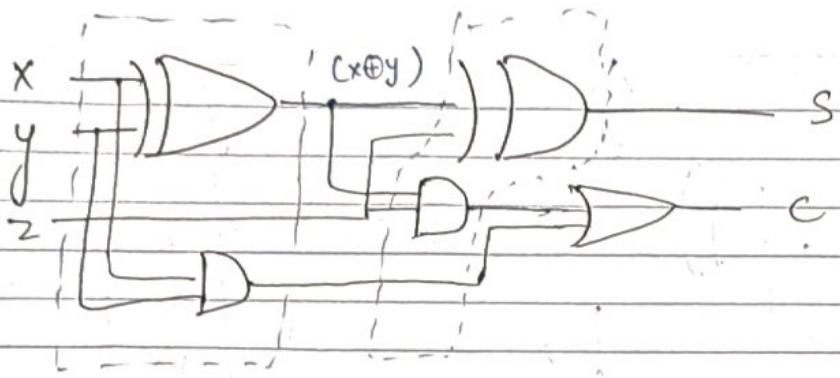
$$\begin{aligned}
 & \cancel{x} \cancel{y} \quad x'y + \bar{x}y'z + x'y'z \\
 & \cancel{y} \cancel{z} \quad xy + 2(x'y + x'y') \\
 & \quad xy + z(x \oplus y)
 \end{aligned}$$

$$\bar{x}\bar{y}z + \bar{x}\bar{y}z + xyz$$

$$+ \bar{x}yz + x(yz + yz) + x(\bar{y}z + yz)$$

$$x(y \oplus z)' + \bar{x}(y \oplus z)$$

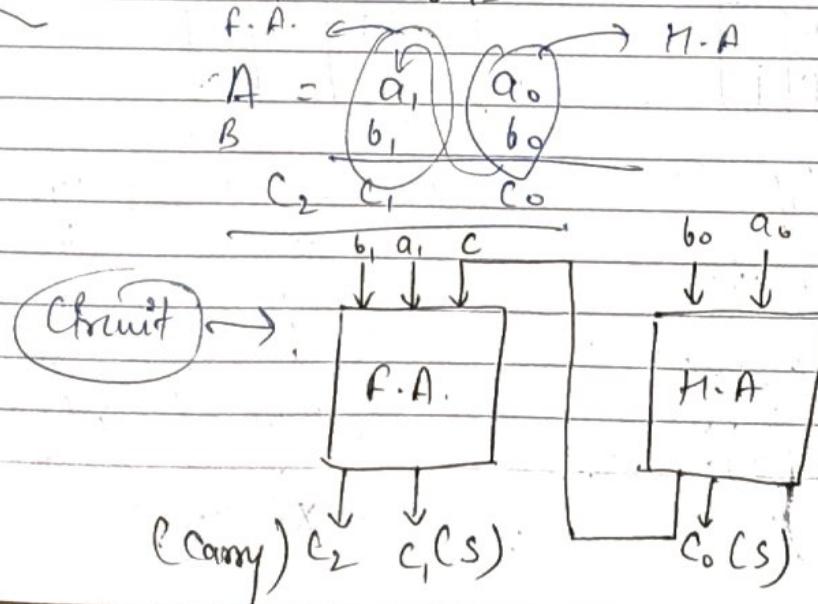
$$x \oplus y \oplus z$$

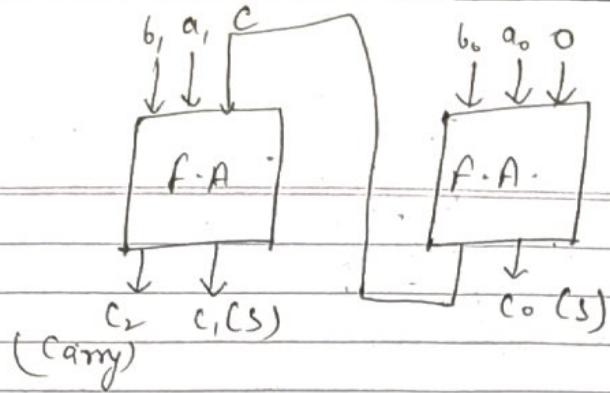


### \* PARALLEL ADDER

$$\text{Ex. } \begin{array}{r} A = a_1 \quad a_0 \\ B = + b_1 \quad b_0 \\ \hline C_2 \quad C_1 \quad C_0 \end{array} \quad \begin{array}{r} A = 11 \\ B = 11 \\ \hline 110 \end{array}$$

Q) Design a combinational circuit which is used to perform addition of two binary no. of two bits.

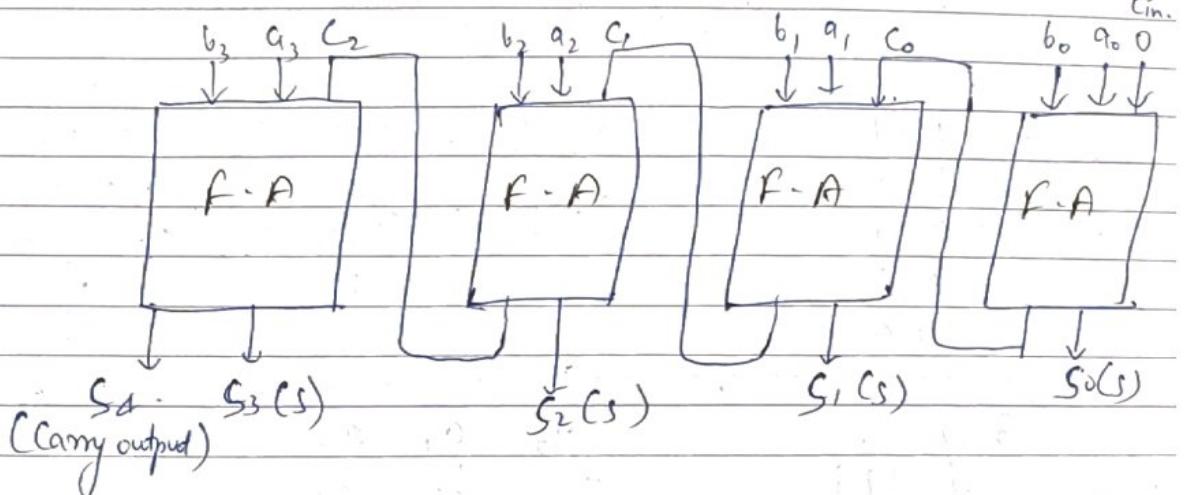




(23)

\* Emp. 4-bit PARALLEL ADDER.

