Energy-Efficient Subcarrier Assignment and Power Allocation in OFDMA Systems With Max-Min Fairness Guarantees

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***Abstract*—In next-generation wireless networks, energy efficiency optimization needs to take individual link fairness into account. In this paper, we investigate a max-min energy efficiency-optimal problem (MEP) to ensure fairness among links in terms of energy efficiency in OFDMA systems. In particular, we maximize the energy efficiency of the worst-case link subject to the rate requirements, transmit power, and subcarrier assignment constraints. Due to the nonsmooth and mixed combinatorial features of the formulation, we focus on low-complexity sub-optimal algorithms design. Using a generalized fractional programming theory and the Lagrangian dual decomposition, we first propose an iterative algorithm to solve the problem. We then devise algorithms to separate the subcarrier assignment and power allocation to further reduce the computational cost. Our simulation results verify the convergence performance and the fairness achieved among links, and particularly reveal a new tradeoff between the network energy efficiency and fairness by comparing the MEP with the existing algorithms.**

***Index Terms*—Energy efficiency, subcarrier assignment, power allocation, max-min fairness, OFDMA.**

I. Introduction

ORTHOGONAL frequency division multiple access (OFDMA) has been adopted as the leading multiaccess technology in 4G broadband wireless communication systems, such as WiMAX [1] and the 3GPP LTE/LTE-A [2]. In particular, by exploiting multiuser diversity and adaptively performing resource allocation, e.g., subcarrier assignment and power allocation, OFDMA can significantly enhance the overall system performance such as spectral efficiency [3].

There have been extensive researches on dynamic resource allocation in multiuser OFDMA systems in the literature, and they belong mostly to three major categories:

- Rate adaptive as in [4], [5], which aims to maximize the sum rates subject to a total transmit power constraint.
- Margin adaptive as in [6], [7], which aims to minimize the total transmit power subject to users' rate constraints.
- Utility adaptive as in [8], [9], which aims to maximize a utility subject to a total transmit power constraint.

Due to the recent interest in energy-efficient communications to reduce energy consumption and greenhouse gas emission [10]–[12], energy efficiency has been proposed as a new metric to measure how efficiently the energy is consumed. This is defined as the ratio of the achieved sum rates to the total energy consumption [10]–[12]. There are also other considerations that can affect the amount of energy needed to meet the quality-of-service requirements of the users such as the utility of link transmissions [8], [9]. Energy efficiency, as a single metric, conveniently combines the spectral efficiency [4], [5] and energy consumption [6], [7] together.

Some recent works have begun to explore the energy efficiency optimization issues by devising resource allocation strategies to ensure prudent use of the scarce radio resources. A sum energy efficiency maximization problem was addressed in [14], where the authors proposed a heuristic scheduling metric to reject users when the problem becomes infeasible. Under the power budget constraints, [15] revealed that OFDMA is the optimal access scheme for single-input single-output OFDM downlink systems in terms of energy efficiency. The authors in [16], [17] studied the tradeoff between energy efficiency and spectral efficiency. The authors in [18] took into account the base station circuit and user-equipment circuit energy consumption. The authors in [19], [20] studied energy efficiency maximization with various system or link tradeoffs.

Thus far, [14]–[19] only focused on the energy efficiency of the entire system, and did not consider the energy efficiency of individual links. We call this the network energy efficiency-optimal problems (NEPs) in the paper. It can happen that the network energy efficiency improves at the cost of the energy efficiency of individual links especially in bad channel conditions for some links. This can lead to severe unfairness among

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links in terms of energy efficiency. There are related works that address the energy-efficiency of individual links, e.g., in [20]–[22], but they do not provision for the energy efficiency of the worst-case links. In this paper, we formulate and study the max-min energy efficiency-optimal problem (MEP) that specifically maximizes the energy efficiency of the worst-case link in an uplink OFDMA system. We also take into account the constraints of rate requirements, transmit powers, and the subcarrier assignment. To reduce the computational cost to solve this problem, we propose suboptimal algorithms with good performance and low complexity.

The main contributions of this work are as follows:

- Different from existing works in OFDMA systems [14]–[19], we formulate and study an energy-efficient optimization problem to maximize the worst-case link energy efficiency.
- Using a generalized fractional programming theory and the Lagrangian dual decomposition, we propose a simplified iterative algorithm to solve this mixed-integer nonlinear programming problem.
- To further reduce the computational cost, we separate subcarrier assignment and power allocation. After the subcarrier assignment, the power allocation is optimized by leveraging decomposability to solve simpler subproblems.
- We provide extensive simulation results to compare the MEP with the existing algorithms in OFDMA systems. More importantly, the obtained results show a new tradeoff between the network energy efficiency and fairness, similarly to the tradeoff between the sum rates and fairness [25], [26].

The remainder of this paper is organized as follows. In Section II, we describe the system model and the optimization formulation. The iterative subcarrier assignment and power allocation algorithm is described in Section III. In Section IV, we solve the problem by separating the subcarrier assignment and power allocation. We evaluate the performance of the proposed algorithms by simulations in Section V. Finally, we conclude the paper in Section VI.

II. System Scenario and Problem Formulation

In this section, we first describe the system model and problem parameters, and then formulate an optimization problem to maximize the energy efficiency of the worst-case link with both subcarrier assignment and power allocation.

A. Description of the Concerned Scenario

We consider a single-cell uplink OFDMA communication system with K active users and N subcarriers, in which both the base station and the users are equipped with a single antenna[1]. There are thus K different communication links from the K active users to the base station. The total bandwidth is B and thus each subcarrier has a bandwidth of B/N. As in [4]–[9], [14]–[20], we assume that subcarriers are exclusively assigned, i.e., each subcarrier can be assigned to at most one link, to avoid interference among links.

Let $\boldsymbol{P} = (P_{k,n})$, where $P_{k,n}$ denotes the transmit power of link k on subcarrier n. Then, the transmit rate of link k on subcarrier n (in unit of bit/s/Hz), $r_{k,n}$, is given by

$$r_{k,n} = \log_2\left(1 + \frac{P_{k,n}|h_{k,n}|^2}{N_0 \frac{B}{N}}\right) = \log_2(1 + P_{k,n}g_{k,n}) \quad (1)$$

where $g_{k,n} = |h_{k,n}|^2/\left(N_0\frac{B}{N}\right)$, N_0 is the single-sided spectral density of additive white Gaussian noise, and $h_{k,n}$ is the frequency response of link k on subcarrier n, which is assumed to be accurately known at the transmitter [4]–[9], [14]–[20].

Accordingly, the sum transmit rates and the total transmit power on link k are respectively given by

$$R_k(\boldsymbol{\rho}, \boldsymbol{P}) = \sum_{n=1}^{N} \rho_{k,n} r_{k,n}, \forall k \quad (2)$$

$$P_k(\boldsymbol{\rho}, \boldsymbol{P}) = \sum_{n=1}^{N} \rho_{k,n} P_{k,n}, \forall k. \quad (3)$$

In (2) and (3), $\boldsymbol{\rho} = (\rho_{k,n})$, where $\rho_{k,n}$ is an indicator variable that is 1 if subcarrier n is allocated to link k and 0 otherwise.

Furthermore, the power consumption of link k is modeled as

$$\mathrm{PC}_k(\boldsymbol{\rho}, \boldsymbol{P}) = \xi_k P_k(\boldsymbol{\rho}, \boldsymbol{P}) + P_k^C, \forall k \quad (4)$$

where $\xi_k \geq 1$ and $P_k^C \geq 0$ are constants that model the inefficiency of the power amplifier and the circuit power consumption at terminal k, respectively [14], [15], [28], [29].

From (2) and (4), the sum transmit rate and aggregate power consumption of the uplink system are respectively given by

$$R_{\mathrm{tot}}(\boldsymbol{\rho}, \boldsymbol{P}) = \sum_{k=1}^{K} R_k(\boldsymbol{\rho}, \boldsymbol{P}) \quad (5)$$

$$\mathrm{PC}_{\mathrm{tot}}(\boldsymbol{\rho}, \boldsymbol{P}) = \sum_{k=1}^{K} \mathrm{PC}_k(\boldsymbol{\rho}, \boldsymbol{P}). \quad (6)$$

Furthermore, the link energy efficiency and the network energy efficiency [14]–[20] are respectively defined as

$$\eta_k^{\mathrm{EE}}(\boldsymbol{\rho}, \boldsymbol{P}) = \frac{R_k(\boldsymbol{\rho}, \boldsymbol{P})}{\mathrm{PC}_k(\boldsymbol{\rho}, \boldsymbol{P})}, \forall k \quad (7)$$

$$\eta^{\mathrm{EE}}(\boldsymbol{\rho}, \boldsymbol{P}) = \frac{R_{\mathrm{tot}}(\boldsymbol{\rho}, \boldsymbol{P})}{\mathrm{PC}_{\mathrm{tot}}(\boldsymbol{\rho}, \boldsymbol{P})}. \quad (8)$$

B. Problem Formulation

To maximize the energy efficiency of the worst-case link and still satisfy the basic rate requirements of all the users, we jointly optimize the subcarrier assignment and power allocation

[1] It is straightforward to extend our work to multiple transmit/receive antennas scenarios by modifying the transmit rate expression (1) [27].

to maximize the energy efficiency of the worst-case link by the following optimization problem[2]

$$\begin{aligned}\max_{\boldsymbol{\rho},\boldsymbol{P}}\min_{k}\quad & \eta_k^{\mathrm{EE}}(\boldsymbol{\rho},\boldsymbol{P})\\ \text{s.t.}\quad & \text{C1:}\ \sum_{n=1}^{N}\rho_{k,n}r_{k,n}\geq R_k^{\mathrm{req}},\forall k\\ & \text{C2:}\ \sum_{k=1}^{K}\rho_{k,n}\leq 1,\forall n\\ & \text{C3:}\ \sum_{n=1}^{N}\rho_{k,n}P_{k,n}\leq P_k^{\max},\forall k\\ & \text{C4:}\ P_{k,n}\geq 0,\forall k,n\\ & \text{C5:}\ \rho_{k,n}\in\{0,1\},\ \forall k,n.\end{aligned} \tag{9}$$

In (9), R_k^{req} and $P_k^{\max}$ are the required transmit rate and the maximum transmit power of user k, respectively. C1 guarantees the rate requirements of users. C2 and C5 indicate that each subcarrier can be allocated to at most one user. C3 is the peak transmit power constraint.

