# EQUIVALENCE CLASS:-

Consider an equivalence relation $R$ in set $A$. The Equivalence class of an element $a \in A$ is the set of elements of $A$ to which element 'a' is related.

It is denoted by $\bar{a}$ or $[a]$.

Eg. Let $A = \{a, b, c\}$.

$R = \{(a, a)\ (b, b)\ (c, c)\ (a, b)\ (b, a)\}$

Find classes of elements of A.

Ans: $[a] = \{a, b\}$

$[b] = \{b, a\} = [a]$.

$[c] = \{c\}$.

Rank = 2

↓ No. of Induced equivalence classes.

Let $A = \{1, 2, 3, 4\}$

$R = \{(1,1)\ (1,2)\ (1,3)\ (2,1)\ (2,2)\ (3,1)$
$(2,3)\ (3,2)\ (3,3)\ (4,4)\}$

Equivalence classes of A :-

$[1] = \{1, 2, 3\}$

$[2] = \{1, 2, 3\} = [1]$

$[3] = \{3, 1, 2\} = [1]$.

$[4] = \{4\}$.

Rank = 2

## Covering of a set :-

collection of all ~~unique~~ classes of elements of A under relation R is denoted by

$$A/R = \{ [x] : x \in A \}$$

It is called the Quotient set of A by R.

Q) $A = \{1, 2, 3, 4, 5, 6\}$
Relation R is equivalence relation
$R = \{ (1,1)\ (4,1)\ (3,2)\ (3,2)\ (5,2)\ (2,3)$
$(3,3)\ (5,3)\ (1,4)\ (4,4)\ (3,5)\ (3,5)$
$(5,5)\ (6,6) \}$

Partition of A :-

$\{ (1,4)\ (3,5,2\}\ \{6\}\}$.

NOTE: let $A_1\ A_2 \ldots A_n$ are partitions of set A then
$A_1 \cup A_2 \cup \ldots A_n = A$.
and $A_i \cap A_j = \phi$.

The sets in A are called blocks or cells of the partition

Also set of all distinct equivalence classes forms a partition of A

* If P is given (Partion) of set A then, we can find the equivalence relation R on A :— (PARTITION INDUCED BY EQUIVALENCE RELATION)

" Each element in a block is related to every other element in the same block and only to those elements".

Eg. Let $A = \{1, 2, 3, 4\}$ $P = \{\{1,2,3\}, \{4\}\}$
Find Eq. Relation (R) by P.

Sol$^n$ Blocks of P are :—
$\{1,2,3\}$ and $\{4\}$.

$R = \{(1,1)\ (1,2)\ (1,3)\ (2,1)\ (2,2)\ (2,3)\ (3,1)\ (3,2)$
$(3,3)\ (4,4)\}$

) Let $X = \{a, b, c, d, e\}$ and
$P = \{\{a,b\}\ \{c\}\ \{d,e\}\}$. Show that partition P defines an equivalence relation on X.

Sol$^n$) We know P induces relation R on X

So, $R = \{(a,a)\ (a,b)\ (b,a)\ (c,c)\ (d,d)\ (d,e)$
$(e,e)\ (e,d)\}$.

) R is reflexive since $aRa$, $bRb$, $cRc$
$dRd$ etc.

) " " Symmetric since $aRb$ & $bRa$

iii) Transitive :— $aRb$ and $bRa \Rightarrow aRa$
$dRe$ and $eRd \Rightarrow dRd$.

