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Unit - 4 Statistical Techniques - II

Theory of Probability

a) Trial and event - Let an experiment be repeated under essentially the same conditions and let it result in any one of the several possible outcomes. Then, the experiment is called a trial and the possible outcomes are known as events or cases.

b) Exhaustive events - The total no. of all possible outcomes in any trial is known as exhaustive events.

c) Favourable events or cases - The cases which entail the happening of an event are said to be favourable to the event.

d) Mutually exclusive events - Events are said to be mutually exclusive or incompatible if the happening of any one of them precludes (or rules out) the happening of all others.

e) Equally likely events - Events are said to be equally likely if there is no reason to expect any one in preference to any other.

f) Independent and dependent events - Two or more events are said to be independent if the happening or non-happening of any one does not depend by the happening or non-happening of any other. Otherwise they are said to be dependent.

Mathematical / Classical definition of Probability

If a trial results in $m+n$ exhaustive, mutually exclusive and equally likely cases and m of them are favourable to the happening of an event E, then the probability of happening of E is given by

$$p \text{ or } P(E) = \frac{\text{Favourable no. of cases}}{\text{Exhaustive no. of cases}} = \frac{m}{m+n}$$

$P(\bar{E})$ = probability that the event E will not happen is given by

$$q \text{ or } P(\bar{E}) = \frac{\text{unfavourable no. of cases}}{\text{Exhaustive no. of cases}} = \frac{n}{m+n}$$

$$= 1 - \frac{m}{m+n} = 1 - p$$

$$p + q = 1 \quad ; \text{ i.e. } P(E) + P(\bar{E}) = 1$$

$$0 \le p \le 1, \quad 0 \le q \le 1$$

* If $P(E) = 1$, E is called a certain event.
  $P(E) = 0$, then E is an impossible event.

Statistical / Empirical definition of probability

If in n trials, an event E happens m times, then the probability of happening of E is given by

$$p = P(E) = \lim_{n \to \infty} \frac{m}{n}$$

Ex→1. A bag contains 7 white, 6 red and 5 black balls. Two balls are drawn at random. Find the probability that they will both be white.

Solⁿ→ Total no. of balls $= 7+6+5 = 18$

Probability of getting both balls white $= \dfrac{{}^7C_2}{{}^{18}C_2}$

$$= \frac{\frac{7!}{2!\,5!}}{\frac{18!}{2!\,16!}} = \frac{7\times 6}{18\times 17} = \frac{7}{51}$$

Ex→2. Four cards are drawn from a pack of cards. Find the probability that (i) all are diamonds.

(ii) there are one card of each suit.

(iii) there are two spades and two hearts.

Sol→(i) There are 13 diamond cards.

Probability of getting all four cards of diamond

$$= \frac{{}^{13}C_4}{{}^{52}C_4} = \frac{\frac{13!}{4!\,9!}}{\frac{52!}{4!\,48!}} = \frac{13\times 12\times 11\times 10}{52\times 51\times 50\times 49}$$

$$= \frac{11}{4165}$$

(ii) There are 4 suits, each containing 13 cards

probability of getting one card of each suit

$$= \frac{{}^{13}C_1\ {}^{13}C_1\ {}^{13}C_1\ {}^{13}C_1}{{}^{52}C_4} = \frac{13\times 13\times 13\times 13}{\frac{52!}{4!\,48!}} = \frac{13\times 13\times 13\times 13}{\frac{52\times 51\times 50\times 49}{4\times 3\times 2}}$$

$$= \frac{\cancel{13} \times 13 \times 13 \times 13 \times \cancel{4} \times \cancel{3} \times \cancel{2}}{\underset{4}{\cancel{52}} \times \underset{17}{\cancel{51}} \times \underset{25}{\cancel{50}} \times 49} = \frac{2197}{20825}$$

(iii) 2 spades and 2 hearts

$$P = \frac{^{13}C_2 \times {}^{13}C_2}{^{52}C_4} = \frac{\frac{13!}{2!\,11!} \times \frac{13!}{2!\,11!}}{\frac{52!}{4!\,48!}}$$

$$= \frac{\frac{13 \times 12}{2} \times \frac{13 \times 12}{2}}{\frac{52 \times 51 \times 50 \times 49}{\cancel{4} \times 3 \times 2}} = \frac{\cancel{13} \times \overset{3}{\cancel{12}} \times 13 \times \overset{6}{\cancel{12}} \times 3 \times 2}{\underset{4}{\cancel{52}} \times \underset{17}{\cancel{51}} \times \underset{25}{\cancel{50}} \times 49}$$

$$= \frac{13 \times 6 \times 3 \times 2}{17 \times 25 \times 49} = \frac{468}{20825}$$

Q3. A bag contains 50 tickets numbered 1, 2, 3, ..., 50 of which five are drawn at random and arranged in ascending order of magnitude ($x_1 < x_2 < x_3 < x_4 < x_5$). What is the probability that $x_3 = 30$?

Sol→ If $x_3 = 30$, then the two tickets with no. x_1 and x_2 must come out of 29 tickets numbered 1 to 29 and this can be done in $^{29}C_2$ ways. The other two tickets with x_4 and x_5 must come out of 20 tickets numbered 31 to 50 and this can be done in $^{20}C_2$ ways.

$$\text{Required probability} = \frac{^{29}C_2 \times {}^{20}C_2}{^{50}C_5} = \frac{551}{15134}$$

Some Important Formulae

* $P(\phi) = 0$
* $P(\bar{A}) = 1 - P(A)$
* $P(\bar{A} \cap B) = P(B) - P(A \cap B)$
* If $B \subseteq A$, then
  (i) $P(A \cap \bar{B}) = P(A) - P(B)$
  (ii) $P(B) \leq P(A)$
* $P(A \cap B) \leq P(A)$ and $P(A \cap B) \leq P(B)$

Addition Theorem of Probabilities

If A and B are two events, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If A and B are two mutually disjoint events; $(A \cap B = \phi)$

$$P(A \cup B) = P(A) + P(B)$$

If A, B and C are three events, then

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

Q.4. In a given race, the odds in favour of four horses A, B, C and D are 1:3, 1:4, 1:5 and 1:6 respectively. Assuming that a dead heat is impossible, find the chances that one of them wins the race.

Sol→ let P_1, P_2, P_3 and P_4 be the probabilities of winning of the horses ~~res~~ A, B, C and D respectively.

Odds in favour of A are 1:3 $\Rightarrow P_1 = \frac{1}{1+3} = \frac{1}{4}$

Similarly; $P_2 = \frac{1}{5}$, $P_3 = \frac{1}{6}$, $P_4 = \frac{1}{7}$.

If P is the chance that one of them wins, then

$$P = P_1 + P_2 + P_3 + P_4 = \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7}$$

$$= \frac{105 + 84 + 70 + 60}{420}$$

$$= \frac{319}{420}$$

Conditional Probability:

The probability of the happening of an event E_1 when another event E_2 is known to have already happened is called Conditional Probability and is denoted by $P(E_1/E_2)$.