Remark 1: We point out that (9) might be infeasible without appropriate parameter setting on R_k^{req} and $P_k^{\max}$. In this paper, we assume that (9) is always feasible as in [4]–[9], [15]–[20]. In addition, to avoid the issue of infeasibility, it is necessary to introduce admission control in addition to the subcarrier assignment and power allocation into the formulation, but this is beyond the scope of this paper.

Remark 2: Our formulation can be readily extended to multi-cell scenarios, but this will impose challenges on algorithm design from two aspects. First, mutual interference among links due to the frequency reuse leads to nonconvex problems that are hard to solve even when the optimal subcarrier assignment is given [30]. Second, the design of algorithms necessitates that the radio resource management module collects global knowledge, such as channel gains of communication and interference links, if centralized methods are adopted.

III. Iterative Subcarrier Assignment and Power Allocation Algorithm

In this section, we solve (9) by generalized fractional programming and the Lagrangian dual decomposition.

A. Iterative Algorithm Design

Since (9) involves both continuous variables $P_{k,n}$ and binary variables $\rho_{k,n}$, this mixed-integer nonlinear programming problem is in general hard to solve. Even if we relax $\rho_{k,n}$ to [0,1], (9) is still nonconvex owing to the nonconvexity of the objective function. We first exploit the problem structure of $\eta_k^{\mathrm{EE}}(\boldsymbol{\rho},\boldsymbol{P})$ to reformulate (9).

Without loss of generality, we assume that $R_k(\boldsymbol{\rho},\boldsymbol{P})>0$ and $\mathrm{PC}_k(\boldsymbol{\rho},\boldsymbol{P})>0$. For notational simplicity, we denote the feasible region of C1–C5 in (9) by $\mathscr{F}$. Let

$$\eta_{\mathrm{EE}}^{\mathrm{opt}}=\max_{\{\boldsymbol{\rho},\boldsymbol{P}\}\in\mathscr{F}}\min_k\frac{R_k(\boldsymbol{\rho},\boldsymbol{P})}{\mathrm{PC}_k(\boldsymbol{\rho},\boldsymbol{P})}=\min_k\frac{R_k(\boldsymbol{\rho}^{\mathrm{opt}},\boldsymbol{P}^{\mathrm{opt}})}{\mathrm{PC}_k(\boldsymbol{\rho}^{\mathrm{opt}},\boldsymbol{P}^{\mathrm{opt}})} \tag{10}$$

where $\{\boldsymbol{\rho}^{\mathrm{opt}},\boldsymbol{P}^{\mathrm{opt}}\}$ and $\eta_{\mathrm{EE}}^{\mathrm{opt}}$ are the optimal solution and value of (9), respectively.

To solve (9), we give the following proposition. The proof uses a standard result in the generalized fractional programming theory [31].

Proposition 1: The optimal solution $\{\boldsymbol{\rho}^{\mathrm{opt}},\boldsymbol{P}^{\mathrm{opt}}\}$ is achieved if and only if

$$\begin{aligned}&\max_{\{\boldsymbol{\rho},\boldsymbol{P}\}\in\mathscr{F}}\min_k\left[R_k(\boldsymbol{\rho},\boldsymbol{P})-\eta_{EE}^{opt}PC_k(\boldsymbol{\rho},\boldsymbol{P})\right]\\ &\quad=\min_k\left[R_k(\boldsymbol{\rho}^{opt},\boldsymbol{P}^{opt})-\eta_{EE}^{opt}PC_k(\boldsymbol{\rho}^{opt},\boldsymbol{P}^{opt})\right]\\ &\quad=0.\end{aligned} \tag{11}$$

Theorem 1 indicates that we can solve (9) via its equivalent problem (11). However, $\eta_{\mathrm{EE}}^{\mathrm{opt}}$ is generally unknown in advance. From [31], the optimal solution of (9) can be obtained by solving (11) with $\eta_{\mathrm{EE}}^{\mathrm{opt}}$ replaced by an update parameter η. The details of the procedure is shown in Algorithm 1. Note that the optimization problem that needs to be solved at line 3 in Algorithm 1 for a given η (e.g., η^i at iteration i) is

$$\begin{aligned}\max_{\boldsymbol{\rho},\boldsymbol{P}}\min_k\quad & R_k(\boldsymbol{\rho},\boldsymbol{P})-\eta\mathrm{PC}_k(\boldsymbol{\rho},\boldsymbol{P})\\ \text{s.t.}\quad & \text{C1, C2, C3, C4, C5.}\end{aligned} \tag{12}$$

Algorithm 1 Iterative Subcarrier Assignment and Power Allocation Algorithm

1: **Initialization**
- Set the maximum iteration number $i_{\max}$ and error tolerance threshold ε.
- Set iteration index $i=0$ and the maximum energy efficiency $\eta^i=0$.

2: **repeat**
3: Solve (12) for the given η^i to obtain $\{\boldsymbol{\rho}^i,\boldsymbol{P}^i\}$.
4: **if** $\left|\min_k[R_k(\boldsymbol{\rho}^i,\boldsymbol{P}^i)-\eta^i\mathrm{PC}_k(\boldsymbol{\rho}^i,\boldsymbol{P}^i)]\right|<\varepsilon$ **then**
5: $\{\boldsymbol{\rho}^{\mathrm{opt}},\boldsymbol{P}^{\mathrm{opt}}\}=\{\boldsymbol{\rho}^i,\boldsymbol{P}^i\}$.
6: $\eta_{\mathrm{EE}}^{\mathrm{opt}}=\min_k\frac{R_k(\boldsymbol{\rho}^i,\boldsymbol{P}^i)}{\mathrm{PC}_k(\boldsymbol{\rho}^i,\boldsymbol{P}^i)}$.
7: break.
8: **else**
9: Set $\eta^{i+1}=\min_k\frac{R_k(\boldsymbol{\rho}^i,\boldsymbol{P}^i)}{\mathrm{PC}_k(\boldsymbol{\rho}^i,\boldsymbol{P}^i)}$.
10: $i=i+1$.
11: **end if**
12: **until** $i>i_{\max}$.

[2]Note that our analysis can be readily applied to downlink OFDMA systems [20]–[23]. Specifically, we can simply change C3 in (9) to C3′ : $\sum_{k=1}^{K}\sum_{n=1}^{N}\rho_{k,n}P_{k,n}\leq P_{\max}$, where $P_{\max}$ denotes the maximum transmit power of the base station.

B. Suboptimal Subcarrier Assignment and Power Allocation

We in this subsection describe how to solve (12). First, we introduce a new variable φ to transform the originally nonsmooth optimization problem (12) (due to the nonsmoothness of the objective function) to a smooth one. Specifically, (12) can be equivalently recast as

$$\begin{aligned} \max_{\boldsymbol{\rho},\boldsymbol{P},\varphi} \quad & \varphi \\ \text{s.t.} \quad & \text{C1, C2, C3, C4, C5} \\ & \text{C6:} \quad R_k(\boldsymbol{\rho},\boldsymbol{P}) - \eta \text{PC}_k(\boldsymbol{\rho},\boldsymbol{P}) \geq \varphi, \forall k. \end{aligned} \tag{13}$$

Regarding the property of φ, we present the following proposition, which follows directly from Proposition 1 and will be used in (26).

Proposition 2: For all feasible $\boldsymbol{\rho}$ and $\boldsymbol{P}$, $\varphi \geq 0$ when $0 \leq \eta \leq \eta_{\text{EE}}^{\text{opt}}$.

However, solving (13) is still hard for a given η. In fact, it is necessary to exhaustively search all the K^N cases, and this can be prohibitive in term of computational complexity. Thus, fast-convergent suboptimal algorithms are preferred in practice.

We next apply the Lagrangian dual decomposition to (13), which is a common approach to tackle resource allocation problems in OFDMA systems with different objectives and constraints [4], [16], [17]. Specifically, we relax $\rho_{k,n}$ to [0,1] (this can be interpreted as a sharing factor of link k using the subcarrier n), introduce a new variable $s_{k,n} = \rho_{k,n} P_{k,n}$, and let $\boldsymbol{S} = (s_{k,n})$. We then rearrange (13) as

$$\begin{aligned} \max_{\boldsymbol{\rho},\boldsymbol{S},\varphi} \quad & \varphi \\ \text{s.t.} \quad & \text{C1:} \quad \sum_{n=1}^{N} \rho_{k,n} \log_2\left(1+\frac{s_{k,n}g_{k,n}}{\rho_{k,n}}\right) \geq R_k^{\text{req}}, \forall k \\ & \text{C2:} \quad \sum_{k=1}^{K} \rho_{k,n} \leq 1, \forall n \\ & \text{C3:} \quad \sum_{n=1}^{N} s_{k,n} \leq P_k^{\max}, \forall k \\ & \text{C4:} \quad 0 \leq \rho_{k,n} \leq 1, \forall k, n \\ & \text{C5:} \quad s_{k,n} \geq 0, \forall k, n \\ & \text{C6:} \quad \sum_{n=1}^{N} \rho_{k,n} \log_2\left(1+\frac{s_{k,n}g_{k,n}}{\rho_{k,n}}\right) \\ & \qquad - \eta\left(\xi_k \sum_{n=1}^{N} s_{k,n} + P_k^C\right) \geq \varphi, \forall k. \end{aligned} \tag{14}$$

In particular, we set $\rho_{k,n}\log_2\left(1+\frac{s_{k,n}g_{k,n}}{\rho_{k,n}}\right) = 0$ when $\rho_{k,n} = 0$ in (14). In addition, we have the following proposition, proved in Appendix A, regarding the feature of (14).

Proposition 3: Optimization problem (14) is jointly convex in $\boldsymbol{S}$, $\boldsymbol{\rho}$, and φ.

