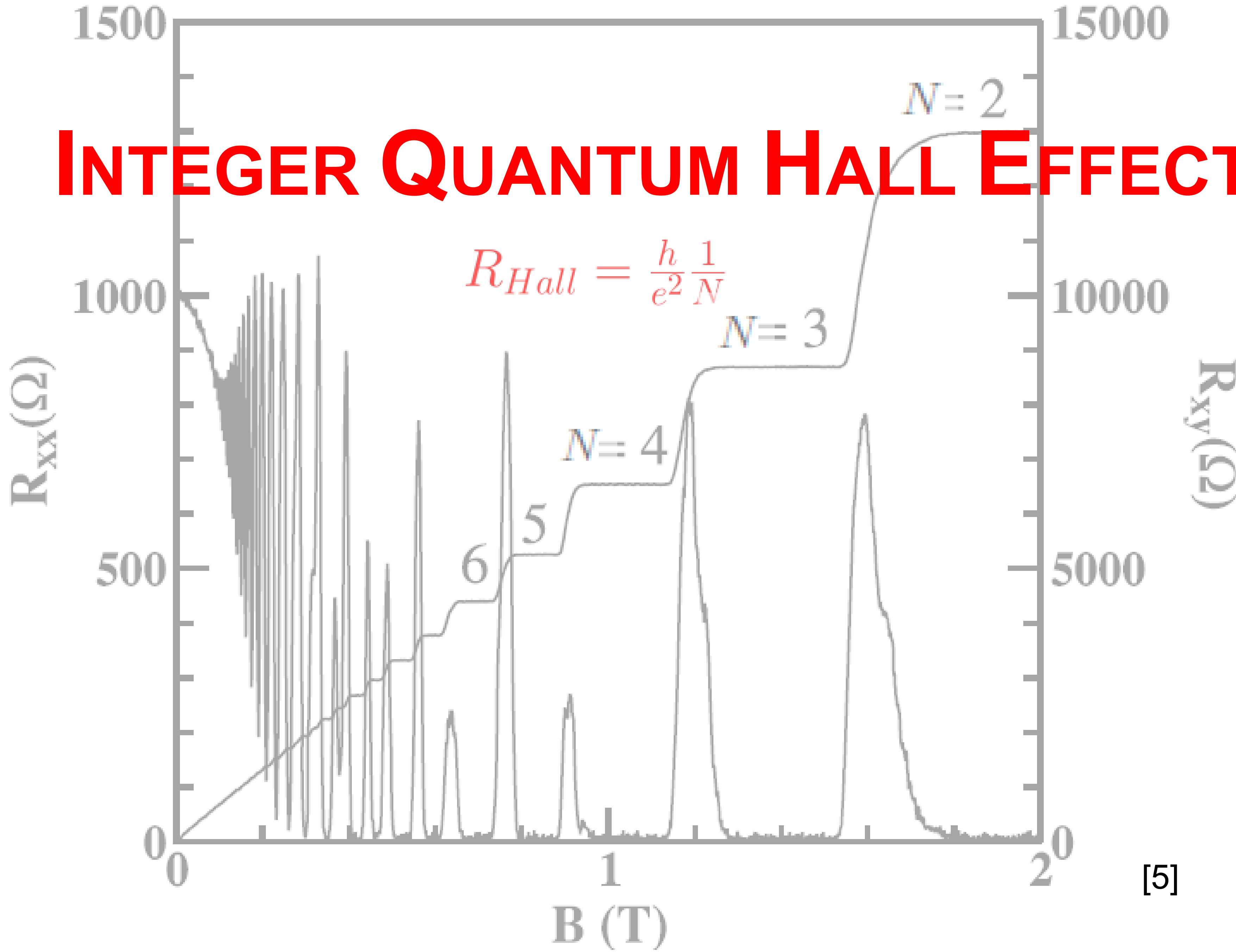
Integer Quantum Hall Effect



[5]

Reminder: (Classical) Hall Effect in 2 dimensions

Drude Model:

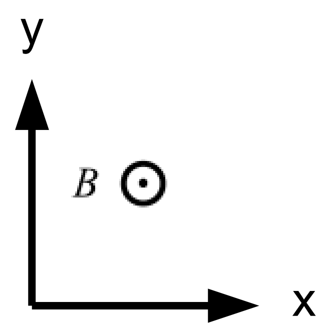
$$F = m \cdot a \quad \rightarrow \quad \frac{m \cdot \vec{v}_d}{\tau} = e \cdot \left(\vec{E} + \frac{\vec{v}_d}{c} \, x \, \vec{B} \right)$$

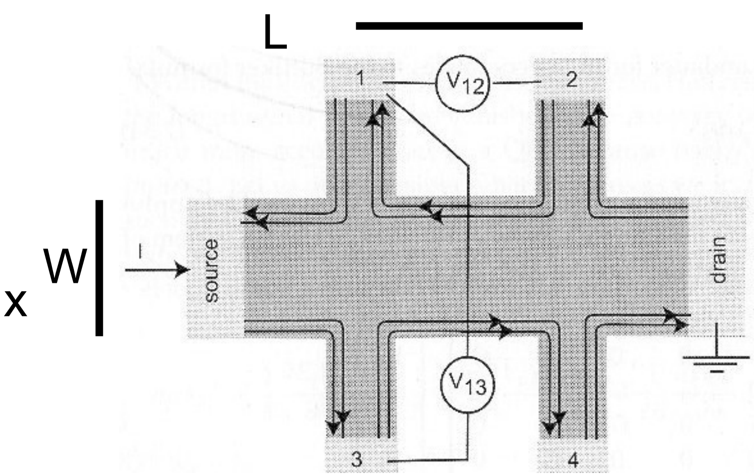
with drift velocity $\vec{v}_d$ and scattering time τ

$$\rightarrow \quad \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} \frac{m}{e\tau} & \frac{-B}{c} \\ \frac{B}{c} & \frac{m}{e\tau} \end{pmatrix} \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

with current density $\vec{j} = e n_e \vec{v}_d = \vec{E} \cdot \sigma$ and Drude conductivity $\sigma = \frac{e^2 \tau n_e}{m}$

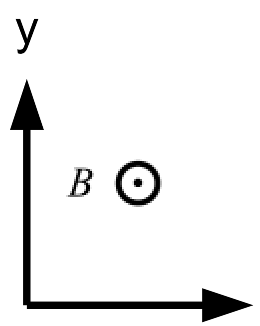
$$\rightarrow \quad \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} \sigma^{-1} & \frac{-B}{c e n_e} \\ \frac{B}{c e n_e} & \sigma^{-1} \end{pmatrix} \begin{pmatrix} j_x \\ j_y \end{pmatrix}$$

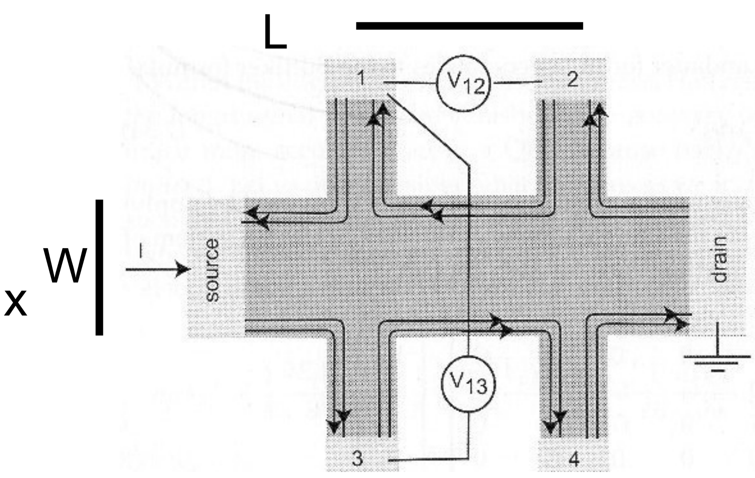




[4]

Reminder: (Classical) Hall Effect in 2 dimensions





[4]

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$$I_x = W \cdot j_x \qquad j_y = 0$$

$$\rightarrow E_x = \sigma^{-1} j_x = \sigma^{-1} \frac{I_x}{W}$$

$$\rightarrow V_{12} = V_x = L \cdot E_x = \sigma^{-1} \frac{L}{W} I_x$$

$$\rightarrow R_x = \sigma^{-1} \frac{L}{W}$$

$$\rightarrow E_y = \frac{B}{c e n_e} j_x = \frac{B}{c e n_e} \frac{I_x}{W}$$

$$\rightarrow V_{Hall} = V_y = W \cdot E_y = \frac{B}{c e n_e} I_x$$

$$\rightarrow \boxed{R_{Hall} = \frac{B}{c e n_e}}$$

→ **linear** increase of the Hall resistance with the applied magnetic field !?!

