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CUS1188
Problem Set 6

1

$T(1) = 1$
$T(n) = 2T(\frac{n}{2}) + 1000n \quad \forall n \geq 2$

$= 2[2T(\frac{n}{2^2}) + 1000(\frac{n}{2})] + 1000n$

$= 2^2T(\frac{n}{2^2}) + 2(\frac{1000n}{2}) + 1000n$

$= 2^2[2T(\frac{n}{2^3}) + 1000(\frac{n}{2^2})] + 2(\frac{1000n}{n}) + 1000n$

$= 2^3T(\frac{n}{2^3}) + 2^2(\frac{1000n}{2^2}) + 2(\frac{1000n}{2}) + 1000n$

$T(n) = 2^kT(\frac{n}{2^k}) + \frac{2^{k-1}}{2^{k-1}}1000n + \frac{2^{k-2}}{2^{k-2}}1000n + ... + \frac{2^0}{2^0}1000n$

$= 2^kT(\frac{n}{2^k}) + \sum_{i=0}^{k-1} 1000n$

$\frac{n}{2^k} = 1 \qquad k = logn$

$T(n) = 2^k(1) + \sum_{i=0}^{k-1} 1000n$

$T(n) = n + 1000n(logn - 1)$

$T(n) = 1000nlogn - 999n$

$T(n) \in O(nlogn)$

2

$T(1) = 1$
$T(n) = 7T(\frac{n}{2}) + 18n^2 \quad \forall n \geq 2$

$= 7[7T(\frac{n}{2^2}) + 18(\frac{n}{2})^2] + 18n^2$

$= 7^2T(\frac{n}{2^2}) + 7 \cdot 18(\frac{n}{2})^2 + 18n^2$

$= 7^2[7T(\frac{n}{2^3}) + 18(\frac{n}{2})^2] + 7 \cdot 18(\frac{n}{2})^2 + 18n^2$

$= 7^3T(\frac{n}{2^3}) + 7^2 \cdot 18(\frac{n}{2^2})^2 + 7 \cdot 18(\frac{n}{2})^2 + 18n^2$

$T(n) = 7^kT(\frac{n}{2^k}) + (\frac{7}{4})^{k-1}18n^2 + (\frac{7}{4})^{k-2}18n^2 + ... + (\frac{7}{4})^0 18n^2$

$= 7^k T(\frac{n}{2^k}) + 18n^2 \sum_{i=0}^{k-1} (\frac{7}{4})^i$

$\frac{n}{2^k} = 1 \qquad k = logn$

$7^{logn}(1) + 18n^2 \frac{1-(\frac{7}{4})^k}{1-\frac{7}{4}}$

$n^{log7} + 18n^2 \frac{1-(\frac{7}{4})^{logn}}{1-\frac{7}{4}}$

$n^{log7} + 18n^2 \frac{1-n^{log\frac{7}{4}}}{1-\frac{7}{4}}$

$n^{log7} + 18n^2(-\frac{4}{3} + \frac{4}{3}n^{log\frac{7}{4}})$

$= 24n^{2+log\frac{7}{4}} - 24n^2 + n^{log7}$

$= 24n^{2+(log7-log4)} - 24n^2 + n^{log7}$

$= 24n^{2+log7-2} - 24n^2 + n^{log7}$

$= 24n^{log7} - 24n^2 + n^{log7}$

$T(n) \in O(n^{log7})$

3

$T(1) = 3$
$T(n) = T(\frac{n}{2}) + n + 2 \quad \forall n \geq 2$

$= [T(\frac{n}{2^2}) + \frac{n}{2} + 2] + n + 2$

$= [T(\frac{n}{2^3}) + \frac{n}{2^2} + 2] + \frac{n}{2} + 2 + n + 2$

$= [T(\frac{n}{2^4}) + \frac{n}{2^3} + 2] + \frac{n}{2^2} + 2 + \frac{n}{2} + 2 + n + 2$

$T(n) = T(\frac{n}{2^k}) + \frac{n}{2^{k-1}} + \frac{n}{2^{k-2}} + ... + \frac{2}{n^0} + \sum_{i=0}^{k-1} 2$

$T(n) = T(\frac{n}{2^k}) + n \sum_{i=0}^{k-1} (\frac{1}{2})^i + \sum_{i=0}^{k-1} 2$

$\frac{n}{2^k} = 1 \qquad k = logn$

$T(n) = T(1) + n\frac{1-(\frac{1}{2})^{logn}}{1-\frac{1}{2}} + 2(logn - 1)$

$= 3 + n\frac{1-\frac{1}{n}}{\frac{1}{2}} + 2(logn - 1)$

$= 3 + n(2 - \frac{2}{n}) + 2logn - 1$

$= 2n + 2logn$

$T(n) \in O(n)$