Newton's Iteration using Divide and Conquer

Presented:

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Background

The problem: How can we calculate the roots of a function?

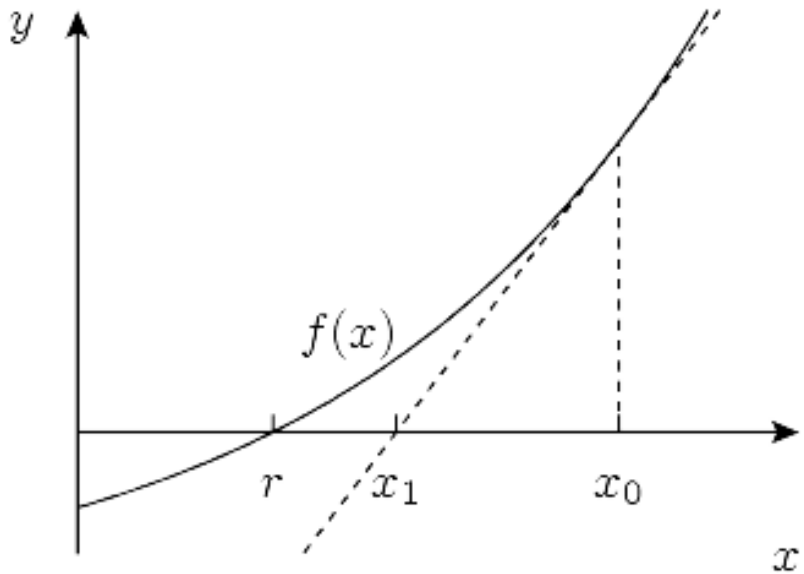
Newton's Iteration (also known as Newton's Method or the Newton-Raphson Method) is a numerical analysis method of approximating the roots of a function.

Algorithm

1. Start with a given estimate, x_0.
2. Find the tangent line of the function at x_0.
3. The point at which this tangent line intersects with the x-axis becomes the next estimate, x_1.
4. Repeat the process until the estimate starts to converge.

This process can be described with the following equation:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$



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Deriving the equation

- The slope of the tangent line at (x0, f(x0)) can be expressed as the following equation: f'(x0)=f(x0) – 0 / x0-x1, obtained by inserting the points (x0, f(x0)) and (x1, 0) into the slope formula m = y2-y1 / x2-x1.
- Solve the slope formula for x1:
- (x0-x1) f'(x0)=f(x0) - 0
- x0-x1=f(x0)f'(x1)
- x1=x0-f(x0)f'(x0)
- If we derive the equation for x2, we will end up with x2=x1-f(x1)f'(x1)and so on for x3,x4…. The sequence can be summarized with the Newton Iteration equation.

Example

Code

Time Complexity

Issues

Example

Example Content.