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CUS1188
Problem Set 3

1

$$T(n) = \sum_{i=1}^{\log_2 n + 1} 2i + 2 = (\log_2 n)^2 + 5\log_2 n + 6$$

2

- a) 57
- b) $T(n) = 2n + 4$
- c)

$$\begin{aligned} T(n) &= 2n + 4 \in O(n) \\ 2n + 4 &\leq cn \\ \frac{2n+4}{n} &\leq c \\ \lim_{n \rightarrow \infty} \frac{2n+4}{n} &\leq c \\ 2 &\leq c \\ T(n) &\in O(n) \end{aligned}$$

3

$$f(n) = n^2 + 3n^3 \in \Theta(n^3)$$

$$\begin{aligned} n^2 + 3n^3 &\in O(n^3) \\ n^2 + 3n^3 &\leq cn^3 \\ \frac{n^2+3n^3}{n^3} &\leq c \\ \lim_{n \rightarrow \infty} \frac{n^2+3n^3}{n^3} &\leq c \\ 3 &\leq c \end{aligned}$$

$$\begin{aligned} n^2 + 3n^3 &\in \Omega(n^3) \\ n^2 + 3n^3 &\geq cn^3 \\ \frac{n^2+3n^3}{n^3} &\geq c \\ \lim_{n \rightarrow \infty} \frac{n^2+3n^3}{n^3} &\geq c \\ 3 &\geq c \end{aligned}$$