Mutually Independent events – An event E_1 is said to be independent of an event E_2 if $P(E_1/E_2) = P(E_1)$

Multiplicative Law of Probability (or theorem of compound probability)

The probability of simultaneous occurence of two events is equal to the probability of one of the events multiplied by the conditional probability of the others.

$$P(A \cap B) = P(A) \times P(B/A)$$

If A and B are independent events

$$P(A \cap B) = P(A) \times P(B)$$

Q.5. A Problem in mechanics is given to three students A, B, C whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. What is the probability that the problem will be solved.

Sol → The probabilities of A, B, C not solving the problem are $1-\frac{1}{2}, 1-\frac{1}{3}, 1-\frac{1}{4}$

$$= \frac{1}{2}, \frac{2}{3}, \frac{3}{4}$$

The probability that the problem is not solved by any of them is $\frac{1}{\cancel{2}} \times \frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{3}}{4} = \frac{1}{4}$

Hence, the probability that the problem is solved by atleast one of them $= 1 - \frac{1}{4} = \frac{3}{4}$.

Q→8. A has 2 shares in a lottery in which there are 3 prizes and 5 blanks. B has 3 shares in a lottery in which there are 4 prizes and 6 blanks. Show that A's chance of success is to B's as 27 : 35

Sol→ A can draw 2 tickets out of 8 in $^8C_2$ ways = 28 ways.
A will get blanks in $^5C_2$ ways = 10 ways.
∴ A can win a prize in 28 − 10 = 18 ways

$$A\text{'s chance of success} = \frac{18}{28} = \frac{9}{14}$$

B can draw 3 tickets out of 10 in $^{10}C_3 = \frac{10 \times \cancel{9}^{3} \times \cancel{8}^{4}}{\cancel{3} \times \cancel{2}} = 120$ ways

B will get blanks in $^6C_3 = \frac{6!}{3!\,3!} = \frac{6 \times 5 \times 4}{\cancel{3} \times \cancel{2}} = 20$ ways

∴ B can win a prize in 120 − 20 = 100 ways

$$B\text{'s chance of success} = \frac{10\cancel{0}}{12\cancel{0}} = \frac{5}{6}$$

$$A\text{'s chance} : B\text{'s chance} = \frac{9}{14} : \frac{5}{6}$$

$$= \frac{9}{\cancel{14}_{7}} \times \frac{\cancel{6}^{3}}{5} = \frac{27}{35}$$

Theoretical Probability Distributions

Discrete probability distribution → Binomial, Poisson, geometric, negative binomial, hypergeometric, multinomial, multivariate hypergeometric distributions.

Continuous probability distributions → Uniform, normal Gamma, exponential, χ^2, Beta, bivariate normal, t, F-distributions.

- Binomial Distribution
- Poisson's Distribution
- Normal Distribution

Probability function of Binomial

$$P(X=r) = {}^nC_r\, p^r q^{n-r}$$

for single trial, $p+q=1$

* n and p occuring in the binomial distribution are called 'parameters' of the distribution.

* In a binomial distribution -
  (i) n, the number of trials is finite
  (ii) each trial has only two possible outcomes usually called success and failure.
  (iii) all trials are independent.
  (iv) p and q is constant for all trials.

Recurrence relation for binomial distribution.

'Imp'

Q. Prove that $\boxed{P(r+1) = \frac{n-r}{r+1} \frac{p}{q} \cdot P(r)}$

$$P(r) = {}^nC_r\, p^r \cdot q^{n-r} = \frac{n!}{r!\,(n-r)!} p^r q^{n-r}$$

$$P(r+1) = {}^nC_{r+1} \cdot p^{r+1} q^{n-r-1} = \frac{n!}{(r+1)!\,(n-r-1)!} p^{r+1} q^{n-r-1}$$

$$\frac{P(r+1)}{P(r)} = \frac{n!}{(r+1)!\,(n-r-1)!} p^{r+1} \cdot q^{n-r-1} \times \frac{r!\,(n-r)!}{n!} \times \frac{1}{p^r q^{n-r}}$$

$$= \frac{(n-r)}{(r+1)} \times \frac{p}{q}$$

$$\boxed{P(r+1) = \frac{n-r}{r+1} \times \frac{p}{q} \cdot P(r)}$$

Mean $= np$

Variance $= npq$

M.G.F $= (q+pe^t)^n$

Moments about mean of Binomial Distribution

$$\mu_2 = npq = \sigma^2$$
$$\mu_3 = npq\,(p-q)$$

$$\beta_1 = \frac{(q-p)^2}{npq} = \frac{(1-2p)^2}{npq}, \quad \gamma_1 = \frac{1-2p}{\sqrt{npq}}$$

$$\beta_2 = 3 + \frac{1-6pq}{npq}, \quad \gamma_2 = \frac{1-6pq}{npq}$$

Q. If 10% of the bolts produced by a machine are defective, determine the probability that out of 10 bolts chosen at random
(i) 1 (ii) None (iii) at most 2 will be defective.

Sol: $p = \frac{10}{100} = \frac{1}{10}$ (defective)

$q = 1 - \frac{1}{10} = \frac{9}{10}$ (non-defective)

$n = 10$

(i) Here, $r = 1$

$$P(X=1) = {}^{10}C_1 \left(\frac{1}{10}\right)^1 \left(\frac{9}{10}\right)^9 = 10 \times \frac{1}{10} \times \left(\frac{9}{10}\right)^9 = 0.3874$$

(ii) Here, $r = 0$

$$P(0) = {}^{10}C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^{10} = \left(\frac{9}{10}\right)^{10} = 0.3486$$

(iii) $P(r \leq 2) = P(0) + P(1) + P(2)$

$$P(2) = {}^{10}C_2 \left(\frac{1}{10}\right)^2 \left(\frac{9}{10}\right)^8 = 45 \times \frac{1}{100} \times \left(\frac{9}{10}\right)^8 = 0.1937$$

$\therefore$ Required probability $= P(0) + P(1) + P(2)$
$= 0.3486 + 0.3874 + 0.1937$
$= 0.9297$.

Q. A binomial variable X satisfies the relation $9P(X=4) = P(X=2)$ when $n=6$. Find the value of the parameter p and $P(X=1)$.

Sol. $P(X=r) = {}^nC_r\, p^r q^{n-r}$

$n=6$

$9P(X=4) = P(X=2)$

$9 \times \cancel{{}^6C_4}\, p^4 \cdot q^2 = \cancel{{}^6C_2} \cdot p^2 \cdot q^4$

$9p^2 = q^2$

$9p^2 = (1-p)^2$

$9p^2 = 1 + p^2 - 2p$

$8p^2 + 2p - 1 = 0$

$8p^2 + 4p - 2p - 1 = 0$

$4p(2p+1) - (2p+1) = 0$

$(2p+1)(4p-1) = 0$

$p = -\frac{1}{2}, \frac{1}{4}$

$\because$ p cannot be negative

So, $\boxed{p = \frac{1}{4}}$

$P(X=1) = {}^6C_1 \cdot \left(\frac{1}{4}\right)^1 \cdot \left(\frac{3}{4}\right)^5 = 0.3559$

Q Fit a binomial distribution to the following frequency data:

x:	0	1	2	3	4
f:	30	62	46	10	2

Sol:

x	f	fx
0	30	0
1	62	62
2	46	92
3	10	30
4	2	8
	$\Sigma f = 150$	$\Sigma fx = 192$

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{192}{150} = 1.28$$

$$np = 1.28$$

$$4p = 1.28 \qquad (n = 4 \text{ ; 0 is not taken})$$

$$p = \frac{1.28}{4} = 0.32$$

$$q = 1 - p = 1 - 0.32 = 0.68$$

$$N = 150$$

$\therefore$ binomial distribution is $N(q+p)^n = 150(0.68 + 0.32)^4$

Poisson Distribution

If the parameters n and p of the binomial distribution are known, we can find the distribution in terms of poisson distribution which is limiting case of binomial distribution (n is very large as compare to p means p is very small).