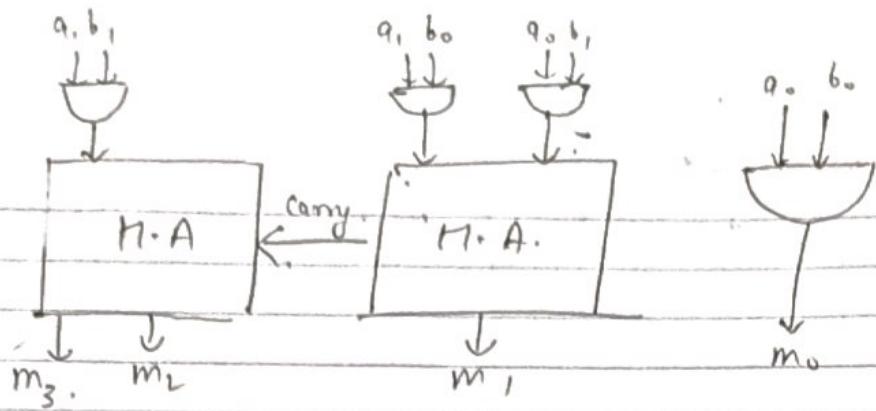
$$\begin{array}{r}
 A = 1 \ 1 \ 1 \ 1 \\
 B = + 1 \ 1 \ 1 \ 1 \\
 \hline
 1 \ 0 \ 1 \ 1 \ 0
 \end{array}
 \quad \text{(9 inputs/5 outputs / 4-F.A.)}$$



$$\begin{array}{r}
 A = a_3 \ a_2 \ a_1 \ a_0 \\
 B = + b_3 \ b_2 \ b_1 \ b_0 \\
 \hline
 S_3 \ S_2 \ S_1 \ S_0
 \end{array}$$

\* 2-bit binary Multiplier (2 bits by 2 bits).  
 $(2 \times 2)$  array multiplier ( $2 \times 2$ )

$$\begin{array}{r}
 A = 1 \ 1 \ (3) \\
 B = x \ 1 \ 1 \ (3) \\
 \hline
 1 \ 1
 \end{array}
 \quad +
 \begin{array}{r}
 1 \ 1 \\
 \hline
 1 \ 0 \ 0 \ 1 \ (9)
 \end{array}
 \quad \left| \begin{array}{r}
 a_1 \ a_0 \\
 b_1 \ b_0 \\
 \hline
 a_1 b_1 \ a_0 b_1 \\
 \hline
 m_3 \ m_2 \ m_1 \ m_0
 \end{array} \right.$$



Q       $M = 13$        $m = 11$       ( $m - N$ ) using 2's complement

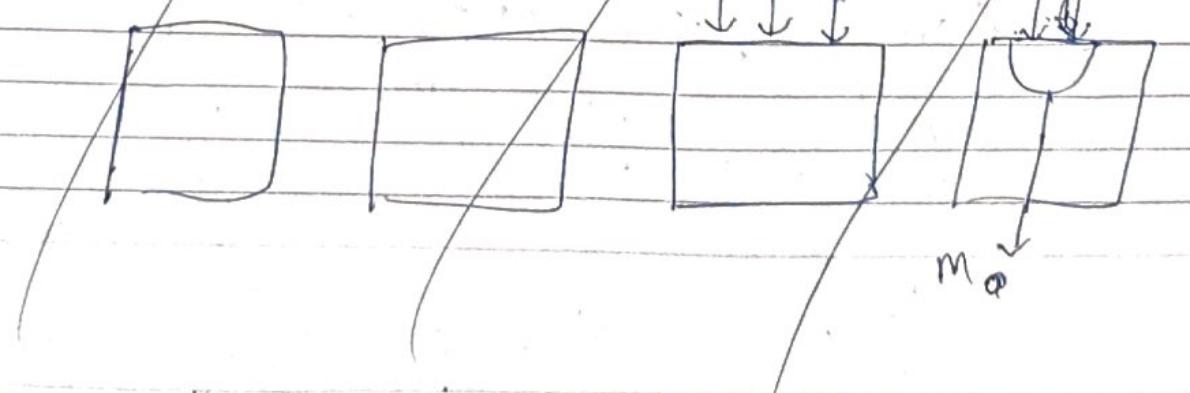
$$\begin{array}{r}
 M = 1101 \\
 N = 1011
 \end{array}
 \rightarrow
 \begin{array}{r}
 1101 \\
 + 0101 \\
 \hline
 0010
 \end{array}
 \rightarrow
 \begin{array}{r}
 0010 \\
 \text{Discard}
 \end{array}$$

$$\begin{array}{r}
 M = 1101 \\
 N = 0100
 \end{array}
 \rightarrow
 \begin{array}{r}
 10111 \quad (\text{Gray}) \\
 \text{Binary}
 \end{array}$$

$$\begin{array}{r}
 10001 \\
 + 1 \\
 \hline
 0010
 \end{array}
 \rightarrow
 \begin{array}{r}
 11010 \quad (\text{Gray}) \\
 \text{Binary}
 \end{array}$$

3 by 2.

$$\begin{array}{r}
 111 \\
 \times 11 \\
 \hline
 111 \\
 111 \\
 \hline
 111 \quad M_4 \quad M_3 \quad M_2 \quad M_1 \quad a_0 b_0 m_0
 \end{array}
 \begin{array}{r}
 a_2 \quad a_1 \quad a_0 \\
 \times \quad b_1 \quad b_0 \\
 \hline
 a_2 b_0 \quad a_1 b_0 \quad a_0 b_0 \\
 a_2 b_1 \quad a_1 b_1 \quad a_0 b_1 \\
 \hline
 a_2 b_1 \quad a_1 b_1 \quad a_0 b_1
 \end{array}$$



\* 3 bits BINARY TO GRAY CONVERTOR  
 Binput Input      Gray Output

A	B	C	x	y	z
---	---	---	---	---	---

0	0	0	0	0	0
---	---	---	---	---	---

0	0	1	0	0	1
---	---	---	---	---	---

0	1	0	0	1	1
---	---	---	---	---	---

0	1	1	0	1	0
---	---	---	---	---	---

1	0	0	1	1	0
---	---	---	---	---	---

1	0	1	1	1	1
---	---	---	---	---	---

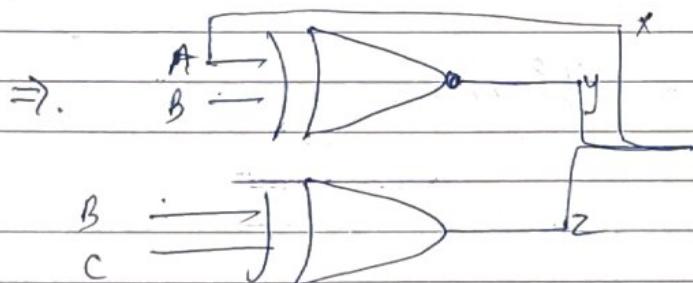
1	1	0	1	0	1
---	---	---	---	---	---

1	1	1	1	0	0
---	---	---	---	---	---

$$x = \begin{array}{|c|c|c|c|} \hline & \textcircled{B} & & \\ \hline 0 & \textcircled{1} & \textcircled{1} & \textcircled{1} \\ \hline 1 & \textcircled{1} & \textcircled{1} & \textcircled{1} \\ \hline \end{array} = \bar{A}\bar{B} + AB = (\cancel{A\oplus B}) A.$$