Due to the convexity of (14), we can obtain its optimal solutions by assuming a zero duality gap and using the Lagrangian dual decomposition. The Lagrangian of (14) is given by (15), shown at the bottom of the page, where $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_K) \succeq 0$, $\boldsymbol{\nu} = (\nu_1, \ldots, \nu_N) \succeq 0$, $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_K) \succeq 0$, and $\boldsymbol{\gamma} = (\gamma_1, \ldots, \gamma_K) \succeq 0$ are the Lagrange multipliers corresponding to C1, C2, C3, and C6 in (14), respectively.

1) Resource Allocation Update: The Lagrange dual function is

$$D(\boldsymbol{\beta},\boldsymbol{\mu},\boldsymbol{\nu},\boldsymbol{\gamma}) = \max_{\{\boldsymbol{\rho},\boldsymbol{S},\varphi\}\in\{\text{C4,C5}\}} L(\boldsymbol{\rho},\boldsymbol{S},\varphi,\boldsymbol{\beta},\boldsymbol{\mu},\boldsymbol{\nu},\boldsymbol{\gamma}). \tag{16}$$

From (16), we maximize $L(\boldsymbol{\rho},\boldsymbol{S},\varphi,\boldsymbol{\beta},\boldsymbol{\mu},\boldsymbol{\nu},\boldsymbol{\gamma})$ for a given $\boldsymbol{\beta}$, $\boldsymbol{\mu}$, $\boldsymbol{\nu}$, and $\boldsymbol{\gamma}$ by solving the following two sub-problems.

1. Adaptive subcarrier assignment and power allocation

$$\begin{aligned} \max \sum_{k=1}^{K} \beta_k & \left[\sum_{n=1}^{N} \rho_{k,n} \log_2\left(1+\frac{s_{k,n}g_{k,n}}{\rho_{k,n}}\right)\right] \\ & - \sum_{n=1}^{N} \nu_n \sum_{k=1}^{K} \rho_{k,n} - \sum_{k=1}^{K} \mu_k \sum_{n=1}^{N} s_{k,n} \\ & + \sum_{k=1}^{K} \gamma_k \left\{\sum_{n=1}^{N}\left[\rho_{k,n}\log_2\left(1+\frac{s_{k,n}g_{k,n}}{\rho_{k,n}}\right) - \eta\xi_k s_{k,n}\right]\right\} \\ \text{s.t.} \quad & \text{C4:} \quad 0 \leq \rho_{k,n} \leq 1, \forall k, n \\ & \text{C5:} \quad s_{k,n} \geq 0, \forall k, n. \end{aligned} \tag{17}$$

Denote the optimal solutions of (17) by $\rho_{k,n}^*$ and $s_{k,n}^*$. Then, the Karush-Kuhn-Tucker (KKT) conditions specify that

$$\frac{\partial L(\boldsymbol{\rho},\boldsymbol{S},\varphi,\boldsymbol{\beta},\boldsymbol{\mu},\boldsymbol{\nu},\boldsymbol{\gamma})}{\partial s_{k,n}^*} \begin{cases} = 0, & s_{k,n}^* > 0 \\ < 0, & s_{k,n}^* = 0 \end{cases}, \forall k, n \tag{18}$$

$$\frac{\partial L(\boldsymbol{\rho},\boldsymbol{S},\varphi,\boldsymbol{\beta},\boldsymbol{\mu},\boldsymbol{\nu},\boldsymbol{\gamma})}{\partial \rho_{k,n}^*} \begin{cases} < 0, & \rho_{k,n}^* = 0 \\ = 0, & 0 < \rho_{k,n}^* < 1 \\ > 0, & \rho_{k,n}^* = 1 \end{cases}, \forall k, n. \tag{19}$$

♦ Optimal power allocation: For a given subcarrier assignment $\boldsymbol{\rho}$, differentiating $L(\boldsymbol{\rho},\boldsymbol{S},\varphi,\boldsymbol{\beta},\boldsymbol{\mu},\boldsymbol{\nu},\boldsymbol{\gamma})$ in (17) with respect to $s_{k,n}$ and substituting it into (18) yields

$$P_{k,n}^* = \frac{s_{k,n}^*}{\rho_{k,n}} = \left[\frac{\beta_k+\gamma_k}{(\mu_k+\eta\xi_k\gamma_k)\ln 2} - \frac{1}{g_{k,n}}\right]^+, \forall k, n. \tag{20}$$

$$\begin{aligned} & L(\boldsymbol{\rho},\boldsymbol{S},\varphi,\boldsymbol{\beta},\boldsymbol{\mu},\boldsymbol{\nu},\boldsymbol{\gamma}) \\ & = \varphi + \sum_{k=1}^{K}\beta_k\left[\sum_{n=1}^{N}\rho_{k,n}\log_2\left(1+\frac{s_{k,n}g_{k,n}}{\rho_{k,n}}\right) - R_k^{\text{req}}\right] + \sum_{n=1}^{N}\nu_n\left(1-\sum_{k=1}^{K}\rho_{k,n}\right) + \sum_{k=1}^{K}\mu_k\left(P_k^{\max}-\sum_{n=1}^{N}s_{k,n}\right) \\ & \quad + \sum_{k=1}^{K}\gamma_k\left\{\sum_{n=1}^{N}\left[\rho_{k,n}\log_2\left(1+\frac{s_{k,n}g_{k,n}}{\rho_{k,n}}\right) - \eta\xi_k s_{k,n}\right] - \eta P_k^C - \varphi\right\} \end{aligned} \tag{15}$$

Note that $[x]^+ \triangleq \max[0, x]$. Observe that the optimal power allocation (20) is the standard water-filling algorithm.

⧫ Optimal subcarrier assignment: On the other hand, after obtaining the optimal power allocation, we can derive the optimal subcarrier assignment as follows. By taking the partial derivative of $L(\boldsymbol{\rho}, \boldsymbol{S}, \varphi, \boldsymbol{\beta}, \boldsymbol{\mu}, \boldsymbol{\nu}, \boldsymbol{\gamma})$ in (17) with respect to $\rho_{k,n}$, we have

$$\frac{\partial L(\boldsymbol{\rho}, \boldsymbol{S}, \varphi, \boldsymbol{\beta}, \boldsymbol{\mu}, \boldsymbol{\nu}, \boldsymbol{\gamma})}{\partial \rho_{k,n}} = (\beta_k+\gamma_k)\left[\log_2\left(1+\frac{s_{k,n}g_{k,n}}{\rho_{k,n}}\right) - \frac{s_{k,n}g_{k,n}}{(\rho_{k,n}+s_{k,n}g_{k,n})\ln 2}\right] - \nu_n \tag{21}$$

for all k and n.

Substituting the optimal power allocation (20) into (21) and exploiting (19), we further obtain

$$\rho^*_{k,n} = \begin{cases} 1, & \nu_n < H_{k,n} \\ 0, & \nu_n > H_{k,n} \end{cases} \tag{22}$$

where

$$H_{k,n} = (\beta_k + \gamma_k)\left\{\left[\log_2\left(g_{k,n}\frac{\beta_k+\gamma_k}{(\mu_k+\eta\xi_k\gamma_k)\ln 2}\right)\right]^+ - \frac{1}{\ln 2}\left[1 - \frac{1}{g_{k,n}\frac{\beta_k+\gamma_k}{(\mu_k+\eta\xi_k\gamma_k)\ln 2}}\right]^+\right\}. \tag{23}$$

In general, for any given $\boldsymbol{\beta}$, $\boldsymbol{\mu}$, and $\boldsymbol{\gamma}$, $H_{k,n}$ among different links k cannot be the same for a given subcarrier n due to the mutually independent random channel conditions $g_{k,n}$. In particular, the authors in [32] proved that the event of two links having the same $H_{k,n}$ has Lebesgue-measure zero when the fading process has a continuous cumulative distribution function. Thus, the subcarrier assignment scheme can be given by:

$$\rho^*_{k(n),n} = 1, \quad \rho^*_{k,n} = 0, \quad \forall\, k \neq k(n), n \tag{24}$$

where $k(n) = \arg\max_{1\le k\le K} H_{k,n}$. If there are more than one links with the same $H_{k,n}$ on a given subcarrier, then we randomly choose a link and assign the subcarrier to it.

2. Adaptive φ Selection

$$\begin{aligned} \max \quad & \left(1-\sum_{k=1}^{K}\gamma_k\right)\varphi \\ \text{s.t.} \quad & 0 \le \varphi \le \sum_{n=1}^{N}\rho_{k,n}\log_2\left(1+\frac{s_{k,n}g_{k,n}}{\rho_{k,n}}\right) \\ & - \eta\left(\xi_k\sum_{n=1}^{N}s_{k,n} + P^C_k\right), \forall k. \end{aligned} \tag{25}$$

Observe that, from (25), the optimal solution of φ denoted by φ^* takes the following form

$$\varphi^* = \begin{cases} 0, & \text{if } \sum_{k=1}^{K}\gamma_k > 1 \\ G^*_{k,n}, & \text{if } \sum_{k=1}^{K}\gamma_k \le 1. \end{cases} \tag{26}$$

where

$$G^*_{k,n} = \min_k\left\{\sum_{n=1}^{N}\rho^*_{k,n}\log_2\left(1+\frac{s^*_{k,n}g_{k,n}}{\rho^*_{k,n}}\right) - \eta\left(\xi_k\sum_{n=1}^{N}s^*_{k,n}+P^C_k\right)\right\}. \tag{27}$$

Note that we get $\varphi^* = -\infty$ without the condition $\varphi \ge 0$ presented in Proposition 2 when $\sum_{k=1}^{K}\gamma_k > 1$.

2) Lagrange Multipliers Update: Observe that, from (20) and (24), we need to know $\boldsymbol{\beta}$, $\boldsymbol{\mu}$, and $\boldsymbol{\gamma}$ to allocate power and assign subcarriers. We address this as follows.