**Example 19:** *Find the relation determine by*

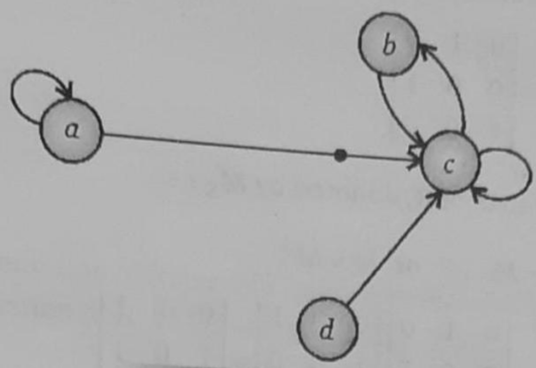


Fig. 2.3

**Solution:** The relation $R$ of the digraph is

$$R = \{(a, a), (a, c), (b, c), (c, b), (c, c), (d, c)\}$$

**Example 20:** *Let* $R = \{(1, 2), (2, 3), (3, 1)\}$ *and* $A = \{1, 2, 3\}$, *find the reflexive, symmetric and transitive closure of R, using*

(i) *Composition of relation R*

(ii) *Composition of matrix relation R*

(iii) *Graphical representation of R*

**[R.G.P.V. (B.E.) Raipur 2005, 2009]**

**[P.T.U. (B.E.) Punjab 2002, 2006, 2009; M.K.U. (B.E.) 2005, 2008; Osmania (B.E.) 2003]**

**Solution:** (i) The reflexive closure of $R$ is denoted by $R_1$ and given by

$$R_1 = R \cup \Delta \text{ or } R \cup I_A$$

$$I_A = \text{identity relation}$$

$$R_1 = \{(1, 2), (2, 3), (3, 1)\} \cup \{(1, 1), (2, 2), (3, 3)\}$$

$$= \{(1, 1), (1, 2), (2, 2), (2, 3), (3, 1), (3, 3)\}$$

The symmetric closure of $R$ is denoted by $R*$ is given by

$$R* = R \cup R^{-1}$$

$$= \{(1, 2), (2, 3), (3, 1)\} \cup \{(2, 1), (3, 2), (1, 3)\}$$

$$= \{(1, 2), (1, 3), (2, 1), (2, 3), (3, 1), (3, 2)\}$$

The transitive closure of $R$ is denoted by $R^+$

Now $$RoR = \{(1, 2), (2, 3), (3, 1)\} \, o \, \{(1, 2), (2, 3), (3, 1)\}$$

$\Rightarrow$ $$R^2 = \{(1, 3), (2, 1), (3, 2)\}$$

$$R^3 = R^2 oR = \{(1, 3), (2, 1), (3, 2)\} \, o \, \{(1, 2), (2, 3), (3, 1)\}$$

$$= \{(1, 1), (2, 2), (3, 3)\}$$

$$R^4 = R^3 oR = \{(1, 1), (2, 2), (3, 3)\} \, o \, \{(1, 2), (2, 3), (3, 1)\}$$

$$= \{(1, 2), (2, 3), (3, 1)\} = R$$

Thus $$R^+ = R \cup R^2 \cup R^3 = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$$

**Example 26:** *Let A be the set {1, 2, 3}, define the following types of binary relation on A.*

(i) *A relation that is both symmetric and anti-symmetric*

(ii) *A relation that is neither symmetric nor anti-symmetric*

[U.P.T.U. (M.C.A.) 2004]

**Solution:** We have $A = \{1, 2, 3\}$

(i) For a binary relation $R$ on $A$ to be symmetric, we have

$${}_aR_b \Rightarrow {}_bR_a \;\; \forall \;\; a, b \in A \;\; i.e. \text{ if } (a, b) \in R \text{ then } (b, a) \in R$$

For a binary relation $R$ on $A$ to be anti-symmetric, we have

$${}_aR_b \text{ and } {}_bR_a \Rightarrow a = b \;\; i.e. \; (a, b) \in R \text{ and } (b, a) \in R \text{ only when } a = b$$

which means that if $a \neq b$ then either ${}_a\not{R}_b$ or ${}_b\not{R}_a$

The binary relation on $A$ is symmetric as well as anti-symmetric is given by

$$R = \{(1, 1), (2, 2), (3, 3)\}$$

(ii) A relation $R$ on $A$ shall be

(*a*) neither symmetric *i.e.* $(a, b) \in R$ and $(b, a) \not{R}$

(*b*) nor anti-symmetric *i.e.* $(a, b) \in R$ and $(b, a) \in R$ even when $a \neq b$.

