$f(x) = x^2$

$f(x_1) = f(x_2)$

$x_1^2 = x_2^2$ Not one-one.

$\pm x_1 \neq \pm x_2$.

3) Check bijective $f: R \to R$.

$$f(x) = \frac{2x-5}{x-2}$$

<u>Sol^n</u> (i) <u>For one-one:</u>

$$f(x_1) = f(x_2)$$

$$\frac{2x_1 - 5}{x_1 - 2} = \frac{2x_2 - 5}{x_2 - 2}$$

$$\cancel{2x_1x_2} - 4x_1 - 5x_2 + \cancel{10} = \cancel{2x_2x_1} - 4x_2 - 5x_1 + \cancel{10}$$

$$x_1 = x_2$$

$\therefore$ It is one-one f^n.

(ii) <u>For onto :-</u>

Putting $f(x) = y$

$$\frac{2x-5}{x-2} = y$$

$$xy - 2y = 2x - 5$$

$$x(y-2) = 2y - 5$$

$$x = \frac{2y-5}{y-2} \quad \text{— (A)}$$

For $y = 2$ we don't get any pre-image x from eq^n (A) $\therefore$ It is not onto.

8) show $f(x) = \frac{1}{x}, x \neq 0$ $f: R \to R$.
Bijective.

Soln (i) $f(x_1) = f(x_2)$

$$\frac{1}{x_1} = \frac{1}{x_2}$$

$$x_1 = x_2.$$

$\therefore$ one-one mapping.

(ii) $f(x) = y$

$$y = \frac{1}{x}$$

$$x = 1/y \quad \therefore y \neq 0.$$

for every y there exist a pre-image

$$f(1/y) = \frac{1}{(1/y)} = y$$

proved.

Hence it is onto.

$\therefore$ It is bijective fn.

Q) Find inverse of $f(x) = 3x+2$ $x \in Q$
(i) $f: Q \to Q$.

Sol^n $f^{-1}(y) = x$.

$$x = \frac{1}{3}(y-2)$$

$$\boxed{f^{-1} = \{(y, x) : x = \frac{1}{3}(y-2)\}}$$

(ii) $x^2 - 3 = y$.

$f(x) = y$

$x^2 - 3 = y$

$x^2 = y+3$

$x = (y+3)^{1/2}$.

thus, $f^{-1}(y) = x = (y+3)^{1/2}$.

Q) If $f: I \to I$ where $f(x) = x^2+1$
then find $f^{-1}(3)$, $f^{-1}(\{10, 37\})$

Sol^n $x^2 + 1 = 3$ $\therefore f^{-1}(3) = \{-\sqrt{2}, \sqrt{2}\}$.

$x^2 = 2$

$x = \pm\sqrt{2}$.

Q) $f^{-1}(\{10, 37\})$

$$x^2+1=10 \qquad x^2+1=37$$
$$x^2=9$$
$$x=\pm 3 \qquad x^2=36$$
$$x=\pm 6.$$

Hence $f^{-1}(\{10,37\}) = \{-6, -3, 3, 6\}$.

Q) If mapping f is defined $f: I \to I$ defined by $f(x)=x^2$. Evaluate $f^{-1}(16)$ & $f^{-1}(8)$

Solⁿ ϕ. as it not invertible.

Q). $f: R \to R$ $f(x) = ax+b$. $a, b, x \in R$ & $a \neq 0$.
Find inverse of f.

Solⁿ (i) ONE-ONE:

$$ax_1+b = ax_2+b.$$
$$ax_1 = ax_2$$
$$x_1 = x_2$$

Yes

(ii) ONTO:

$$f(x) = y$$
$$ax+b = y$$
$$ax = y-b$$
$$x = \frac{y-b}{a} \quad a \neq 0$$

∴ It's onto fⁿ.

$$\therefore f^{-1}(y) = \frac{y-b}{a}$$

2) $f: A \to B$. set of real nos. is given by formula.

