Detecting Multicollinearity in R

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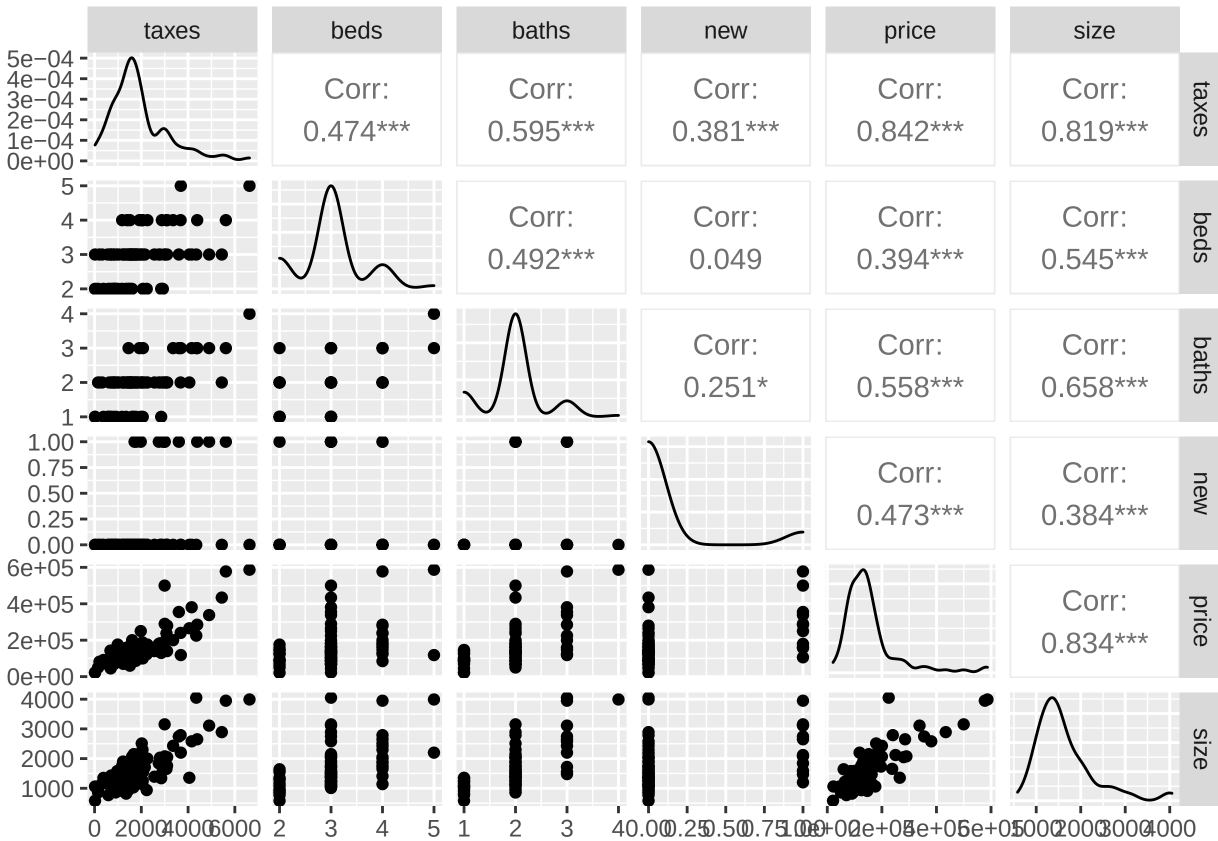
```
library(tidyverse)
library(broom)
library(GGally)
library(fastDummies)
library(car)
```

```
Houses <- read_table("https://users.stat.ufl.edu/~aa/smss/data/Houses.dat",
                     col_types = cols(X7 = col_skip()))
names(Houses)[2] <- 'beds'
glimpse(Houses)
```

```
## Rows: 100
## Columns: 6
## $ taxes <dbl> 3104, 1173, 3076, 1608, 1454, 2997, 4054, 3002, 6627, 320, 630, ~
## $ beds  <dbl> 4, 2, 4, 3, 3, 3, 3, 3, 5, 3, 3, 3, 3, 3, 3, 2, 3, 3, 2, 3, 2, 4~
## $ baths <dbl> 2, 1, 2, 2, 3, 2, 2, 2, 4, 2, 2, 2, 2, 2, 1, 1, 2, 1, 2, 2, 1, 3~
## $ new   <dbl> 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1~
## $ price <dbl> 279900, 146500, 237700, 200000, 159900, 499900, 265500, 289900, ~
## $ size  <dbl> 2048, 912, 1654, 2068, 1477, 3153, 1355, 2075, 3990, 1160, 1220,~
```

