Question Set

Composition of 2 Functions

Example 1.

Functions f and g are give by
$f(x) = \sqrt{(x + 2)}$ and $g(x) = \ln(1 - x^2)$
Find the composite function defined by $(g \circ f)(x)$ and describe its domain.

Solution: 1

- $(g_o f)(x) = g(f(x))$
  $= \ln(1 - f(x)^2)$
  $= \ln(1 - \sqrt{(x + 2)}^2)$
  $= \ln(1 - (x + 2))$
  $= \ln(- x - 1)$
- The domain of $g_o f$ is the set of all values of x so that

a) x is in the domain of f

b) f(x) is in the domain of g

a) is written as follows: $x + 2 \geq 0$

or $x \geq -2$ or in interval form $[-2 , + \infty)$

b) is written as follows: $1 - f(x)^2 > 0$

or $-x - 1 > 0$
or $x < -1$ or in interval form $(- \infty , -1)$

- The domain of $g_o f$ is given by the intersection of the sets $[-2 , + \infty)$ and $(- \infty , -1)$ and is given by
  $[-2 , -1)$

Example 2

Functions f and g are as sets of ordered pairs
$f = \{(-2,1),(0,3),(4,5)\}$
and
$g = \{(1,1),(3,3),(7,9)\}$
Find the composite function defined by $g_o f$ and describe its domain and range.

Solution 2

- $(g_o f)(-2) = g(f(-2)) = g(1) = 1$
  $(g_o f)(0) = g(f(0)) = g(3) = 3$
  $(g_o f)(4) = g(f(4)) = g(5) =$ undefined

- Hence $g_o f$ is given by
  $g_o f = \{ (-2, 1), (0, 3) \}$

- The domain d and range r of $g_o f$ are given by
  $d = \{-2, 0\}$ and $r = \{1, 3\}$

Example 3. Find $(f \circ g)(x)$ and the domain of $f \circ g$ given that

$f(x) = (x - 1) / (x + 2)$ and $g(x) = (x + 1) / (x - 2)$

Solution 3

- First find (fog)(x)
  (f o g)(x) = f(g(x)) = (g(x) - 1)/(g(x) + 2)
  =[(x + 1)/(x - 2) - 1] / [(x + 1)/(x - 2) + 2]
  = 3 / (3x - 3)

  First find domain of f and g
  domain of f : x not equal to -2
  domain of g : x not equal to 2
  g(x) has to be in the domain of f.
  g(x) not equal to -2
  solve for x the equation g(x) = -2
  (x + 1)/(x - 2) = -2
  x + 1 = -2x + 4
  3x = 3
  x = 1
  for g(x) to be different from - 2, x has to be different from 1.
  conclusion: The domain of f o g is: (- ∞ , 1) U (1 , 2) U (2 , + ∞)

Ex. Questions 1

1. Evaluate f(g(3)) given that
   $f(x) = | x - 6 | + x^2 - 1$ and $g(x) = 2x$

2. Find f(x) and g(x) if the composite function
   $f(g(x)) = 2 \sec(2x + 1)$

3. Find the domain of the composite function
   $g_o f$ if $f(x) = \sqrt{x}$ and $g(x) = 1 / x$.

4. Find the range of the composite function f(g(x)) given that
   $f(x) = x + 4$ and $g(x) = x^2 + 2$

5. Find the composite function (f o g)(x) given that
   f = {(3,6) , (5,7) , (9,0)} and g = {(2,3) , (4,5) , (6,7)}

6. Find the composite function (f o g)(x) given that
   f = {(1,6) , (4,7) , (5,0)} and g = {(6,1) , (7,4) , (0,5)}

Solutions

1. 35

2. One possibility: $f(x) = 2 \sec (x)$ and $g(x) = 2x + 1$.

3. $[0 , 4) \cup (4 , + \infty)$

4. $[6 , + \infty)$

5. f o g) = {(2 , 6) , (4 , 7)}

6. f o g = {(6 , 6) , (7 , 7) , (0 , 0)}

Functions

- Operations on Functions

Example 1 : Does the equation

$y^2 + x = 1$

represents a function y in terms of x?