$f(x) = 2x^3 - 1$ and $g: B \to A$.

$g(y) = \sqrt[3]{\frac{1}{2}y + \frac{1}{2}}$. show f is bijection b/w A & B and g is bijection b/w B and A.

Sol

$f(x) = 2x^3 - 1$

Let $f(a) = f(b)$

$2a^3 - 1 = 2b^3 - 1$

$2a^3 = 2b^3$

$a = b$ $\therefore$ one one f^n

Now put $f(x) = y$

$y = 2x^3 - 1$

$2x^3 = y + 1$

$x = \sqrt[3]{\frac{y+1}{2}}$.

$\left[f(y) = 2\left(\sqrt[3]{\frac{y+1}{2}}\right)^3 - 1 = y \right]$ checking

For each value $y \in B$. there exist preimage in A. $\therefore$ it is onto.

Hence ~~surje~~ bijective f^n.

Now for $g(y) = \sqrt[3]{y/2 + 1/2}$

Let $a, b \in B$ such that $g(a) = g(b)$

$$\sqrt[3]{a/2 + 1/2} = \sqrt[3]{b/2 + 1/2}$$

$$\frac{1}{2} + \frac{a}{2} = \frac{1}{2} + b/2$$

$$a = b$$

$\therefore$ One one f^n.

Put $f(x) = y$

$$\sqrt[3]{y/2 + 1/2} = x$$

$$\frac{y}{2} + 1/2 = x^3.$$

$$x^3 - 1/2 = \frac{y}{2}$$

$$y = 2x^3 - 1$$

For each x there is corresponding y in B. $\therefore$ g is onto.

$\therefore$ g is bijective from B to A.

Q. $f: R \to R$ $g: R \to R$ defined as

$$f(x) = x+2 \quad \forall (x) \in R$$

$$g(x) = x^2 \quad \forall (x) \in R.$$

Find gof and fog.

Solⁿ

$$gof(x) = g(f(x)) = g(x+2)$$
$$= (x+2)^2$$

$$fog(x) = f(g(x)) = f(x^2)$$
$$= x^2+2.$$

$$gof \neq fog$$

Q). $f(x) = \cos x$ $g(x) = e^x$.

$$fog = f(g(x)) = \cos e^x$$
$$gof = g(f(x)) = e^{\cos x}.$$

Ques Prove if $f: X \to Y$ is one-one & onto then

$$f^{-1} o f = I_X$$
$$f o f^{-1} = I_Y$$

Solⁿ we have, $f^{-1} of = x$.

LHS $f^{-1}(f(x)) = f^{-1}(y) = x$.

$f^{-1} o f$ is an identity fⁿ of X.

Now $f(f^{-1}(y)) = f(x)$

$= y$

$fof^{-1}(y) = y = I_y$ Proved.

Q) If $f: A \to B$ is bijective

$g: B \to C$ is "

then prove (i) gof is also bijective and

(ii) $(gof)^{-1} = f^{-1} o g^{-1}$.

Solⁿ

gof is one-one :-

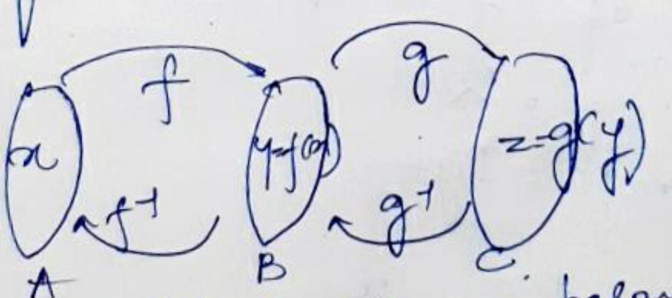
let $x_1 \neq x_2$

$f(x_1) \neq f(x_2)$

$g(f(x_1)) \neq g(f(x_2))$

$gof(x_1) \neq gof(x_2)$.

gof is onto :-



For every value of z belonging to C

there exist $\forall z \in C$, $\exists y$

such that, $z = g(y)$

i.e $y = g^{-1}(z)$

$\forall y \in B$, $\exists x$.

such that, $y = f(x)$

i.e $\forall z \in C$ $\exists x$ such that
$g o f(x) = z$ also $(g o f)^{-1}(z) = x$.