Correlation Pair Matrix

```
ggpairs(Houses)
```



Creating the Model With All Variables

```r
model <- lm(price ~ taxes + beds + baths + new + size, Houses)
glance(model)
```

```
## # A tibble: 1 x 12
##   r.squared adj.r.squared  sigma statistic  p.value    df logLik   AIC   BIC
##       <dbl>         <dbl>  <dbl>     <dbl>    <dbl> <dbl>  <dbl> <dbl> <dbl>
## 1     0.793         0.782 47238.      72.2 1.17e-30     5 -1215. 2444. 2462.
## # ... with 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

```r
tidy(model)
```

```
## # A tibble: 6 x 5
##   term        estimate std.error statistic     p.value
##   <chr>          <dbl>     <dbl>     <dbl>       <dbl>
## 1 (Intercept)   4526.    24474.      0.185 0.854
## 2 taxes           38.1       6.82    5.60  0.000000216
## 3 beds        -11259.     9115.     -1.24  0.220
## 4 baths        -2114.    11465.     -0.184 0.854
## 5 new          41711.    16887.      2.47  0.0153
## 6 size            68.4      13.9     4.90  0.00000392
```

$\widehat{price} = 4525.75 + 38.13 \cdot taxes - 11259.06 \cdot beds - 2114.37 \cdot baths + 41711.43 \cdot new + 68.35 \cdot size$

```r
new_model <- dummy_cols(Houses, select_columns = "new", remove_selected_columns = TRUE)
dummy_model <- lm(price ~ taxes + beds + baths + new_1 + size, data = new_model)
```

```
tidy(dummy_model)
```

```
## # A tibble: 6 x 5
##   term        estimate std.error statistic     p.value
##   <chr>          <dbl>     <dbl>     <dbl>       <dbl>
## 1 (Intercept)   4526.   24474.       0.185 0.854
## 2 taxes           38.1      6.82     5.60  0.000000216
## 3 beds        -11259.    9115.      -1.24  0.220
## 4 baths        -2114.   11465.      -0.184 0.854
## 5 new_1        41711.   16887.       2.47  0.0153
## 6 size            68.4     13.9      4.90  0.00000392
```

```
glancedummy <- glance(dummy_model)
glancedummy
```

```
## # A tibble: 1 x 12
##   r.squared adj.r.squared  sigma statistic  p.value    df logLik   AIC   BIC
##       <dbl>         <dbl>  <dbl>     <dbl>    <dbl> <dbl>  <dbl> <dbl> <dbl>
## 1     0.793         0.782 47238.      72.2 1.17e-30     5 -1215. 2444. 2462.
## # ... with 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

Finding the VIF Values

```
taxes <- lm(taxes ~ 1 + beds + baths + new_1 + size, data = new_model)
taxes_g <- glance(taxes)
taxes_g
```

Calculating Taxes VIF

```
## # A tibble: 1 x 12
##   r.squared adj.r.squared sigma statistic  p.value    df logLik   AIC   BIC
##       <dbl>         <dbl> <dbl>     <dbl>    <dbl> <dbl>  <dbl> <dbl> <dbl>
## 1     0.682         0.669  711.      51.0 7.46e-23     4  -796. 1604. 1620.
## # ... with 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

```
taxesvif <- 1/(1 - taxes_g[[1]])
taxesvif
```

```
## [1] 3.147119
```

```
beds <- lm(beds ~ taxes + 1 + baths + new_1 + size, data = new_model)
beds_g <- glance(beds)
beds_g
```

Calculating beds VIF

```
## # A tibble: 1 x 12
##   r.squared adj.r.squared sigma statistic      p.value    df logLik   AIC   BIC
##       <dbl>         <dbl> <dbl>     <dbl>        <dbl> <dbl>  <dbl> <dbl> <dbl>
## 1     0.361         0.334 0.532      13.4 0.0000000108     4  -76.2  164.  180.
## # ... with 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

```
bedsvif <- 1/(1 - beds_g[[1]])
bedsvif
```

```
## [1] 1.563795
```

```r
baths <- lm(baths ~ taxes + beds + 1 + new_1 + size, data = new_model)
baths_g <- glance(baths)
baths_g
```

