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CUS1188
Problem Set 3

1

$T(1) = 1$
$T(n) = 2T(\frac{n}{2}) + 1000n \quad \forall n \geq 2$
$T(n) \in O(nlogn)$, thus, $T(n) \leq cnlogn$
Base case: $n = 2$

$$T(2) = 2(1) + 1000(2) = 2002 \quad , (2)log(2) = 2c$$
$$\text{Valid for } c \geq 1001$$

Assumption that statement is true for k, $k < n$

$$T(k) \leq cklogk$$

Induction

$$T(n) = 2T(\frac{n}{2}) + 1000n \leq 2c\frac{n}{2}log\frac{n}{2} + 1000n =$$
$$cnlog\frac{n}{2} + 1000n =$$
$$cnlogn - cnlog2 + 1000n =$$
$$cnlogn - cn + 1000n =$$
$$cnlogn - (cn - 1000n) \leq cnlogn \text{ for } c \geq 1000$$

2

$T(1) = 1$
$T(n) = 7T(\frac{n}{2}) + 18n^2 \forall n \geq 2$

c)

$$T(n) = 2n + 4 \in O(n)$$
$$2n + 4 \leq cn$$
$$2n + 4 \leq 2n + 4n$$
$$2n + 4 \leq 6n$$
$$6 = c$$
$$T(n) \in O(n)$$

3

$$f(n) = n^2 + 3n^3 \in \Theta(n^3)$$

$$n^2 + 3n^3 \in O(n^3)$$
$$n^2 + 3n^3 \leq cn^3$$
$$n^2 + 3n^3 \leq 2n^3 + 3n^3$$
$$n^2 + 3n^3 \leq 5n^3$$
$$c = 5$$

$$f(n) \in O(n^3)$$

$$n^2 + 3n^3 \in \Omega(n^3)$$
$$n^2 + 3n^3 \geq cn^3$$
$$n^2 + 3n^3 \geq 3n^3$$
$$c = 3$$
$$f(n) \in \Omega(n^3)$$

$$f(n) \in \Theta(n^3) = O(n^3) \cap \Omega(n^3)$$