MIT Integration Bee: Quarterfinals

(Time limit per integral: 3 minutes)

Quarterfinal #1 Problem 1

$$\int_1^\infty x^5 e^{-x}\,dx$$

Quarterfinal #1 Problem 1

$$\int_1^\infty x^5 e^{-x} \, dx = \boxed{\frac{326}{e}}$$

Quarterfinal #1 Problem 2

$$\int_0^{100} \left(\left\lceil \tfrac{x-1}{3} \right\rceil - \left\lfloor \tfrac{x+1}{3} \right\rfloor \right) \left(\left\lceil \tfrac{x-1}{5} \right\rceil - \left\lfloor \tfrac{x+1}{5} \right\rfloor \right) \left(\left\lceil \tfrac{x-1}{7} \right\rceil - \left\lfloor \tfrac{x+1}{7} \right\rfloor \right) dx$$

Quarterfinal #1 Problem 2

$$\int_0^{100} \left(\left\lceil \tfrac{x-1}{3} \right\rceil - \left\lfloor \tfrac{x+1}{3} \right\rfloor \right) \left(\left\lceil \tfrac{x-1}{5} \right\rceil - \left\lfloor \tfrac{x+1}{5} \right\rfloor \right) \left(\left\lceil \tfrac{x-1}{7} \right\rceil - \left\lfloor \tfrac{x+1}{7} \right\rfloor \right) dx = \boxed{14}$$

Quarterfinal #1 Problem 3

$$\int \frac{x^2}{\sqrt{4e^{2x} + (x^2 + 2x + 2)^2}} \, dx$$

Quarterfinal #1 Problem 3

$$\int \frac{x^2}{\sqrt{4e^{2x} + (x^2 + 2x + 2)^2}}\, dx$$

$$= \boxed{-\operatorname{arcsinh}\left(\frac{x^2 + 2x + 2}{2e^x}\right)}$$

MIT Integration Bee: Quarterfinal Tiebreakers

(Time limit per integral: 3 minutes)

Quarterfinal Tiebreakers Problem 1

$$\int_{-2024}^{2026} x \left(1 + \cos\left(\frac{x-1}{2025} \cdot \pi \right) \right) \, dx$$

Quarterfinal Tiebreakers Problem 1

$$\int_{-2024}^{2026} x \left(1 + \cos \left(\frac{x-1}{2025} \cdot \pi \right) \right) dx = \boxed{4050}$$

Quarterfinal Tiebreakers Problem 2

$$\int_0^2 \left\lfloor e^x \right\rfloor \, dx$$

Quarterfinal Tiebreakers Problem 2

$$\int_0^2 \lfloor e^x \rfloor \, dx = \boxed{14 - \log 5040}$$

Quarterfinal Tiebreakers Problem 3

$$\int_0^{2025} \frac{\lfloor x \rfloor}{\lceil \sqrt{x} \rceil} \, dx$$

Quarterfinal Tiebreakers Problem 3

$$\int_0^{2025} \frac{\lfloor x \rfloor}{\lceil \sqrt{x} \rceil} \, dx = \boxed{59730}$$

MIT Integration Bee: Lightning Round

(Time limit per integral: 1 minute)

Lightning Round Problem 1

$$\int \frac{\arctan(x) - x\arctan(x)}{1 - x + x^2 - x^3} \, dx$$

Lightning Round Problem 1

$$\int \frac{\arctan(x) - x\arctan(x)}{1 - x + x^2 - x^3}\, dx = \boxed{\frac{1}{2}(\arctan(x))^2}$$

Quarterfinal #2 Problem 1

$$\lim_{A \to \infty} \int_{-\infty}^{\infty} \frac{A}{A^2(x^3 - 3x)^2 + 1} \, dx$$

Quarterfinal #2 Problem 1

$$\lim_{A \to \infty} \int_{-\infty}^{\infty} \frac{A}{A^2(x^3 - 3x)^2 + 1} \, dx = \boxed{\frac{2\pi}{3}}$$

Quarterfinal #2 Problem 2

$$\int \frac{dx}{(\cos(x) \cos(x + \frac{2\pi}{3}) \cos(x - \frac{2\pi}{3}))^2}$$

Quarterfinal #2 Problem 2

$$\int \frac{dx}{(\cos(x)\cos(x+\frac{2\pi}{3})\cos(x-\frac{2\pi}{3}))^2} = \boxed{\frac{16}{3}\tan(3x)}$$

Quarterfinal #2 Problem 3

$$\int_1^{2025} \left(\left\lceil \frac{2025}{\lfloor x \rfloor} \right\rceil - \left\lfloor \frac{2025}{\lceil x \rceil} \right\rfloor \right) dx$$

Quarterfinal #2 Problem 3

$$\int_1^{2025} \left(\left\lceil \frac{2025}{\lfloor x \rfloor} \right\rceil - \left\lfloor \frac{2025}{\lceil x \rceil} \right\rfloor \right) dx = \boxed{4034}$$

Quarterfinal Tiebreakers Problem 4

$$\int (x+1)e^x \log(x)\, dx$$

Quarterfinal Tiebreakers Problem 4

$$\int (x+1)e^x \log(x)\, dx = \boxed{e^x(x\log x - 1)}$$

Quarterfinal #3 Problem 1

$$\int_{-\pi/2}^{\pi/2} \sqrt{\sec(x) - \cos(x)} \, dx$$

Quarterfinal #3 Problem 1

$$\int_{-\pi/2}^{\pi/2} \sqrt{\sec(x) - \cos(x)}\, dx = \boxed{4}$$

Quarterfinal #3 Problem 2

$$\int \frac{x}{\sqrt[3]{x^3 - 3x - 2}} \, dx$$

Quarterfinal #3 Problem 2

$$\int \frac{x}{\sqrt[3]{x^3 - 3x - 2}} \, dx = \boxed{\sqrt[3]{(x+1)(x-2)^2}}$$

Quarterfinal #3 Problem 3

$$\int_0^{2\pi} \left(\sum_{n=0}^{\infty} \frac{\cos(2^n x)}{2^n} \right)^2 dx$$

Quarterfinal #3 Problem 3

$$\int_0^{2\pi} \left(\sum_{n=0}^{\infty} \frac{\cos(2^n x)}{2^n} \right)^2 dx = \boxed{\frac{4\pi}{3}}$$

Quarterfinal #4 Problem 1

$$\int_{x=0}^{x=10} x^2 \, d\left\{ x + \frac{1}{2} \right\}$$

Quarterfinal #4 Problem 1

$$\int_{x=0}^{x=10} x^2 \, d\left\{ x + \frac{1}{2} \right\} = \boxed{\frac{5}{6}}$$

Quarterfinal #4 Problem 2

$$\int_0^1 \frac{x^2}{\sqrt{x(1-x)}}\, dx$$

Quarterfinal #4 Problem 2

$$\int_0^1 \frac{x^2}{\sqrt{x(1-x)}} \, dx = \boxed{\frac{3\pi}{8}}$$

Quarterfinal #4 Problem 3

$$\int \frac{dx}{x^8 - x^6}$$

Quarterfinal #4 Problem 3

$$\int \frac{dx}{x^8 - x^6} = \boxed{\frac{1}{2} \log \left(\frac{x-1}{x+1} \right) + \frac{1}{x} + \frac{1}{3x^3} + \frac{1}{5x^5}}$$