$$y = \begin{array}{|c|c|c|c|} \hline & \textcircled{B} & & \\ \hline 0 & \textcircled{1} & \textcircled{1} & \textcircled{1} \\ \hline 1 & \textcircled{1} & \textcircled{1} & \textcircled{1} \\ \hline \end{array} = A\bar{B} + \bar{A}B = (A\oplus B).$$

$$z = \begin{array}{|c|c|c|c|} \hline & \textcircled{C} & & \\ \hline 0 & \textcircled{1} & \textcircled{1} & \textcircled{1} \\ \hline 1 & \textcircled{1} & \textcircled{1} & \textcircled{1} \\ \hline \end{array} = \bar{B}C + BC = B\oplus C.$$



## # HALF SUBTRACTOR (A combinational circuit for subtracting 2 binary bits)

Truth Table      Borrow      Difference.

X	y	B	D
0	0	0	0
0	1	1	1
1	0	0	1
1	1	0	0

0 - 1

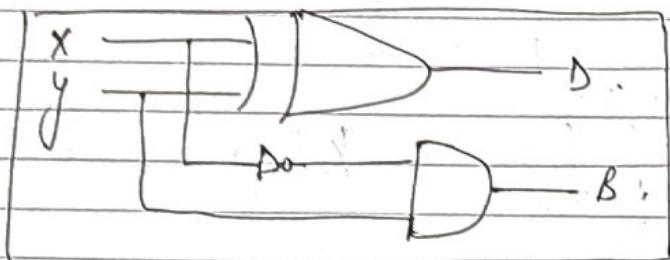
2 - 1 = 1.

$$D = \bar{x}y + x\bar{y} = x \oplus y$$

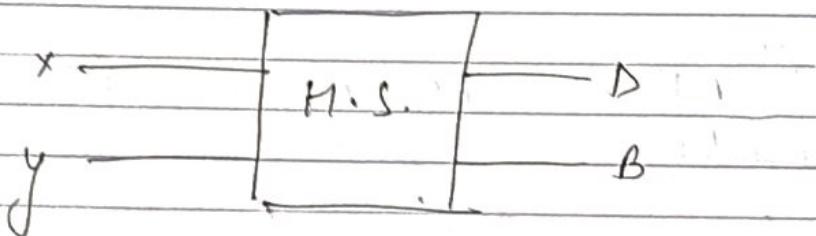
$$B = \bar{xy}$$

Boolean expression.

Half Subtractor.



Logic Diagram.



Block Diagram.

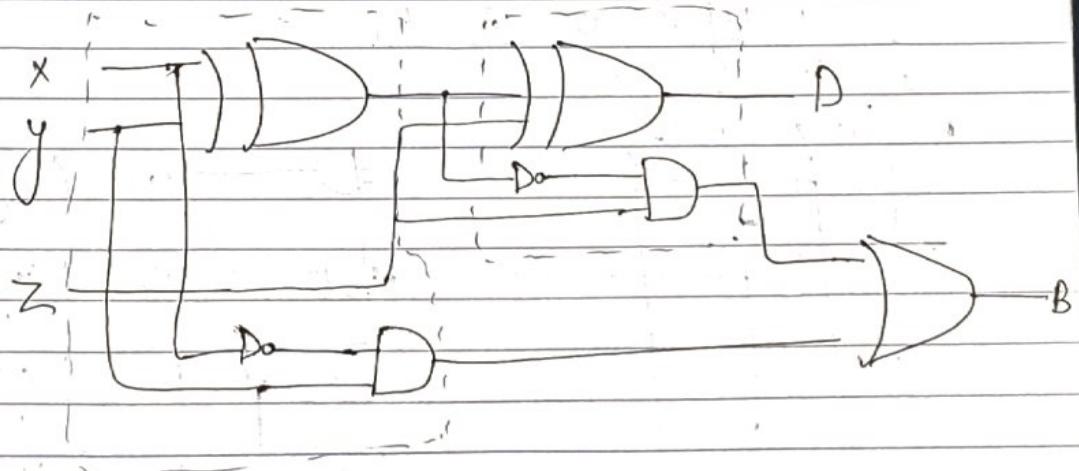
← L.R.

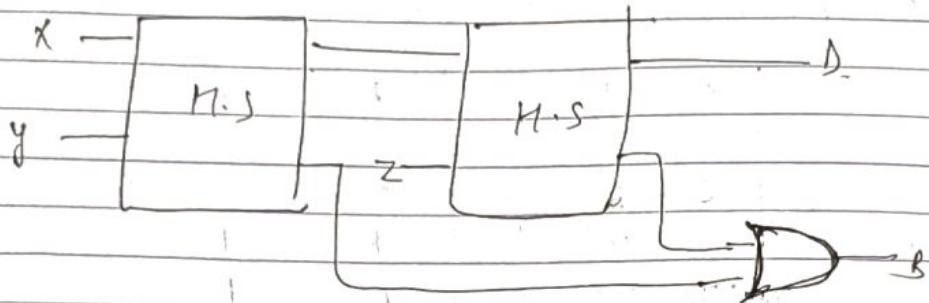
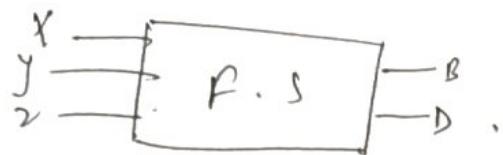
(IMP)

## # FULL SUBTRACTOR.

x	y	z	B	D.
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	1	0
1	0	0	0	1
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

$$B = \begin{array}{|c|c|c|c|c|} \hline & x & y & z & \\ \hline 0 & 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 & 0 \\ \hline \end{array} = \begin{aligned} & \bar{x} + \bar{y}z, \bar{x}y + \bar{y}z + \bar{x}z \\ & = \bar{x}y + \bar{x}\bar{y}z + xyz \\ & = \bar{x}y + z(x \oplus y)' \end{aligned}$$
$$D = \begin{array}{|c|c|c|c|c|} \hline & x & y & z & \\ \hline 0 & 0 & 0 & 0 & 0 \\ \hline 1 & 1 & 1 & 1 & 0 \\ \hline \end{array} = \begin{aligned} & \bar{x}\bar{y}z + \bar{x}\bar{z} + \bar{y}\bar{z} \\ & = \bar{x} \oplus y \oplus z. \end{aligned}$$





## # DECODER

- $n$  inputs —  $2^n$  (max) output
- binary to other code converter
- a combinational circuit.