If we assume $n \to \infty$ and $p \to 0$ such that np always remains finite say λ (mean of poisson distribution).

Probability function of Poisson distribution

$$P(X=r) = \sum \frac{e^{-\lambda} \cdot \lambda^r}{r!} \quad (r = 0, 1, 2, 3, \ldots)$$

where λ is a finite no $= np$

It is called poisson probability distribution.

Mean, Variance and MGF of Poisson distribution.

Mean = Variance = $np = \lambda$.

$$M.G.F = M_x(t) = e^{\lambda(e^t - 1)}$$

Recurrence relation for the poisson distribution

Imp $\boxed{P(r+1) = \frac{\lambda}{r+1} P(r)}$, $r = 0, 1, 2, 3, \ldots$

Proof → $P(r) = \frac{e^{-\lambda} \cdot \lambda^{r}}{r!}$, $P(r+1) = \frac{e^{-\lambda} \cdot \lambda^{r+1}}{(r+1)!}$

$$\frac{P(r+1)}{P(r)} = \frac{e^{-\lambda} \cdot \lambda^{r+1}}{(r+1)!} \times \frac{r!}{e^{-\lambda} \cdot \lambda^{r}}$$

$$\boxed{P(r+1) = \frac{\lambda}{r+1} P(r)}$$

Hence, Proved

$\beta_1 = \frac{1}{\lambda}$, $\beta_2 = 3 + \frac{1}{\lambda}$

$\gamma_1 = \frac{1}{\sqrt{\lambda}}$, $\gamma_2 = \frac{1}{\lambda}$

Mode of Poisson Distribution

* If λ is a positive integer, there are two modes $\lambda - 1$ and λ.
* If λ is not a positive integer, there is one mode and is the integral value between $\lambda - 1$ and λ.

Q.1 If the variance of the Poisson distribution is 2, find the probabilities for $r = 1, 2, 3, 4$ from the recurrence relation of the Poisson distribution. Also find $P(r \geq 4)$.

Sol. variance $= \lambda = 2$

$$P(r+1) = \frac{\lambda}{r+1} P(r) = \frac{2}{r+1} P(r)$$

$$P(r) = \frac{e^{-\lambda} \cdot \lambda^r}{r!} \Rightarrow P(0) = \frac{e^{-2} \cdot (2)^0}{0!} = e^{-2} = 0.1353$$

$$P(1) = \frac{2}{1} \times P(0) = 2 \times 0.1353 = 0.2706$$

$$P(2) = \frac{2}{2} P(1) = 0.2706$$

$$P(3) = \frac{2}{3} P(2) = \cancel{0.66}\ \frac{2}{3} \times 0.2706 = 0.1804$$

$$P(4) = \frac{2}{4} P(3) = \frac{1}{2} \times 0.1804 = 0.0902.$$

$$P(r \geq 4) = 1 - [P(0) + P(1) + P(2) + P(3)]$$

$$= 1 - [0.1353 + 0.2706 + 0.2706 + 0.1804]$$

$$= 1 - 0.8569 = 0.1431$$

Q2 (i) Fit a poisson distribution to the following data and calculate theoretical frequencies.

Deaths :	0	1	2	3	4
Frequencies :	122	60	15	2	1

Sol → Mean $= \frac{\Sigma fx}{\Sigma f} = \frac{60+30+6+4}{200} = 0.5$

Required Poisson distribution $= N \cdot \frac{e^{-\lambda} \cdot \lambda^{r}}{r!} = 200 \cdot \frac{e^{-0.5} \cdot (0.5)^{r}}{r!}$

r	$N \cdot P(r)$	Theoretical frequency
0	$121.306 \times \frac{(0.5)^0}{0!} = 121.306$	121
1	$121.306 \times \frac{(0.5)^1}{1!} = 60.653$	61
2	$121.306 \times \frac{(0.5)^2}{2!} = 15.163$	15
3	$121.306 \times \frac{(0.5)^3}{3!} = 2.527$	3
4	$121.306 \times \frac{(0.5)^4}{4!} = 0.3159$	0

Total = 200

(ii) The frequency of accidents per shift in a factory is shown in the following table:

Accident per shift	Frequency
0	192
1	100
2	24
3	3
4	1
Total	320

Calculate the mean number of accidents per shift. Fit a poisson distribution and calculate theoretical frequencies.

Sol: Mean $= \lambda = \frac{\Sigma fx}{\Sigma f} = \frac{100+48+9+4}{320} = 0.5031$

$\therefore$ Required Poisson distribution

$= N \cdot \frac{e^{-\lambda} \cdot \lambda^{r}}{r!} = 320 \cdot \frac{e^{-0.5031} \cdot (0.5031)^{r}}{r!} = \frac{(193.48)(0.5031)^{r}}{r!}$

r	$N \cdot P(r)$	Theoretical frequency
0	$193.48 \cdot \frac{(0.5031)^0}{0!} = 193.48$	194
1	$193.48 \cdot \frac{(0.5031)^1}{1!} = 97.34$	97
2	$193.48 \cdot \frac{(0.5031)^2}{2!} = 24.38$	24
3	$193.48 \cdot \frac{(0.5031)^3}{3!} = 4.10$	4
4	$193.48 \cdot \frac{(0.5031)^4}{4!} = 0.51$	1
		Total = 320

Q3 A manufacturer knows that the condensors he makes contain on an average 1% of defectives. He packs them in boxes of 100. What is the probability that a box picked at random will contain 4 or more faulty condensors?

Sol: $n = 100$, $p = \frac{1}{100} = 0.01$

p is very small in comparision to n so, poisson probability distribution will be applied.

$\lambda = np = 1$

$$P(r) = \frac{e^{-\lambda} \cdot \lambda^r}{r!} = \frac{e^{-1}}{r!}$$

$$P(r \geq 4) = 1 - [P(0) + P(1) + P(2) + P(3)]$$

$$= 1 - \left[\frac{e^{-1}}{0!} + \frac{e^{-1}}{1!} + \frac{e^{-1}}{2!} + \frac{e^{-1}}{3!}\right]$$

$$= 1 - \left[\frac{6e^{-1} + 6e^{-1} + 3e^{-1} + e^{-1}}{6}\right]$$

$$= 1 - \frac{16e^{-1}}{6}$$

$$= 1 - \frac{8}{3}e^{-1} = 1 - 0.981 = 0.019$$

Q.4 (i) If the probabilities of a bad reaction from a certain injection is 0.0002. determine the chance that out of 1000 individuals more than two will get a bad reaction.