From (14), its dual problem is

$$\begin{aligned} \min \quad & D(\boldsymbol{\beta}, \boldsymbol{\mu}, \boldsymbol{\nu}, \boldsymbol{\gamma}) \\ \text{s.t.} \quad & \boldsymbol{\beta} \succeq 0, \boldsymbol{\mu} \succeq 0, \boldsymbol{\nu} \succeq 0, \boldsymbol{\gamma} \succeq 0. \end{aligned} \tag{28}$$

From (15) and (16), (28) is always convex as $D(\boldsymbol{\beta}, \boldsymbol{\mu}, \boldsymbol{\nu}, \boldsymbol{\gamma})$ is the maximum of linear functions with respect to $\boldsymbol{\beta}$, $\boldsymbol{\mu}$, $\boldsymbol{\nu}$, and $\boldsymbol{\gamma}$. We thus use the subgradient projection method to solve (28). We first give the following proposition, proved in Appendix B, to compute one of its subgradients.

Proposition 4: For the dual problem (28) with the primal defined in (14), a subgradient of $D(\boldsymbol{\beta}, \boldsymbol{\mu}, \boldsymbol{\nu}, \boldsymbol{\gamma})$ is given by

$$\Delta\beta_k = \sum_{n=1}^{N}\rho^*_{k,n}\log_2\left(1+\frac{s^*_{k,n}g_{k,n}}{\rho^*_{k,n}}\right) - R^{\text{req}}_k, \forall k \tag{29}$$

$$\Delta\mu_k = P^{\max}_k - \sum_{n=1}^{N}s^*_{k,n} \tag{30}$$

$$\Delta\gamma_k = \sum_{n=1}^{N}\left[\rho^*_{k,n}\log_2\left(1+\frac{s^*_{k,n}g_{k,n}}{\rho^*_{k,n}}\right) - \eta\xi_k s^*_{k,n}\right] - \eta P^C_k - \varphi \tag{31}$$

where $\rho^*_{k,n}$ and $s^*_{k,n}$ are the optimal solutions of (16) for a given $\boldsymbol{\beta}$, $\boldsymbol{\mu}$, and $\boldsymbol{\gamma}$.

Thus, the subgradient projection method for (28) is specified as

$$\beta_k(t+1) = [\beta_k(t) - \alpha(t)\Delta\beta_k(t)]^+ \tag{32}$$

$$\mu_k(t+1) = [\mu_k(t) - \lambda(t)\Delta\mu_k(t)]^+ \tag{33}$$

$$\gamma_k(t+1) = [\gamma_k(t) - \tau(t)\Delta\gamma_k(t)]^+ \tag{34}$$

where t is the iteration index and $\alpha(t)$, $\lambda(t)$, and $\tau(t)$ are sufficiently small positive step sizes. Typical step size rules are constant step size and square summable but not summable[3] [33]. In the paper, we choose the latter and set $\alpha(t) = \lambda(t) =$

[3]It refers to the step size that satisfies $\alpha(t) \ge 0, \sum_{t=1}^{\infty}\alpha(t)^2 < \infty$, $\sum_{t=1}^{\infty}\alpha(t) = \infty$. One example is $\alpha(t) = \frac{a}{b+t}$, where $a > 0$ and $b \ge 0$.

$\tau(t) = \frac{0.1}{t}$. We can set $\boldsymbol{\nu}$ to a constant value in practice. The whole procedure to solve (12) is summarized in Algorithm 2.

Algorithm 2 Suboptimal Subcarrier Assignment and Power Allocation Algorithm

Initialization
- Set $\boldsymbol{\beta}$, $\boldsymbol{\mu}$, $\boldsymbol{\gamma}$, α, λ, τ, $t_{\max}$, and δ.
- Set $t = 0$.

repeat
  Allocate transmit power $P_{k,n}$ from (20).
  Assign subcarrier $\rho_{k,n}$ from (24).
  Update $\boldsymbol{\beta}$, $\boldsymbol{\mu}$, and $\boldsymbol{\gamma}$ from (32), (33), and (34), respectively.
  $t = t + 1$.
  if $\|\boldsymbol{\beta}(t+1) - \boldsymbol{\beta}(t)\|_2 < \delta$, $\|\boldsymbol{\mu}(t+1) - \boldsymbol{\mu}(t)\|_2 < \delta$, and $\|\boldsymbol{\gamma}(t+1) - \boldsymbol{\gamma}(t)\|_2 < \delta$ **then**
    break.
  end if
until $t > t_{\max}$.

IV. Separate Subcarrier Assignment and Power Allocation Algorithms

To further reduce the complexity cost, we separate subcarrier assignment and power allocation to devise simpler algorithms.

A. Subcarrier Assignment

To determine the subcarrier assignment, we assume that power is equally distributed across all subcarriers. We propose Algorithm 3 to determine Ω_k, which denotes the set of subcarriers assigned to user k. Note that we denote the number of elements in Ω_k by $|\Omega_k|$. This algorithm runs in two stages.

In the first stage (lines 1-6), it aims to satisfy the basic rate requirements of users, i.e., R_k^{req} for all k. In each iteration of this stage, among the unsatisfied users, the user whose current transmit rate R_k deviates most from its target rate R_k^{req} is allowed to pick a new subcarrier from the remaining subcarriers. Then the subcarrier with the best channel condition from this particular user to the base station is assigned to the user. Thus, R_k^{req} for all the users are ensured after this stage.

In the second stage (lines 7–19), we attempt to improve the energy efficiency of the worst-cast link. In each iteration of this stage, we first choose the user with the minimum η_k^{EE} and find the subcarrier with the best channel condition from this user to the base station. We then assign the subcarrier to the user if the η_k^{EE} can be improved by this newly-found subcarrier. Otherwise, the algorithm terminates.

B. Power Allocation

After determining the subcarrier assignment Ω_k for all k, (9) can be decomposed into K subproblems due to the independent power constraints among links (see C3). Each of these subproblems optimizes power allocation in parallel to maximize the energy efficiency of individual links, which, say k, is given by

$$\begin{aligned} \max_{\boldsymbol{P}} \quad & \frac{\sum_{n\in\Omega_k} \log_2\left(1 + P_{k,n} g_{k,n}\right)}{\xi_k \sum_{n\in\Omega_k} P_{k,n} + P_k^C} \\ \text{s.t. C1:} \quad & \sum_{n\in\Omega_k} \log_2\left(1 + P_{k,n} g_{k,n}\right) \geq R_k^{\text{req}} \\ \text{C3:} \quad & \sum_{n\in\Omega_k} P_{k,n} \leq P_k^{\max} \\ \text{C4:} \quad & P_{k,n} \geq 0, \forall n \in \Omega_k. \end{aligned} \tag{35}$$

We first present the following proposition, proved in Appendix C, to yield a property of (35), which will be used in Proposition 6 to design the algorithm.

Algorithm 3 Subcarrier Assignment Algorithm

1: **Initialization**
- Set $R_k = 0$ and $\Omega_k = \emptyset$ for all $k = 1, \cdots, K$.
- Set $\mathcal{A} = \{1, 2, \ldots, N\}$.

2: **repeat**
3: Find k^* with $R_{k^*} < R_{k^*}^{\text{req}}$ and $R_{k^*} - R_{k^*}^{\text{req}} \leq R_k - R_k^{\text{req}}$ for all $1 \leq k \leq K$.
4: Find n^* satisfying $g_{k^*,n^*} \geq g_{k^*,n}$ for $n \in \mathcal{A}$ for the found k^*
5: Update $\Omega_{k^*} = \Omega_{k^*} \cup \{n^*\}$, $\mathcal{A} = \mathcal{A} - \{n^*\}$, and $R_{k^*} = R_{k^*} + \log_2\left(1 + \frac{g_{k^*,n^*} P_{k^*}^{\max}}{N}\right)$.
6: **until** $\mathcal{A} = \emptyset$ or $R_k \geq R_k^{\text{req}}$ for all $1 \leq k \leq K$.
7: Compute $P_k = |\Omega_k| \frac{P_k^{\max}}{N}$ and $\eta_k^{\text{EE}} = \frac{R_k}{\xi_k P_k + P_k^C}$ for all k.
8: **while** $\mathcal{A} \neq \emptyset$ **do**
9: Find k^* with $\eta_{k^*}^{\text{EE}} \leq \eta_k^{\text{EE}}$ for all $1 \leq k \leq K$.
10: Find n^* satisfying $g_{k^*,n^*} \geq g_{k^*,n}$ for $n \in \mathcal{A}$ for the found k^*.
11: **if** $\frac{R_{k^*} + \log_2\left(1 + \frac{g_{k^*,n^*} P_{k^*}^{\max}}{N}\right)}{\xi_{k^*}\left(P_{k^*} + \frac{P_{k^*}^{\max}}{N}\right) + P_{k^*}^C} \geq \eta_{k^*}^{\text{EE}}$ **then**
12: Update $\Omega_{k^*} = \Omega_{k^*} \cup \{n^*\}$ and $\mathcal{A} = \mathcal{A} - \{n^*\}$.
13: Set $R_{k^*} = R_{k^*} + \log_2\left(1 + \frac{g_{k^*,n^*} P_{k^*}^{\max}}{N}\right)$.
14: Set $P_{k^*} = P_{k^*} + \frac{P_{k^*}^{\max}}{N}$.
15: Set $\eta_{k^*}^{\text{EE}} = \frac{R_{k^*}}{\xi_{k^*} P_{k^*} + P_{k^*}^C}$.
16: **else**
17: break.
18: **endif**
19: **end while**

Proposition 5: Problem (35) is quasiconcave in $\boldsymbol{P}$.

It is known that the bisection method can be exploited to optimally solve quasiconcave problems in an iterative manner [34]. Assume that w_l and w_u are respectively the lower and upper bounds of the optimal value of problem (35). At each iteration, we first bisect w_l and w_u to get $w = \frac{w_l + w_u}{2}$, and then solve a convex feasibility problem or equivalently solve the following convex optimization problem until convergence of the bisection process.

$$\begin{aligned}\max_{\boldsymbol{P}} \; & \sum_{n\in\Omega_k} \log_2\left(1+P_{k,n}g_{k,n}\right) - w\left(\xi_k \sum_{n\in\Omega_k} P_{k,n} + P_k^C\right)\\ \text{s.t. } & \text{C1: } \sum_{n\in\Omega_k} \log_2\left(1+P_{k,n}g_{k,n}\right) \geq R_k^{\text{req}}\\ & \text{C3: } \sum_{n\in\Omega_k} P_{k,n} \leq P_k^{\max}\\ & \text{C4: } P_{k,n} \geq 0, \forall n \in \Omega_k. \end{aligned} \tag{36}$$

Due to the convexity of (36), we can apply standard convex optimization techniques such as the interior-point method [34] to optimally solve it using numerical solvers such as the CVX [35]. The overall computational time also depends on the number of bisections, which in general requires solving a number of (typically 8–12 [28]) problem (36) before convergence. As a result, the computation complexity becomes higher when the numbers of subcarriers and users are larger. For this reason, we exploit the special structure of (35) to devise a more efficient algorithm, which requires only solving at most two simpler subproblems. The following proposition, proved in Appendix D, characterizes the solution of (35) by connecting it with these two subproblems.