The relation is given by

$$R = \{(1, 2), (2, 1), (1, 3)\} \text{ in which } (1, 3) \in R \text{ but } (3, 1) \notin \not{R}$$

$\Rightarrow$ It is not symmetric and $(1, 2) \in \not{R}$ as well as $(2, 1) \in R$ but $1 \neq 2$, showing that $R$ is not anti-symmetric.

**Example 27:** *If R is an equivalence relation on A, then prove that $R^{-1}$ is also equivalence relation on A.*

[U.P.T.U. (M.C.A.) 2002-2003, 2005-2006]

**Solution:** (i) Let $x \in A$. Since $R$ is a reflexive relation, $(x, x) \in R$

$\Rightarrow \quad (x, x) \in R^{-1}$.

So $\quad R^{-1}$ is reflexive

(ii) Let $x, y \in R$, as $R$ is symmetric relation

$(x, y) \in R \Rightarrow (y, x) \in R$

$\Rightarrow \quad (y, x) \in R^{-1}$ and $(x, y) \in R^{-1}$

So $\quad (y, x) \in R^{-1} \Rightarrow (x, y) \in R^{-1}$

So $\quad R^{-1}$ is symmetric

(iii) Let $x, y, z \in A$, as $R$ is a transitive relation

$(x, y) \in R$ and $(y, z) \in R \quad \Rightarrow (x, z) \in R$

which means that $(y, x) \in R^{-1}$ and $(z, y) \in R^{-1} \Rightarrow (z, x) \in R^{-1}$

or $\quad (z, y) \in R^{-1}$ and $(y, x) \in R^{-1} \Rightarrow (z, x) \in R^{-1}$

So $\quad R^{-1}$ is transitive

Hence, $R^{-1}$ is an equivalence relation.

$\Rightarrow \quad a b'' = a'' b$

$\Rightarrow \quad (a, b) R (a'', b'')$

Hence it is transitive

Therefore, $R$ is an equivalence relation.

**Example 37:** *If R and S are equivalence relations on the set A, show that the following are equivalence relation.*

(i) $R \cap S$ (ii) $R \cup S$ [U.P.T.U. (B.Tech.) 2003]

**Solution:** (i) $R \cap S$ is an equivalence relation, if

**(a) Reflexive:** $\forall a \in A$, $(a, a) \in R$ and $(a, a) \in S$, since R and S are equivalence relations. This implies $\forall a \in A,\ (a, a) \in R \cap S$

Hence, $R \cap S$ is reflexive

**(b) Symmetric:** Let $(a, b) \in R \cap S \Rightarrow (a, b) \in R$ and $(a, b) \in S$

$\Rightarrow (b, a) \in R$ and $(b, a) \in S$ as R, S is symmetric

$\Rightarrow (b, a) \in R \cap S$

**(c) Transitive:** Let $(a, b) \in R \cap S, (b, c) \in R \cap S$

$\Rightarrow \quad (a, b) \in R, (a, b) \in S$ and $(b, c) \in R, (b, c) \in S$

$\therefore \quad (a, b) \in R, (b, c) \in R$ and R is transitive $\Rightarrow (a, c) \in R$

and $\quad (a, b) \in S, (b, c) \in S$ and S is transitive $\Rightarrow (a, c) \in S$

$\therefore \quad (a, c) \in R, (a, c) \in S \Rightarrow (a, c) \in R \cap S$

Hence $R \cap S$ is an equivalence relation.

(ii) The union of two equivalence relation on a set is not necessarily an equivalence relation. F example

$A = \{a, b, c\}$ and $R, S$ be two relation on $A$ given as

$R = \{(a, a), (b, b), (c, c), (a, b), (b, a)\}$

and $\quad S = \{(a, a), (b, b) (c, c), (b, c), (c, b)\}$

Each $R$ and $S$ is an equivalence relation on $A$. But $R \cup S$ is not transitive, because $(a, b) \in R \cup S$ a $(b, c) \in R \cup S \Rightarrow (a, c) \notin R \cup S$

Hence $R \cup S$ is not equivalence relation.