$n = 1000$, $p = 0.0002$

Poisson probability distribution,

$\lambda = np = 0.2$

$$P(r) = \frac{e^{-\lambda} \cdot \lambda^r}{r!} = \frac{e^{-0.2} \cdot (0.2)^r}{r!}$$

$$P(r > 2) = 1 - [P(r \leq 2)] = 1 - [P(0) + P(1) + P(2)]$$

$$= 1 - \left[\frac{e^{-0.2} \cdot (0.2)^0}{0!} + \frac{e^{-0.2} \cdot (0.2)^1}{1!} + \frac{e^{-0.2} \cdot (0.2)^2}{2!}\right]$$

$$= 1 - [0.8187 + 0.1637 + 0.01637] = 0.0012$$

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(ii) The probability that a man aged 50 years will die within a year is 0.01125. What is the probability that of 12 such men, at least 11 will reach their 51st birthday?.
(Given: $e^{-.135} = 0.87366$)

Sol→ $n = 12, \quad p = 0.01125$

$\lambda = 12 \times 0.01125 = 0.135$

P(at least 11 should stay live) =

P (at most 1 die) $= P(0) + P(1)$

$$= \frac{e^{-0.135} \cdot (0.135)^0}{0!} + \frac{e^{-0.135} \cdot (0.135)^1}{1!}$$

$$= 0.87366 + 0.11794$$

$$= 0.9916$$

Continuous Distribution → given interval or range

Probability Density function-

let the probability of the variate x falling in the infinitesimal $\left(x - \frac{1}{2}dx, x + \frac{1}{2}dx\right)$ be expressed in the form

$f(x)\,dx$, where $f(x)$ is a continuous function of x.
Then, $f(x)$ is called probability density function.

$$P\left(x - \frac{1}{2}dx \le X \le x + \frac{1}{2}dx\right) = f(x)\,dx$$

$f(x) = 0, \quad x < a$

$f(x) = \phi(x), \quad a \le x \le b$

$f(x) = 0, \quad x > b$

Imp

The density function possesses the following two properties:-

* $f(x) \geq 0$ for every x, as negative probability has no meaning.

* $\int_{-\infty}^{\infty} f(x)\,dx = 1$, this corresponds to the fact that the probability of an event that is sure to happen is equal to unity.

Q If the function $f(x)$ is defined by $f(x) = ce^{-x}$, $0 \leq x \leq \infty$ find the value of c which changes $f(x)$ to a probability density function.

Sol: In order that $f(x)$ may be density function, we should have

a) $f(x) \geq 0$ for every x b) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

Since e^{-x} is always positive for value of x lying between 0 and ∞, the condition will be satisfies iff. $c \geq 0$.

The second condition will be satisfied ~~to~~ if

$$\int_0^{\infty} ce^{-x}\,dx = 1$$

$$\left[-ce^{-x}\right]_0^{\infty} = 1$$

$$\left[-c \cdot e^{-\infty} + ce^{-0}\right] = 1$$

$$0 + c = 1$$

$$\Rightarrow c = 1.$$

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Q If $f(x)$ has probability density cx^2, $0<x<1$, determine c and find the probability that $\frac{1}{3}<x<\frac{1}{2}$

i.e. $P\left(\frac{1}{3}<x<\frac{1}{2}\right)$

Sol: $f(x)$ will have a probability density if $\int_0^1 cx^2 dx = 1$

$$\left[\frac{cx^3}{3}\right]_0^1 = 1 \quad = \quad \frac{c}{3} - 0 = 1$$

$$\boxed{c=3}$$

$$P\left(\frac{1}{3}<x<\frac{1}{2}\right) = \int_{\frac{1}{3}}^{\frac{1}{2}} 3x^2 dx$$

$$= \left[x^3\right]_{\frac{1}{3}}^{\frac{1}{2}} = \left(\frac{1}{8} - \frac{1}{27}\right) = \frac{19}{216}$$

Conditional Probability

The probability of the happening of an event E_1 when another event E_2 is known to have already happened is called Conditional probability and is denoted by $P(E_1/E_2)$.

* If E_1 and E_2 are mutually independent events.
$\Rightarrow P(E_1/E_2) = P(E_1)$

Multiplicative Law of Probability

The probability of simultaneous occurence of two events is equal to the probability of one of the events multiplied by the conditional probability of the other,

$$P(A \cap B) = P(A) \times P(B/A)$$

Q. An urn contains 10 white and 3 black balls, while another urn contains 3 white and 5 black balls. Two balls are drawn from the first urn and put into the second urn and then a ball is drawn from the latter. What is the probability that it is a white ball?

Solⁿ→ The two balls drawn from the first urn may be (i) both white (ii) both black (iii) one white and one black.

Let these events be denoted by A, B and C respectively.

$$P(A) = \frac{^{10}C_2}{^{13}C_2} = \frac{10 \times 9}{13 \times 12} = \frac{15}{26}$$

$$P(B) = \frac{^{3}C_2}{^{13}C_2} = \frac{3 \times 2}{13 \times 12} = \frac{1}{26}$$

$$P(C) = \frac{^{10}C_1 \cdot {}^{3}C_1}{^{13}C_2} = \frac{10 \times 3}{\frac{13 \times 12}{2}} = \frac{10}{26}$$

When two balls are transferred from first urn to second urn, the second urn will contain

(i) 5 white and 5 black balls (ii) 3 white and 7 black balls
(iii) 4 white and 6 black balls

Let W denote the event of drawing a white ball from the second urn in the three cases (i), (ii) and (iii).

$$P(W/A) = \frac{5}{10}, \quad P(W/B) = \frac{3}{10}$$

$$P(W/C) = \frac{4}{10}$$

$\therefore$ Required probability $= P(A) \cdot P(W/A) + P(B) \cdot P(W/B) + P(C) \cdot P(W/C)$

$$= \frac{15}{26} \times \frac{5}{10} + \frac{1}{26} \times \frac{3}{10} + \frac{10}{26} \times \frac{4}{10}$$

$$= \frac{75 + 3 + 40}{260} = \frac{118}{260} = \frac{59}{130}$$

Baye's Theorem

If $E_1, E_2, \ldots, E_n$ are mutually exclusive and exhaustive events with $P(E_i) \neq 0$, $(i = 1, 2, \ldots, n)$ of a random experiment then for any arbitrary event A of the sample space of the above experiment with $P(A) > 0$, we have.

$$P(E_i/A) = \frac{P(E_i)\, P(A/E_i)}{\sum_{i=1}^{n} P(E_i) P(A/E_i)}$$

Q. A bag X contains 2 white and 3 red balls and bag Y contains 4 white and 5 red balls. One ball is drawn at ~~fo~~ random from one of the bags and is found to be red. Find the probability that it was drawn from bag Y.

Sol^n → Let E_1 : the ball is drawn from bag X

E_2 : the ball is drawn from bag Y.

A : the ball is red

We have to find ~~$P(A/E_2)$~~ $P(E_2/A)$, by Baye's theorem

$$P(E_2/A) = \frac{P(E_2) \cdot P(A/E_2)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)}$$

$$P(E_1) = P(E_2) = \frac{1}{2}$$

$$P(A/E_1) = \frac{3}{5}, \qquad P(A/E_2) = \frac{5}{9}$$

$$P(E_2/A) = \frac{\frac{1}{2} \times \frac{5}{9}}{\frac{1}{2} \times \frac{3}{5} + \frac{1}{2} \times \frac{5}{9}}$$

$$= \frac{5/18}{\frac{3}{10} + \frac{5}{18}} = \frac{5/18}{\frac{27+25}{90}}$$

$$= \frac{5/18}{52/90_5} = \frac{25}{52}$$

Q. In a bolt factory, machines A, B and C manufacture respectively 25%, 35% and 40% of the total. Of their output 5, 4 and 2 percent are defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by machine B?

