Algebraic Structures

1. Define group and prove that if every element of a group G is its own inverse then G is abelian group.
    [U.P.T.U. (B.Tech.) 2009]

2. Write short notes on groups and semi groups
    P.T.U. (B.E.) Punjab 2004, 2006, 2009; R.G.P.V. (B.E.) Bhopal 2003, Pune (B.E.) 2005, 2007
    Nagpur (B.E.) 2008; M.K.U. (B.E.) 2007]

3. Define normal subgroup, homomorphism, isomorphism.   [R.G.P.V. (B.E.) Bhopal 2002]

4. Write short notes on:
    (i)   Groups and semi groups,   [Rohtak (M.C.A.) 2005, 2009; M.K.U. (B.E.) 2008; R.G.P.V. (B.E.) 2003;
    Kurukshetra (B.E.) 2008]
    (ii)   Cosets   [P.T.U. (B.E.) Punjab 2008, R.G.P.V. (B.E.) Bhopal 2003]
    (iii) Isomorphism and homomorphism   [U.P.T.U. (B.Tech.) 2007; Rohotak (B.E.) 2001, 2005, 2007,
    R.G. P.V. (B.E.) Bhopal 2002]
    (iv)   Homomorphism   [Pune (B.E.) 2006; Nagpur (B.E.) 2004
    (v)   Rings   [R.G.P.V. (B.E.) Bhopal 2002; M.K.U. (M.C.A.) 2007]

5. (i)   Differentiate between semigroup and subgroup with example.   [U.P.T.U. (B.Tech.) 2008]
    (ii)   How monoid, ring and field are related to each other?   [U.P.T.U. (B.Tech. 2008]

6. Let $A = \{a, b\}$. Which of the following tables define a semigroup? Which define a monoid on A?
    [U.P.T.U. (B.Tech.) 2005]

(i)

*	a	b
a	b	a
b	a	b

(ii)

*	a	b
a	b	b
b	a	a

(iii)

*	a	b
a	b	b
b	b	a

7. Let $(\{p, q\}*)$ be a semigroup where $p*q = q$. Show that
    (i) $p*q = q*p$
    (ii) $q*q = q$
    [U.P.T.U. (B.Tech.) 2006; P.T.U. (B.E.) Punjab 2007; M.K. U. (B.E.) 2009; Nagpur 2003]

8. Find all the subgroup of
    (i) $(Z_{12}, +_{12})$
    (ii) $(Z_5, +_5)$ [U.P.T.U. (B.Tech.) 2006]

9. Define the symmetric group of degree n denoted by S_n.   [U.P.T.U. (B.Tech.) 2006]

10. Define algebraic systems with examples.   [M.K.U. (M.C.A.) 2005, 2008]

11. Define subgroups and cyclic groups.   [M.K.U. (B.E.) Tamil Nadu 2004, 2007]

12. Show that every group of order 2 is isomorphic to every other group of order 2.

13. If G is cyclic group of order n then show $o(a) = n$.

14. If $(G, *)$ is a group of order 4, then show that G is abelian.

15. Show that the additive group Z_4 is isomorphic to multiplicative group of non-zero elements of Z_5.

16. Find all subgroup of a cyclic group of order 10 and 12.

17. Define and explain the following with suitable example

    (i)   cyclic group   (ii)   zero divisor of a ring

    (iii) order of an element of a group   (iv)   field   **[U.P.T.U. (B.Tech.) 2009]**

18. If G is group of order n then order of any element $a \in G$ is a factor of n, prove .

                                                        [U.P.T.U. (B.Tech.) 2009]

19. State and prove Lagrange's theorem.   **[U.P.T.U. (B.Tech.) 2009; P.T.U. (B.E.) Punjab 2004, 2007, 2009;**

                                                R.G.T.U. (B.Tech.) 2005, 2008; Raipur (B.E.) 2006]

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