**Example 38:** *Let $A = R \times R$ (R be the set of real numbers) and define the following relation on A.*

$$(a, b) R (c, d) \Rightarrow a^2 + b^2 = c^2 + d^2$$

(i) *Verify that $(A, R)$ is an equivalence relation.*

(ii) *Describe geometrically what the equivalence classes are for this relation (justify).*

[U.P.T.U. (B.Tech.) 2002; R.G.P.V. (B.E.) B

**Solution: Reflexive:** Let $(a, b)R(a, b) \Rightarrow a^2 + b^2 = a^2 + b^2$ which is true.

Hence $R$ is reflexive.

**Symmetric:** $(a, b)R(c, d) \Rightarrow a^2 + b^2 = c^2 + d^2$

$$\Rightarrow c^2 + d^2 = a^2 + b^2$$

$$\Rightarrow (c, d)R(a, b)$$

$$\Rightarrow R \text{ is symmetric}$$

**Transitive:** Let $(a, b)R(c, d) \Rightarrow a^2 + b^2 = c^2 + d^2$

and $(c, d)R(e, f) \Rightarrow c^2 + d^2 = e^2 + f^2$

$\therefore \quad a^2 + b^2 = e^2 + f^2 \Rightarrow (a, b)R(e, f)$

$$\Rightarrow R \text{ is transitive}$$

Hence, R is an equivalence relation.

**Example 39:** *Let $A = \{1, 2, 3, 4, 6, 7, 8, 9\}$ and let $\sim$ be the relation on $A \times A$ defined a $a + d = b + c$. Prove that*

(i) $\sim$ is an equivalence relation

(ii) Find $[(2, 5)]$, the equivalence class of $(2, 5)$

**Solution:** $\sim$ is an equivalence, if

(a) **Reflexive:** $(a, b) \sim (a, b)$ *i.e.* $a + b = a + b$ which is true. Hence $\sim$ is reflexive

(b) **Symmetric:** $(a, b) \sim (c, d) \Rightarrow a + d = b + c$

$$\Rightarrow b + c = a + d \Rightarrow c + b = d + a \Rightarrow (c, d) \sim (a, b)$$

Hence relation is symmetric.

(c) **Transitive:** Let $(a, b) \sim (c, d)$ and $(c, d) \sim (e, f)$ then $a + d = b + c$ and $c +$ $a + d + c + f = b + c + d + e$ or $a + f = b + e$ or $(a, b) \sim (e, f)$ then $\sim$ is equivalenc

(i) $R[(2, 5)] = \{(2, 5), (1, 4), (3, 6), (4, 7), (5, 8), (6, 9)\}$

**Example 40:** Let $A = \{1, 2, 3, 4, 5, 6\}$, *construct description of relation R on A for the*

(i) $R = \{(j, k) : k \text{ is multiple of } j\}$

(ii) $R = \{(j, k) : (J - k)^2 \in A\}$

(iii) $R = \{(j, k) : j \text{ divide } k\}$

(iv) $R = \{(j, k) : j \times k \text{ is prime}\}$

**Example 44:** *Let A be the set of all integers and a relation R is defined as*

*$R = \{(x, y): x \equiv y \pmod{m}$, m divide $(x - y)$ where m is positive integer. Prove that R is an equivalence relation.* [U.P.T.U. (M.C.A.) 2008]

**Solution:** (i) Since $(x - x)$ is divisible by $m$, therefore

$$x \equiv x \ (\text{mod m}) \ \text{i.e. } {}_xR_x.$$

$\Rightarrow R$ is reflexive

(ii) If $x, y \in A$ and $(x - y)$ is divisible by $m$, then $(y - x) = -(x - y)$ is also divisible by $m$.