$g o f$ is an onto f^n.

(iii) $\boxed{(T.P): (g o f)^{-1} = f^{-1} o g^{-1}}$

RHS $f^{-1} o g^{-1}(z)$

$= f^{-1}[g^{-1}(z)]$

$= f^{-1}(y) = x$.

$LHS = (g o f)^{-1} z = x$. LHS = RHS.

Theorem: Associative law of f^n composition

Sol^n Let $f: A \to B$, $g: B \to C$ and $h: C \to D$
then, $h o (g o f) = (h o g) o f$.

Proof:

$f: A \to B$ $g: B \to C$

$g o f : A \to C$.

also $h: C \to D$.

$h o g: B \to D$.

$h o (g o f) : A \to D$ and
$(h o g) o f : A \to D$.

$\therefore$ mapping is equal.

Let $x \in A$, $y \in B$ and $z \in C$ such that

$$f(x) = y \quad g(y) = z.$$

RHS then,

$$[(h \circ g) \circ f]\,x = h \circ g[f(x)] = h(g(f(x)))$$

$$h \circ g(f(y)) = h(z) \quad \text{— (1)}$$

LHS $\quad (h \circ (g \circ f))\,x = h[g \circ f(x)]$

$$= h[g(f(x))] = h(g(y))$$

$$= h(z) \quad \text{— (2)}$$

from (1) & (2)

$$\begin{tikzcd} A \arrow[r, "f"] \arrow[rrr, bend left, "h \circ (g \circ f)"] \arrow[rr, bend right, "g \circ f"'] & B \arrow[r, "g"] & C \arrow[r, "h"] & D \end{tikzcd}.$$

$$\begin{tikzcd} A \arrow[r, "f"] & B \arrow[r, "g"] \arrow[rr, bend right, "h \circ g."'] & C \arrow[r, "h"] & D \end{tikzcd}$$

THEOREM: (Identity Law).

The composition of any f^n with its identity f^n is the f^n itself.

$$(f \circ I_A)\,x = (I_A \circ f)\,x = f(x)$$

$\underline{\underline{Sol^n}}$

$$f : A \to A.$$
$$f(x) = x. \; ; \; \forall (x) \in A$$

let A and B are two non-empty sets, let I_A & I_A be identity f^ns.

$$I_A : A \to A$$
$$I_B : B \to B.$$

let $x \in A$, $y \in B$ such that $y = f(x)$

$I_A(x) = x$.

So, ~~$x \in A$~~, $I_A : A \to A$ i.e $\forall (x) \in A$.