Ex - (2 to 4 Line decoder).

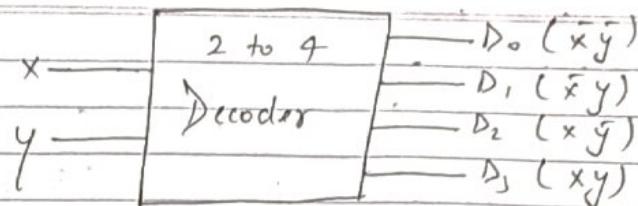
(Inputs)	(Outputs)	TRUTH TABLE
$x \ y$	$D_0 \ D_1 \ D_2 \ D_3$	
0 0	1 0 0 0	1 1 1 1
0 1	0 1 0 0	1 0 1 0
1 0	0 0 1 0	0 1 0 1
1 1	0 0 0 1	0 0 1 1



logic Diagram.

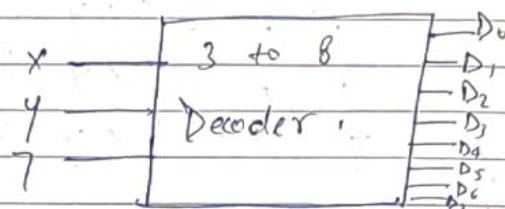
Every output of decoder represent minterms.

(2)



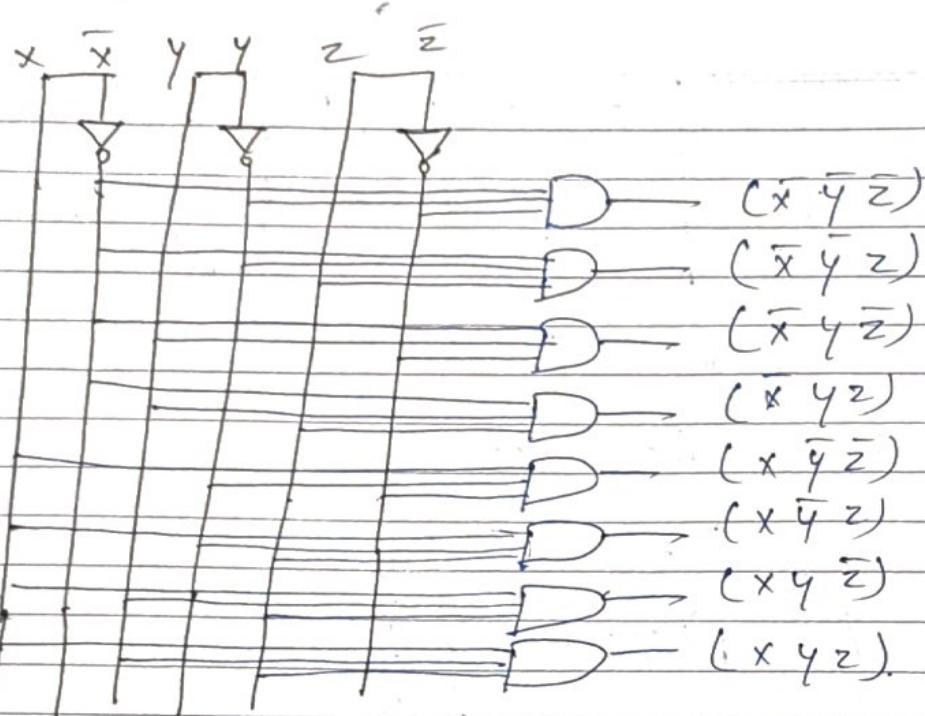
Block Diagram

Q 3 to 8 line decoder.

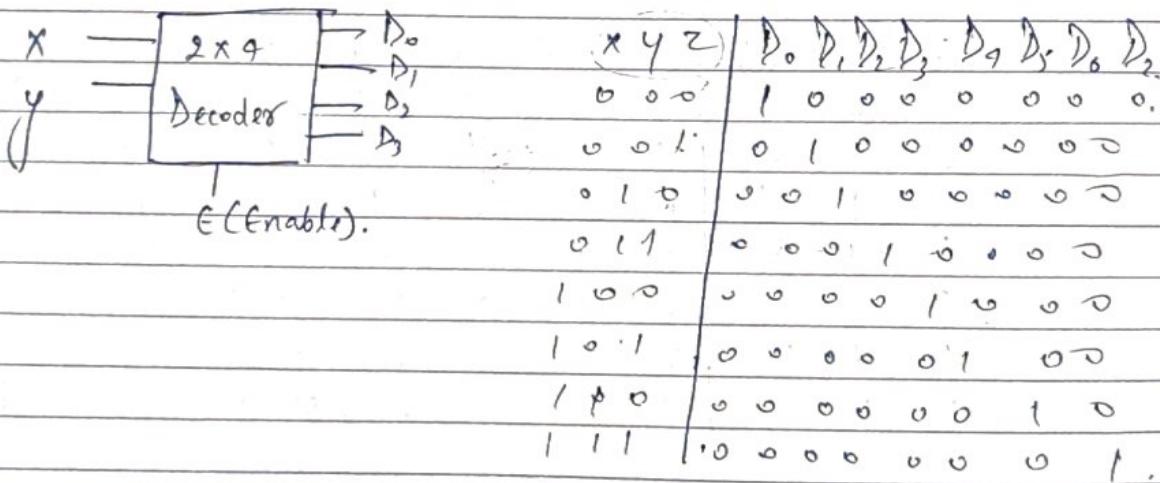


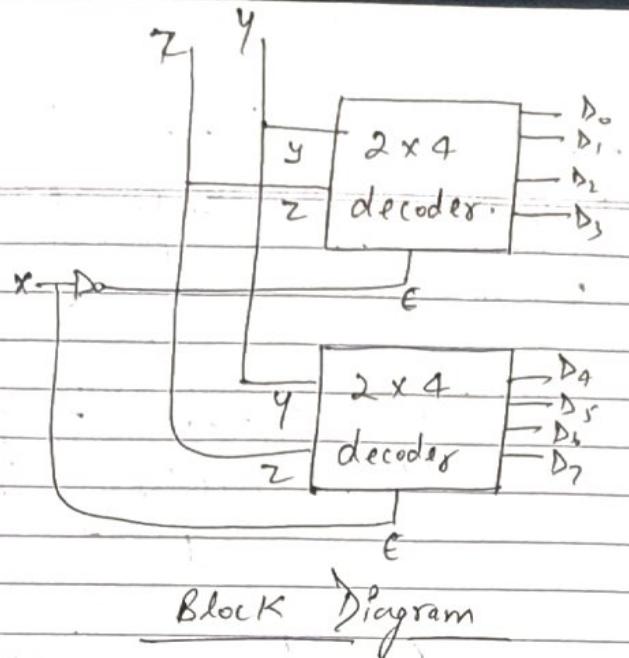
Truth Table

$x$	$y$	$z$	$D_0$	$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	$D_6$	$D_7$
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1



Q) Design a 3 cross decoder with a help of  $2 \times 4$  decoder chips.