Calculating baths VIF

```
## # A tibble: 1 x 12
##   r.squared adj.r.squared sigma statistic  p.value    df logLik   AIC   BIC
##       <dbl>         <dbl> <dbl>     <dbl>    <dbl> <dbl>  <dbl> <dbl> <dbl>
## 1     0.467         0.444 0.423      20.8 2.46e-12     4  -53.2  118.  134.
## # ... with 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

```r
bathsvif <- 1/(1 - baths_g[[1]])
bathsvif
```

```
## [1] 1.875628
```

```r
new_1 <- lm(new_1 ~ taxes + beds + baths + 1 + size, data = new_model)
new_1_g <- glance(new_1)
new_1_g
```

Calculating new__1 VIF

```
## # A tibble: 1 x 12
##   r.squared adj.r.squared sigma statistic  p.value    df logLik   AIC   BIC
##       <dbl>         <dbl> <dbl>     <dbl>    <dbl> <dbl>  <dbl> <dbl> <dbl>
## 1     0.201         0.167 0.287      5.97 0.000251     4  -14.5  41.0  56.6
## # ... with 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

```r
new_1vif <- 1/(1 - new_1_g[[1]])
new_1vif
```

```
## [1] 1.251166
```

```r
size <- lm(size ~ taxes + beds + baths + new_1 + 1, data = new_model)
size_g <- glance(size)
size_g
```

Calculating size VIF

```
## # A tibble: 1 x 12
##   r.squared adj.r.squared sigma statistic  p.value    df logLik   AIC   BIC
##       <dbl>         <dbl> <dbl>     <dbl>    <dbl> <dbl>  <dbl> <dbl> <dbl>
## 1     0.739         0.728  348.      67.3 6.91e-27     4  -724. 1461. 1477.
## # ... with 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

```r
sizevif <- 1/(1 - size_g[[1]])
sizevif
```

```
## [1] 3.832948
```

```r
vif(dummy_model)
```

We can use `vif()` function from package `{car}` to see all variables' VIF

```
##    taxes     beds    baths    new_1     size
## 3.147119 1.563795 1.875628 1.251166 3.832948
```

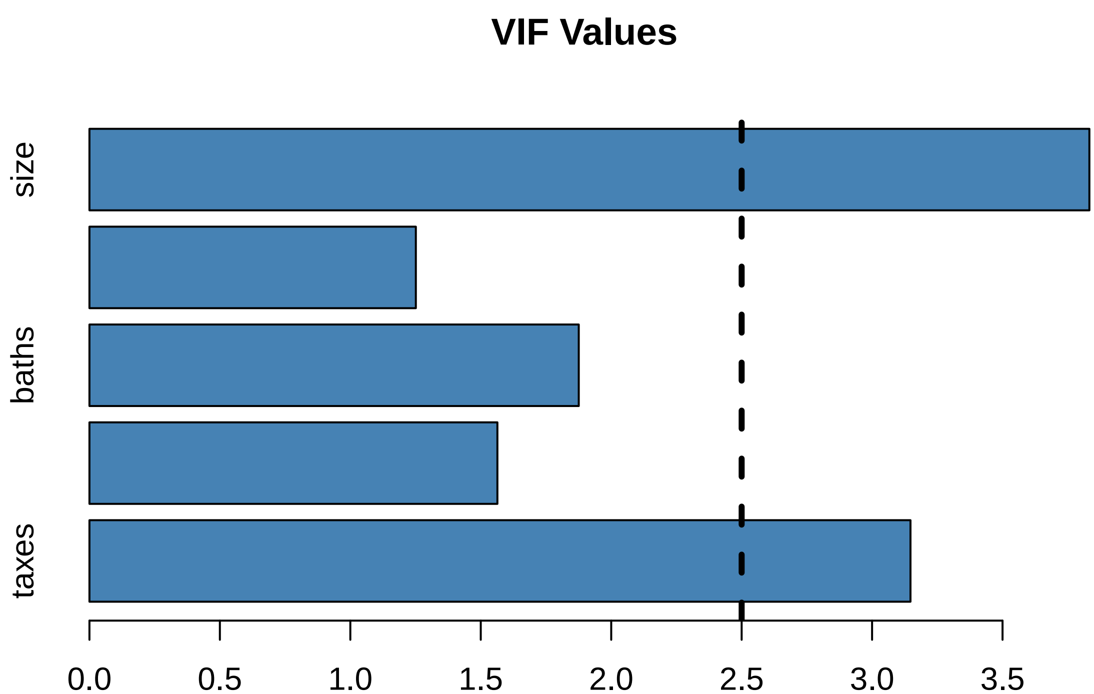
Correlation Matrix