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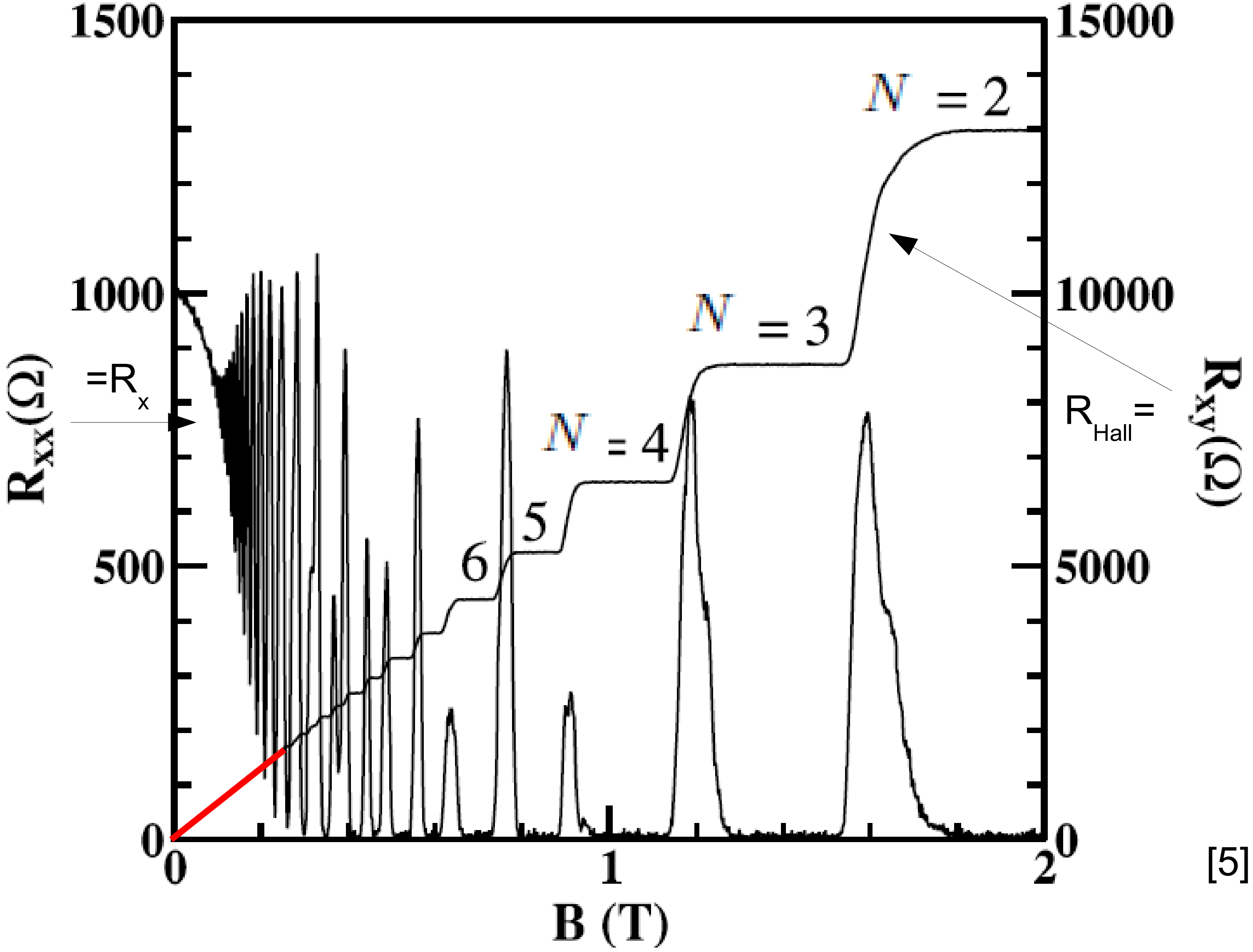
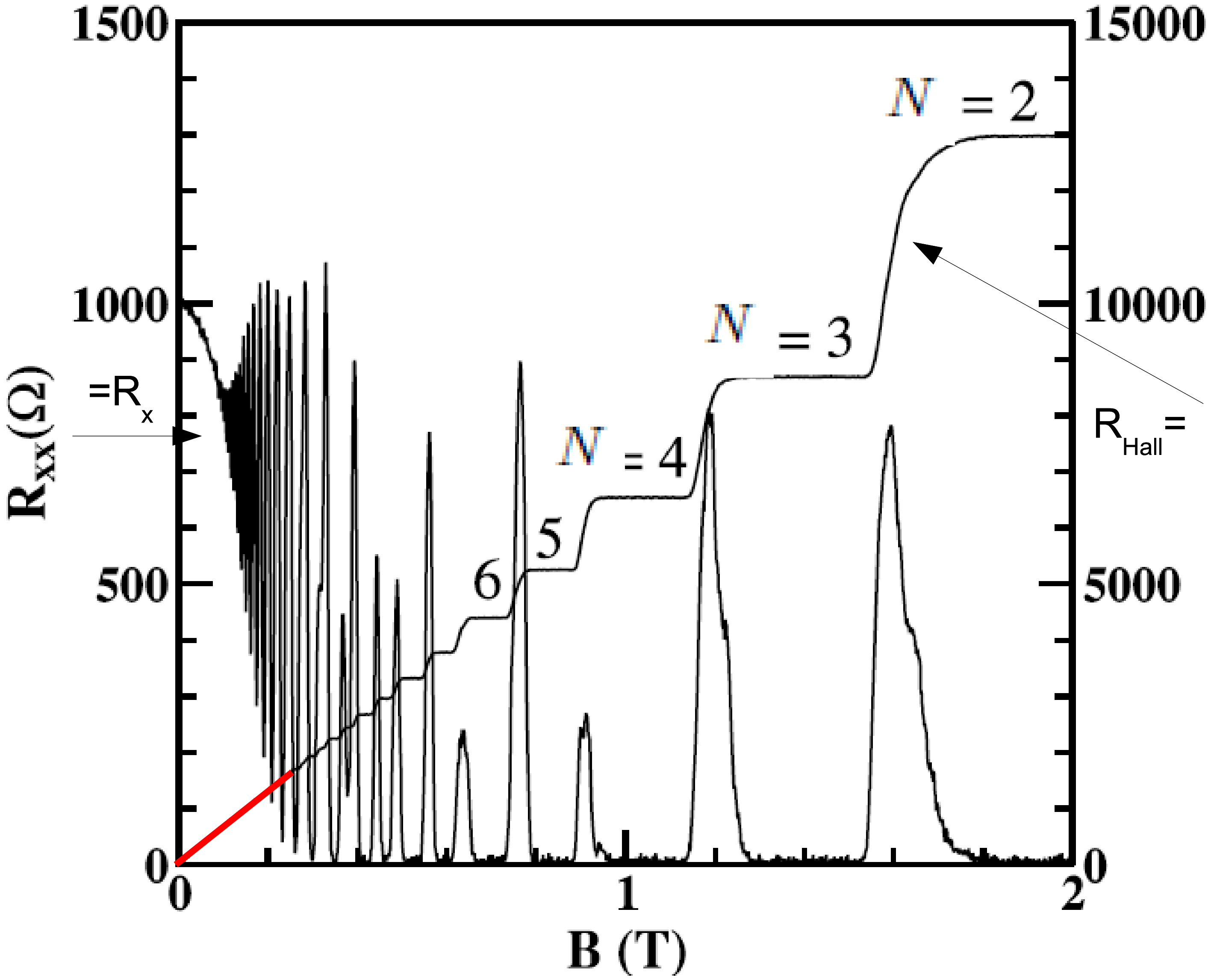