Proposition 6: If the optimal solution $P_{k,n}^*$ of the problem[4]

$$\begin{aligned}\max_{\boldsymbol{P}} \quad & \frac{\sum_{n\in\Omega_k} \log_2\left(1+P_{k,n}g_{k,n}\right)}{\xi_k \sum_{n\in\Omega_k} P_{k,n} + P_k^C}\\ \text{s.t.} \quad & \text{C1: } \sum_{n\in\Omega_k} \log_2\left(1+P_{k,n}g_{k,n}\right) \geq R_k^{\text{req}}\\ & \text{C4: } P_{k,n} \geq 0, \forall n \in \Omega_k \end{aligned} \tag{37}$$

satisfies the total power constraint C3, i.e., $\sum_{n\in\Omega_k} P_{k,n}^* \leq P_k^{\max}$, then it is also optimal to (35). Otherwise (i.e., $\sum_{n\in\Omega_k} P_{k,n}^* > P_k^{\max}$), the optimal solution of (35) satisfies the total power constraint C3 with equality, and also solves

$$\begin{aligned}\max_{\boldsymbol{P}} \quad & \sum_{n\in\Omega_k} \log_2\left(1+P_{k,n}g_{k,n}\right)\\ \text{s.t.} \quad & \text{C3: } \sum_{n\in\Omega_k} P_{k,n} = P_k^{\max}\\ & \text{C4: } P_{k,n} \geq 0, \forall n \in \Omega_k \end{aligned} \tag{38}$$

given the feasibility of (35).

Remark 3: 1) It is required to solve either (37) or (38) to obtain the optimal solution of (35) from Proposition 6. Specifically, (37) is required if $\sum_{n\in\Omega_k} P_{k,n}^* \leq P_k^{\max}$, otherwise (38). 2) Similar to (35), (37) is a quasiconcave optimization problem. Thus, we can use the bisection method to solve it. Specifically, at each iteration of the bisection, we solve

$$\begin{aligned}\max_{\boldsymbol{P}} \quad & \sum_{n\in\Omega_k} \log_2\left(1+P_{k,n}g_{k,n}\right) - \eta\left(\xi_k \sum_{n\in\Omega_k} P_{k,n} + P_k^C\right)\\ \text{s.t.} \quad & \text{C1: } \sum_{n\in\Omega_k} \log_2\left(1+P_{k,n}g_{k,n}\right) \geq R_k^{\text{req}}\\ & \text{C4: } P_{k,n} \geq 0, \forall n \in \Omega_k \end{aligned} \tag{39}$$

[4]We denote the optimal power allocation by $P_{k,n}^*$ throughout the paper.

where $\eta = \frac{\eta_l+\eta_u}{2}$, and η_l and η_u denote the lower and upper bounds of the optimal value of (37), respectively. 3) Compared to directly tackling (35), (37) relaxes a constraint C3. However, the improvement in the computational complexity is quite considerable if $\sum_{n\in\Omega_k} P_{k,n}^* > P_k^{\max}$, as we can obtain its optimal solution by solving the convex optimization problem (38) (e.g., by interior-point methods [34]) once without iteratively solving (36) in the bisection method for (35).

The key issue is thus to find out the case when solving (37) or (38) holds. Proposition 7, proved in Appendix E, simplifies this process.

Proposition 7: The total transmit power $P_k = \sum_{n\in\Omega_k} P_{k,n}^*$ achieved by solving (39) is a non-increasing function in η.

By using Proposition 7, it is easy to find out the case when the bisection method is used to solve (37). Specifically, it is required to solve (38), if $P_k \geq P_k^{\max}$ and $\frac{\sum_{n\in\Omega_k} \log_2(1+P_{k,n}^* g_{k,n})}{\xi_k \sum_{n\in\Omega_k} P_{k,n}^* + P_k^C} < \eta$ occur at the same time during the iterations, otherwise (37). This is because we will reduce η if the bisection method is used to solve (37), when $\frac{\sum_{n\in\Omega_k} \log_2(1+P_{k,n}^* g_{k,n})}{\xi_k \sum_{n\in\Omega_k} P_{k,n}^* + P_k^C} < \eta$, thus leading to a further increased P_k from Proposition 7. The details of the power allocation strategy to optimally solve (35) is presented in Algorithm 4. As compared to the algorithm that directly uses the bisection method to solve (35), Algorithm 4 requires solving only at most two simpler subproblems and thus is more efficient.

Algorithm 4 Power Allocation Algorithm

Input:
- Set $i = 0$ and the maximum tolerance $\delta > 0$.
- Set $\eta_l = 0$ and $\eta_u = \frac{\max_{n\in\Omega_k} g_{k,n}}{\xi_k}$.

repeat
  $\eta = \frac{\eta_l+\eta_u}{2}$.
  Solve (39) to obtain $P_{k,n}^*$.
  Set $P_k = \sum_{n\in\Omega_k} P_{k,n}^*$.
  Set $\eta_i = \frac{\sum_{n\in\Omega_k} \log_2\left(1+P_{k,n}^* g_{k,n}\right)}{\xi_k \sum_{n\in\Omega_k} P_{k,n}^* + P_k^C}$.
  if $P_k \geq P_k^{\max}$ and $\eta_i < \eta$ **then**
    Solve (38) to obtain $P_{k,n}^*$.
    Compute $R_k = \sum_{n\in\Omega_k} \log_2\left(1+P_{k,n}^* g_{k,n}\right)$.
    if $R_k < R_k^{\text{req}}$ **then**
      Problem (35) is infeasible.
    end if
    break.
  end if
  if $\eta_i > \eta$ **then**
    $\eta_l = \eta$.
  else
    $\eta_u = \eta$.
  end if
until $|\eta_u - \eta_l| \leq \delta$.
Output: The optimal power allocation $P_{k,n}^*$.

TABLE I
THE RELATIVE GAP BY USING THE LAGRANGIAN DUAL DECOMPOSITION TO SOLVE THE RELAXED (13) (I.E., (14)) INSTEAD OF DIRECTLY TACKLING (13) UNDER DIFFERENT SCENARIOS. NOTE THAT THIS TABLE IS PLOTTED BASED ON ONE RANDOM SAMPLE

Scenarios	d^*	$\widehat{\varphi}$	$\frac{d^*-\widehat{\varphi}}{\widehat{\varphi}}$
$K=8, N=64$	46.5740	45.4180	0.02545
$K=8, N=128$	100.7850	100.2441	0.00540

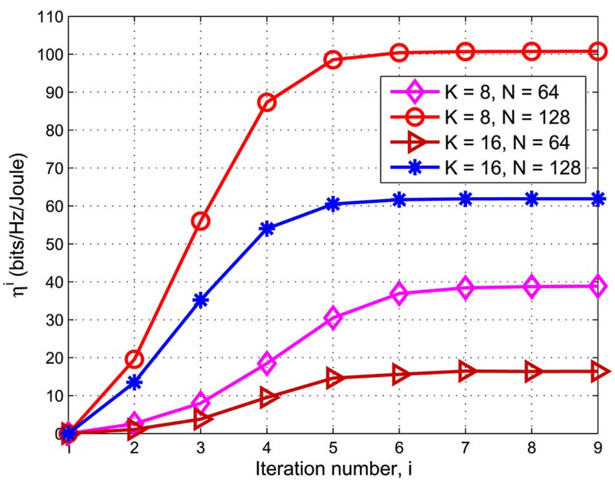

Fig. 1. Convergence of the outer loop of Algorithm 1 under different K and N with $P_k^C = 0.4$ W, $\xi_k = 18$ [37], $R_k^{\mathrm{req}} = 15$ bits/s/Hz, and $P_k^{\max} = 0.2$ W for all k.

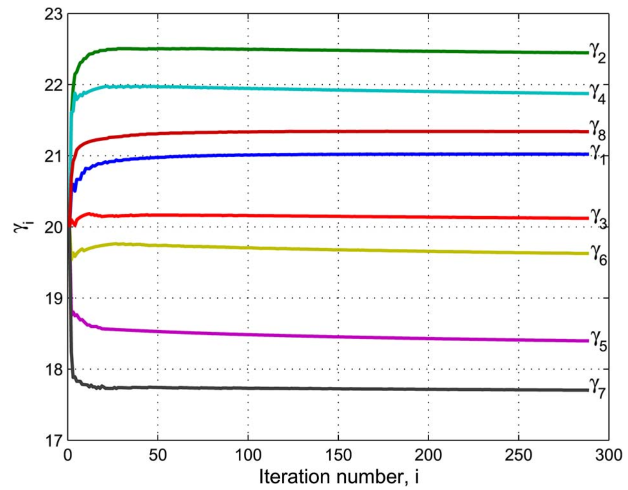

Fig. 2. Convergence of the inner loop of Algorithm 1, i.e., Algorithm 2, for different links with $P_k^C = 0.4$ W, $\xi_k = 18$ [37], $R_k^{\mathrm{req}} = 15$ bits/s/Hz, and $P_k^{\max} = 0.2$ W for all k. Note that this figure is plotted based on one random sample.

V. SIMULATION RESULTS

In this section, we present simulation results to evaluate the performance of the proposed algorithms. In the simulations, the wireless channel is modeled as a frequency-selective channel consisting of twelve independent Rayleigh multipaths. Each multipath component is modeled by the Clarke's flat fading model, and the relative power of the twelve multipath components are $[-4, -3, 0, -2.6, -3.0, -5, -7.0, -5.0, -6.5, -8.6, -11, -10]$ dB [36]. We set $B = 1$ MHz, $\varepsilon = \delta = 10^{-2}$, and $N_0 = 1.1565 \times 10^{-8}$ W/Hz. Note that each point in the following figures is based on the average values of 5000 runs (except Fig. 2, Table I, and Fig. 5).