Sol: Let E_1, E_2 and E_3 denote the events that the bolt selected at random is manufacture by the machines A, B and C respectively and let H denote the event of its being defective.

$P(E_1) = 0.25$, $P(E_2) = 0.35$, $P(E_3) = 0.40$

$P(H/E_1) = 0.05$, $P(H/E_2) = 0.04$, $P(H/E_3) = 0.02$

By Baye's theorem,

$$P(E_2/H) = \frac{P(E_2)P(H/E_2)}{P(E_1).P(H/E_1) + P(E_2).P(H/E_2) + P(E_3).P(H/E_3)}$$

$$= \frac{0.35 \times 0.04}{0.25 \times 0.05 + 0.35 \times 0.04 + 0.40 \times 0.02}$$

$$= \frac{0.0140}{0.0345} = 0.41$$

Q. The contents of urn I, II and III are as follows:
1 white, 2 black and 3 red balls,
2 white, 1 black and 1 red balls, and
4 white, 5 black and 3 red balls
One urn is choosen at random and two balls drawn. They happen to be white and red. What is the probability that they come from urns I, II or III?

Sol: Let E_1: urn I is chosen,
E_2: urn II is chosen;
E_3: urn III is chosen.
A: the two balls chosen are white and red.

$$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

$$P(A/E_1) = \frac{{}^1C_1 \times {}^3C_1}{{}^6C_2} = \frac{1 \times 3}{15} = \frac{1}{5}$$

$$P(A/E_2) = \frac{{}^2C_1 \times {}^1C_1}{{}^4C_2} = \frac{2 \times 1}{6} = \frac{1}{3}$$

$$P(A/E_3) = \frac{{}^4C_1 \times {}^3C_1}{{}^{12}C_2} = \frac{4 \times 3}{6 \times 11} = \frac{2}{11}$$

$$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) + P(E_3) \cdot P(A/E_3)}$$

$$= \frac{\frac{1}{3}\times\frac{1}{5}}{\frac{1}{3}\times\frac{1}{5} + \frac{1}{3}\times\frac{1}{3} + \frac{1}{3}\times\frac{2}{11}} = \frac{\frac{1}{15}}{\frac{1}{15}+\frac{1}{9}+\frac{2}{33}}$$

$$= \frac{33}{118}$$

$$P(E_2/A) = \frac{\frac{1}{3}\times\frac{1}{3}}{\frac{1}{15}+\frac{1}{9}+\frac{2}{33}} = \frac{55}{118}$$

$$P(E_3/A) = \frac{\frac{1}{3}\times\frac{2}{11}}{\frac{1}{15}+\frac{1}{9}+\frac{2}{33}} = \frac{15}{59}$$

Normal Distribution

The normal distribution is a continuous distribution. It can be derived from the binomial distribution in the limiting case when n, the number of trials is very large and p, the probability of a success, is close to $\frac{1}{2}$.

The general eq^n of the normal distribution is given by

$$\boxed{f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}$$

If a variable x has the normal distribution with mean μ and standard deviation σ, we briefly write

$x : N(\mu, \sigma^2)$

The graph of the normal distribution is called the normal curve.

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In the normal distribution mean, median and mode coincide each other.
The total area under the normal curve above the x-axis is 1.

Basic Properties of the Normal Distribution

The probability density function of the normal distribution is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

(i) $f(x) \geq 0$ (ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

(iii) The normal distribution is symmetrical about its mean.
(iv) It is a unimodal distribution. The mean, mode and median of this distribution coincide.

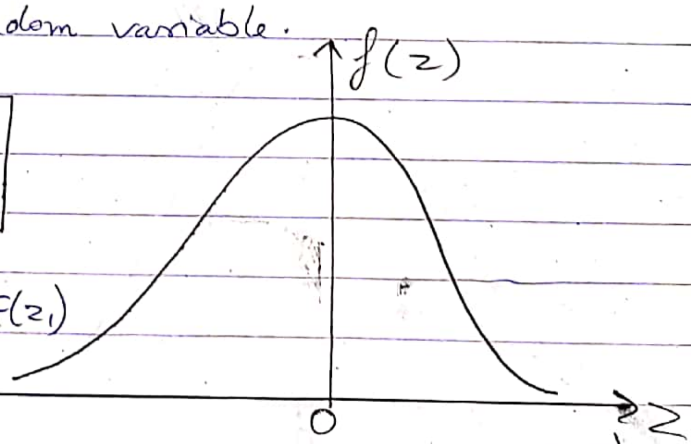
Standard form of the normal distribution.

If X is a normal random variable with mean μ and standard deviation σ, then the random variable $Z = \frac{X-\mu}{\sigma}$ has the normal distribution with mean 0 and standard deviation 1.

$Z \rightarrow$ Standard normal random variable.

Probability density function $\rightarrow$

$$\boxed{f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}}$$



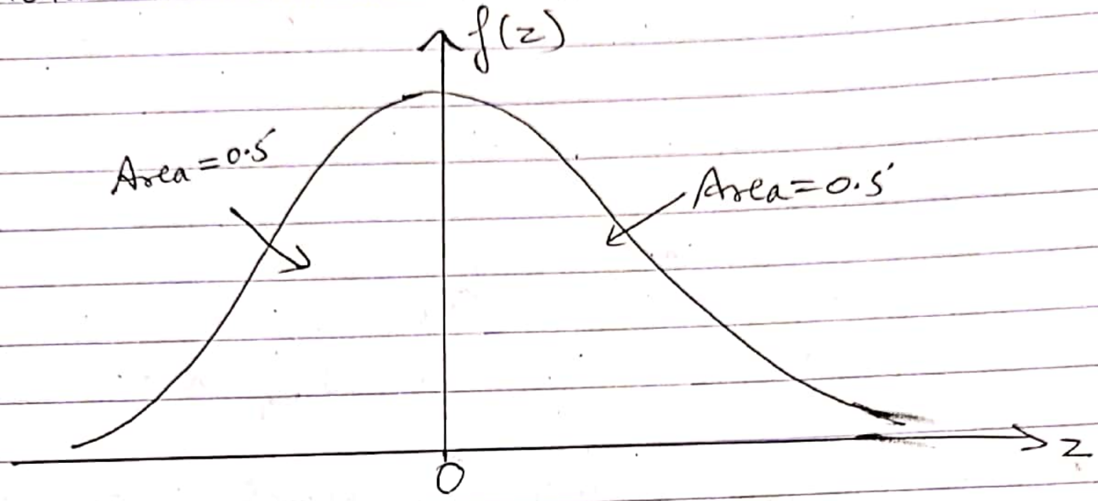
$(-\infty < z < \infty)$

$$P(z_1 < Z < z_2) = \int_{z_1}^{z_2} f(z)\,dz = F(z_2) - F(z_1)$$

Area under the normal curve

By taking $z = \frac{x-\mu}{\sigma}$, standard normal curve is formed.