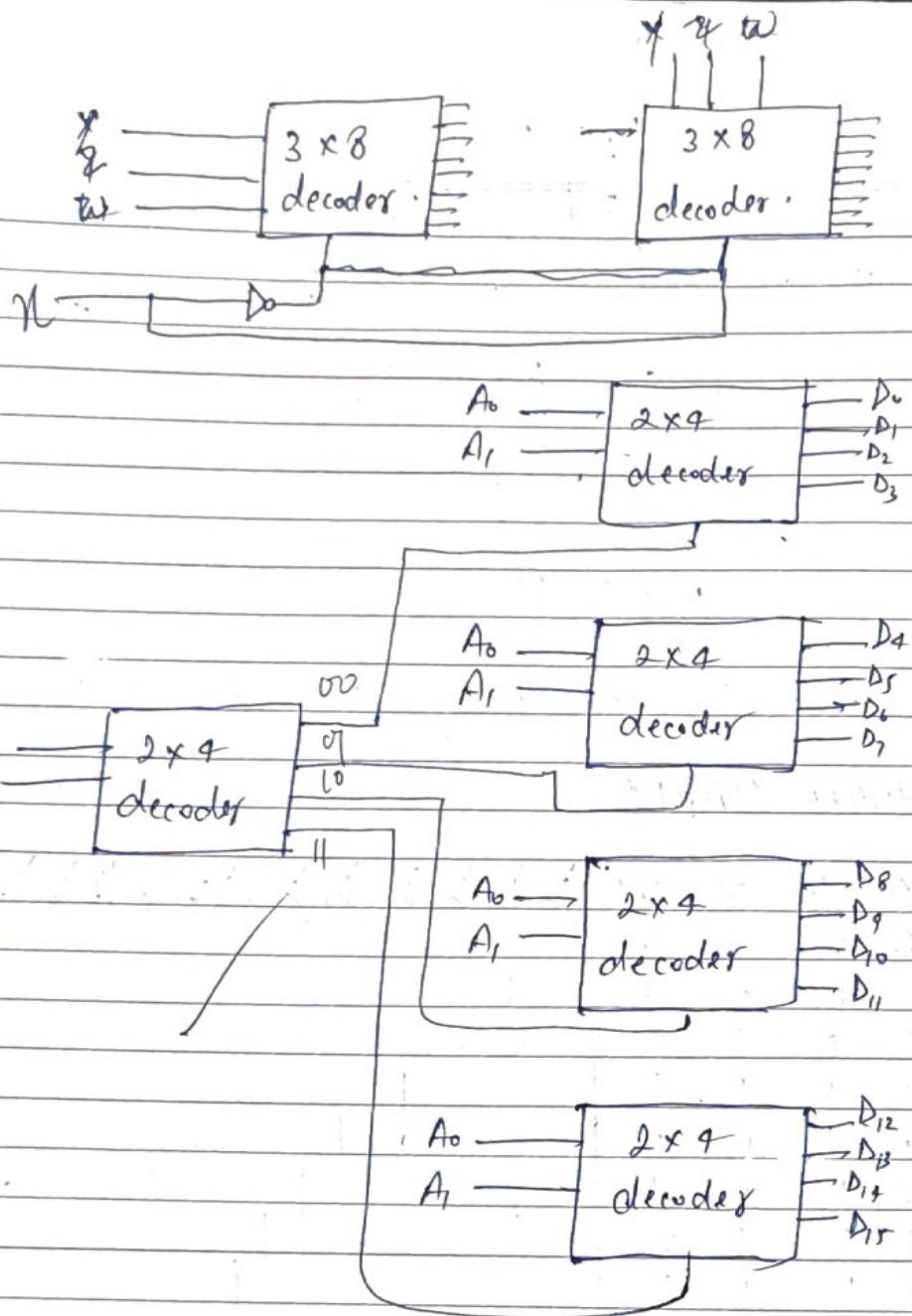


Block Diagram

Q = 4x16 Decoder using 3x8 decoder, 2x4 decoder

TRUTH TABLE -

X	Y	Z	W	D <sub>0</sub>	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>	D <sub>6</sub>	D <sub>7</sub>	D <sub>8</sub>	D <sub>9</sub>	D <sub>10</sub>	D <sub>11</sub>	D <sub>12</sub>	D <sub>13</sub>	D <sub>14</sub>	D <sub>15</sub>
0	0	0	0	1															
0	0	0	1		1														
0	0	1	0			1													
0	0	1	1				1												
0	1	0	0					1											
0	1	0	1						1										
0	1	1	0							1									
0	1	1	1								1								
1	0	0	0																
1	0	0	1																
1	0	1	0																
1	0	1	1																
1	1	0	0																
1	1	0	1																
1	1	1	0																
1	1	1	1																



## # ENCODER.

- $n$  inputs  $\rightarrow 2^n$  inputs (max.) output.
- other code to binary code conversion.
- a combination circuit.

$\Rightarrow$  4 to 2 encoder

Truth Table.

$D_0$	$D_1$	$D_2$	$D_3$	x	y
1	0	0	0	0	0
0	1	0	0	0	1
0	0	1	0	1	0
0	0	0	1	1	1

$D_0$  —

$D_1$  —

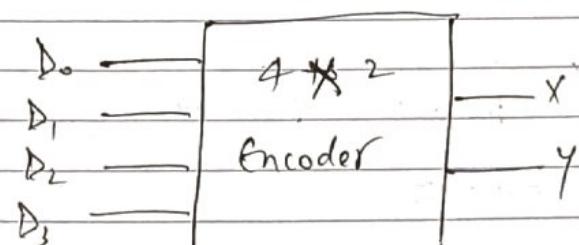
$D_2$  —

$D_3$  —

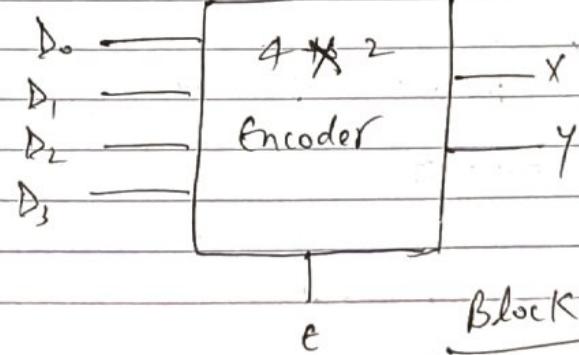
$$x = D_2 + D_3$$

$$y = D_1 + D_3$$

logic diagram



Block Diagram.