[5]

→ **linear** increase of the Hall resistance with the applied magnetic field !?!
→ Drude Modell fails for **higher** magnetic fields



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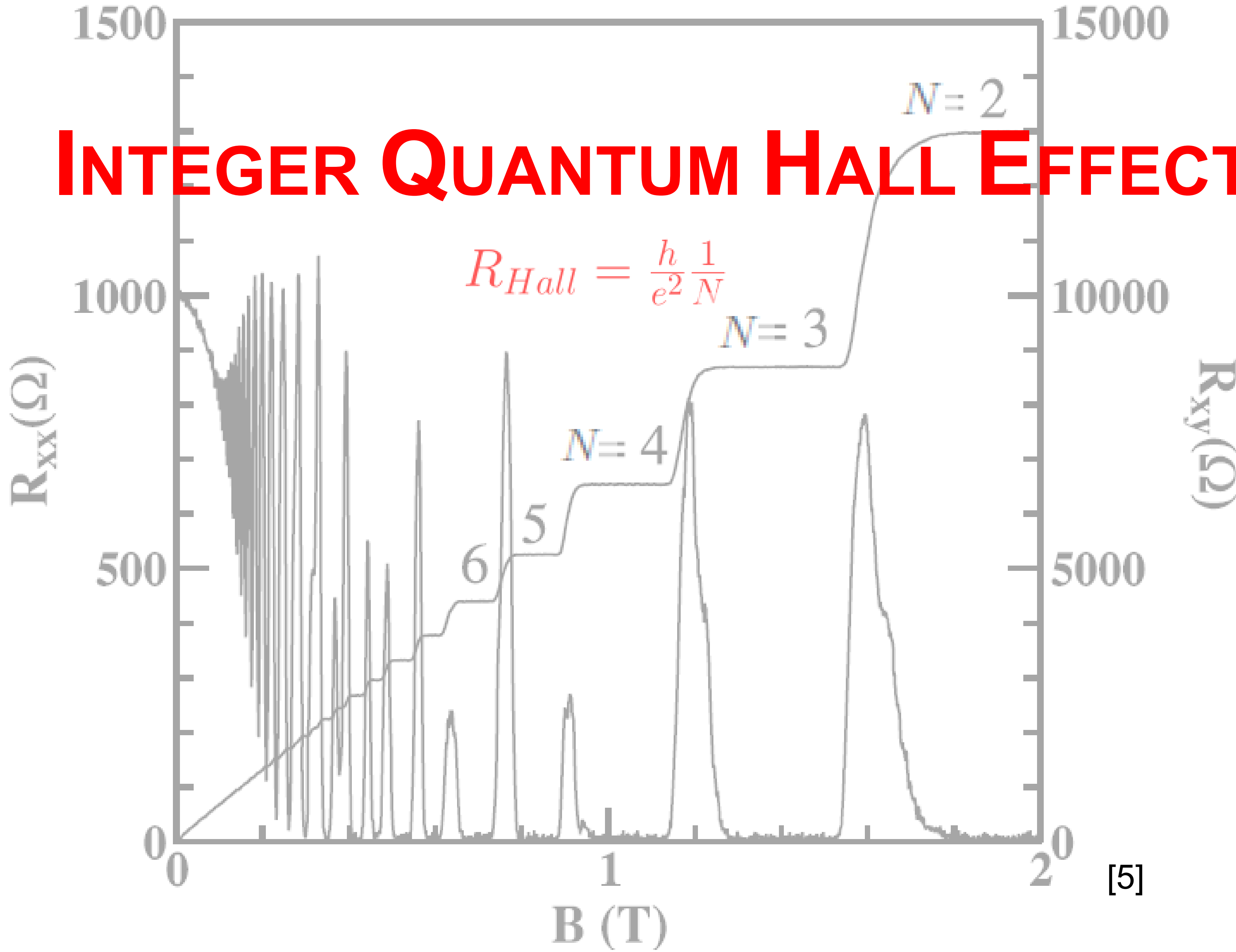
INTEGER QUANTUM HALL EFFECT

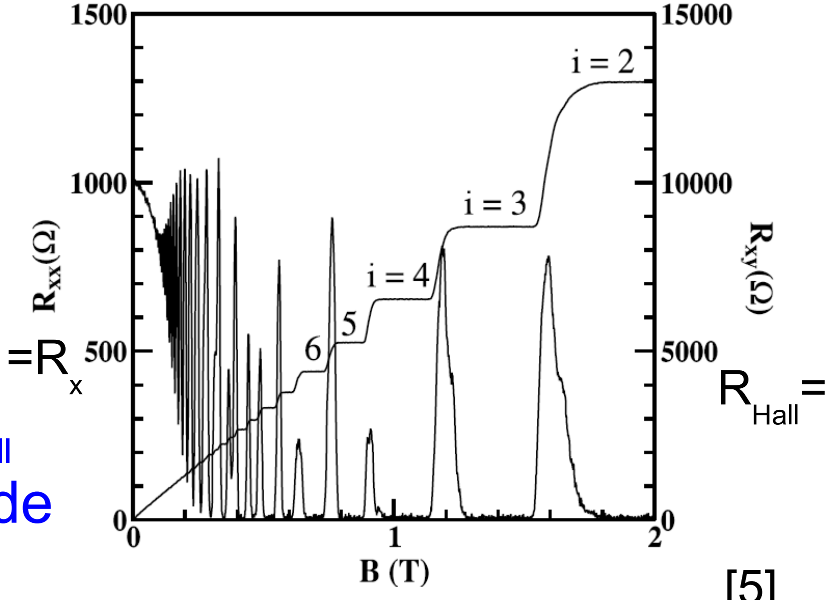


[5]

Observations

1) Longitudinal resistance R_x

- vanishes at plateau regions of R_{Hall}
- jumps about 13 orders of magnitude when R_{Hall} changes



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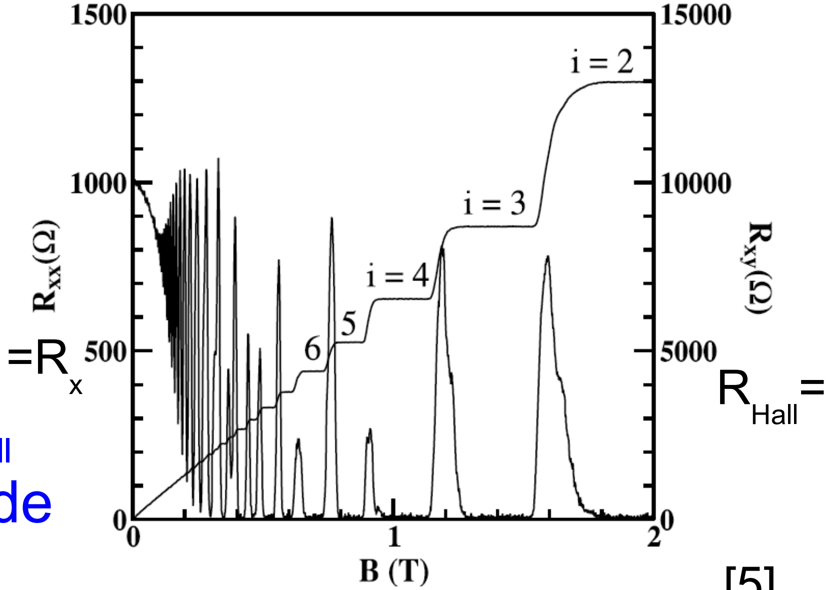

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1) Longitudinal resistance R_x

- vanishes at plateau regions of R_{Hall}
- jumps about 13 orders of magnitude when R_{Hall} changes