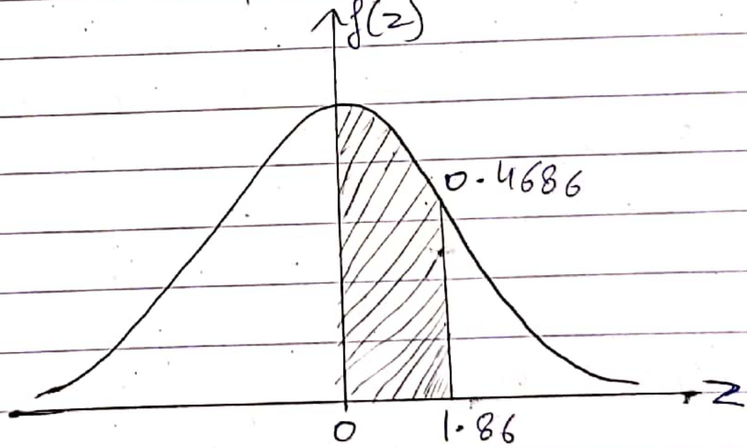
The total area under this curve is 1.



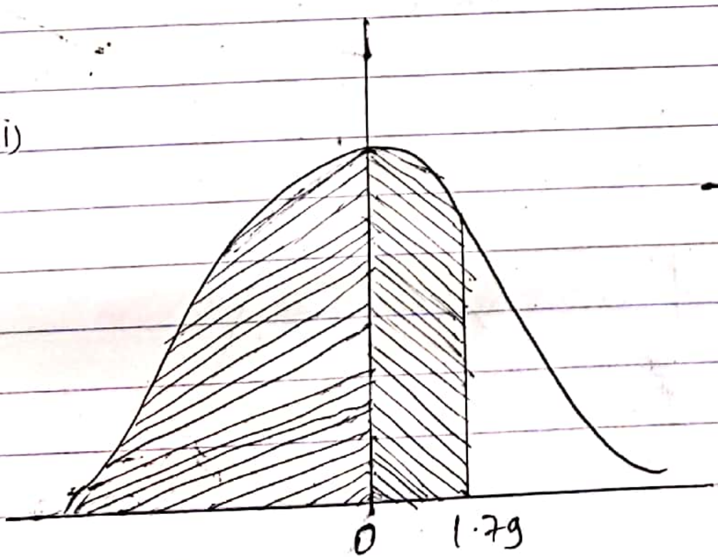
Q. Find the area under the normal curve (i) between $z=0$ and $z=1.86$ (ii) area to the left of $z=1.79$ and (iii) area between $z=-0.5$ and $z=0.8$.

Sol: (i) $P(0 \le z \le 1.86)$

$= 0.4686$



(ii)

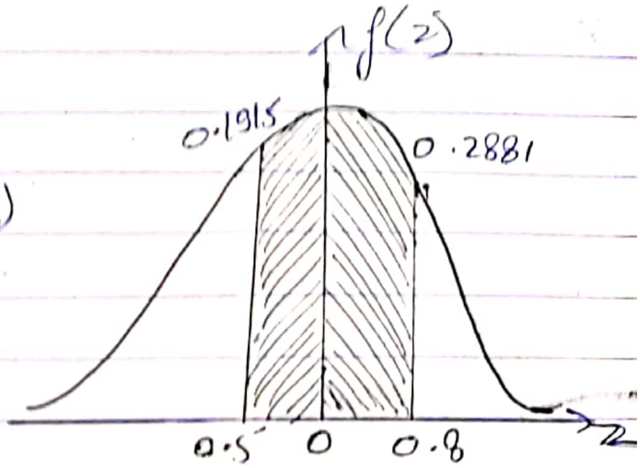


$P(-\infty < z < 1.79)$

$= P(-\infty < z < 0) + P(0 < z < 1.79)$

$= 0.5 + 0.4633 = 0.9633$

(iii) $z = -0.5$ and $z = 0.8$



$P(-0.5 < z < 0.8)$

$= P(-0.5 < z < 0) + P(0 < z < 0.8)$

$= 0.1915 + 0.2881$

$= 0.4796$

Q. A sample of 100 dry battery cells tested to find the length of life produced the following results:

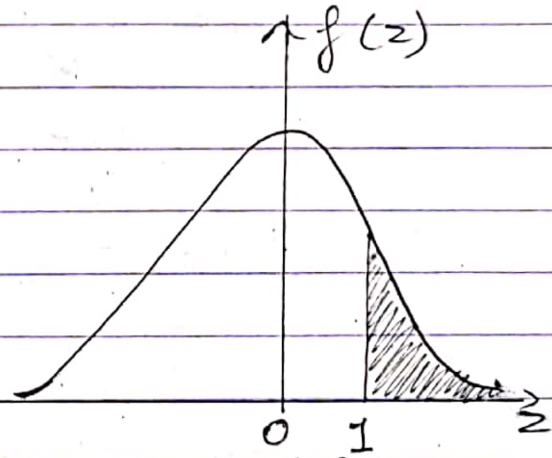
$\bar{x} = 12$ hrs, $\sigma = 3$ hrs

Assume the data to be normally distributed, what percentage of battery cells are expected to have life.

(i) more than 15 hrs (ii) less than 6 hrs (iii) between 10 and 14 hrs?

Sol: x denotes the length of life of dry battery cells.

$$z = \frac{x - \bar{x}}{\sigma} = \frac{x - 12}{3}$$



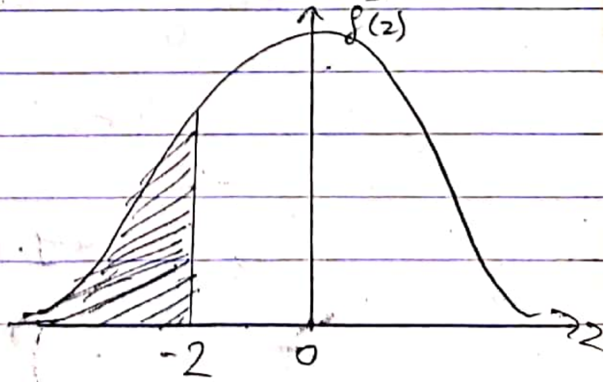
(i) when $x = 15$, $z = 1$

$\therefore P(x > 15) = P(z > 1)$

$= P(0 < z < \infty) - P(0 < z < 1)$

$= 0.5 - 0.3413$

$= 0.1587 = 15.87\%$



(ii) when $x = 6$, $z = -2$

$\therefore P(x < 6) = P(z < -2)$

$= P(-\infty < z < 0) - P(-2 < z < 0)$

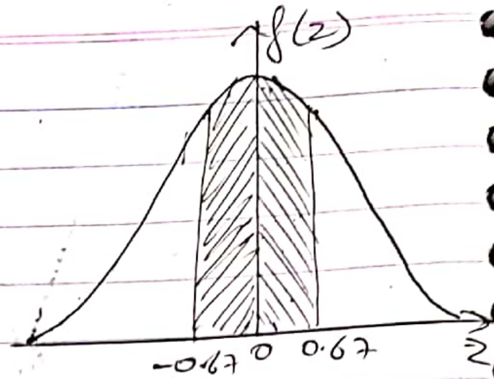
$= 0.5 - 0.4772$

$= 0.0228 = 2.28\%$

(iii) When $x = 10$, $z = -\frac{2}{3} = -0.67$

when $x = 14$, $z = \frac{2}{3} = 0.67$

$P(10 < x < 14) = P(-0.67 < z < 0.67)$



$= 2 \times 0.2485 = 0.4970$

$= 49.70\%$.