A. Convergence of Algorithms 1 and 2

In Fig. 1, we plot the convergence evolution of the outer loop of Algorithm 1. It is observed that it converges typically in seven steps. To show the overall convergence rate of Algorithm 1, we further take $\boldsymbol{\gamma} = (\gamma_k)$ as an example to display the convergence evolution of its inner loop, i.e., Algorithm 2. Observe that it has a fast convergence rate. Thus, Algorithm 1 is cost-efficient in the computational complexity.

In addition, we find that, from Fig. 1, for a given user number K, say $K = 16$, the energy efficiency of the worst-case link increases as the number of subcarriers N increases. This is because it is more likely for the controller to assign subcarriers to the users with good channel conditions and then optimize the power allocation. On the contrary, the energy efficiency of the worst-case link decreases as the number of users K increases for a given N, say $N = 128$, because it becomes harder for the controller to satisfy the rate requirements of all the users by subcarrier assignment and power allocation. Observe that the energy efficiency generated for K decreasing from 16 to 8 when $N = 64$ is less than that in the case when $N = 128$ and K decreases from 16 to 8.

Furthermore, Table I illustrates the performance of Algorithm 2 by computing the relative duality gap, which is obtained from the Lagrangian dual decomposition on the relaxed (13) (i.e., (14)) instead of directly tackling (13). In Table I, we denote the optimal values of the primal problem (13), relaxed problem (14), and dual problem (28) by φ^*, $\widehat{\varphi}$, and d^*, respectively. Using the Lagrangian dual decomposition on the relaxed problem (14), the relative duality (optimality) gap $\frac{d^*-\varphi^*}{\varphi^*}$ is upper bounded by $\frac{d^*-\widehat{\varphi}}{\widehat{\varphi}}$ [38]. Observe that, from Table I, the relative duality (optimality) gap $\frac{d^*-\varphi^*}{\varphi^*}$ can be negligible, as its upper bounds $\frac{d^*-\widehat{\varphi}}{\widehat{\varphi}}$ are already small. In other words, we can achieve a good performance of (13) by using the Lagrangian dual decomposition to solve its relaxed problem (14).

B. Energy Efficiency Improvement From Joint Resource Allocation

In Fig. 3, we show how resource allocation affects the worst-case link energy efficiency. "Subcarrier assignment" denotes that only Algorithm 3 is used, where power is equally distributed. In addition, "Subcarrier assignment + Power allocation" denotes that we first call Algorithm 3 to assign subcarrier and then Algorithm 4 to allocate power. Thus, a joint resource allocation can improve the energy efficiency of the worst-case link as compared with only a subcarrier assignment scheme.

C. Impacts of Maximum Transmit Power $P_k^{\max}$ on Energy Efficiency

Fig. 4 illustrates how the maximum transmit power $P_k^{\max}$ affects the worst-cast link energy efficiency. The energy

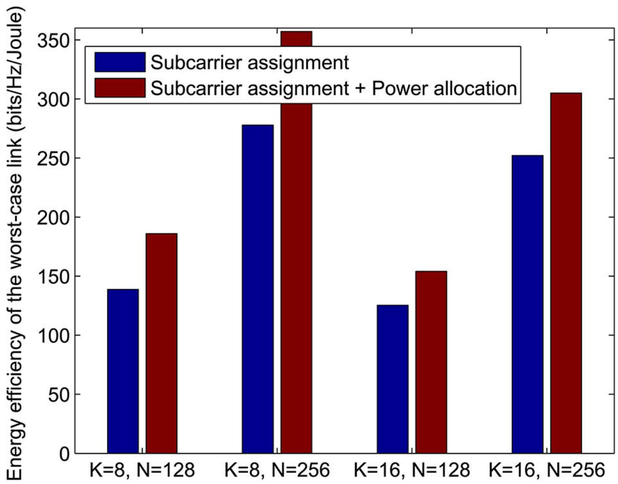

Fig. 3. Comparison of the energy efficiency of the worst-case link under different schemes with $P_k^C = 0.1$ W, $\xi_k = 18$, $R_k^{\text{req}} = 15$ bits/s/Hz, and $P_k^{\max} = 0.2$ W for all k.

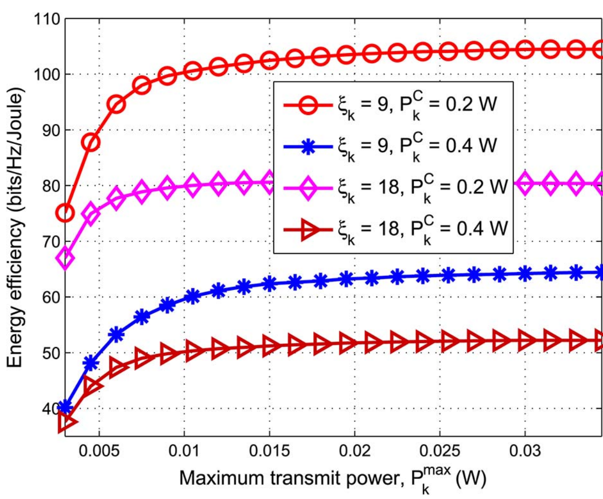

Fig. 4. Energy efficiency against maximum transmit power $P_k^{\max}$ under different ξ_k and P_k^C with $R_k^{\text{req}} = 5$ bits/s/Hz for all k, $K = 8$, and $N = 64$.

efficiency decreases when P_k^C or ξ_k increases from (4) and (7). Further, it is observed that energy efficiency increases with $P_k^{\max}$ only when $P_k^{\max}$ is below a certain threshold and then keeps unchanged once $P_k^{\max}$ is larger than this threshold. This is because, in an ideal case, i.e., no circuit power consumption at the base station ($P_k^C = 0$), energy efficiency is monotonically decreasing in the transmit power due to the diminishing slope of the logarithmic rate-power function (see (1)). However, in realistic systems, the circuit power does not have such monotonic characteristics [13].

D. Performance Comparisons of the MEP and Other Existing Algorithms

In Figs. 5 and 6, we provide performance comparisons among the six schemes, namely the NEP [16], [39], the MEP with Algorithms 1 and 2, the MEP with Algorithms 3 and 4, the rate adaptive [4], [5], the margin adaptive [6], [7], and the utility adaptive [8], [9], in terms of energy efficiency from different perspectives.

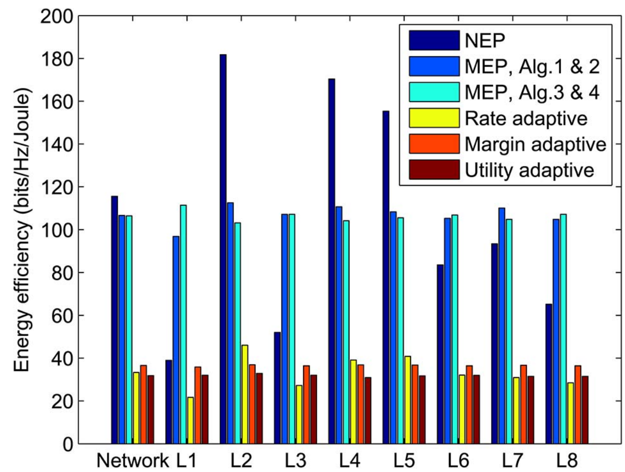

Fig. 5. Comparisons of the energy efficiency of the network and each link among the NEP [16], [39], the MEP with Algorithms 1 and 2, the MEP with Algorithms 3 and 4, the rate adaptive [4], [5], the margin adaptive [6], [7], and the utility adaptive [8], [9] under $K = 8$ and $N = 128$ with $P_k^C = 0.4$ W, $\xi_k = 18$ [37], $R_k^{\text{req}} = 15$ bits/s/Hz, and $P_k^{\max} = 0.2$ W for all k. Note that Lk in the figure denotes the k-th link and this figure is plotted based on one random sample.

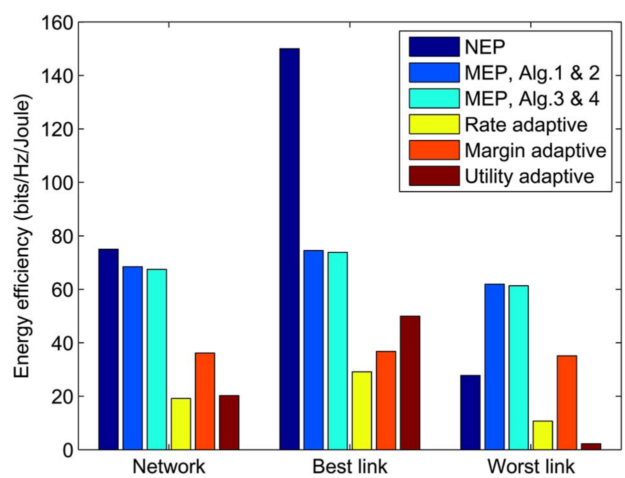

Fig. 6. Comparisons of the energy efficiency of the network, the best link, and the worst link among the NEP [16], [39], the MEP with Algorithms 1 and 2, the MEP with Algorithms 3 and 4, the rate adaptive [4], [5], the margin adaptive [6], [7], and the utility adaptive [8], [9] under $K = 16$ and $N = 128$ with $P_k^C = 0.4$ W, $\xi_k = 18$ [37], $R_k^{\text{req}} = 15$ bits/s/Hz, and $P_k^{\max} = 0.2$ W for all k. Note that the energy efficiency of the best/worst link is obtained by saving the energy efficiency of the link with the respective highest/lowest energy efficiency in each sample and then taking an average.

Fig. 5 compares the energy efficiency of the network and each link among the above six schemes based on one random sample. The NEP improves the network energy efficiency at the cost of each link energy efficiency in bad channel conditions. On the other hand, the MEP, regardless of Algorithms 1 and 2 or Algorithms 3 and 4, balances the link energy efficiency performance.