```r
x_vari <- new_model[,c("taxes", "beds", "new_1", "baths", "size")]
cor(x_vari)
```

```
##           taxes       beds      new_1     baths      size
## taxes 1.0000000 0.47392873 0.38087410 0.5948543 0.8187958
## beds  0.4739287 1.00000000 0.04931556 0.4922224 0.5447831
## new_1 0.3808741 0.04931556 1.00000000 0.2514810 0.3843277
## baths 0.5948543 0.49222235 0.25148095 1.0000000 0.6582247
## size  0.8187958 0.54478311 0.38432773 0.6582247 1.0000000
```

Visualize Predictor VIFs

```r
vif_vals <- vif(dummy_model)
barplot(vif_vals, main = "VIF Values", horiz = TRUE, col = "steelblue")
abline(v = 2.5, lwd = 3, lty = 2)
```



Without taxes, as taxes and size are highly correlated

```r
model_2 <- lm(price ~ beds + baths + new_1 + size, new_model)
tidy(model_2)
```

```
## # A tibble: 5 x 5
##   term        estimate std.error statistic  p.value
##   <chr>          <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)  -28849.    27261.    -1.06  2.93e- 1
## 2 beds          -8202.    10450.    -0.785 4.34e- 1
## 3 baths          5274.    13080.     0.403 6.88e- 1
```

```
## 4 new_1         54562.   19215.       2.84  5.53e- 3
## 5 size            118.      12.3      9.59  1.27e-15
```

Original Model

$\widehat{price} = 4525.75 + 38.13 \cdot taxes - 11259.06 \cdot beds - 2114.37 \cdot baths + 41711.43 \cdot new + 68.35 \cdot size$

Model Without "taxes"

$\widehat{price} = -28849.217 - 8202.38 \cdot beds + 5273.78 \cdot baths + 5273.77 \cdot new + 118.12 \cdot size$

Detecting Multicollinearity in R - Second Data Set

The variables for this data set are violent crime rate (number of violent crimes per 100,000 population), murder rate, percent in metropolitan areas, percent white, percent high school graduates,percent below the poverty level, and percent of families headed by a single parent. The data are from StatisticalAbstract of the United States for 2005.

