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CUS1188
Problem Set 3

1

$T(1) = 1$
 $T(n) = 2T(\frac{n}{2}) + 1000n \quad \forall n \geq 2$
 $T(n) \in O(n \log n)$, thus, $T(n) \leq cn \log n$
 Base case: $n = 2$

$$T(2) = 2(1) + 1000(2) = 2002, (2) \log(2) = 2c$$

Assumption that statement is true for $k, k < n$

$$T(k) \leq ck \log k$$

Induction

$$\begin{aligned} T(n) &= 2T(\frac{n}{2}) + 1000n \leq 2c\frac{n}{2} \log \frac{n}{2} + 1000n = \\ &cn \log \frac{n}{2} + 1000n = \\ &cn \log n - cn \log 2 + 1000n = \\ &cn \log n - cn + 1000n = \\ &cn \log n - (cn - 1000n) \leq cn \log n \end{aligned}$$

2

$T(1) = 1$
 $T(n) = 7T(\frac{n}{2}) + 18n^2 \quad \forall n \geq 2$
 $T(n) \in O(n^2 \log n)$, thus, $T(n) \leq cn^2 \log n$
 Base case: $n=2$

$$T(2) = 7(1) + 18(4) = 79, \quad c(2^2) \log(2) = 4c$$

Assumption that statement is true for $k, k < n$

$$T(k) \leq ck^2 \log k$$

Induction

$$\begin{aligned} T(n) &= 7T(\frac{n}{2}) + 18n^2 \leq 7c(\frac{n}{2})^2 \log \frac{n}{2} + 18n^2 = \\ &\frac{7}{4}cn^2 \log \frac{n}{2} + 18n^2 = \\ &\frac{7}{4}cn^2 \log n - \frac{7}{4}cn^2 \log 2 + 18n^2 = \\ &\frac{7}{4}cn^2 \log n - \frac{7}{4}cn^2 + 18n^2 = \\ &\frac{7}{4}cn^2 \log n - \frac{7}{4}cn^2 + 18n^2 - \frac{3}{4}cn^2 \log n + \frac{3}{4}cn^2 \log n = \\ &cn^2 \log n - \frac{7}{4}cn^2 + 18n^2 + \frac{3}{4}cn^2 \log n = \\ &\frac{7}{4}cn^2 \log n - (\frac{7}{4}cn^2 - 18n^2) \leq cn^2 \log n \end{aligned}$$

3

$$T(1) = 3$$

$$T(n) = T(\frac{n}{2}) + n + 2 \quad \forall n \geq 2$$

$$T(n) \in O(n \log n), \text{ thus, } T(n) \leq cn \log n$$

Base case: $n=2$

$$T(2) = 5 \quad , \quad c(2)(\log 2) = 2c$$

Assumption that statement is true for k , $k < n$

$$T(k) = ck \log k$$

Induction

$$T(n) = T(\frac{n}{2}) + n + 2 \leq c \frac{n}{2} \log \frac{n}{2} + n + 2 =$$

$$\frac{1}{2}cn \log n - \frac{1}{2}cn \log 2 + n + 2 =$$

$$\frac{1}{2}cn \log n + \frac{1}{2}cn + n + 2 =$$

$$\frac{1}{2}cn \log n + \frac{1}{2}cn + n + 2 + \frac{1}{2}cn \log n - \frac{1}{2}cn \log n =$$

$$cn \log n + \frac{1}{2}cn + n + 2 - \frac{1}{2}cn \log n =$$

$$cn \log n - (\frac{1}{2}cn \log n - \frac{1}{2}cn - n - 2) \leq cn \log n$$