2) Hall resistance R_{Hall}

- shows plateau regions
- varies stepwise with step height given by $\boxed{R_{Hall} = \underbrace{\frac{h}{e^2}}_{R_K} \cdot \frac{1}{N}}$ $N \in \mathbb{N}$

R_K =25812.807Ω

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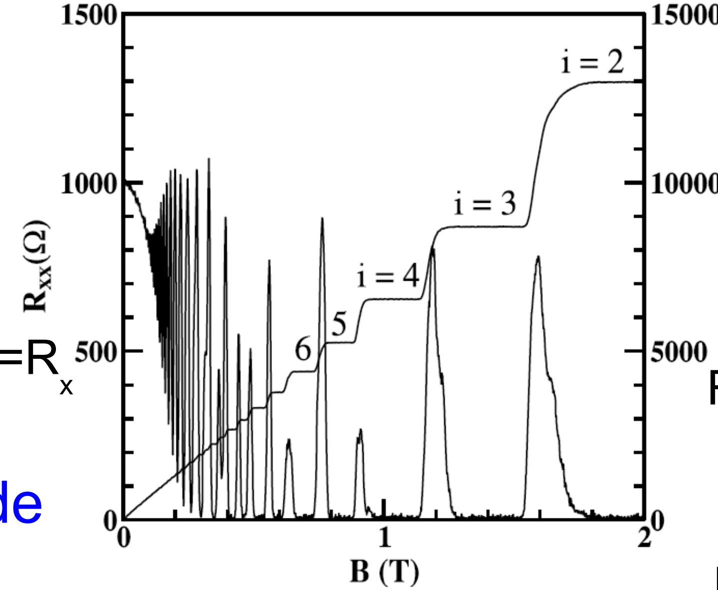

$=R_x$ $R_{Hall}=$

[5]

2) Hall resistance R_{Hall}

- shows plateau regions
- varies stepwise with step height given by $\boxed{R_{Hall} = \underbrace{\frac{h}{e^2}}_{R_K = 25812.807\Omega} \cdot \frac{1}{N}}$ $N \in \mathbb{N}$

3) difference of plateau values

- of different samples and
- between different plateaus

is smaller than 10^{-10} times the quantized value

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III Integer Quantum Hall Effect – key ingredients

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- Effect of finite sample size: edge states

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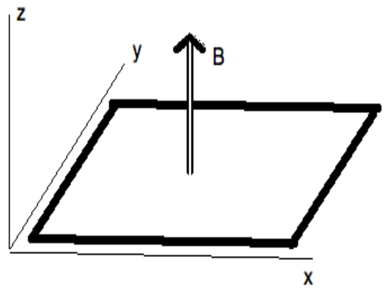
- # Quantum mechanical derivation of Landau Levels

$$H=\frac{1}{2\mathrm{m}}\left|\vec{p}+\frac{e}{c}\vec{A}\right|^2=\frac{1}{2\mathrm{m}}(\hat{p}_x-\frac{eB}{c}\hat{y})^2+\frac{\hat{p}_y^2}{2\mathrm{m}}$$ with Landau Gauge $\vec{A}=-B\begin{pmatrix}y\\0\\0\end{pmatrix}$



$$H\Psi(x,y)=E\Psi(x,y)$$ with $$\Psi(x,y)=\frac{1}{\sqrt{L_x}}\exp(ik_x x)\Psi(y)$$

[7]

$\rightarrow$ get effective 1 dimensional Schrödinger equation for $\Psi(y)$ and plane wave in x-direction

$$H=\frac{\hat{p}_y^2}{2\mathrm{m}}+\frac{1}{2}m w_c^2(\hat{y}-y_0)^2$$

$$H\Psi(y)=E\Psi(y)$$

$$\boxed{\begin{aligned}&\rightarrow E_N=(N+\frac{1}{2})\hbar w_c\\&\rightarrow \Psi_N(y)=\sqrt{\frac{1}{2^N N!}}\left(\frac{mw_c}{\pi\hbar}\right)^{\frac{1}{4}}\exp(-\frac{mw_c}{2\hbar}(y-y_0)^2)H_N\left(\sqrt{\frac{mw_c}{\hbar}}(y-y_0)\right)\\&\text{with } w_c^2=(\frac{eB}{mc})^2 \text{ and } y_0=\frac{\hbar c}{eB}k_x=l^2k_x\,,\ l=\sqrt{\frac{\hbar c}{eB}}\ \text{magnetic length}\end{aligned}}$$

finite sample size $L_x=L$ and $L_y=W$ $\rightarrow$ quantized momentum $k_x=\frac{2\pi}{L_x}m$ $m=$ integer

• Quantum mechanical derivation of Landau Levels

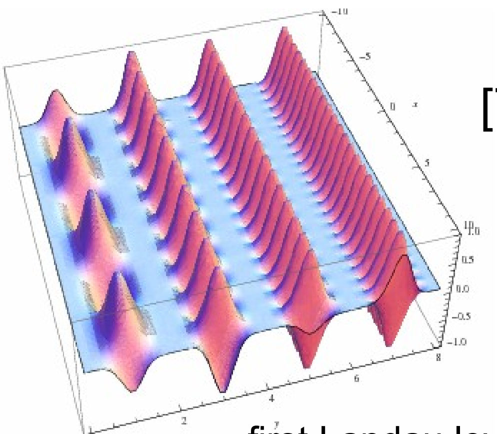
$$H\Psi(x,y)=E\Psi(x,y) \quad \text{with} \quad \Psi_{N,k_x}(x,y)=\frac{1}{\sqrt{L_x}}\exp(ik_x x)\Psi_N(y) \quad \text{and} \quad H=\frac{\hat{p}_y^{\,2}}{2\mathrm{m}}+\frac{1}{2}m w_c^{\,2}(\hat{y}-y_0)^2$$

$$\rightarrow E_N=(N+\frac{1}{2})\hbar w_c \qquad N \mathrel{\hat{=}} \text{Landau Levels}$$

$$\text{with } w_c^{\,2}=(\frac{eB}{mc})^2 \text{ and } y_0=\frac{\hbar c}{eB}k_x=l^2 k_x \ , \ l=\sqrt{\frac{\hbar c}{eB}} \quad \text{magnetic length}$$

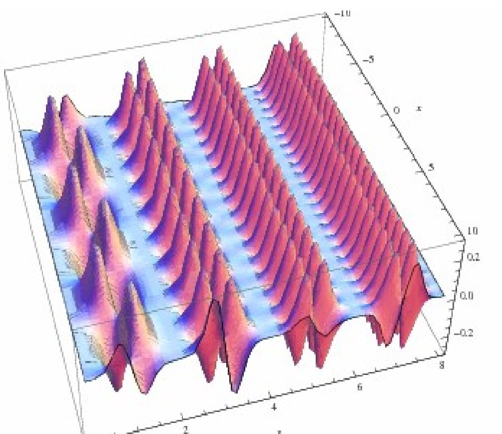
quantized momentum $k_x=\frac{2\pi}{L_x}m$

→ plane waves in x-direction, localized around $y_0(k_x)$



first Landau level

[7]



Second Landau Level

• Quantum mechanical derivation of Landau Levels

$$H\Psi(x,y)=E\Psi(x,y) \quad \text{with} \quad \Psi_{N,k_x}(x,y)=\frac{1}{\sqrt{L_x}}\exp(ik_x x)\Psi_N(y) \quad \text{and} \quad H=\frac{\hat{p}_y^{\,2}}{2\text{m}}+\frac{1}{2}m w_c^{\,2}(\hat{y}-y_0)^2$$

$$\rightarrow E_N=(N+\frac{1}{2})\hbar w_c \qquad N \,\hat{=}\, \text{Landau Levels}$$