In Fig. 6, we further compare the performance among the schemes from three aspects: the energy efficiency of the

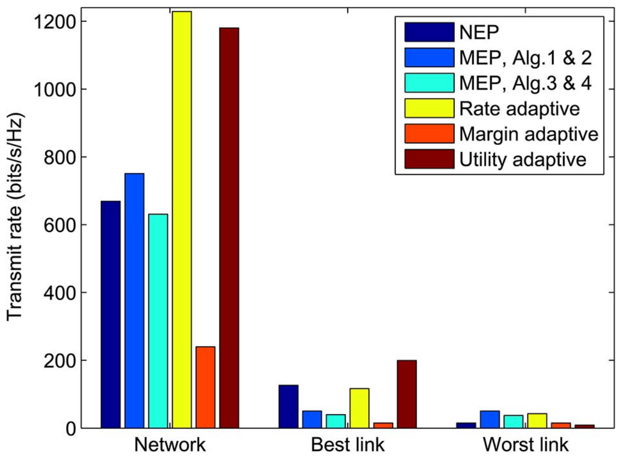

Fig. 7. Comparisons of the transmit rate of the network, the best link, and the worst link among the NEP [16], [39], the MEP with Algorithms 1 and 2, the MEP with Algorithms 3 and 4, the rate adaptive [4], [5], the margin adaptive [6], [7], and the utility adaptive [8], [9] under $K = 16$ and $N = 128$ with $P_k^C = 0.4$ W, $\xi_k = 18$ [37], $R_k^{\text{req}} = 15$ bits/s/Hz, and $P_k^{\max} = 0.2$ W for all k. Note that the transmit rate of the best/worst link is obtained by saving the transmit rate of the link that has the respective highest/lowest energy efficiency in each sample and then taking an average.

network, the best link and the worst-case link. Observe that there is a considerable difference in the energy efficiency between the best and worst links in the NEP, while the energy efficiency of each link in the MEP is well-balanced with a slightly reduced loss in network energy efficiency. This tradeoff between the network energy efficiency and individual link fairness is similar to the tradeoff between the sum rates and fairness [25], [26].

In addition, we plot Fig. 7 to show the performance of MEP in comparison to the other four algorithms. It is observed from Figs. 6 and 7 that the energy efficiency improves at the cost of the total sum rates for both the NEP and the MEP as compared with the rate and utility adaptive schemes. Furthermore, MEP ensures a link rate fairness in comparison with the other four schemes, as the achieved rate of the best and worst-case links is almost the same. This is due to MEP increasing the transmit rate of the worst-case link to ensure the energy efficiency fairness among links (see (7)). As the gap between the energy efficiency of the best and worst link widens, the gap between their transmit rates in the NEP increases as well.

VI. Conclusion

In this paper, we formulated and studied a max-min energy efficiency-optimal problem (MEP) for an uplink OFDMA system so as to provision the fairness of link energy efficiency by integrating both the subcarrier assignment and power allocation. To solve this nonsmooth and mixed-integer fractional programming problem, we first proposed an iterative algorithm using a generalized fractional programming theory and the Lagrangian dual decomposition. We then exploited the problem structure to decompose and simplify the subcarrier assignment and power allocation to further reduce the computational cost. Our simulation results verified the convergence performance of the proposed algorithms in providing fairness to links in terms of energy efficiency.

Appendix A
Proof of Proposition 3

If $f(x)$ is concave, then its perspective function $tf(x/t)$ is concave in (x, t) [34]. Hence, $\rho_{k,n} \log_2\left(1 + \frac{s_{k,n} g_{k,n}}{\rho_{k,n}}\right)$ is concave, because it is the perspective function of the concave function $\log_2(1 + s_{k,n} g_{k,n})$. Since the sum of concave functions preserves concavity, $\sum_{n=1}^{N} \rho_{k,n} \log_2\left(1 + \frac{s_{k,n} g_{k,n}}{\rho_{k,n}}\right)$ is concave. Since the superlevel set of a concave function is convex [34], C1 and C6 are convex. In addition, C2–C5 are all linear constraints. Therefore, (14) is a convex optimization problem that maximizes a concave function over a convex set.

Appendix B
Proof of Proposition 4

From the definition of $D(\boldsymbol{\beta}, \mu, \boldsymbol{\nu}, \boldsymbol{\gamma})$ in (16), we have

$$\begin{aligned}
&D(\boldsymbol{\beta}', \boldsymbol{\mu}', \boldsymbol{\nu}, \boldsymbol{\gamma}') \\
&\geq \varphi + \sum_{k=1}^{K} \beta_k' \left[\sum_{n=1}^{N} \rho_{k,n}^* \log_2\left(1 + \frac{s_{k,n}^* g_{k,n}}{\rho_{k,n}^*}\right) - R_k^{\text{req}} \right] \\
&\quad + \sum_{n=1}^{N} \nu_n \left(1 - \sum_{k=1}^{K} \rho_{k,n}^*\right) + \sum_{k=1}^{K} \mu_k' \left(P_k^{\max} - \sum_{n=1}^{N} s_{k,n}^*\right) \\
&\quad + \sum_{k=1}^{K} \gamma_k' \left\{ \sum_{n=1}^{N} \left[\rho_{k,n}^* \log_2\left(1 + \frac{s_{k,n}^* g_{k,n}}{\rho_{k,n}^*}\right) - \eta \xi_k s_{k,n}^* \right] - \eta P_k^C - \varphi \right\}
\end{aligned} \tag{40}$$

where the inequality is due to the fact that $\rho_{k,n}^*$ and $P_{k,n}^*$ are the optimal solutions corresponding to $\boldsymbol{\beta}$, μ, $\boldsymbol{\nu}$, and $\boldsymbol{\gamma}$.

Rearranging (40) yields

$$\begin{aligned}
&D(\boldsymbol{\beta}', \mu', \boldsymbol{\nu}, \boldsymbol{\gamma}') \geq D(\boldsymbol{\beta}, \mu, \boldsymbol{\nu}, \boldsymbol{\gamma}) \\
&+ \sum_{k=1}^{K} (\mu_k' - \mu_k) \left(P_k^{\max} - \sum_{n=1}^{N} s_{k,n}^*\right) \\
&+ \sum_{k=1}^{K} (\beta_k' - \beta_k) \left[\sum_{n=1}^{N} \rho_{k,n}^* \log_2\left(1 + \frac{s_{k,n}^* g_{k,n}}{\rho_{k,n}^*}\right) - R_k^{\text{req}} \right] \\
&+ \sum_{k=1}^{K} (\gamma_k' - \gamma_k) \left\{ \sum_{n=1}^{N} \left[\rho_{k,n}^* \log_2\left(1 + \frac{s_{k,n}^* g_{k,n}}{\rho_{k,n}^*}\right) - \eta \xi_k s_{k,n}^* \right] \right. \\
&\qquad \left. - \eta P_k^C - \varphi \right\}.
\end{aligned} \tag{41}$$

Note that z is defined as a subgradient of a convex function $f(\cdot)$ if $f(x) \geq f(y) + z^T(x - y)$ holds for all x and y in the domain. Hence, Proposition 4 holds.

APPENDIX C
PROOF OF PROPOSITION 5

Let

$$f_k(\boldsymbol{P}) = \frac{\sum_{n\in\Omega_k} \log_2(1 + P_{k,n}g_{k,n})}{\xi_k \sum_{n\in\Omega_k} P_{k,n} + P_k^C}. \tag{42}$$

Since it is the ratio of a strictly concave over an affine function, it follows that $f_k(\boldsymbol{P})$ is strictly quasiconcave [34, Page 95]. Besides, the feasible region from C1, C3, and C4 is convex. Therefore, (35) is a quasiconcave optimization problem as it maximizes a quasiconcave function over a convex set [34].

APPENDIX D
PROOF OF PROPOSITION D

Observe from (37) and (35) that both have the same optimal value, but (37) has a larger feasible region. Thus, they achieve the same optimal solution when $\sum_{n\in\Omega_k} P_{k,n}^* \le P_k^{\max}$ (i.e., C3 in (35) is an inactive constraint at this point). However, C3 in (35) is satisfied with equality at optimality (i.e., C3 is an active constraint) if $\sum_{n\in\Omega_k} P_{k,n}^* > P_k^{\max}$, due to the strict quasiconcavity of $f_k(\boldsymbol{P})$ from Appendix C. Thus, (35) becomes

$$\begin{aligned} \max_{\boldsymbol{P}} \quad & \frac{\sum_{n\in\Omega_k} \log_2(1 + P_{k,n}g_{k,n})}{\xi_k P_k^{\max} + P_k^C} \\ \text{s.t.} \quad & \text{C1: } \sum_{n\in\Omega_k} \log_2(1 + P_{k,n}g_{k,n}) \ge R_k^{\text{req}} \\ & \text{C3: } \sum_{n\in\Omega_k} P_{k,n} = P_k^{\max} \\ & \text{C4: } P_{k,n} \ge 0, \ \forall n \in \Omega_k. \end{aligned} \tag{43}$$

Due to the constant denominator in the objective, we can equivalently recast (43) as

$$\begin{aligned} \max_{\boldsymbol{P}} \quad & \sum_{n\in\Omega_k} \log_2(1 + P_{k,n}g_{k,n}) \\ \text{s.t.} \quad & \text{C1: } \sum_{n\in\Omega_k} \log_2\left(1 + P_{k,n}g_{k,n}\right) \ge R_k^{\text{req}} \\ & \text{C3: } \sum_{n\in\Omega_k} P_{k,n} = P_k^{\max} \\ & \text{C4: } P_{k,n} \ge 0, \forall n \in \Omega_k. \end{aligned} \tag{44}$$

Furthermore, (44) is equivalent to (38) under the feasible assumption of (35), because (44) and (38) have the same optimal solutions and values given the feasibility of (35). In other words, C1 is redundant since we maximize the user rate (if the maximum rate achieved by (38) is less than R_k^{req}, then (35) is infeasible).