```
Crime <- read_table("https://users.stat.ufl.edu/~aa/smss/data/Crime2.dat",
                    col_types = cols(X9 = col_skip()))
```

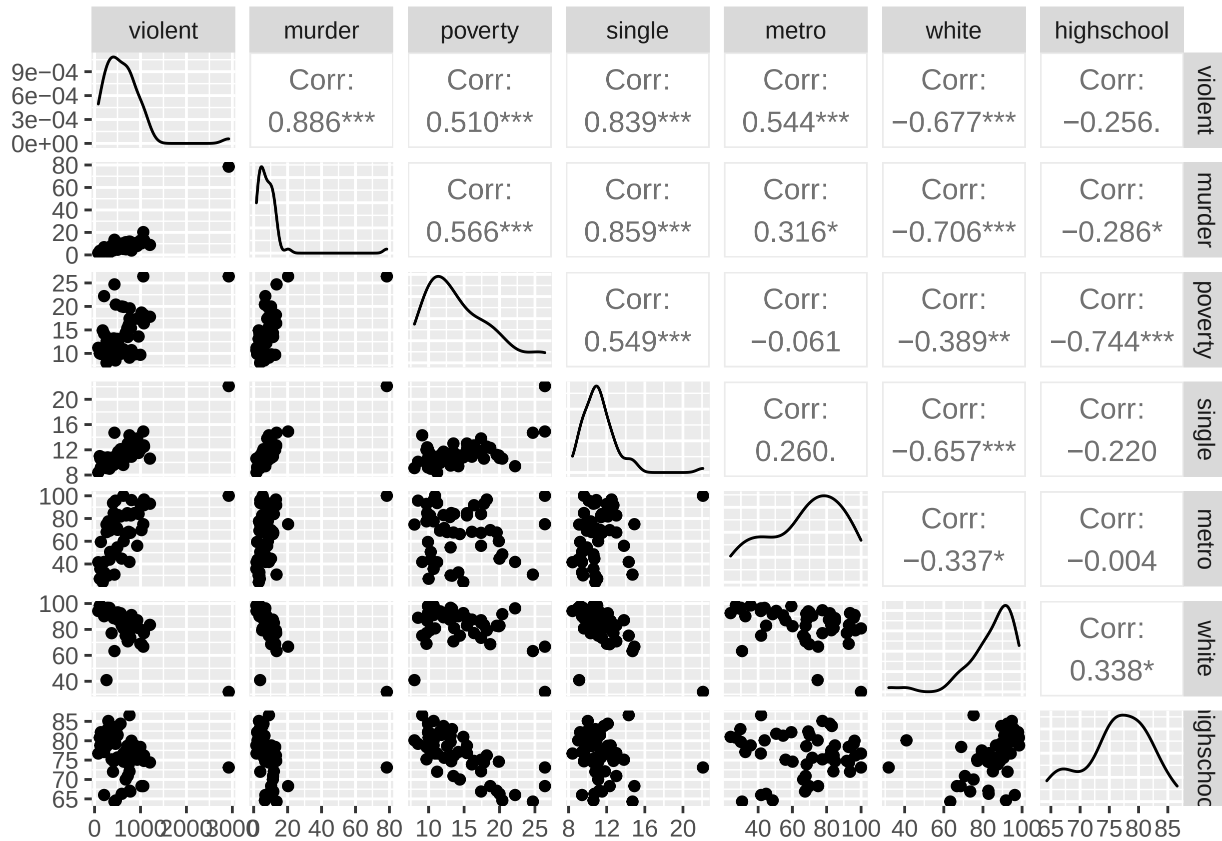
```
## Warning: Missing column names filled in: 'X9' [9]
```

```
glimpse(Crime)
```

```
## Rows: 51
## Columns: 8
## $ State      <chr> "AK", "AL", "AR", "AZ", "CA", "CO", "CT", "DE", "FL", "GA",~
## $ violent    <dbl> 761, 780, 593, 715, 1078, 567, 456, 686, 1206, 723, 261, 32~
## $ murder     <dbl> 9.0, 11.6, 10.2, 8.6, 13.1, 5.8, 6.3, 5.0, 8.9, 11.4, 3.8, ~
## $ poverty    <dbl> 9.1, 17.4, 20.0, 15.4, 18.2, 9.9, 8.5, 10.2, 17.8, 13.5, 8.~
## $ single     <dbl> 14.3, 11.5, 10.7, 12.1, 12.5, 12.1, 10.1, 11.4, 10.6, 13.0,~
## $ metro      <dbl> 41.8, 67.4, 44.7, 84.7, 96.7, 81.8, 95.7, 82.7, 93.0, 67.7,~
## $ white      <dbl> 75.2, 73.5, 82.9, 88.6, 79.3, 92.5, 89.0, 79.4, 83.5, 70.8,~
## $ highschool <dbl> 86.6, 66.9, 66.3, 78.7, 76.2, 84.4, 79.2, 77.5, 74.4, 70.9,~
```

Correlation Pair Matrix

```
ggpairs(Crime[2:8])
```



Creating the Model With All Variables

```r
model2 <- lm(violent ~ murder + poverty + single + metro + white + highschool, Crime)
glance(model2)
```