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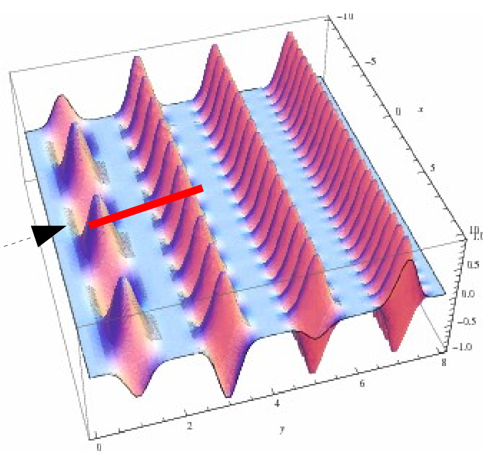

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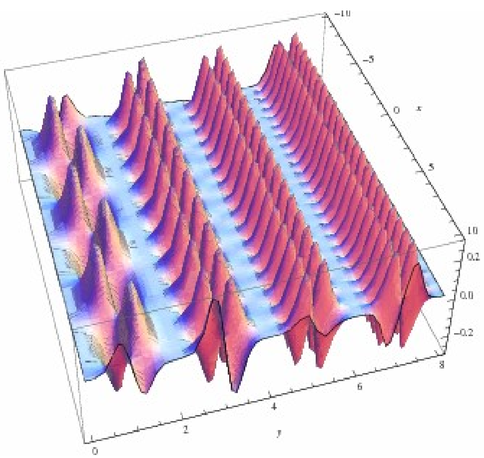

$$\Delta y_0=\frac{2\pi l^2}{L_x} \qquad \rightarrow \text{number of different wave functions for one Landau Level } \Omega_N=\frac{L_y}{\Delta y_0}=\frac{Lx\cdot Ly}{2\pi l^2}$$

$$\text{filling factor } \upsilon=\frac{\text{number of electrons}}{\Omega_N}=2\pi l^2 n_e$$

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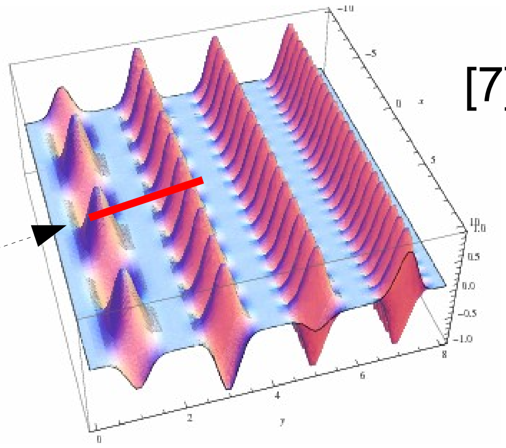

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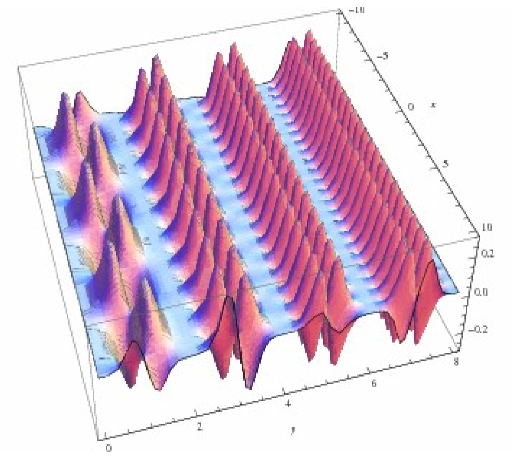

$$\Delta y_0 = \frac{2\pi l^2}{L_y} \quad \rightarrow \text{number of different wave functions for one Landau Level } \Omega_N = \frac{L_y}{\Delta y_0} = \frac{Lx \cdot Ly}{2\pi l^2}$$

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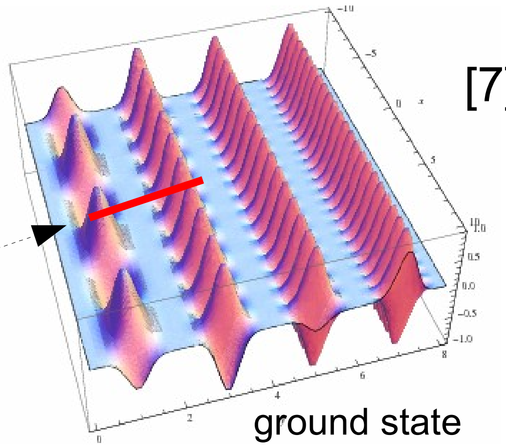

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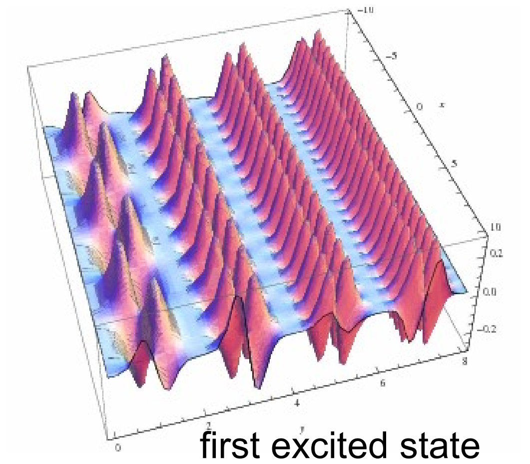

$\Delta y_0 = \frac{2\pi l^2}{L_x}$ → number of different wave functions for one Landau Level $\Omega_N = \frac{L_y}{\Delta y_0} = \frac{Lx \cdot Ly}{2\pi l^2}$

filling factor $\upsilon = \boxed{\frac{\text{number of electrons}}{\Omega_N} = 2\pi l^2 n_e = n_e \frac{hc}{eB}}$

$$n_e = \frac{eB}{hc}\upsilon$$

$$R_{Hall} = \frac{B}{cen_e} = \frac{h}{e^2}\frac{1}{\upsilon} = \frac{h}{e^2}\frac{1}{N}$$

→ Drude Hall resistance ---------------------->IQHresistance

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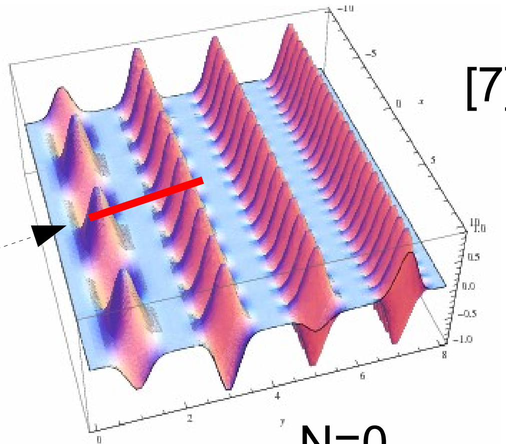

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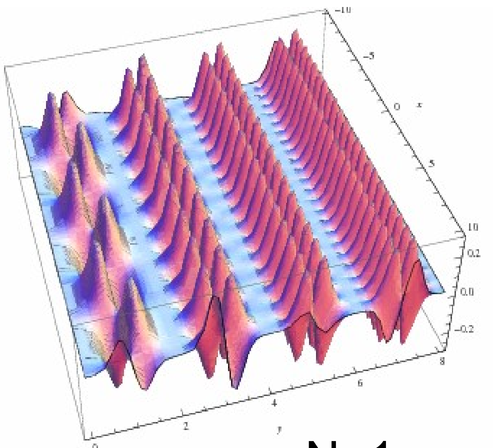
N=0



N=1

$$\Delta y_0 = \frac{2\pi l^2}{L_y} \quad \rightarrow \text{number of different wave functions for one Landau Level } \Omega_N = \frac{L_y}{\Delta y_0} = \frac{Lx \cdot Ly}{2\pi l^2}$$

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$$R_{Hall} = \frac{B}{cen_e} = \frac{h}{e^2}\frac{1}{\upsilon} \overset{?}{=} \frac{h}{e^2}\frac{1}{N}$$

→ Drude Hall resistance ---------------------->IQHresistance

but only for integer filling υ=N → no plateaus !?!