$\therefore \quad x \equiv y \ (\text{mod } m) \ \Rightarrow y \equiv x \ (\text{mod } y)$

or $\quad {}_xR_y \Rightarrow {}_yR_x.$

So $R$ is symmetric

(iii) If $x, y, z \in A$ and $x - y$, $y - z$ are divisible by $m$

$\therefore \ x - z = (x - y) + (y - z)$ is also divisible by $m$

$\Rightarrow x \equiv z \ (\text{mod m})$

$\therefore \quad {}_xR_y, {}_yR_z \Rightarrow {}_xR_z.$

So $R$ is transitive. Hence $R$ is an equivalence relation.

## Exercise

### Relation

1. Give an example of a relation which is:
   (i) reflexive and transitive but not symmetric.
   (ii) symmetric and transitive but not reflexive.
   (iii) reflexive and symmetric but not transitive.
   (iv) reflexive and transitive but neither symmetric nor anti-symmetric.

2. Prove that if a relation $R$ on set $A$ is transitive and irreflexive, then it is symmetric

3. If $R$ be a relation in the set of integer $I$ defined by $R = \{(x, y): x \in I, y \in I, (x - y) = 8k$ or $(x - y)$ is divisible by $8\}$. Prove that $R$ is an equivalence relation.

4. List the order pairs in the relation $R$ from $A = \{0, 1, 2, 3, 4\}$ to $B = \{0, 1, 2, 3\}$ where $(a, b) \in R$ if and only if
   (i) $a = b$ (ii) $a + b = 3$ (iii) $a \times b$
   (iv) $a \perp b$ (v) $g.c.d\,(a, b) = 1$ (vi) $\text{lcm}\,(a, b) = 2$

5. Let $A = \{1, 2, 3, 4\}$, determine whether the relation are reflexive, symmetric, anti-symmetric or transitive
   (i) $R = \{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)\}$
   (ii) $R = \{(1, 3), (4, 2), (2, 4), (3, 1), (2, 2)\}$
   (iii) $R = \{(1, 2), (1, 3), (3, 1), (1, 1), (3, 3), (3, 2), (1, 4), (4, 2), (3, 4)\}$

6. Write down the relations in the square of the set $\{1, 2, 4, 8, 16, 32, 64\}$

7. Let $f$ be a mapping of a set $X$ onto a set $B$. Then if we define $(a, b) \in R$ for $a, b \in X$ provided $f(a) = f(b)$. Prove that $R$ is an equivalence relation.

8. Let $R$ be the relation in the natural numbers $N = \{1, 2, 3 \ldots\}$

   Define by $x + 2y = 10$ *i.e.* Let $R = \{(x, y) 1\, x, y \in N, x + 2y = 10\}$

   Find (*a*) The domain and range of $R$ (*b*) $R^{-1}$

9. Show that the relation is congruent module 4 to on the set of integers $\{0, 1, 2, \ldots 10\}$ is an equivalence relation.

10. Determine which of the following are equivalence relations and or partial ordering relations for the given sets.

    (*a*) $A = \{$lines in the plane$\}$: ${}_xR_y$ iff $x$ is parallel to $y$

    (*b*) $A = \{$the set of real numbers: ${}_xR_y$ iff $|x - y| \le 7$

11. Let $A = \{1, 2, 3, 6\}$. If for $x, y \in A$

    $R = \{(x, y) : x \le y\}$

    $S = \{(x, y) : x \text{ divides } y\}$

    write $R$ and $S$ as sets and find $R \cap S$

12. For a set $X$ with $n$ elements, find how many relations on $X$, which are

    [U.P.T.U. (M.C. A.) 2004]

    (i) Symmetric [P.T.U. (B.E.) Punjab 2008]

    (ii) Anti-symmetric [U.P.T.U. (B.Tech.) 2009]

    (iii) reflexive

    (iv) irreflexive [Pune (B.E.) 2007]

    (v) reflexive and symmetric [Rohtak (M.C.A.) 2007]

    (vi) reflexive and not symmetric [Rohtak (M.C.A.) 2007]

    (vii) symmetric but not reflexive

    (viii) neither reflexive nor irreflexive.