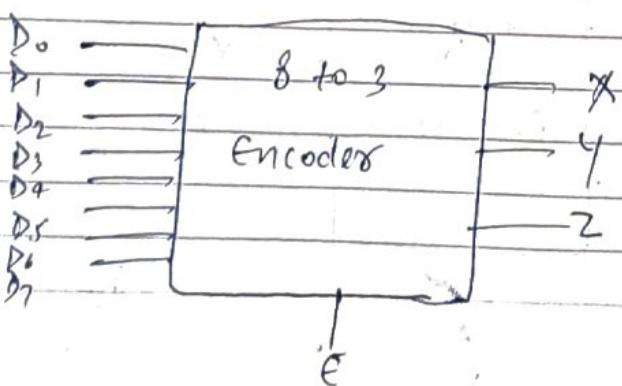
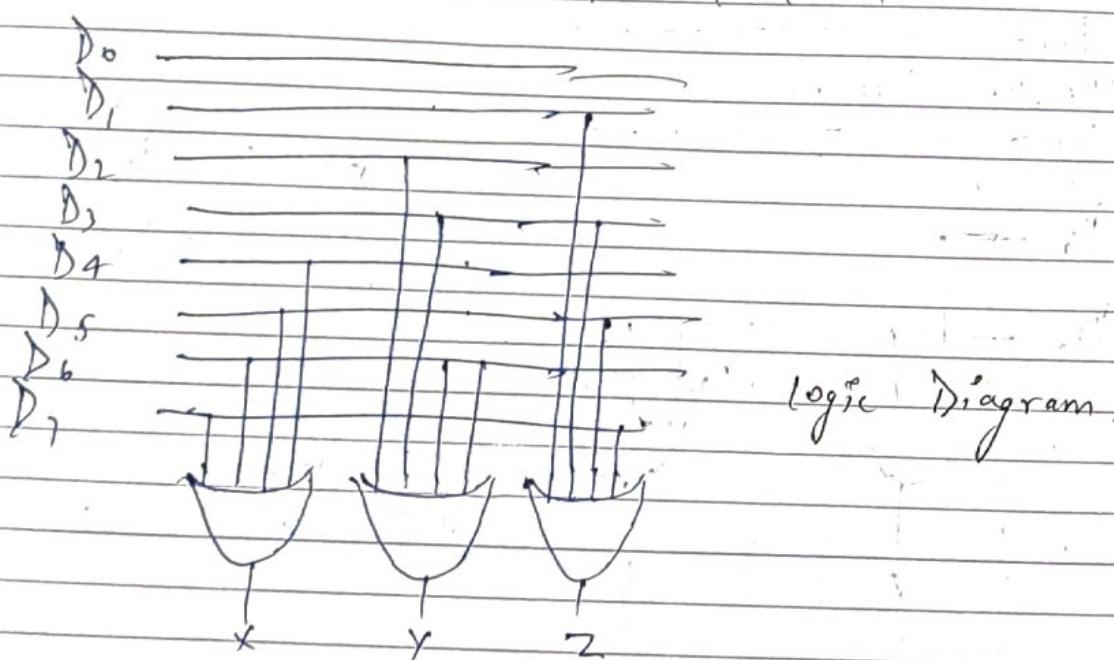


8 to 3 encoder. (Octal to Binary).

Truth Table -

$D_0 \ D_1 \ D_2 \ D_3 \ D_4 \ D_5 \ D_6 \ D_7 \ x \ y \ z$

1	0	0		0	0	0
	1			0	0	1
		1		0	1	0
			1	0	1	1
				1	0	1
					1	1
						1



$X \rightarrow$  don't care

(3)

\* PRIORITY ENCODER - High priority  
↓  
 $D_3, D_2, D_1, D_0$ .  
↑ Lowest priority  
 $D_3 > D_1 > D_2 > D_0$ .

Truth Table.

Valid output indicator.

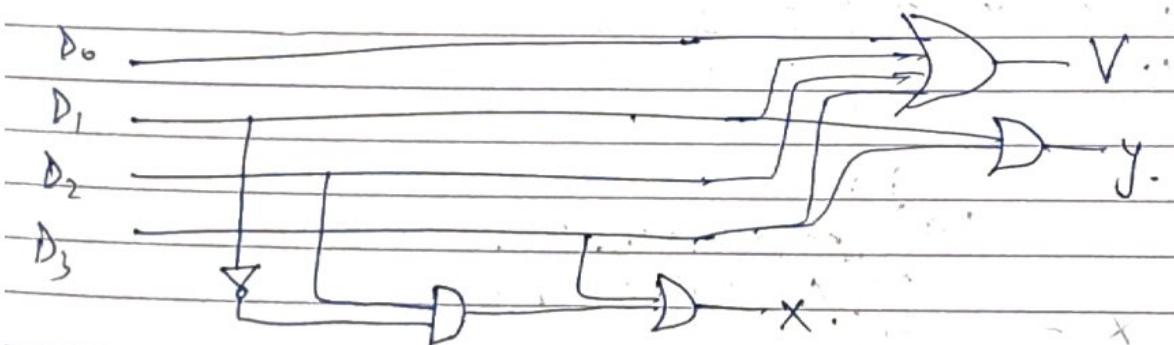
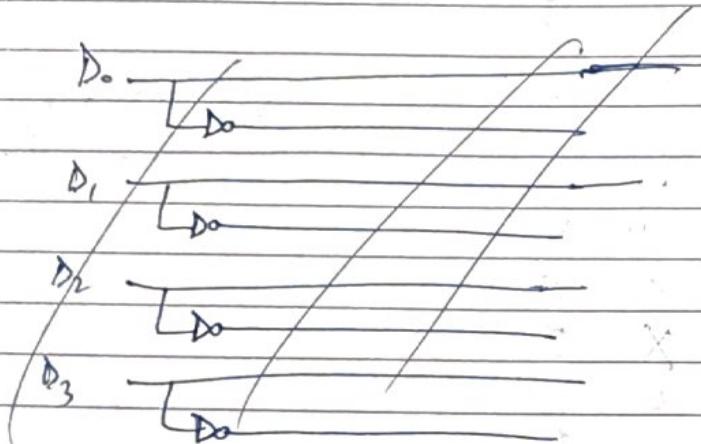
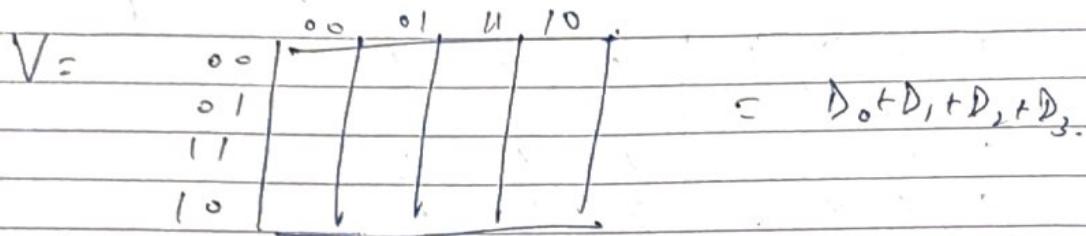
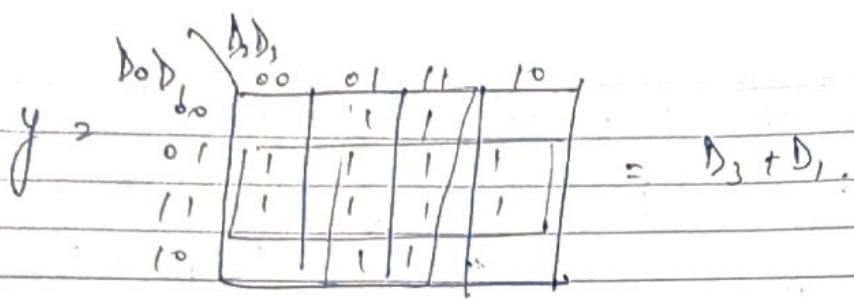
$D_0$	$D_1$	$D_2$	$D_3$	$x$	$y$	$v$
0	0	0	0	x	x	0
x	x	x	1	1	1	1
x	1	x	0	0	1	1
x	0	1	0	1	0	1
1	0	0	0	0	0	1

