

Q) Prove the following by M.I method:-

$$P(n) = 7 + 77 + 777 + \dots + 7 \dots n \text{ times} \\ = \frac{7}{81} (10^{n+1} - 9n - 10)$$

$\forall n \in \mathbb{N}$.

Solⁿ Step Basic Step:-

For $n=1$ LHS $P(1) = 1$

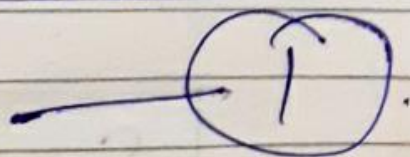
$$\text{RHS} = \frac{7}{81} (100 - 9 - 10) = \frac{7}{81} \times 81 = 7$$

$$\text{LHS} = \text{RHS}.$$

Step II Let statement $P(n)$ be true for $n=k$ i.e

$$7 + 77 + \dots + 77 \dots k \text{ times} =$$

$$\frac{7}{81} (10^{k+1} - 9k - 10)$$



Step 3 To prove $P(k+1)$ is true we have to show

$$7 + 77 + 777 + \dots + 7 \dots (m+1) \text{ times}$$

$$= \frac{7}{81} (10^{m+2} - 9(m+1) - 10)$$

We can write,

$$7 = 10^0 \times 7$$

$$77 = 10^1 \times 7 + 7$$

⋮

$$\begin{aligned} 77 \dots 7 &= 10^k \times 7 + \\ (k+1) \text{ times} &10^{k-1} \times 7 + \\ &10 \times 7 + 7 \end{aligned}$$

$$\Rightarrow 7(1 + 10 + 10^2 + \dots + 10^k)$$

AP with $r = 10$
($r > 0$)

$$\boxed{S_n = \frac{a(r^n - 1)}{r - 1}}$$

$$= 7 \left(\frac{10^{k+1} - 1}{10 - 1} \right)$$

$$= \frac{7}{9} \times (10^{k+1} - 1)$$

LHS :-

$$\frac{7}{81} (10^{k+1} - 9k - 10) + \frac{7}{9}$$

from eqⁿ (1)

$$\frac{7}{81} (10^{k+1} - 1)$$

$$= \frac{7}{81} (10^{k+1} - 9k - 10) + 7 \times 9$$

$$\frac{7}{81} (10^{k+1} - 1)$$

$$= \frac{7}{81} \times \left(\frac{10^{k+1} + 9 \times 10^{k+1} - 9k - 10}{9 - 10} \right)$$

$$= \frac{7}{81} (10^{k+1} - 9(k+1) - 10)$$

= RHS.

$\therefore P(k+1)$ is also true.