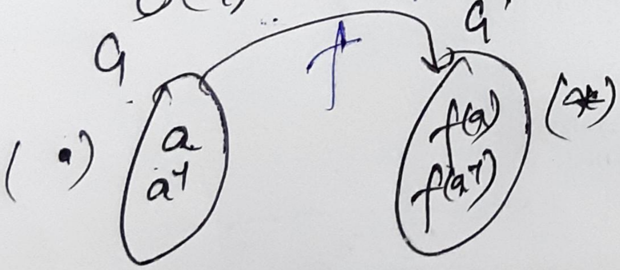
The order of f-image of an element 'a' of 'G' is inverse of f-image of a i.e.

$\boxed{\cancel{f(a^{-1}) = [f(a)]^{-1}}}$ $\quad O(f(a)) = O(a).$

Proof 6 :- Let 'e' be identity of 'G' then f(e) is be identity of G'. Let order of $a \in G$ be finite and it equal of 'n' i.e.

$$O(a) = n.$$



Then, $a^n = e.$

$f(a^n) = f(e).$ (images will also be equal)

$f(a \cdot a \cdot a \cdots n \text{ times}) = f(e).$

$f(a) \cdot f(a) \cdot f(a) \cdots\cdots n \text{ times} = f(e)$

(becoz they are homomorphic).

(f is one one also. from part 1). $f(e) = e'$

$\therefore [f(a)]^n = e'$ (least +ve integer)

$\Rightarrow \boxed{O[f(a)] \leq n}$ — (1).

$m \leq n$

ow, $o(f(a)) = m.$

$[f(a)]^m = e'.$

$[f(a)]^m = f(e).$ (As $f(e) = e'$)

$\Rightarrow f(a) * f(a) * f(a) \dots m \text{ times} = f(e).$

$f(a \cdot a \cdot a \dots m \text{ times}) = f(e).$ (f is homomorphic)

$f(a^m) = f(e).$

(Also f is one to one).

$\boxed{a^m = e}$

$o(a) \leq m.$

$n \leq m$ — (2)

from (1) & (2) $\boxed{n = m}$