Q. In a sample of 1000 cases, the mean of a certain test is 14 and S.D. is 2.5. Assuming the distribution to be normal, find
(i) how many students score between 12 and 15?
(ii) how many score above 18?
(iii) how many score below 8?
(iv) how many score 16?

Sol: (i) $z_1 = \frac{x_1 - \bar{x}}{\sigma} = \frac{12 - 14}{2.5} = -0.8$

$z_2 = \frac{x_2 - \bar{x}}{\sigma} = \frac{15 - 14}{2.5} = 0.4$

$P(-0.8 < z < 0.4) = P(-0.8 < z < 0) + P(0 < z < 0.4)$

$= 0.2881 + 0.1554$

$= 0.4435$

Req. no. of students $= 1000 \times 0.4435 = 444$ (app.)

(ii) $z = \frac{18 - 14}{2.5} = 1.6$

$P(z > 1.6)$ ~~$= P(-1.6 < z < 0) + P(0 < z < \infty)$~~

$0.5 - P(0 < z < 1.6) = 0.5 - 0.4452 = 0.0548$

Req. no. of students $= 1000 \times 0.0548 = 55$ (app.)

(iii) $z = \frac{8-14}{2.5} = -2.4$

$P(z < -2.4)$

$= 0.5 - P(-2.4 < z < 0)$

$= 0.5 - 0.4918$

$= 0.0082$

Req. no. of students $= 1000 \times 0.0082 = 8$ (app.)

(iv) $z_1 = \frac{15.5 - 14}{2.5} = 0.6$

$z_2 = \frac{16.5 - 14}{2.5} = 1$

Area between 0.6 and 1 $= P(0.6 < z < 1)$

$= P(0 < z < 1) - P(0 < z < 0.6)$

$= 0.3413 - 0.2257 = 0.1157$

Req. no. of students $= 1000 \times 0.1157 = 116$ (app.)

Q. Assume mean height of soldiers to be 68.22 inches with a variance of 10.8 inches square. How many soldiers in a regiment of 1000 would you expect to be over 6 feet tall, given that the area under the standard normal curve between $z=0$ and $z=0.35$ is 0.1368 and between $z=0$ and $z=1.15$ is 0.3746.

Sol> $x = 6$ feets $= 72$ inches

$z = \frac{x - \bar{x}}{\sigma} = \frac{72 - 68.22}{\sqrt{10.8}} = 1.15$

$P(x > 72) = P(z > 1.15) = 0.5 - P(0 < z < 1.15)$

$= 0.5 - 0.3746 = 0.1254$

$\therefore$ Expected no. of soldiers $= 1000 \times 0.1254 = 125$ (app.)

Q. A large number of measurement is normally distributed with a mean 65.5" and SD of 6.2". Find the percentage of measurements that fall between 54.8" and 68.8".

Sol: Mean = 65.5, $\sigma = 6.2$

$x_1 = 54.8$, $x_2 = 68.8$

$$z_1 = \frac{x_1 - \bar{x}}{\sigma} = \frac{54.8 - 65.5}{6.2} = -1.73$$

$$z_2 = \frac{x_2 - \bar{x}}{\sigma} = \frac{68.8 - 65.5}{6.2} = 0.53$$

$$P(-1.73 \le z \le 0.53) = P(-1.73 \le z < 0) + P(0 < z < 0.53)$$

$$= 0.4582 + 0.2019 = 0.6601$$

$\therefore$ Req. percentage of meas. = 66.01%

Q. In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution. It is given that if $f(t) = \frac{1}{\sqrt{2\pi}} \int_0^t e^{-\frac{1}{2}x^2} dx$ then $f(0.5) = 0.19$ and $f(1.4) = 0.42$.

Sol: let $\bar{x}$ and σ be the mean and S.D.

$P(x < 45) = 0.31$, $P(x > 64) = 0.08$

$$P(x < 45) = 0.5 + P(0 < x < 45) = 0.31$$

$$P(0 < x < 45) = 0.31 - 0.5$$

$$= -0.19 = f(-0.5)$$

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$P(z < 0.5) = 0.19 \Rightarrow z = -0.5$

$$-0.5 = \frac{45 - \bar{x}}{\sigma}$$

$$\bar{x} = 45 + 0.5\sigma \quad -(1)$$

$P(x > 64) = 0.08$

$P(x > 64) = 0.5 - P(0 < x < 64) = 0.08$

$P(0 < x < 64) = 0.5 - 0.08$

$= 0.42 = f(1.4)$

$x = 64 \Rightarrow z = 1.4$

$$1.4 = \frac{64 - \bar{x}}{\sigma}$$

$$\bar{x} = 64 - 1.4\sigma \quad -(2)$$

from (1) and (2);

$45 + 0.5\sigma = 64 - 1.4\sigma$

$1.9\sigma = 19$

$\sigma = 10$

$\bar{x} = 45 + 5 = 50$

$\boxed{\sigma = 10}$, $\boxed{\bar{x} = 50}$

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Various Measures for continuous probability distributions -

Let $f_X(x)$ or $f(x)$ be the p.d.f of a random variable X where X is defined from a to b. Then,

(i) Arithmetic mean $= \int_a^b x f(x)\,dx$

(ii) Harmonic mean $\Rightarrow \frac{1}{H} = \int_a^b \frac{1}{x} \cdot f(x)\,dx$

(iii) Geometric mean $\Rightarrow \log G = \int_a^b \log x \cdot f(x)\,dx$

(iv) v_r (about origin) $= \int_a^b x^r \cdot f(x)\,dx$

μ_r' (about the point $x = A$) $= \int_a^b (x-A)^r \cdot f(x)\,dx$

and μ_r (about mean) $= \int_a^b (x - \text{mean})^r \cdot f(x)\,dx$

v_1 (about origin) $=$ Mean $= \int_a^b x f(x)\,dx$ and $v_2 = \int_a^b x^2 f(x)\,dx$

Hence, $\mu_2 = (v_2 - v_1^2) = \int_a^b x^2 f(x)\,dx - \left(\int_a^b x f(x)\,dx\right)^2$

(v) Median $\rightarrow \int_a^M f(x)\,dx = \int_M^b f(x)\,dx = \frac{1}{2}$

(vi) Mean deviation $\rightarrow$ M.D. $= \int_a^b |x - \text{mean}|\, f(x)\,dx$

(vii) Mode $\rightarrow$ Mode is the value of x for which $f(x)$ is max.

$f'(x) = 0$ and $f''(x) < 0$.

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Cumulative Distribution Function

If $F(x) = \int_{-\infty}^{x} f(x)\,dx = P(X \le x)$, then the function $F(x)$ is the probability that the value of the variate X will be $\le x$.

$$F(b) = P(X \le b)$$
$$F(b) - F(a) = \int_a^b f(x)\,dx = P(a \le X \le b)$$

and

$F(x)$ is called the cumulative distribution function of x or simply the distribution function.

