

Q1 Give a simple description of the language generated by the grammar with productions

$$S \rightarrow aA \quad p_1$$

$$A \rightarrow bS \quad p_2$$

$$S \rightarrow \Lambda \quad p_3$$

Sol:

$$S \xRightarrow{p_1} aA \xRightarrow{p_2} abS \xRightarrow{p_1} ab aS \xRightarrow{p_2} ab ab S$$

$$\dots \Rightarrow ab \cdot ab \cdot ab \dots S$$

$$\xRightarrow{p_3} (ab)^n \in L(G)$$

$$n \in L(G)$$

Therefore $L(G) = \{(ab)^n : n \geq 0\}$

Q 2

Let $\Sigma = \{a, b\}$ find G

$$L = \{a^n b^m : n \geq 0, m > n\}$$

Sol:

Generate an equal number of a 's & b 's then one or more b 's as needed.

$$S \rightarrow AB$$

$$A \rightarrow aAb \mid \Lambda$$

$$B \rightarrow bB \mid b$$

Q 3

$$L = \{a^n b^{n-3} : n \geq 3\} \quad \text{find } G$$

Sol

$$\text{put } n = m + 3$$

$$L = \{a^{m+3} b^m : m \geq 0\}$$

$$\text{So } S \rightarrow aa aA$$

$$A \rightarrow aAb \mid \Lambda$$

Q4

$$L = \{w : |w| \bmod 3 = 0\} \quad \text{find } G \quad \text{when } \Sigma = \{a\}$$

Sol

$\therefore |w| \bmod 3 = 0$ shows w is multiple of 3

$$\text{i.e. } L = \{\Lambda, aaa, aaaa, aaaaa, aaaaaa, \dots\}$$

$$\therefore S \rightarrow aaaaS \mid \Lambda$$

Q5: $L = \{w : |w| \bmod 3 > 0\}$ where $w \in \{a,b\}^*$ find G

Sol:

$$\therefore |w| \bmod 3 > 0$$

$$\therefore |w| \bmod 3 = 1 \rightarrow \textcircled{1}$$

$$|w| \bmod 3 = 2 \rightarrow \textcircled{2}$$

First is covered by

$$S_1 \rightarrow a a a S_1 | a$$

Second is covered by

$$S_2 \rightarrow a a a S_2 | a a$$

The two can be combined into a single grammar by

$$S \rightarrow S_1 | S_2$$

Q6: $L = \{w : n_a(w) = n_b(w) \mid w \in \{a,b\}^*\}$

w has equal no of a & b

Sol

using the concept of balanced parenthesis

$$S \rightarrow S.S$$

$$S \rightarrow a S b$$

$$S \rightarrow b S a \quad (\text{change}) \rightarrow \text{differs from balanced parenthesis}$$

$$S \rightarrow \epsilon$$

Q7: $L = \{w : n_a(w) = n_b(w) + 1\}$ find G

$$S \rightarrow a S_1 \quad (\text{if string starts with } a)$$

$$S \rightarrow S_1 S \quad (\text{if string starts with } b)$$

$$S_1 \rightarrow S_1 S_1$$

$$S_1 \rightarrow a S_1 b \mid b S_1 a \mid \epsilon$$

Q7⑥ Find the grammar that generates

$$L = \{ a^n b^m : (n+m) \text{ is even} \}$$

Sol

$$S \rightarrow AeBe \mid A_0 B_0$$

$$Ae \rightarrow aaAe \mid \epsilon$$

$$Be \rightarrow bbBe \mid \epsilon$$

$$A_0 \rightarrow aAe$$

$$B_0 \rightarrow bBe$$

Q7⑦ Find Grammar for $L = \{ w : |w| \bmod 3 = 0 \} \text{ } w \in \{a,b\}^*$

$$S \rightarrow \epsilon \mid aA \mid bB$$

$$A \rightarrow aB \mid bB$$

$$B \rightarrow aS \mid bS$$