Now, by definition of composition

$$f \circ I_A(x) = f[I_A(x)] = f(x) = y \quad \text{— (1)}$$

Again, $I_B : B \to B$; $I_B(y) = y$; $\forall y \in B$

$$f \circ I_B(x) = f(I_B(x)) = f(x) = y \quad \text{— (2)}$$

from (1) & (2) we get,

$$\boxed{f \circ I_B(x) = f \circ I_B(x) = y} = f(x)$$

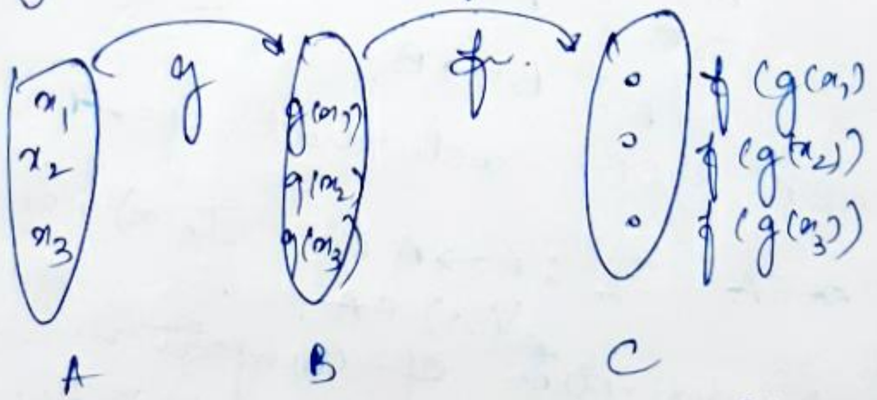
Theorem: Suppose 'g' is a f^n from $A \to B$
f is a f^n from $B \to C$

$g: A \to B$, $f: B \to C$

(i) Prove if f & g are one-one then fog is one-one. $(g \circ f)^{-1} = f^{-1} o g^{-1}$.

(ii) P.T if f & g are onto then fog is also onto. Also Prove

Sol^n (i) $g: A \to B$ $\quad f: B \to C$



Since $g: A \to B$ & g is one-one, it means

$\forall (x)$ $g(x_1) = g(x_2) \Rightarrow x_1 = x_2$.

i.e there is unique mapping from $A \to B$ for each x.

Also f is one-one, $f: B \to C$

$\forall (y)$ $f(y_1) = f(y_2)$

$\Rightarrow y_1 = y_2$

For composition fog also one-one, i.e

$x_1 = g(x_1) \quad x_2 = g(x_2) \quad x_3 = g(x_3)$

$g(x_1) = f(g(x_1))$

assume (2) $g(x_2) = f(g(x_2))$

$g(x_3) = f(g(x_3))$

Hence one-one f^n.

b) Given :- $g : A \to B$ is onto

$f : B \to C$ " "

i.e there is no element in B which is not connected to A.

also $f : B \to C$ is onto :.

every element of C is connected to any element of B.

Hence, comp. fog will also be onto.

NOTE :- Two f^ns f and g are inverses of each other, if $fog(x) = I_X$ and $gof(x) = I_X$.

Q) Show $f(x) = x^3$ and $g(x) = x^{1/3}$ $\forall (x) \in R$ are inverses of each other.

Sol^n $f : A \to B$ and $g : A \to A$ are inverses.

$gof = I_A$ and $fog = I_B$

$fog = f(g(x)) = f(x^{1/3}) = (x^{1/3})^3 = x$

$gof = g(f(x)) = g(x^3) = (x^3)^{1/3} = x$

$\therefore \boxed{gof = fog = I_x.}$

$\Rightarrow$ g is inverse of f

$f = g^{-1}$ and $g = f^{-1}$.

Q) Find domain of real value f^n

1) $f(x) = \sqrt{81 - x^2}$.

Sol^n $\sqrt{81 - x^2} \in R$.

$81 - x^2 \geq 0$.

$x^2 \leq 81$

$|x| \leq 9$

$-9 \leq x \leq 9$.

dom $(f) = [-9, 9]$

For each element $x \in [-9, 9]$ will have unique value in interval $[0, 9]$

2) $f(x) = \frac{1}{x - 3}$

Domain $f(x)$ $= R - (3)$

Also, $y = \frac{1}{x-3}$

$x = \frac{1+3y}{y}$

x is not defined at $y = 0$

Range $f(x) = R - \{0\}$

Q) $X = \{1, 2, 3\}$ as.

$f = \{(1,2)\ (2,3)\ (3,1)\}$

$g = \{(1,2)\ (3,1)\ (3,3)\}$

$h = \{(1,1), (2,2)\ (3,1)\}$

compute fog, gof, fogoh and fohog ?