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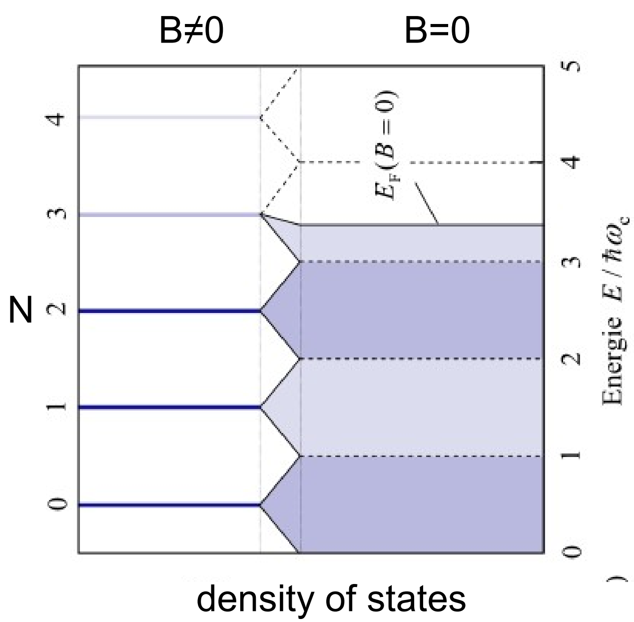
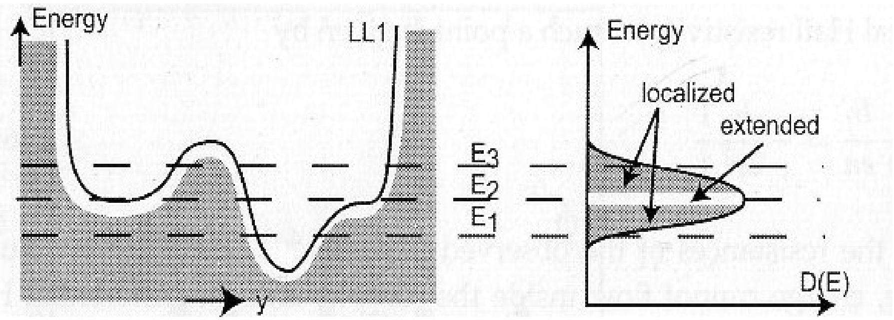


[2]

• Disorder effects: delocalized and localized states

additional disorder potential
→ degeneracy of Landau levels are lifted
→ broadened Landau Levels



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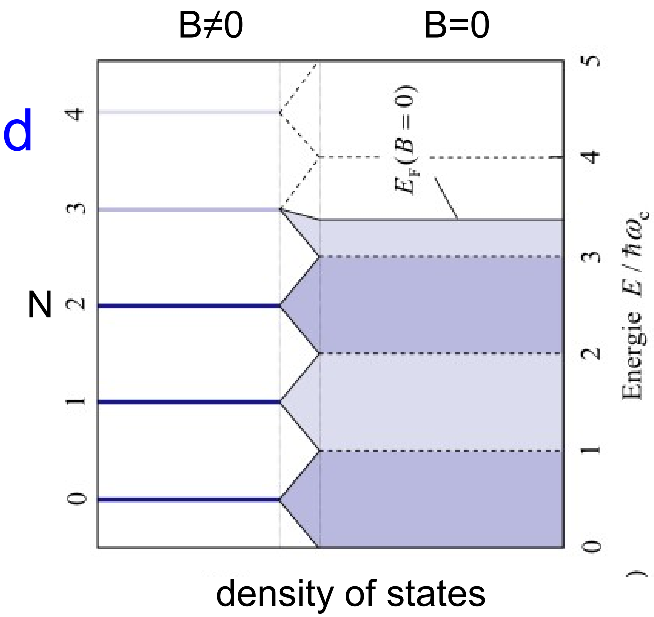

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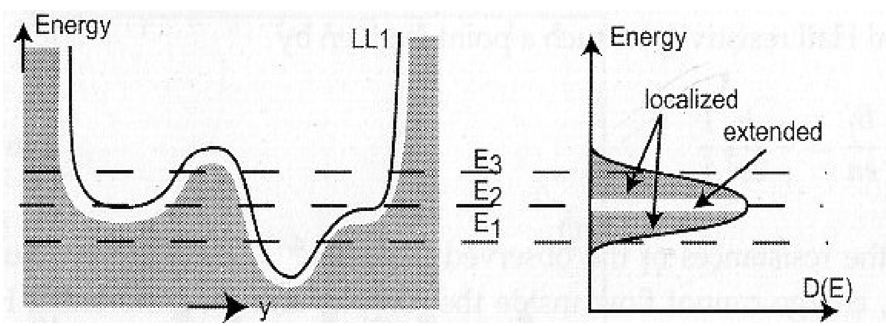

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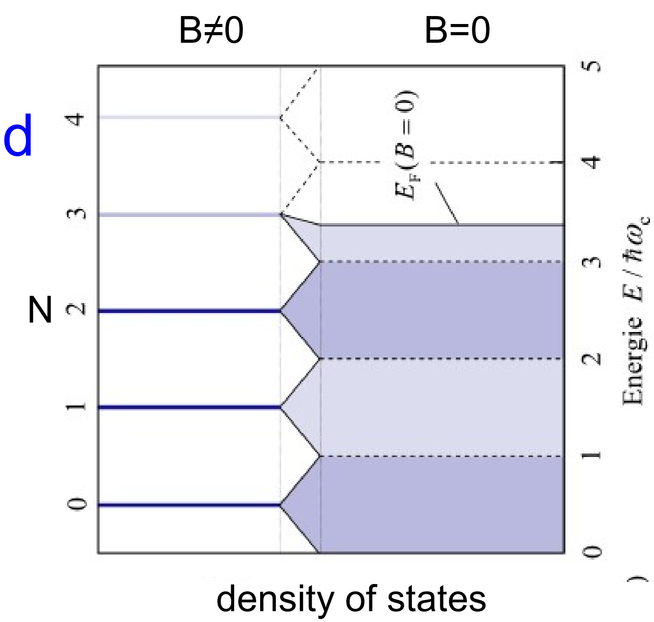
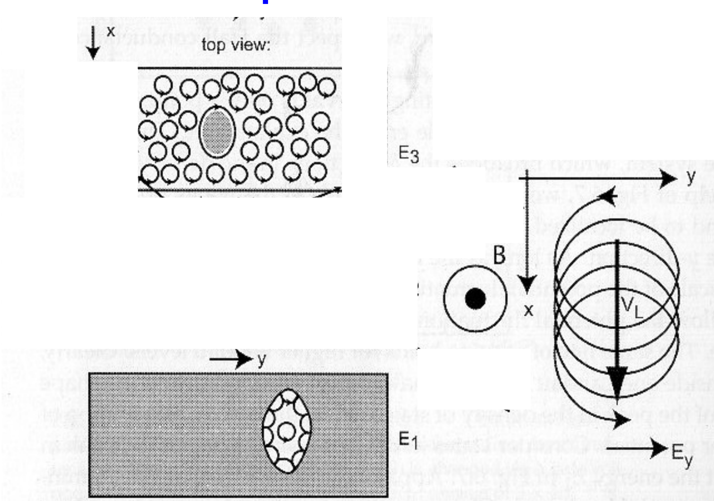


[2]

classical picture



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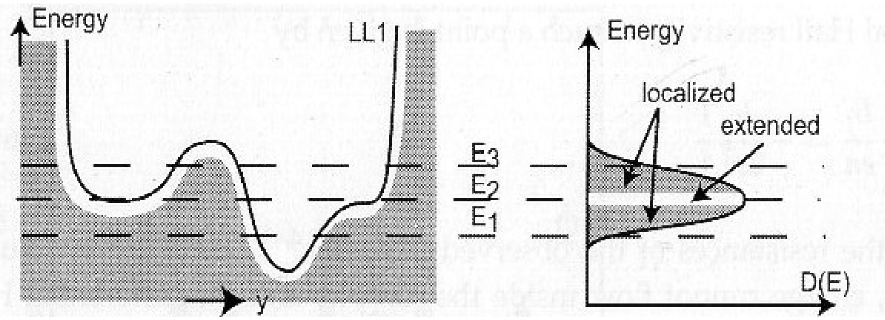