```
## # A tibble: 1 x 12
##   r.squared adj.r.squared sigma statistic  p.value    df logLik   AIC   BIC
##       <dbl>         <dbl> <dbl>     <dbl>    <dbl> <dbl>  <dbl> <dbl> <dbl>
## 1     0.895         0.881  152.      62.5 6.52e-20     6  -325.  666.  681.
## # ... with 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

```r
tidy(model2)
```

```
## # A tibble: 7 x 5
##   term        estimate std.error statistic     p.value
##   <chr>          <dbl>     <dbl>     <dbl>       <dbl>
## 1 (Intercept) -1144.       585.     -1.95  0.0570
## 2 murder         19.3        4.44    4.35  0.0000794
## 3 poverty        15.0        9.72    1.54  0.130
## 4 single         54.9       21.3     2.57  0.0135
## 5 metro           6.62       1.12    5.92  0.000000442
## 6 white          -0.696      2.51   -0.278 0.783
## 7 highschool      4.79       6.68    0.717 0.477
```

$$\widehat{violent} = -1143.8 + 19.33 \cdot murder + 15 \cdot poverty + 54.85 \cdot single + 6.62 \cdot metro - 0.70 \cdot white + 4.79 \cdot highschool$$

Finding the Individual VIF Values

```r
murder <- lm(murder ~ 1 + poverty + single + metro + white + highschool, data = Crime)
murder_g <- glance(murder)
murder_g
```

Calculating murder VIF

```
## # A tibble: 1 x 12
##   r.squared adj.r.squared sigma statistic  p.value    df logLik   AIC   BIC
##       <dbl>         <dbl> <dbl>     <dbl>    <dbl> <dbl>  <dbl> <dbl> <dbl>
## 1     0.795         0.773  5.11      35.0 1.98e-14     5  -152.  319.  332.
## # ... with 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

```r
murdervif <- 1/(1 - murder_g[[1]])
murdervif
```

```
## [1] 4.885397
```

```r
poverty <- lm(poverty ~ murder + 1 + single + metro + white + highschool, data = Crime)
poverty_g <- glance(poverty)

povertyvif <- 1/(1 - poverty_g[[1]])
povertyvif
```

Calculating poverty VIF

```
## [1] 4.278128
```

```r
single <- lm(single ~ murder + poverty + 1 + metro + white + highschool, data = Crime)
single_g <- glance(single)

singlevif <- 1/(1 - single_g[[1]])
singlevif
```

Calculating single VIF

```
## [1] 4.400805
```

```r
metro <- lm(metro ~ murder + poverty + single + 1 + white + highschool, data = Crime)
metro_g <- glance(metro)

metrovif <- 1/(1 - metro_g[[1]])
metrovif
```

Calculating metro VIF

```
## [1] 1.299233
```

```r
white <- lm(white ~ murder + poverty + single + metro + 1 + highschool, data = Crime)
white_g <- glance(white)

whitevif <- 1/(1 - white_g[[1]])
whitevif
```

Calculating white VIF

```
## [1] 2.375882
```

```
highschool <- lm(highschool ~ murder + poverty + single + metro + white + 1, data = Crime)
highschool_g <- glance(highschool)

highschoolvif <- 1/(1 - highschool_g[[1]])
highschoolvif
```

Calculating highschool VIF

```
## [1] 3.002861
```

```
vif(model2)
```

We can use `vif()` function from package `{car}` to see all variables' VIF

