Report: Neural Network Approximation of the Runge Function and Its Derivative

111652042 顏友君

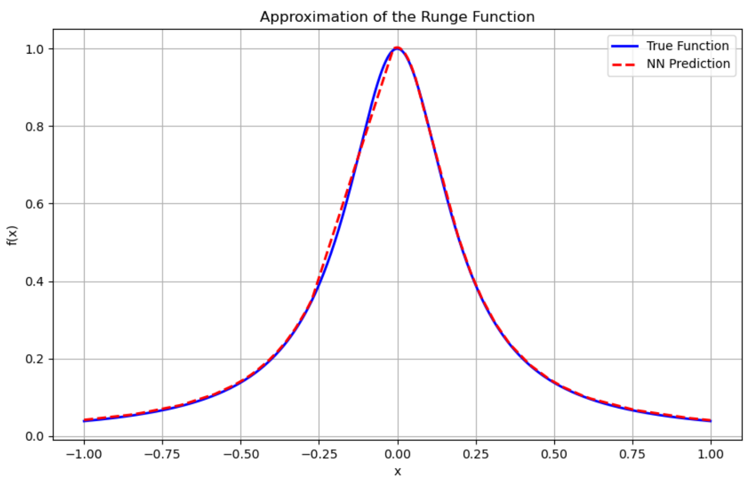
Methodology

This study uses a neural network to simultaneously approximate the Runge function, $f(x) = \frac{1}{1+25x^2}$, and its derivative, $f'(x) = \frac{-50x}{(1+25x^2)^2}$. We trained the model with a **dual-component loss function** composed of a "function loss" and a "derivative loss" to simultaneously optimize its fit to both the function's values and its slopes.

Results and Discussion

1. Function Approximation

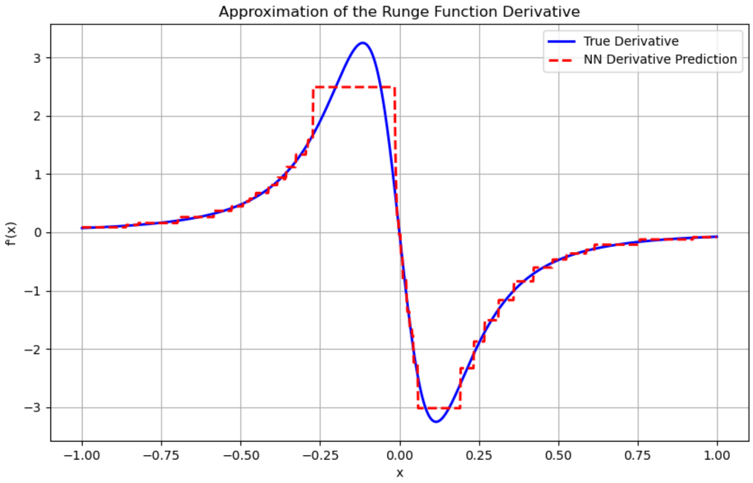
The figure below shows the neural network's predicted curve for the Runge function, which **matches the true function with high accuracy**. This demonstrates the model's exceptional ability to learn the function itself.



2. Derivative Approximation

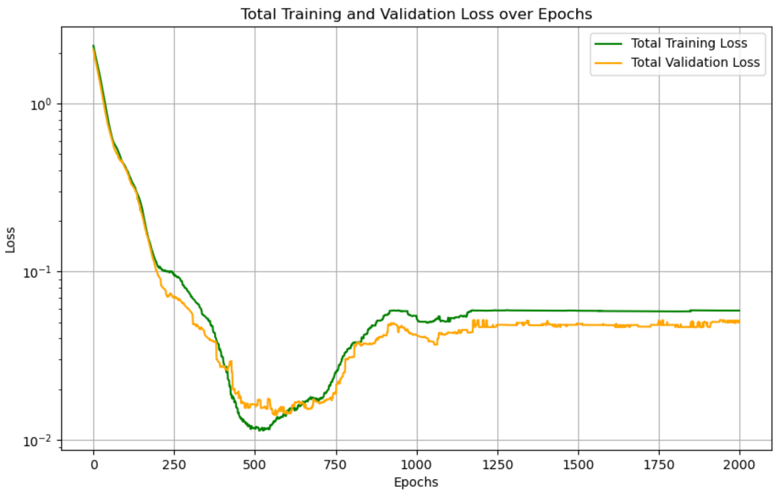
The figure below displays the model's derivative prediction. While the overall trend aligns with the true derivative, the curve shows a step-like pattern, particularly where the derivative changes most rapidly. This indicates that the model still faces challenges in handling sharp changes in slope, likely due to the properties of the

ReLU activation function.



3. Training Performance and Error Analysis

The figure below shows the total training and validation loss during the training process. Both losses decreased steadily, confirming that the model learned effectively without overfitting.



The final errors are as follows:

Function MSE：0.000087

Derivative MSE：0.056801

Although the derivative error is higher, the overall results demonstrate that this dual-loss function method is effective for training a neural network to simultaneously learn both a function and its derivative.