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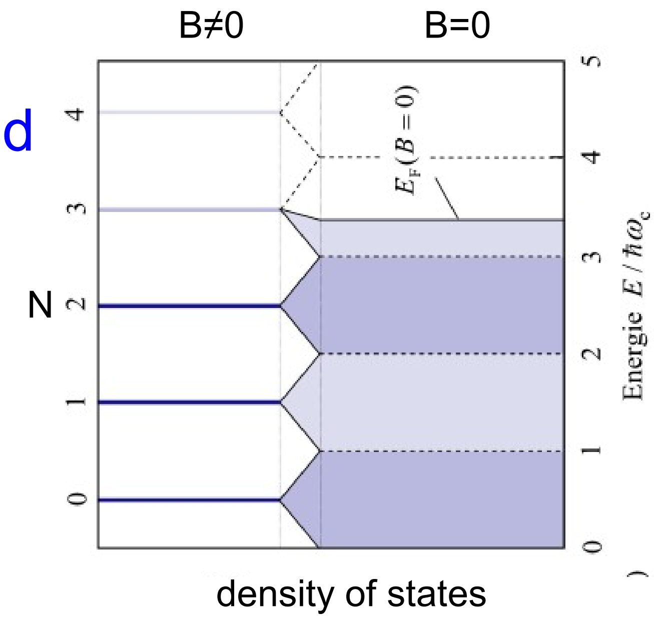

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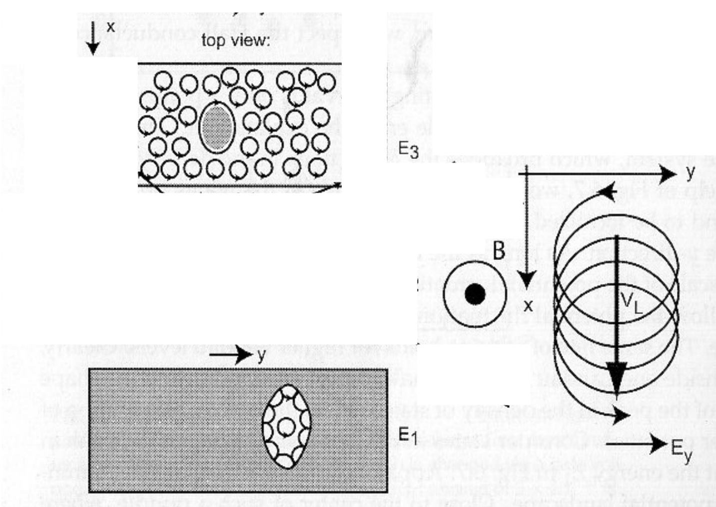

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quantum mechanical picture

plane waves in x-direction localized around $y_0(k_x)$

get additionally localized in x-direction due to the impurity potential

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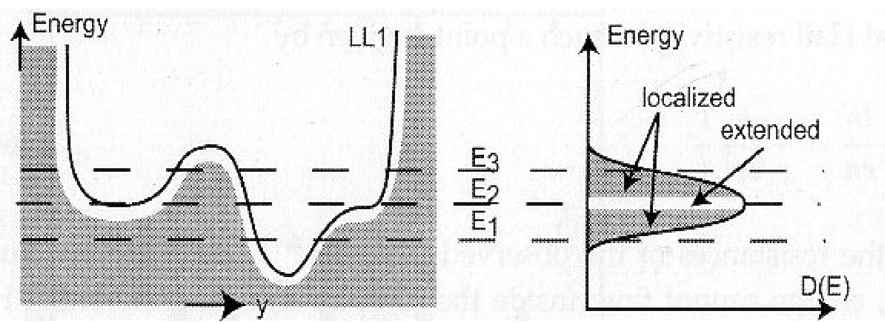

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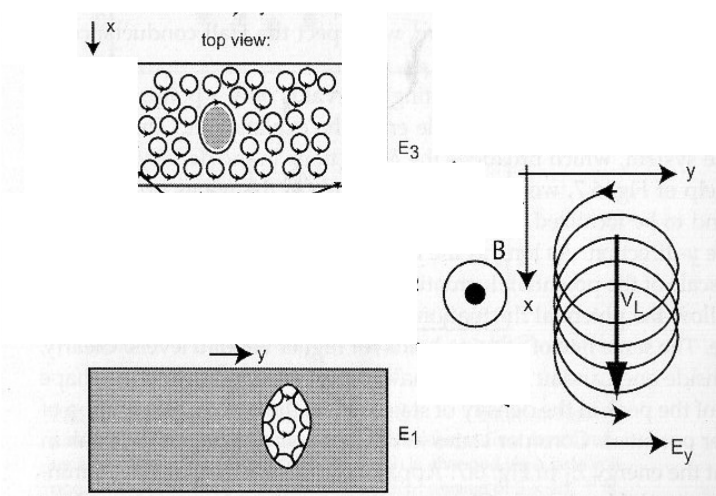

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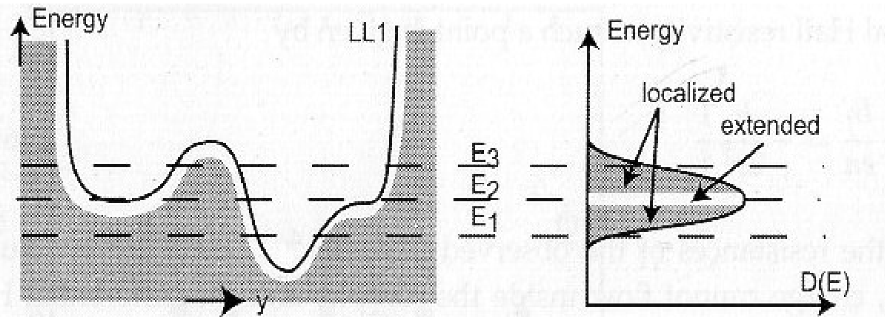

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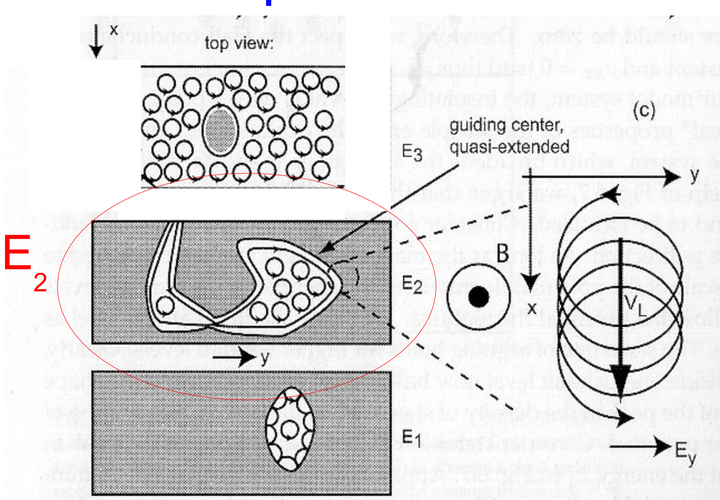
$E_F = E_2$



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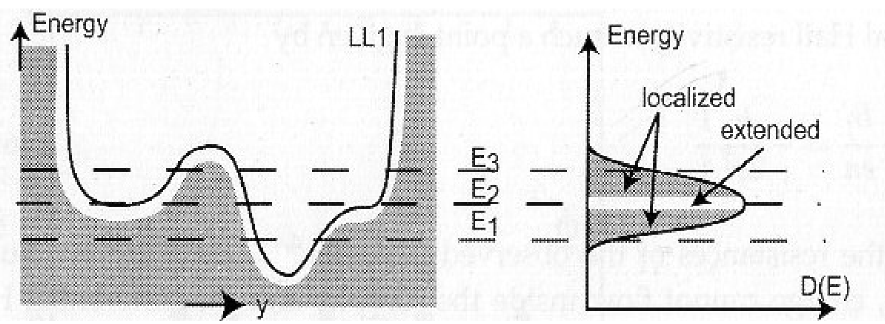

