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CUS1188
Problem Set 3

1

$T(1) = 1$
$T(n) = 2T(\frac{n}{2}) + 1000n \quad \forall n \geq 2$
$T(n) \in O(nlogn)$, thus, $T(n) \leq cnlogn$
Base case: $n = 2$

$$T(2) = 2(1) + 1000(2) = 2002 \quad , (2)log(2) = 2c$$

Assumption that statement is true for k, $k < n$

$$T(k) \leq cklogk$$

Induction

$$\begin{gathered} T(n) = 2T(\tfrac{n}{2}) + 1000n \leq 2c\tfrac{n}{2}log\tfrac{n}{2} + 1000n = \\ cnlog\tfrac{n}{2} + 1000n = \\ cnlogn - cnlog2 + 1000n = \\ cnlogn - cn + 1000n = \\ cnlogn - (cn - 1000n) \leq cnlogn \end{gathered}$$

2

$T(1) = 1$
$T(n) = 7T(\frac{n}{2}) + 18n^2 \quad \forall n \geq 2$
$T(n) \in O(n^3)$, thus, $T(n) \leq cn^3$
Base case: n=2

$$T(2) = 7(1) + 18(4) = 79 \quad , \quad c(2^3) = 8c$$

Assumption that statement is true for k, $k < n$

$$T(k) \leq ck^3$$

Induction

$$T(n) = 7T(\tfrac{n}{2}) + 18n^2 \leq 7c(\tfrac{n}{2})^3 + 18n^2 =$$

$$\tfrac{7}{8}cn^3 + 18n^2 =$$

$$\tfrac{7}{8}cn^3 + 18n^2 + \tfrac{1}{8}cn^3 - \tfrac{1}{8}cn^3 =$$

$$cn^3 + 18n^2 - \tfrac{1}{8}cn^3 =$$

$$cn^3 - (\tfrac{1}{8}cn^3 - 18n^2) \leq cn^3$$

3

$T(1) = 3$
$T(n) = T(\frac{n}{2} + n + 2 \quad \forall n \geq 2$
$T(n) \in O(nlogn)$, thus, $T(n) \leq cnlogn$
Base case: n=2

$$T(2) = 5 \quad , \quad c(2)(log2) = 2c$$

Assumption that statement is true for k, $k < n$

$$T(k) = cklogk$$

Induction

$$T(n) = T(\frac{n}{2}) + n + 2 \leq c\frac{n}{2}log\frac{n}{2} + n + 2 =$$

$$\frac{1}{2}cnlogn - \frac{1}{2}cnlog2 + n + 2 =$$

$$\frac{1}{2}cnlogn + \frac{1}{2}cn + n + 2 =$$

$$\frac{1}{2}cnlogn + \frac{1}{2}cn + n + 2 + \frac{1}{2}cnlogn - \frac{1}{2}cnlogn =$$

$$cnlogn + \frac{1}{2}cn + n + 2 - \frac{1}{2}cnlogn =$$

$$cnlogn - (\frac{1}{2}cnlogn - \frac{1}{2}cn - n - 2) \leq cnlogn$$