$D_0 > D_3 > D_1 > D_2$

$D_0$	$D_1$	$D_2$	$D_3$	$x$	$y$	$v$
0	0	0	0	x	x	0
1	x	x	0	x	x	0
0	x	x	1	0	0	1
0	1	x	0	0	1	1
0	0	1	0	1	0	1

$D_0$	$D_1$	$D_2$	$D_3$	$x$	$y$	$v$
0	0	0	0	x	1	1
x	0	1	1	1	1	1
1	1	0	1	1	1	1
1	0	1	0	1	0	1

$$x = D_3 + \bar{D}_1 D_2$$



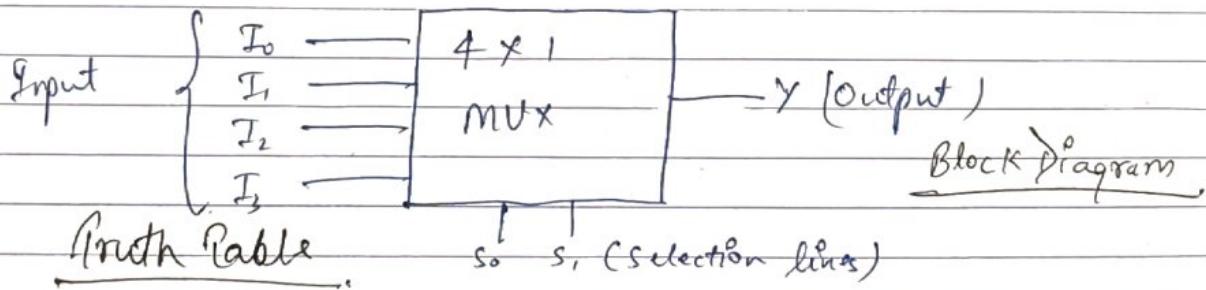
300  $\rightarrow$  nearest  $2^n$  i.e.,  $512 \rightarrow 2^9 = 9 \rightarrow$  selection lines.

## # MULTIPLEXER (MUX) (Data Selector).

$\rightarrow 2^n$  inputs  $\rightarrow 1$  output.  
(or less than).

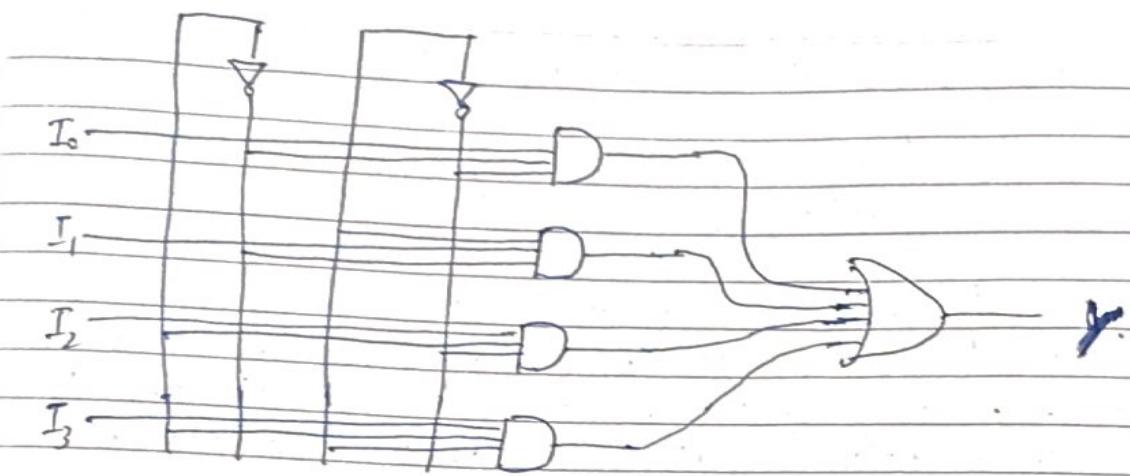
Multiplexing means transmitting a large no. of information units over a smaller no. of channels or lines. A digital multiplexer is a combinational circuit that selects binary information from one of many input lines & directs it to a single output line. The selection of a particular input line is controlled by a set of selection lines. Normally, there are  $2^n$  input lines &  $n$  selection lines, whose bit combination determine which input is selected.

\* 4 to 2 lines MUX -  
 $2^2$  means  $\rightarrow$  2 selection lines.