```
##     murder    poverty     single      metro      white highschool
##   4.885397   4.278128   4.400805   1.299233   2.375882   3.002861
```

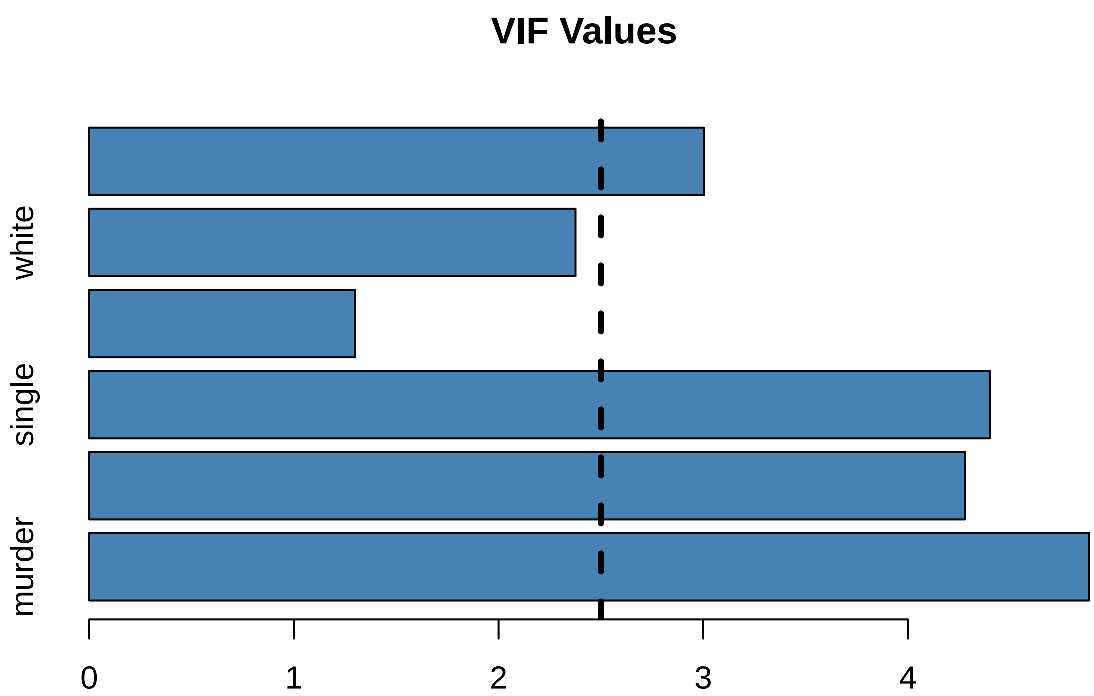
Correlation Matrix

```
x_vari2 <- Crime[ , c("murder", "poverty", "single", "metro", "white", "highschool")]
cor(x_vari2)
```

```
##                murder    poverty     single       metro      white
## murder      1.0000000  0.5658711  0.8589106  0.316114166 -0.7062589
## poverty     0.5658711  1.0000000  0.5485890 -0.060538499 -0.3891346
## single      0.8589106  0.5485890  1.0000000  0.259810085 -0.6567078
## metro       0.3161142 -0.0605385  0.2598101  1.000000000 -0.3374351
## white      -0.7062589 -0.3891346 -0.6567078 -0.337435120  1.0000000
## highschool -0.2860708 -0.7439382 -0.2197829 -0.003977358  0.3381212
##              highschool
## murder     -0.286070828
## poverty    -0.743938249
## single     -0.219782892
## metro      -0.003977358
## white       0.338121236
## highschool  1.000000000
```

Visualize Predictor VIFs

```
vif_vals2 <- vif(model2)
barplot(vif_vals2, main = "VIF Values", horiz = TRUE, col = "steelblue")
abline(v = 2.5, lwd = 3, lty = 2)
```



Without murder, as murder, single, and white are highly correlated

```
model2_1 <- lm(violent ~ poverty + single + metro + white + highschool, Crime)
glance(model2_1)
```

```
## # A tibble: 1 x 12
##   r.squared adj.r.squared sigma statistic  p.value    df logLik   AIC   BIC
##       <dbl>         <dbl> <dbl>     <dbl>    <dbl> <dbl>  <dbl> <dbl> <dbl>
## 1     0.850         0.833  180.      50.9 2.05e-17     5  -334.  682.  696.
## # ... with 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

```
tidy(model2_1)
```

```
## # A tibble: 6 x 5
##   term        estimate std.error statistic     p.value
##   <chr>          <dbl>     <dbl>     <dbl>       <dbl>
## 1 (Intercept) -1796.      669.       -2.69 0.0101
## 2 poverty        26.2      11.1       2.37 0.0222
## 3 single        109.       20.4       5.38 0.00000260
## 4 metro           7.61      1.30      5.87 0.000000480
## 5 white          -4.48      2.78     -1.61 0.114
## 6 highschool      8.65      7.83      1.10 0.275
```

With variable "murder"

$\widehat{violent} = -1143.8 + 19.33 \cdot murder + 15 \cdot poverty + 54.85 \cdot single + 6.62 \cdot metro - 0.70 \cdot white + 4.79 \cdot highschool$

Without Variable "murder"

$\widehat{violent} = -1795.9 + 26.2 \cdot poverty + 109.5 \cdot single + 7.6 \cdot metro - 4.48 \cdot white + 8.65 \cdot highschool$

VIF Values of Model Without "murder"

```
vif(model2_1)
```

```
##    poverty     single      metro      white highschool
##   3.975997   2.873489   1.245801   2.089245   2.949878
```