$fog = \{(1,3)\ (2,2)\ (3,1)\}$

$g(1) = 2$ $\quad f(g(1)) = f(2)$

$g(2) = 1$ $\quad = 3$

$g(3) = 3$

$gof = \{(1,1)\ (2,3)\ (3,2)\}$

$fogoh = \{(1,3)\ (2,2)\ (3,3)\}$ Ans

$fohog = \{(1,3)\ (2,2)\ (3,2)\}$

Q). If f is one-to one & onto then prove:-

$$f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y)$$

Soln Let x be an arbitrary element of $f^{-1}(X \cup Y)$

$\Rightarrow x \in f^{-1}(X \cup Y)$

$\Rightarrow f(x) \in (X \cup Y)$

$\Rightarrow f(x) \in X$ or $f(x) \in Y$

$\Rightarrow x \in f^{-1}(X)$ or $x \in f^{-1}(Y)$

$\Rightarrow x \in f^{-1}(X) \cup f^{-1}(Y)$

$\therefore f^{-1}(X \cup Y) \subseteq f^{-1}(X) \cup f^{-1}(Y)$ —— (1)

Range / K...

Q) $f(A \cap B) \subseteq f(A) \cap f(B)$.

Sol^n Let x be arbitrary element of $f(A \cap B)$. then

$x \in f(A \cap B)$

$\Rightarrow f(x) \in f(A \cap B)$

$\Rightarrow f(x) \in A$ and $f(x) \in B$.

$\Rightarrow f(x) \in (A \cap B)$

$\Rightarrow f(x) \in f(A) \cap f(B)$

$\therefore f(A \cap B) \subseteq f(A) \cap f(B)$ proved.

P.T.O

Q) $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$.

Q) $f^{-1}(X') = [f^{-1}(X)]'$

Sol^n LHS:-

Let $x \in f^{-1}(X')$

$\Rightarrow f(x) \in X'$

$\Rightarrow f(x) \notin X$

$\Rightarrow x \notin f^{-1}(X)$

$\Rightarrow x \in [f^{-1}(X)]'$

LHS $\subseteq$ RHS — (1)

RHS

Let $x \in [f^{-1}(X)]'$

$\Rightarrow x \notin f^{-1}(X)$

$\Rightarrow f(x) \notin X$

$\Rightarrow f(x) \in X'$

$\Rightarrow x \in f^{-1}(X')$

$\therefore$ RHS $\subseteq$ LHS — (2)

LHS = RHS

1. Let x be an arbitrary element of $f^{-1}(X) \cup f^{-1}(Y)$

$\Rightarrow x \in f^{-1}(X) \cup f^{-1}(Y)$

$\Rightarrow x \in f^{-1}(X) \cup x \in f^{-1}(Y)$

$\Rightarrow f(x) \in X \cup f(x) \in Y$

$\Rightarrow f(x) \in (X \cup Y)$

$\Rightarrow \underline{x \in f^{-1}(X \cup Y)}$

$\Rightarrow f^{-1}(X) \cup f^{-1}(Y) \subseteq f^{-1}(X \cup Y)$ — (2)

from (1) & (2) we get

$f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y)$.

---

2) Let f, g, h be f^n from N to N where N is set of natural nos. so that

$f(n) = n+1 \quad g(n) = 2n$ and

$$\begin{bmatrix} h(n) = 0 & \text{if } n \text{ is even} \\ h(n) = 1 & \text{if } n \text{ is odd} \end{bmatrix}$$

Determine :- $f \circ f$, $f \circ g$, $g \circ f$, $g \circ h$, $(f \circ g) h$.

(a) $f \circ f = f(n+1) = n+2$

(b) $f \circ g = f(g(n)) = f(2n) = 2n+1$

(c) $g \circ f = g(n+1) = 2n+2$

(d) $g \circ h = g(h(n)) = 0$ or 1

depending even or odd

(e) $(f \circ g) h = f(g(n)) \circ h$

$= f(g(h(n)))$

$= 0$ if n is even

$= 1$ " " " odd.