13. Let $L$ be the set of lines in the euclidean plane and let $R$ be the relation in $L$ defined by "$x$ is parallel to $y$". Is $R$ a symmetric relation? Why? Is $R$ a transitive relation?

14. Prove that if $R$ is asymmetric relation then

$$R \cap R^{-1} = R$$

15. Find all partitions on

    (i) $A = \{1, 2, 3\}$ (ii) $A = \{a, b, c, d\}$

16. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $R = \{x, y) : x - y$ is divisible by $3\}$

Show that $R$ is an equivalence relation. Draw the graph of $R$.

17. Let $R = \{(1, 2), (3, 4), (2, 2)\}$ and

$$S = \{(4, 2), (2, 5), (3, 1), (1, 3)\}$$

Find RoS, SoR, Ro(SoR), (RoS)oR, RoR, SoS, and RoRoR.

18. Let $R$ and $S$ be any two relations on a set of positive integers.

$R = \{(x, 2x) | x \in I\}$ and $S = \{(x, 7x) | x \in I$, then show

$$RoS = \{(x, 14x) | x \in I\} = SoR$$

$$RoR = \{(x, 4x) | x \in I\}$$

$$RoRoR = \{(x, 8x) | x \in I\}$$

$$RoSoR = \{(x, 2x) | x \in I\}]$$

19. On the set of integers the relation is defined by ${}_aR_b$ "iff $(a - b)$ is even integer". Show that $R$ is an equivalence relation.

20. Suppose $S$ and $T$ are two sets and $f$ is a functions from $S$ to $T$ let $R_1$ be an equivalence relation on $T$. Let $R_2$ be binary relation on $S$ such that $(x, y) \in R_2$ if and only if $(f(x), f(y)) \in R_1$. Show that $R_2$ is also an equivalence relation.

[U.P.T.U. (B.Tech.) 2009]

21. Prove that the relation "congruence modulo m" is given by

$$R = \{(x, y) | (x - y) \text{ is divisible by } m\}$$

Over the set of positive integer is an equivalence relation. Also show that if $x_1 = y_1$, and $x_2 = y_2$ then $(x_1 + x_2) = (y_1 + y_2)$

[U.P.T.U. (M.C.A.) 2008]

22. Given the relational matrices $M_R$ and $M_S$, find $M_{RoS}, M_{(R)^{-1}}, M_{(S)^{-1}}, M_{(RoS)}$ and show that $M_{(RoS)^{-1}} = M_{(S)^{-1}\, o(R)^{-1}}$.

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad \text{and} \quad M_S = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

[Raipur (B.E.) 2005, 2007]

23. Let $A = \{1, 2, 3, 4, 5, 6\}$. Define a relation $R$ on $A$

$R = \{(x, y) : x + y$ is divisor of $24\}$

(i) Find the relational matrix $M$ of $R$

(ii) Compute $M^2$ and use $M$ and $M^2$ whether or not $R$ is transitive

24. A number of binary relations are defined on the set $A = \{0, 1, 2, 3\}$ Fig. Shows some diagrams of relations.

Find the relation each case as a set of ordered pairs. Decide which of the following properties it has

(i) reflexivity (ii) symmetry (iii) transitivity

(a)

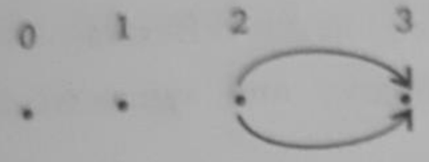


Fig. 2.15

(b)

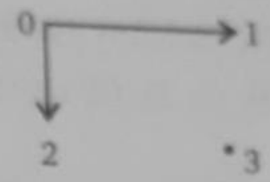


Fig. 2.16

(c)

