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CUS1188
Problem Set 6

1

$T(1) = 1$
$T(n) = 2T(\frac{n}{2}) + 1000n \quad \forall n \geq 2$

$= 2[2T(\frac{n}{2^2}) + 1000(\frac{n}{2})] + 1000n$

$= 2^2 T(\frac{n}{2^2}) + 2(\frac{1000n}{2}) + 1000n$

$= 2^2[2T(\frac{n}{2^3}) + 1000(\frac{n}{2^2})] + 2(\frac{1000n}{n}) + 1000n$

$= 2^3 T(\frac{n}{2^3}) + 2^2(\frac{1000n}{2^2}) + 2(\frac{1000n}{2}) + 1000n$

$T(n) = 2^k T(\frac{n}{2^k}) + \frac{2^{k-1}}{2^{k-1}}1000n + \frac{2^{k-2}}{2^{k-2}}1000n + ... + \frac{2^0}{2^0}1000n$

$= 2^k T(\frac{n}{2^k}) + \sum_{i=0}^{k-1} 1000n$

$\frac{n}{2^k} = 1 \qquad k = logn$

$T(n) = 2^k(1) + \sum_{i=0}^{k-1} 1000n$

$T(n) = n + 1000n(logn - 1)$

$T(n) = 1000nlogn - 999n$

$T(n) \in O(nlogn)$

2

$T(1) = 1$
$T(n) = 7T(\frac{n}{2}) + 18n^2 \quad \forall n \geq 2$

$= 7[7T(\frac{n}{2^2}) + 18(\frac{n}{2})^2] + 18n^2$

$= 7^2 T(\frac{n}{2^2}) + 7 \cdot 18(\frac{n}{2})^2 + 18n^2$

$= 7^2[7T(\frac{n}{2^3}) + 18(\frac{n}{2})^2] + 7 \cdot 18(\frac{n}{2})^2 + 18n^2$

$= 7^3 T(\frac{n}{2^3}) + 7^2 \cdot 18(\frac{n}{2^2})^2 + 7 \cdot 18(\frac{n}{2})^2 + 18n^2$

$T(n) = 7^k T(\frac{n}{2^k}) + (\frac{7}{4})^{k-1}18n^2 + (\frac{7}{4})^{k-2}18n^2 + ... + (\frac{7}{4})^0 18n^2$

$$= 7^k T\left(\tfrac{n}{2^k}\right) + 18n^2 \sum_{i=0}^{k-1} \left(\tfrac{7}{4}\right)^i$$

$$\tfrac{n}{2^k} = 1 \qquad k = logn$$

$$7^{logn}(1) + 18n^2 \frac{1 - \left(\tfrac{7}{4}\right)^k}{1 - \tfrac{7}{4}}$$

$$n^{log7} + 18n^2 \frac{1 - \left(\tfrac{7}{4}\right)^{logn}}{1 - \tfrac{7}{4}}$$

$$n^{log7} + 18n^2 \frac{1 - n^{log\tfrac{7}{4}}}{1 - \tfrac{7}{4}}$$

$$n^{log7} + 18n^2 \left(-\tfrac{4}{3} + \tfrac{4}{3} n^{log\tfrac{7}{4}}\right)$$

$$= 24n^{2 + log\tfrac{7}{4}} - 24n^2 + n^{log7}$$

$$T(n) \in O\left(n^{2 + log\tfrac{7}{4}}\right)$$

3

$$T(1) = 3$$
$$T(n) = T\left(\tfrac{n}{2}\right) + n + 2 \quad \forall n \geq 2$$

$$= \left[T\left(\tfrac{n}{2^2}\right) + \tfrac{n}{2} + 2\right] + n + 2$$

$$= \left[T\left(\tfrac{n}{2^3}\right) + \tfrac{n}{2^2} + 2\right] + \tfrac{n}{2} + 2 + n + 2$$

$$= \left[T\left(\tfrac{n}{2^4}\right) + \tfrac{n}{2^3} + 2\right] + \tfrac{n}{2^2} + 2 + \tfrac{n}{2} + 2 + n + 2$$

$$T(n) = T\left(\tfrac{n}{2^k}\right) + \tfrac{n}{2^{k-1}} + \tfrac{n}{2^{k-2}} + ... + \tfrac{2}{n^0} + \sum_{i=0}^{k-1} 2$$

$$T(n) = T\left(\tfrac{n}{2^k}\right) + n \sum_{i=0}^{k-1} \left(\tfrac{1}{2}\right)^i + \sum_{i=0}^{k-1} 2$$

$$\tfrac{n}{2^k} = 1 \qquad k = logn$$

$$T(n) = T(1) + n \frac{1 - \left(\tfrac{1}{2}\right)^{logn}}{1 - \tfrac{1}{2}} + 2(logn - 1)$$

$$= 3 + n \frac{1 - \tfrac{1}{n}}{\tfrac{1}{2}} + 2(logn - 1)$$

$$= 3 + n\left(2 - \tfrac{2}{n}\right) + 2logn - 1$$

$$= 2n + 2logn$$

$$T(n) \in O(n)$$