2025 Machine Learning Assignment 2_P2

111652042 顏友君

Report: Neural Network Approximation of the Runge Function

This report details the use of a neural network to approximate the Runge function:

$$f(x) = \frac{1}{1 + 25x^2}$$

over the interval $[-1,1]$.

1. Methodology

A dataset was created by sampling 1000 points from the function. We used a simple feedforward neural network with two hidden layers (64 neurons each, ReLU activation) to learn the function's behavior. The model was trained using the **Adam optimizer** and the **Mean Squared Error (MSE)** loss function for 1000 epochs.

2. Results and Discussion

Prediction vs. True Function

The neural network's prediction (dashed red line) closely matches the true function (solid blue line). The model successfully learned the function's complex shape, though slight discrepancies can be seen at the interval boundaries.

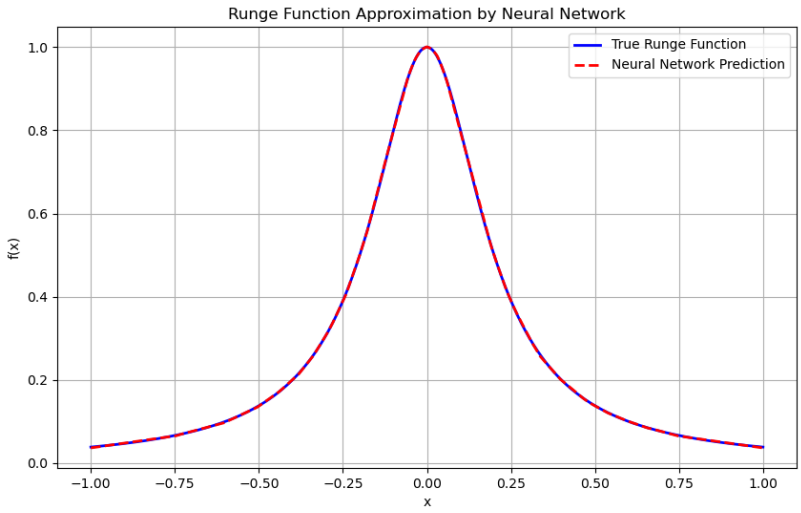

Fig1. Runge Function Approximation by Neural Network

Loss Curves

Both the training and validation loss curves decreased consistently, indicating that the model was learning effectively and **not overfitting**.

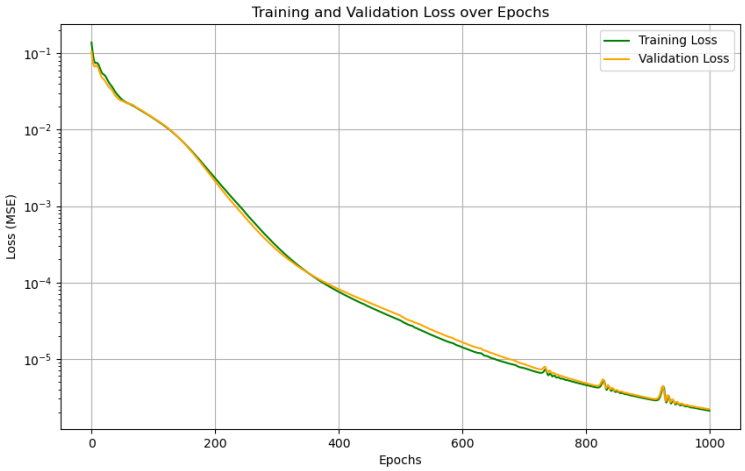

Fig2. Training and Validation Loss over Epochs

Error Analysis

- **Mean Squared Error (MSE)**: 0.000002
- **Max Error**: 0.005744

The low MSE confirms high average accuracy. The Max Error shows the largest deviation, which occurred at the edges of the interval, a common challenge for this function. Overall, the results demonstrate the neural network's effectiveness in function approximation.

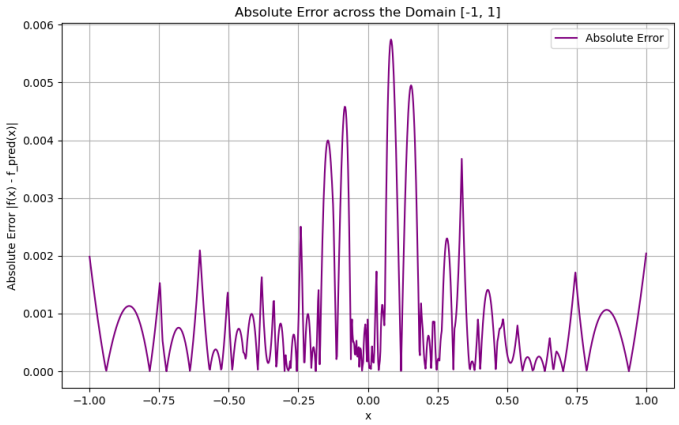

Fig3. Absolute Error across the Domain [-1, 1]