Truth Table

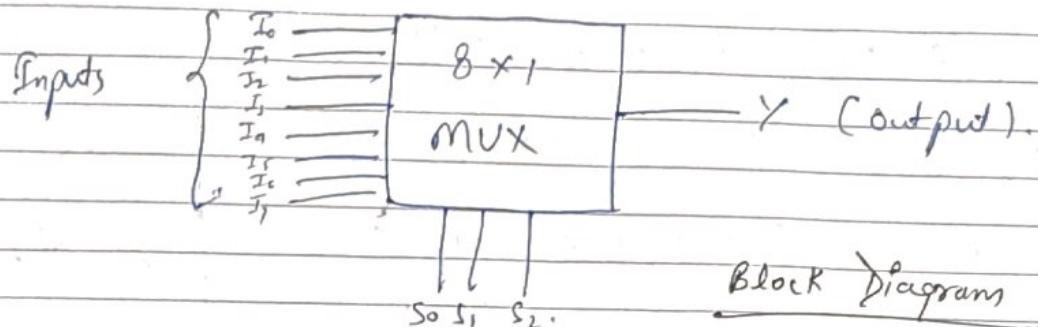
Selection Line			Output
S <sub>0</sub>	S <sub>1</sub>		Y
I <sub>0</sub>	0	0	I <sub>0</sub>
I <sub>1</sub>	0	1	I <sub>1</sub>
I <sub>2</sub>	1	0	I <sub>2</sub>
I <sub>3</sub>	1	1	I <sub>3</sub>



logic Diagram

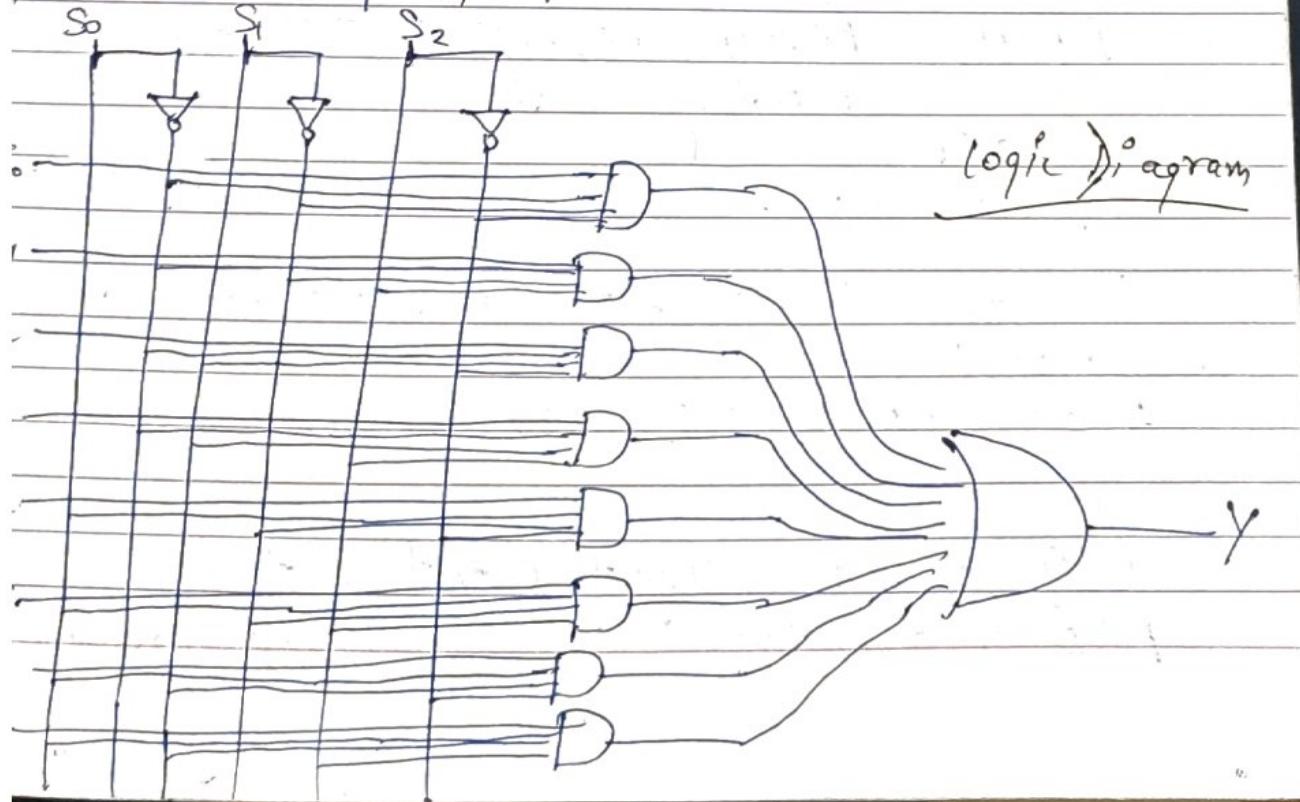
(39)

\*  $8 \times 1$  MUX.



Truth Table,

	$S_0$	$S_1$	$S_2$	$Y$
$I_0$	0	0	0	$I_0$
$I_1$	0	0	1	$I_1$
$I_2$	0	1	0	$I_2$
$I_3$	0	1	1	$I_3$
$I_4$	1	0	0	$I_4$
$I_5$	1	0	1	$I_5$
$I_6$	1	1	0	$I_6$
$I_7$	1	1	1	$I_7$



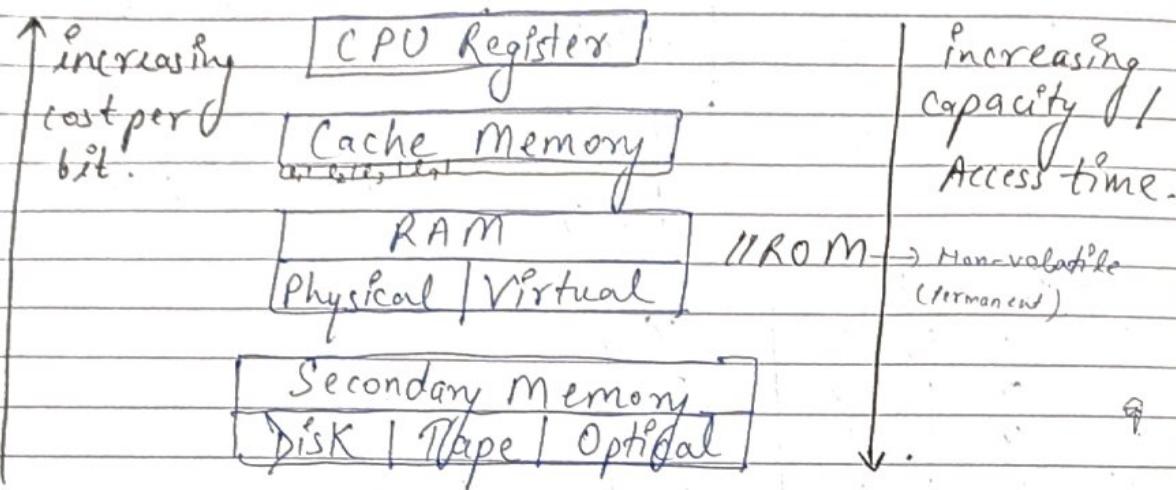
Date - 19/08/19

P. -

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## UNIT-4 MEMORY ORGANIZATION -

### MEMORY HIERARCHY -



- \* CPU Register is the smallest & fastest memory.
- \* Cache memory have very less space.
- \* RAM (Primary Memory).
  - Primary Memory
  - Volatile
  - Semiconductor.

Ram are divided into two types static & dynamic.

static RAM are made from use of flip-flop.

BIOS - Basic Input Output System Programme  
BIOS is also known as boot step loader.