The cumulative distribution function has the following properties:

(1.) $F'(x) = f(x) \ge 0$, so that $F(x)$ is a non decreasing function. This means that $dF(x) = f(x)\,dx$.
This is known as probability differential of X.

(2.) $F(-\infty) = 0$

(3.) $F(\infty) = 1$

(4.) $F(x)$ is a continuous function of x on the right.

distribution function $= \int$ density function

density function $= \frac{d}{dx}$ (distribution function)

Q. The distribution function of a random variable X is given by

$$F(x) = \begin{cases} 1 - (1+x)e^{-x}, & \text{for } x \ge 0 \\ 0, & \text{for } x < 0 \end{cases}$$

find the corresponding density function of random variable X.

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Sol→ The probability density function $= f(x) = \frac{d}{dx} F(x)$

$$f(x) = -e^{-x} + (1+x) \cdot e^{-x}$$
$$= \begin{cases} xe^{-x}, & \text{for } x \geq 0 \\ 0, & \text{for } x < 0. \end{cases}$$

Q. A random variable x has the density function
$$f(x) = k \cdot \frac{1}{1+x^2}, \quad -\infty, x < \infty$$
Determine k and the distribution function.

Sol→ It will be a density function if

$$\int_{-\infty}^{\infty} k \cdot \frac{1}{1+x^2} = 1$$
$$k \cdot \left[\tan^{-1} x\right]_{-\infty}^{\infty} = 1$$
$$k \cdot \left[\frac{\pi}{2} - \left(\frac{-\pi}{2}\right)\right] = 1$$
$$k \cdot \pi = 1 \Rightarrow \boxed{k = \frac{1}{\pi}}$$

$$F(x) = \int \frac{1}{\pi} \cdot \frac{1}{1+x^2} dx = \frac{1}{\pi} \cdot \tan^{-1} x + c$$

But $F(-\infty)$ should be zero for distribution function

$$\therefore \frac{1}{\pi} \cdot \left(\frac{-\pi}{2}\right) + c = 0 \Rightarrow c = \frac{1}{2}$$

$$F(x) = \frac{1}{\pi} \cdot \tan^{-1} x + \frac{1}{2} \quad \text{for } -\infty < x < \infty$$

Mathematical Expectation or Expected Value of a random variable

The expected value of a discrete random variable is a weighted average of all possible values of the random variable, where the weights are the probabilities associated with the corresponding values.

If x, denotes a discrete random variable which can assume the values $x_1, x_2, \dots, x_n$ with respective probabilities $p_1, p_2, \dots, p_n$ where $p_1+p_2+\dots+p_n = 1$, the mathematical expectation of x or simply the expectation of X, denoted by $E(x)$, is defined as

$$E(x) = p_1x_1 + p_2x_2 + \dots + p_nx_n = \sum_{i=1}^{n} p_ix_i = \sum px, \quad \sum p = 1$$

$$E[\phi(x)] = p_1\phi(x_1) + p_2\phi(x_2) + \dots + p_n\phi(x_n), \quad \sum p = 1$$

If $\phi(x) = x^r$, then

$$E(x^r) = p_1x_1^r + p_2x_2^r + \dots + p_nx_n^r$$

Here, if p_i is replaced by $\frac{f_i}{N}$ where $\sum f_i = N$, then

$$E(x) = \frac{\sum fx}{N}, \text{ which is the mean.}$$

$$M_2 = E[\{x - E(x)\}^2] = E(x^2) - [E(x)]^2$$

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Laws of Expectation

1. If c is a constant the $E(c) = c$
2. If a is a constant, then $E(ax) = aE(X)$
3. $E(X+Y) = E(X) + E(Y)$

   (i) $E\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} E(X_i)$

   (ii) $E(aX+b) = aE(X) + b$

4. $E(XY) = E(X) \cdot E(Y)$

5. $E(X-Y) = E(X) - E(Y)$

Q.1. What is the expected value of the number of points that will be obtained in a single throw with an ordinary die? Find variance also.

Sol^n → Variates $= 1, 2, 3, 4, 5, 6$.

Probability of each variate $= \frac{1}{6}$

$$E(x) = p_1x_1 + p_2x_2 + p_3x_3 + \cdots + p_6x_6$$

$$= \frac{1}{6}\times 1 + \frac{1}{6}\times 2 + \frac{1}{6}\times 3 + \frac{1}{6}\times 4 + \frac{1}{6}\times 5 + \frac{1}{6}\times 6$$

$$= \frac{21}{6} = 3.5$$

$$var(x) = E(x^2) - [E(x)]^2 = \frac{1}{6}\left[1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2\right] - \left(\frac{7}{2}\right)^2$$

$$= \frac{35}{12}$$

Q.2. Thirteen cards are drawn simultaneously from a deck of 52. If aces count 1, face cards 10 and other according to denomination, find the expectation of the total score on the 13 cards.

Solⁿ → $x_i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 10, 10, 10$

probability $(p_i) = \frac{1}{13}$

$$E(x_i) = \frac{1}{13}[1+2+3+4+5+6+7+8+9+10+10+10+10]$$

$$= \frac{85}{13}$$

Q.3. In four tosses of a coin, let x be the number of heads. Calculate the expected values of x.

Solⁿ → $P(x=4) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$ (All heads)

$$P(x=3) = \frac{^4C_3}{16} = \frac{4}{16} = \frac{1}{4}$$

$$P(x=2) = \frac{^4C_2}{16} = \frac{6}{16} = \frac{3}{8}$$

$$P(x=1) = \frac{^4C_1}{16} = \frac{4}{16} = \frac{1}{4}$$

$$P(x=0) = \frac{^4C_0}{16} = \frac{1}{16}$$

$$E(x) = 0 \times \frac{1}{16} + 1 \times \frac{1}{4} + 2 \times \frac{3}{8} + 3 \times \frac{1}{4} + 4 \times \frac{1}{16} = 2$$

Q.4. Find $E(x)$, $E(x^2)$, $E\{(x-\bar{x})^2\}$ for the following probability distributions:

x :	8	12	16	20	24
$p(x)$:	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{12}$

Solⁿ → $E(x) = 8\times\frac{1}{8} + 12\times\frac{1}{6} + 16\times\frac{3}{8} + 20\times\frac{1}{4} + 24\times\frac{1}{12}$

$= 1 + 2 + 6 + 5 + 2 = 16 \Rightarrow$ mean

$E(x^2) = 8^2\times\frac{1}{8} + 12^2\times\frac{1}{6} + 16^2\times\frac{3}{8} + 20^2\times\frac{1}{4} + 24^2\times\frac{1}{12}$

$= 8 + 24 + 96 + 100 + 48$

$= 276$

$E\{(x-\bar{x})^2\} = (8-16)^2\times\frac{1}{8} + (12-16)^2\times\frac{1}{6} + (16-16)^2\times\frac{3}{8}$

$+ (20-16)^2\times\frac{1}{4} + (24-16)^2\times\frac{1}{12}$

$= 8 + \frac{16}{6} + 0 + 4 + \frac{64}{12}$

$= 12 + \frac{8}{3} + \cancel{0} + \frac{16}{3} = 12 + \frac{24}{3} = 12 + 8$

$= 20$ Ans