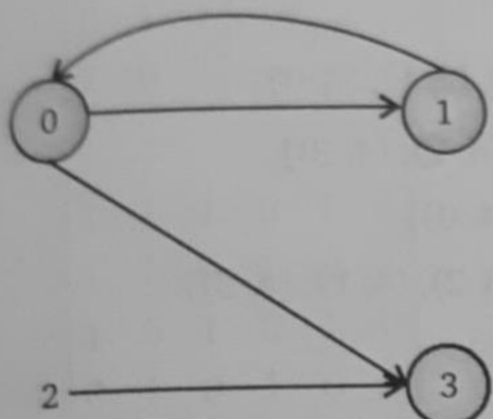


Fig. 2.17

(d)

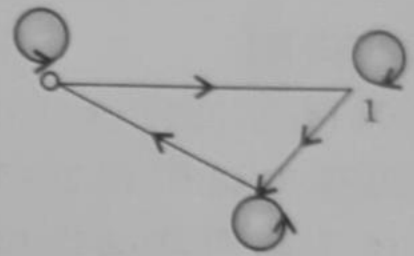


Fig. 2.18

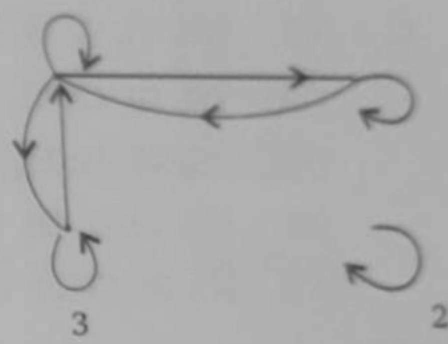


Fig. 2.19

25. The relation $R$ on a set $A = \{1, 2, 3, 4\}$ is defined by

$$R = \{(1, 1), (1, 2), (1, 4), (2, 2), (2, 1), (2, 4), (3, 3), (3, 4), (3, 2), (4, 3), (4, 2), (4, 1)\}.$$

Find the digraph of $R$ and hence find $R^{-1}$

26. Let $A = \{0, 1, 2, 3, 4\}$. Show that the relation

$$R = \{(0, 0), (0, 4), (1, 1), (1, 3), (2, 2), (3, 1), (3, 3), (4, 0), (4, 4)\}$$ is an equivalence relation.

Find the distinct equivalence classes of $R$ [U.P.T.U. (M.C.A.) 2000]

27. Define relation and explain properties of relation. [R.G.P.V. (B.E.) Raipur (B.E.) 2007]

# Answers

1. Let $A = \{1, 2, 3\}$

   (i) $R_1 = \{(1, 1), (2, 2), (3, 3), (1, 3)\}$ is reflexive, transitive but not symmetric.

   (ii) $R_2 = \{(1, 1), (3, 3), (1, 3), (3, 1)\}$ is symmetric and transitive but not reflexive.

   (iii) $R_3 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)\}$ is reflexive and symmetric but not transitive.

   (iv) $Z^*$ be the set of non-zero integers and $R$ be the relation on $Z^*$ given by $(a, b) \in R$ if a is a factor of $b$, $R$ is reflexive and transitive but not symmetric.

4. (i) $\{(0, 0), (1, 1), (2, 2), (3, 3)\}$ (ii) $\{(0, 3), (1, 2), (2, 1), (3, 0)$

   (iii) $\{(1, 0), (2, 0), (3, 0), (4, 0), (2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)\}$

   (iv) $\{(1, 0), (1, 1), (1, 2), (1, 3), (2, 0), (2, 2), (3, 0), (3, 3), (4, 0)\}$

   (v) $\{(0, 1), (1, 0), (1, 1), (1, 2), (1, 3), (2, 1), (2, 3), (3, 1), (3, 2), (4, 1), (4, 3)\}$

   (vi) $\{(1, 2), (2, 1), (2, 2)\}$

5. (i) Transitive (ii) Symmetric (iii) transitive

6. $R = \{(1, 1), (4, 2), (16, 4), (64, 8)\}$

8. (*a* $R = \{(8, 1), (6, 2), (4, 3), (2, 4)\}$ (*b*) $R^{-1} = \{(1, 8), (2, 6), (3, 4), (4, 2)\}$

10. (*a*) It is an equivalence relation but not a partial ordering relation, since $R$ is not anti-symmetric.

