

1

$$T(n) = 2 + \sum_{i=1}^{\log_2 n + 1} 2i + 2 = (\log_2 n)^2 + 5\log_2 n + 8$$

2

- a) 57
- b) $T(n) = 2n + 4$
- c)

$$\begin{aligned} T(n) &= 2n + 4 \in O(n) \\ 2n + 4 &\leq cn \\ 2n + 4 &\leq 2n + 4n \\ 2n + 4 &\leq 6n \\ 6 &= c \\ T(n) &\in O(n) \end{aligned}$$

3

$$f(n) = n^2 + 3n^3 \in \Theta(n^3)$$

$$\begin{aligned} n^2 + 3n^3 &\in O(n^3) \\ n^2 + 3n^3 &\leq cn^3 \\ n^2 + 3n^3 &\leq 2n^3 + 3n^3 \\ n^2 + 3n^3 &\leq 5n^3 \\ c &= 5 \\ f(n) &\in O(n^3) \end{aligned}$$

$$\begin{aligned} n^2 + 3n^3 &\in \Omega(n^3) \\ n^2 + 3n^3 &\geq cn^3 \\ n^2 + 3n^3 &\geq 3n^3 \\ c &= 3 \\ f(n) &\in \Omega(n^3) \end{aligned}$$

$$f(n) \in \Theta(n^3) = O(n^3) \cap \Omega(n^3)$$