Solution : Solve the above equation for y

$y^2 = 1 - x$

$y = + \sqrt{(1 - x)}$ or $y = - \sqrt{(1 - x)}$

For one value of x we have two values of y and this is not a function.

Example 2 : Function f is defined by

$f(x) = - 2 x^2 + 6 x - 3$

find $f(- 2)$.

Example 3 : Function h is defined by

$h(x) = 3 x^2 - 7 x - 5$

find $h(x - 2)$.

Solutions 2. $f(-2) = -23$

3. $h = 3x^2 - 19x + 7$

Example 4: Functions f and g are defined by
$f(x) = 1/x + 3x$ and $g(x) = -1/x + 6x - 4$
find $(f + g)(x)$ and its domain.

Solution 4 :

(f + g)(x) is defined as follows

$(f + g)(x) = f(x) + g(x)$

$= (1/x + 3x) + (-1/x + 6x - 4)$

Group like terms to obtain

$(f + g)(x) = 9 x - 4$

The domain of function f + g is given by the intersection of the domains of f and g

Domain of f + g is given by the interval $(-\infty , 0) \cup (0 , + \infty)$

Example 5: Functions f and g are defined by

$f(x) = x^2 - 2x + 1$ and $g(x) = (x - 1)(x + 3)$

find $(f / g)(x)$ and its domain

Solution 5

- (f / g)(x) is defined as follows
  $(f / g)(x) = f(x) / g(x) = (x^2 - 2x + 1) / [(x - 1)(x + 3)]$
  Factor the numerator of f / g and simplify
  $(f / g)(x) = f(x) / g(x) = (x - 1)^2 / [(x - 1)(x + 3)]$
  $= (x - 1) / (x + 3)$, x not equal to 1
  The domain of f / g is the intersections of the domain of f and g excluding all values of x that make the numerator equal to zero.
  The domain of f / g is given by
  $(-\infty , -3) \cup (-3 , 1) \cup (1 , +\infty)$

Example 6

Find the domain of the real valued function h defined by

$h(x) = \sqrt{(x - 2)}$

Solution 6

- For function h to be real valued, the expression under the square root must be positive or equal to 0. Hence the condition
  $x - 2 \geq 0$
  Solve the above inequality to obtain the domain in inequality form
  $x \geq 2$
  and interval form
  $[2 , + \infty)$

Example 7 Find the range of
$f(x) = -x^2 - 10$

Solution 7

- $-x^2$ is either negative or equal to zero as x takes real values, hence
  $-x^2 <= 0$
  Add -10 to both sides of the above inequality to obtain
  $-x^2 - 10 <= -10$
  The expression on the left side is equal to f(x), hence
  $f(x) <= -10$
  The above inequality gives the range of f as the interval
  $(-\infty , -10]$

Example 8: Find the range of

$$h(x) = x^2 - 4x + 9$$

Solution 8

- h(x) is a quadratic function, so let us first write it in vertex form using completing the square
  $h(x) = x^2 - 4x + 9$
  $= x^2 - 4x + 4 - 4 + 9$
  $= (x - 2)^2 + 5$
  $(x - 2)^2$ is either positive or equal to zero as x takes real values, hence
  $(x - 2)^2 \geq 0$
  Add 5 to both sides of the above inequality to obtain
  $(x - 2)^2 + 5 \geq 5$
  The above inequality gives the range of h as the interval
  $[5 , + \infty)$

Ex. Questions:2

1. Evaluate f(3) given that $f(x) = | x - 6 | + x^2 - 1$
2. Find $f(x + h) - f(x)$ given that $f(x) = a x + b$
3. Find the domain of $f(x) = \sqrt{(-x^2 - x + 2)}$
4. Find the range of $g(x) = - \sqrt{(- x + 2)} - 6$
5. Find $(f \circ g)(x)$ given that $f(x) = \sqrt{(x)}$ and $g(x) = x^2 - 2x + 1$

Solutions: 2

1. $f(3) = 11$
2. $f(x + h) - f(x) = a h$
3. $[-2 , 1]$
4. $(- \infty , - 6]$
5. $(f \circ g)(x) = | x - 1 |$