    (*b*) Not transitive, therefore, it is neither.

11. $R = \{(1, 1), (1, 2), (1, 3), (1, 6), (2, 2), (2, 3), (2, 6), (3, 3), (3, 6), (6, 6)\}$

    $S = \{(1, 2), (1, 3), (1, 6), (2, 6), (3, 6)\}$

    $R \cap S = \{(1, 2), (1, 3), (1, 6), (2, 6), (3, 6)\} = S$

12. (i) $2^{n(n+1)/2}$ (ii) $2^n \cdot 3^{(n-1)/2}$ (iii) $2^{n(n-1)}$

    (iv) $2^{(n^2-n)}$ (v) $2^{(n^2-n)/2}$ (vi) $2^{n^2-n} - 2^{(n^2-n)/2}$

    (vii) $2^{(n^2+n)/2} - 2^{(n^2-n)/2}$ (viii) $2^{n^2} - 2 \cdot 2^{n^2-n}$

13. Symmetric and transitive, since

    (i) $x$ is parallel to $y$ and $y$ parallel to $x$

    (ii) if $x$ is parallel to $y$ and $y$ is parallel to $z$ then $x$ is parallel to $z$

15. (i) $\{\{1, 2, 3\}\}, \{\{1\}, \{2, 33\}, \{\{2\}, \{1, 33\}, \{\{3\}, \{1, 3\}, \{\{13, 52\}, \{3\}\}$

    (ii) The number of different partition on $A$ is 15.

17. $RoS = \{(4, 2), (3, 2), (1, 4)\}$

$SoR = \{(1, 5), (3, 2), (2, 5)\}$

$Ro\,(SoR) = \phi$

$(RoS)\,oR = \{(3, 2)\}$

$RoR = \{1, 2\} = SoS = \{(4, 5), (3, 3)\}$

$RoRoR = \phi$

22. $M_{(R)^{-1}} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, $M_{(S)^{-1}} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ $M_{(RoS)^{-1}} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

23. (i) $M_R = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$, $M^2 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$

Since $M^2 \subseteq M$ is not true. $M$ is not transitive.

24. (a) $\{(2, 3), (3, 2)\}$ Not reflexive and transitive but symmetric

(b) $\{(0, 1), (0, 2)\}$ Not reflexive, symmetric but transitive

(c) $\{(0, 0), (0, 1), (0, 3), (1, 1), (1, 0), (2, 3), (3, 3)\}$ Not reflexive, symmetric and transitive

(d) $\{(0, 0), (0, 1), (1, 1), (1, 2), (2, 2), (2, 0)\}$ Not reflexive, symmetric and transitive

(e) $\{(0, 0), (0, 1), (0, 3), (1, 1), (1, 0), (3, 0)\}$ Reflexive, symmetric but not transitive

25.

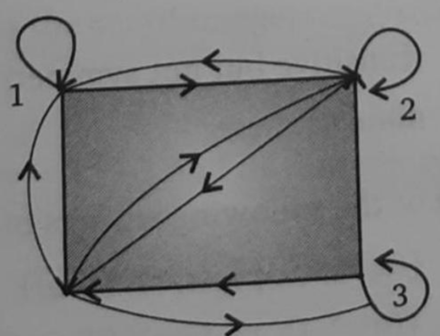


Fig. 2.20

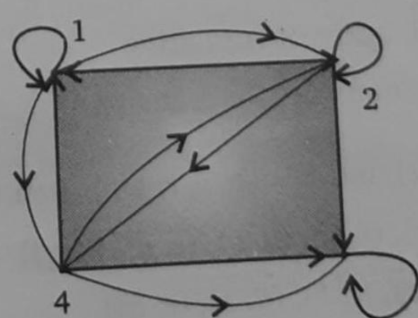


Fig. 2.21