Set Identities

Set Operations

- **Set Identities**

• Identity law	$A \cap U = A$
	$A \cup \emptyset = A$
• Domination law	$A \cup U = U$
	$A \cap \emptyset = \emptyset$
• Idempotent law	$A \cup A = A$
	$A \cap A = A$
• Complementation law	$(\overline{A}) = A$

- **Set Identities**
- Commutative laws

$$A \cup B = B \cup A$$

$$A \cap B = B \cap A$$

- Associative laws

$$A \cup (B \cup C) = (A \cup B) \cup C$$

$$A \cap (B \cap C) = (A \cap B) \cap C$$

- Distributive laws

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

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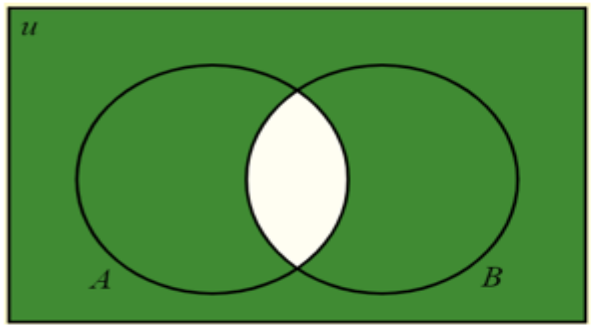
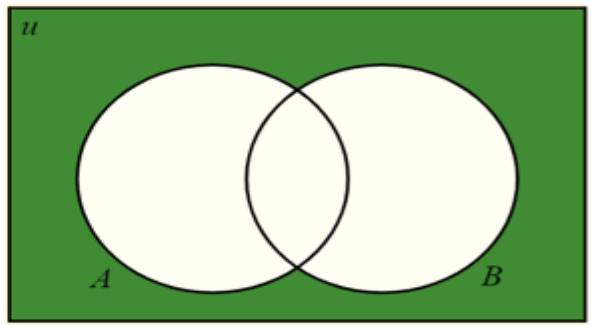
De Morgan's Law



$$(A \cap B)' = A' \cup B'$$



$$(A \cup B)' = A' \cap B'$$

- **Set Identities**
- Absorption laws

$$A \cup (A \cap B) = A$$

$$A \cap (A \cup B) = A$$

Complement laws

$$A \cup \overline{A} = U$$

$$A \cap \overline{A} = \emptyset$$

Using Set Identities

- For all subsets A and B of a universal set U, prove that
  - $(A - B) \cup (A \cap B) = A$
- **PROOF:**

LHS $= (A - B) \cup (A \cap B)$

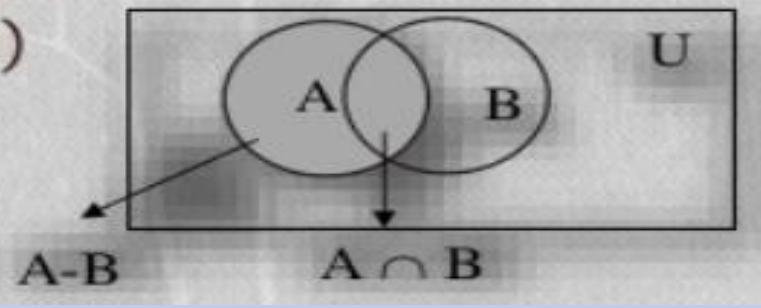
$= (A \cap B^c) \cup (A \cap B)$ (Alternative representation for set difference)

$= A \cap (B^c \cup B)$ Distributive Law

$= A \cap U$ Complement Law

$= A$ Identity Law

$= RHS$ (proved)



Using Set Identities

- **For any two sets A and B prove that $A - (A - B) = A \cap B$**

SOLUTION

$LHS = A - (A - B)$

$= A - (A \cap B^c)$ Alternative representation for set difference

$= A \cap (A \cap B^c)^c$ Alternative representation for set difference

$= A \cap (A^c \cup (B^c)^c)$ DeMorgan's Law

$= A \cap (A^c \cup B)$ Double Complement Law

$= (A \cap A^c) \cup (A \cap B)$ Distributive Law

$= \varnothing \cup (A \cap B)$ Complement Law

$= A \cap B$ Identity Law

$= RHS$ (proved)

