Yuchen Jin
CUS1188
Problem Set 3

1

$$T(n) = 2 + \sum_{i=1}^{log_2n+1} 2i + 2 = 2 \sum_{i=1}^{log_2n+1} i + 2 \sum_{i=1}^{log_2n+1} 1 = (log_2n)^2 + 5log_2n + 8$$

2

a) 57
b) $T(n) = 2n + 4$
c)

$$T(n) = 2n + 4 \in O(n)$$
$$2n + 4 \le cn$$
$$2n + 4 \le 2n + 4n$$
$$2n + 4 \le 6n$$
$$6 = c$$
$$T(n) \in O(n)$$

3

$$f(n) = n^2 + 3n^3 \in \Theta(n^3)$$

$$n^2 + 3n^3 \in O(n^3)$$
$$n^2 + 3n^3 \le cn^3$$
$$n^2 + 3n^3 \le 2n^3 + 3n^3$$
$$n^2 + 3n^3 \le 5n^3$$
$$c = 5$$
$$f(n) \in O(n^3)$$

$$n^2 + 3n^3 \in \Omega(n^3)$$
$$n^2 + 3n^3 \ge cn^3$$
$$n^2 + 3n^3 \ge 3n^3$$
$$c = 3$$
$$f(n) \in \Omega(n^3)$$

$$f(n) \in \Theta(n^3) = O(n^3) \cap \Omega(n^3)$$