APPENDIX E
PROOF OF PROPOSITION 7

From (39), we can interpret η as a weight to reflect the preference of link k on the achieved rate and the power consumption. Specifically, a smaller η implies a higher preference for the achieved rate as compared to the power consumption. Thus, P_k decreases as η increases, but it is lower-bounded by P_k', where $P_k' = \sum_{n\in\Omega_k} P_{k,n}'$ is the minimum transmit power required to satisfy the rate requirement of link k, i.e., R_k^{req}.

REFERENCES

[1] *IEEE Standard for Local and Metropolitan Area Networks Part 16: Air Interface for Broadband Wireless Access Systems Amendment 3: Advanced Air Interface*, IEEE Std. 802.16m-2011 Std., Jun. 2011.
[2] S. Sesia, I. Toufik, and M. Baker, *LTE: The UMTS Long Term Evolution*. New York, NY, USA: Wiley, 2009.
[3] S. Sadr, A. Anpalagan, and K. Raahemifar, “Radio resource allocation algorithms for the downlink of multiuser OFDM communication systems,” *IEEE Commun. Surveys Tuts.*, vol. 11, no. 3, pp. 92–106, 3rd Quart. 2009.
[4] M. Tao, Y.-C. Liang, and F. Zhang, “Resource allocation for delay differentiated traffic in multiuser OFDM systems,” *IEEE Trans. Wireless Commun.*, vol. 7, no. 6, pp. 2190–2201, Jun. 2008.
[5] D. T. Ngo, S. Khakurel, and T. Le-Ngoc, “Joint subchannel assignment and power allocation for OFDMA femtocell networks,” *IEEE Trans. Wireless Commun.*, vol. 13, no. 1, pp. 342–355, Jan. 2014.
[6] C. Y. Wong, R. Cheng, K. Lataief, and R. Murch, “Multiuser OFDM with adaptive subcarrier, bit, and power allocation,” *IEEE J. Sel. Areas Commun.*, vol. 17, no. 10, pp. 1747–1758, Oct. 1999.
[7] D. Lopez-Perez, A. Ladanyi, A. Juttner, H. Rivano, and J. Zhang, “Optimization method for the joint allocation of modulation schemes, coding rates, resource blocks and power in self-organizing LTE networks,” in *Proc. IEEE INFOCOM*, Apr. 2011, pp. 111–115.
[8] H. Ju, B. Liang, J. Li, and X. Yang, “Dynamic joint resource optimization for LTE-advanced relay networks,” *IEEE Trans. Wireless Commun.*, vol. 12, no. 11, pp. 5668–5678, Nov. 2013.
[9] G. Song and Y. Li, “Cross-layer optimization for OFDM wireless networks-part II: Algorithm development,” *IEEE Trans. Wireless Commun.*, vol. 4, no. 2, pp. 625–634, Mar. 2005.
[10] D. Feng *et al.*, “A survey of energy-efficient wireless communications,” *IEEE Commun. Surveys Tuts.*, vol. 15, no. 1, pp. 167–178, 1st Quart. 2013.
[11] J. Rao and A. Fapojuwo, “A survey of energy efficient resource management techniques for multicell cellular networks,” *IEEE Commun. Surveys Tuts.*, vol. 16, no. 1, pp. 154–180, 1st Quart. 2014.
[12] Y. Li, M. Sheng, Y. Shi, X. Ma, and W. Jiao, “Energy efficiency and delay tradeoff for time-varying and interference-free wireless networks,” *IEEE Trans. Wireless Commun.*, vol. 13, no. 11, pp. 5921–5931, Nov. 2014.
[13] Y. Chen, S. Zhang, S. Xu, and G. Li, “Fundamental trade-offs on green wireless networks,” *IEEE Commun. Mag.*, vol. 49, no. 6, pp. 30–37, Jun. 2011.
[14] H. Ye, G. Lim, L. Cimini, and Z. Tan, “Energy-efficient scheduling and resource allocation in uplink OFDMA systems,” *IEEE Commun. Lett.*, vol. 19, no. 3, pp. 439–442, Mar. 2015.
[15] Q. Shi *et al.*, “On the energy efficiency optimality of OFDMA for SISO-OFDM downlink system,” *IEEE Commun. Lett.*, vol. 17, no. 3, pp. 541–544, Mar. 2013.
[16] C. Xiong, G. Li, S. Zhang, Y. Chen, and S. Xu, “Energy- and spectral-efficiency tradeoff in downlink OFDMA networks,” *IEEE Trans. Wireless Commun.*, vol. 10, no. 11, pp. 3874–3886, Nov. 2011.
[17] C. Xiong, G. Li, S. Zhang, Y. Chen, and S. Xu, “Energy-efficient resource allocation in OFDMA networks,” *IEEE Trans. Commun.*, vol. 60, no. 12, pp. 3767–3778, Dec. 2012.
[18] Q. Xu, X. Li, H. Ji, and X. Du, “Energy-efficient resource allocation for heterogeneous services in OFDMA downlink networks: Systematic perspective,” *IEEE Trans. Veh. Technol.*, vol. 63, no. 5, pp. 2071–2082, Jun. 2014.
[19] C. Xiong, G. Li, Y. Liu, Y. Chen, and S. Xu, “Energy-efficient design for downlink OFDMA with delay-sensitive traffic,” *IEEE Trans. Wireless Commun.*, vol. 12, no. 6, pp. 3085–3095, Jun. 2013.
[20] L. Venturino, A. Zappone, C. Risi, and S. Buzzi, “Energy-efficient scheduling and power allocation in downlink OFDMA networks with base station coordination,” *IEEE Trans. Wireless Commun.*, vol. 14, no. 1, pp. 1–14, Jan. 2015.
[21] S. He, Y. Huang, L. Yang, and B. Ottersten, “Coordinated multicell multiuser precoding for maximizing weighted sum energy efficiency,” *IEEE Trans. Signal Process.*, vol. 62, no. 3, pp. 741–751, Feb. 2014.

[22] A. Zappone, Z. Chong, E. Jorswieck, and S. Buzzi, "Energy-aware competitive power control in relay-assisted interference wireless networks," *IEEE Trans. Wireless Commun.*, vol. 12, no. 4, pp. 1860–1871, Apr. 2013.

[23] S. He, Y. Huang, S. Jin, F. Yu, and L. Yang, "Max-min energy efficient beamforming for multicell multiuser joint transmission systems," *IEEE Commun. Lett.*, vol. 17, no. 10, pp. 1956–1959, Oct. 2013.

[24] Y. Li, M. Sheng, X. Wang, Y. Zhang, and J. Wen, "Max-min energy-efficient power allocation in interference-limited wireless networks," *IEEE Trans. Veh. Technol.*, 2015, to be published.

[25] C. Joe-Wong, S. Sen, T. Lan, and M. Chiang, "Multiresource allocation: Fairness-efficiency tradeoffs in a unifying framework," *IEEE/ACM Trans. Netw.*, vol. 21, no. 6, pp. 1785–1798, Dec. 2013.

[26] A. Sediq, R. Gohary, R. Schoenen, and H. Yanikomeroglu, "Optimal tradeoff between sum-rate efficiency and Jain's fairness index in resource allocation," *IEEE Trans. Wireless Commun.*, vol. 12, no. 7, pp. 3496–3509, Jul. 2013.

[27] M. Moretti and A. Perez-Neira, "Efficient margin adaptive scheduling for MIMO-OFDMA systems," *IEEE Trans. Wireless Commun.*, vol. 12, no. 1, pp. 278–287, Jan. 2013.

[28] Y. Li, M. Sheng, C. Yang, and X. Wang, "Energy efficiency and spectral efficiency tradeoff in interference-limited wireless networks," *IEEE Commun. Lett.*, vol. 17, no. 10, pp. 1924–1927, Oct. 2013.

[29] Y. Li, M. Sheng, X. Wang, Y. Shi, and Y. Zhang, "Globally optimal antenna selection and power allocation for energy efficiency maximization in downlink distributed antenna systems," in *Proc. IEEE GLOBECOM*, Austin, TX, USA, Dec. 2014, pp. 3856–3861.

[30] S. Hayashi and Z.-Q. Luo, "Spectrum management for interference-limited multiuser communication systems," *IEEE Trans. Inf. Theory*, vol. 55, no. 3, pp. 1153–1175, Mar. 2009.

[31] J. Crouzeix, J. Ferland, and S. Schaible, "An algorithm for generalized fractional programs," *J. Optim. Theory Appl.*, vol. 47, no. 1, pp. 35–49, Sep. 1985. [Online] Available: http://dx.doi.org/10.1007/BF00941314

[32] X. Wang and G. Giannakis, "Resource allocation for wireless multiuser OFDM networks," *IEEE Trans. Inf. Theory*, vol. 57, no. 7, pp. 4359–4372, Jul. 2011.

[33] S. Boyd. Subgradient methods. [Online] Available: http://stanford.edu/class/ee364b/lectures/subgrad_method_notes.pdf

[34] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2004.

[35] M. Grant, S. Boyd, and Y. Ye, "Cvx: Matlab software for disciplined convex programming," version 2.0 beta, Sep. 2012. [Online]. Available: http://cvxr.com/cvx

[36] Z. Shen, J. Andrews, and B. Evans, "Adaptive resource allocation in multiuser OFDM systems with proportional rate constraints," *IEEE Trans. Wireless Commun.*, vol. 4, no. 6, pp. 2726–2737, Nov. 2005.

[37] O. Munoz-Medina, A. Pascual-Iserte, and J. Vidal, "Optimization of radio and computational resources for energy efficiency in latency-constrained application offloading," *IEEE Trans. Veh. Technol.*, 2015, to be published.

[38] I. Wong and B. Evans, "Optimal resource allocation in the OFDMA downlink with imperfect channel knowledge," *IEEE Trans. Commun.*, vol. 57, no. 1, pp. 232–241, Jan. 2009.

[39] D. Ng, E. Lo, and R. Schober, "Energy-efficient resource allocation in OFDMA systems with large numbers of base station antennas," *IEEE Trans. Wireless Commun.*, vol. 11, no. 9, pp. 3292–3304, Sep. 2012.

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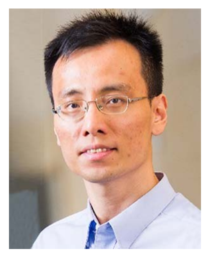

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