Set Operations

- Set Identities
- Prove that $\overline{A \cup (B \cap C)} = (\overline{C} \cup \overline{B}) \cap \overline{A}$

$$\overline{A \cup (B \cap C)} = \overline{A} \cap (\overline{B \cap C})$$

$$= \overline{A} \cap (\overline{B} \cup \overline{C})$$

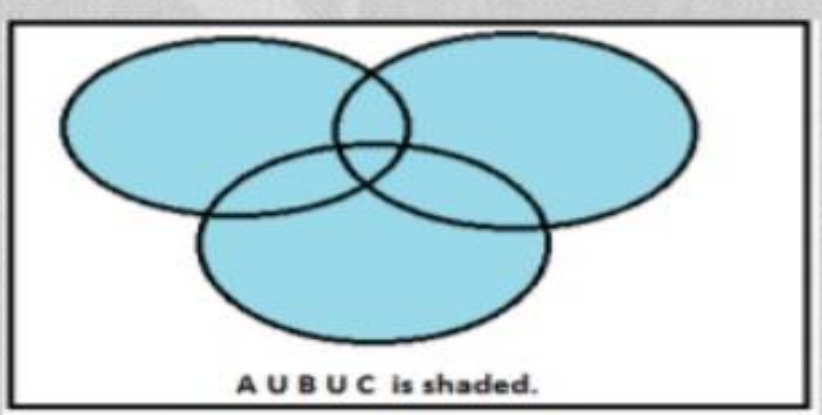
$$= (\overline{B} \cup \overline{C}) \cap \overline{A}$$

$$= (\overline{C} \cup \overline{B}) \cap \overline{A}$$

Set Operations

- **Generalized Unions and Intersections**
- **Generalized Unions**
- The union of a collection of sets is the set that contains those elements that are members of at least one set in the collection.

$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \ldots \cup A_n$$

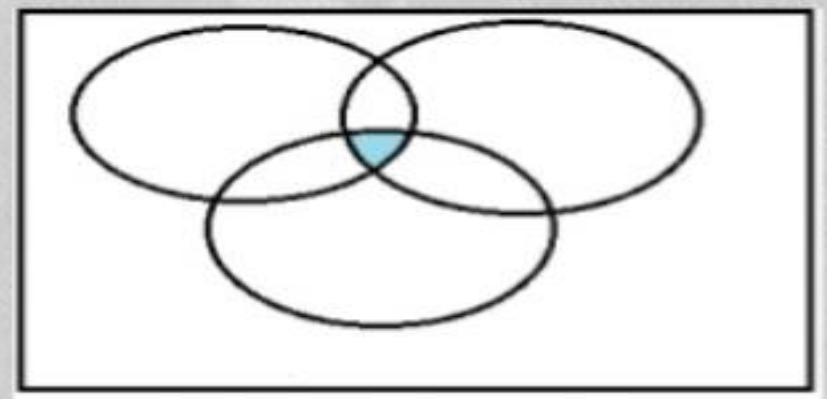


The union of A, B, And C

Set Operations

- **Generalized Intersection**
- The intersection of a collection of sets is the set that contains those elements that are members of every set in the collection.

$$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \ldots \cap A_n$$



The intersection of A, B, And C

Set Operations

- **Let A be the set of students who live within one mile of school and let B be the set of students who walk to classes. Describe the students in each of these sets**
- a) $A \cap B$
- **Ans :** The set of students who live within one mile of school and walk to classes .
- b) $A \cup B$
- **Ans :** The set of students who live within one mile of school or walk to classes (or do both) .

Set Operations

- c) A–B
- **Ans :** The set of students who live within one mile of school but do not walk to classes.
- d) B–A
- **Ans :** The set of students who walk to classes but live more than one mile away from school.
- **Suppose that A is the set of sophomores at your school and B is the set of students in discrete mathematics at your school. Express each of these sets in terms of A and B.**

Set Operations

- a) the set of sophomores taking discrete mathematics in your school.
- **Ans :** $A \cap B$
- b) the set of sophomores at your school who are not taking discrete mathematics.
- **Ans :** $A - B$
- c) the set of students at your school who either are sophomores or are taking discrete mathematics.
- **Ans :** $A \cup B$

Set Operations

- d) the set of students at your school who either are not sophomores or are not taking discrete mathematics.
- **Ans :** $\bar{A} \cup \bar{B}$
- **Let A={1,2,3,4,5} and B={0,3,6}. Find**
- a) $A \cup B$
- **Ans :** {0,1,2,3,4,5,6}
- b) $A \cap B$
- **Ans :** {3}

Set Operations

- c) A–B
- **Ans :** {1,2,4,5}
- d) B–A
- **Ans :** {0,6}
- **Let A={a, b, c, d, e} and B={a, b, c, d, e, f, g, h}.Find**
- a) A∪B
- **Ans :** {a,b,c,d,e,f,g,h}
- b) A∩B
- **Ans :** {a,b,c,d,e}

Set Operations

- **Find the sets A and B if A−B={1,5,7,8}, B−A={2,10}, and A∩B={3,6,9}.**
- **Ans :** A={1, 3, 5, 6, 7, 8, 9 }

  B={2, 3, 6, 9, 10}