Numerically one can show that there is only one extended state for each Landau Level. The energies of the extended states correspond to the centers of the broadened Landau Levels, $=E_N=(N+1/2)\hbar w_c$

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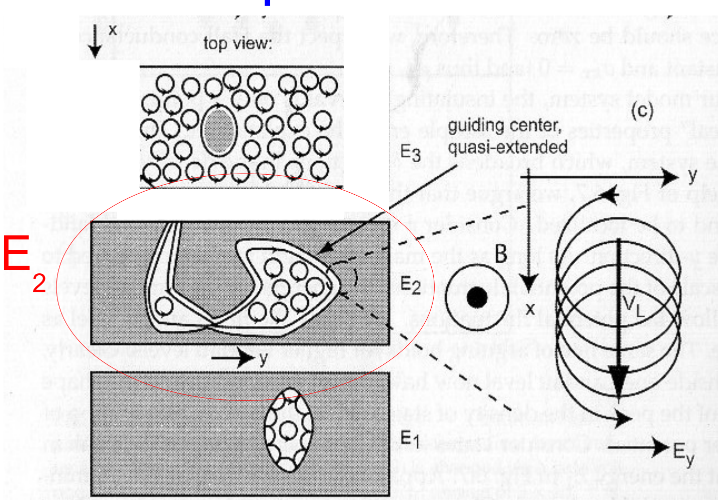

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get additionally localized in x-direction due to the impurity potential

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I Reminder: (Classical) Hall Effect in 2 dimensions

II Integer Quantum Hall Effect - observations

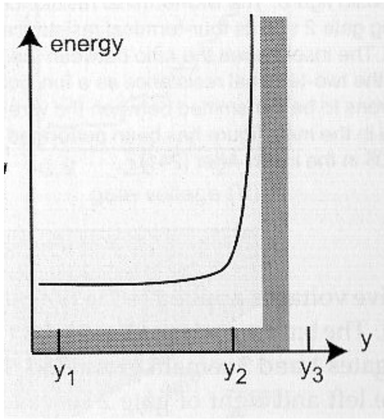
III Integer Quantum Hall Effect – key ingredients

- Quantum mechanical derivation of Landau Levels
- Disorder effects: delocalized and localized states
- Effect of finite sample size: edge states

IV Integer Quantum Hall Effect - explanations

• Effect of finite sample size: edge states

confining potential at edges of sample → high additional energies at edges



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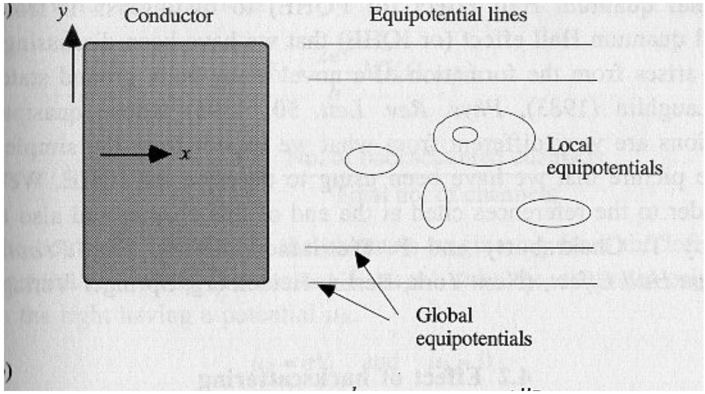

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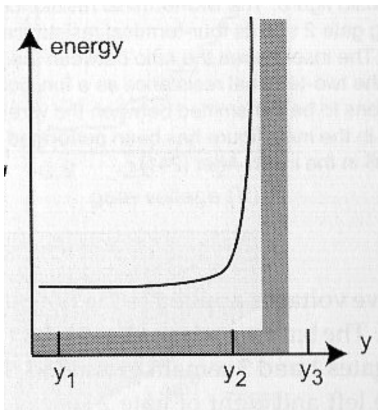

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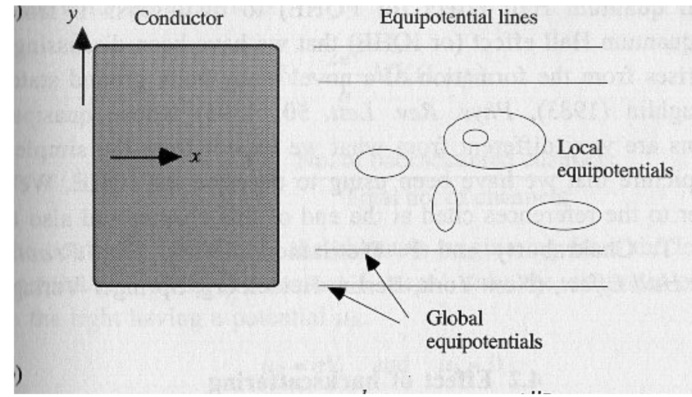

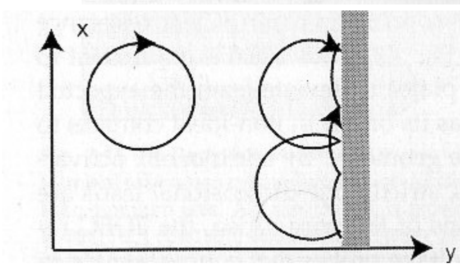

[4]

classical:
skipping orbits along the edges

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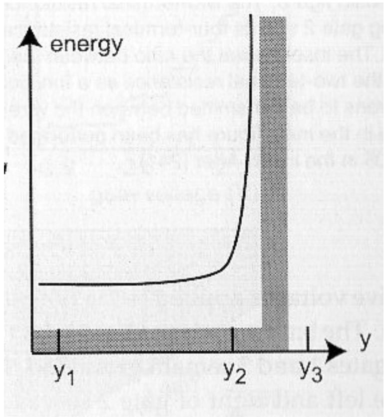

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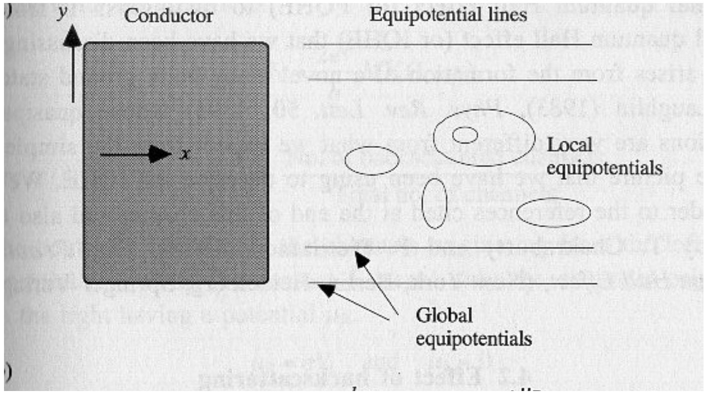

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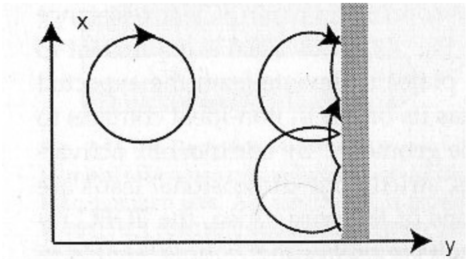

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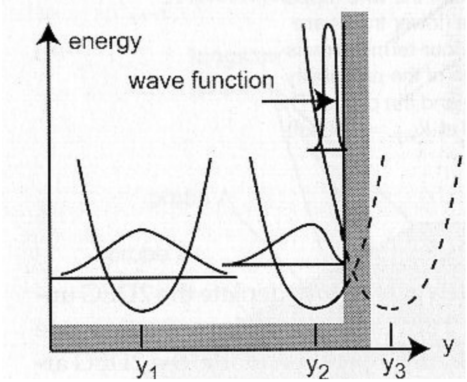


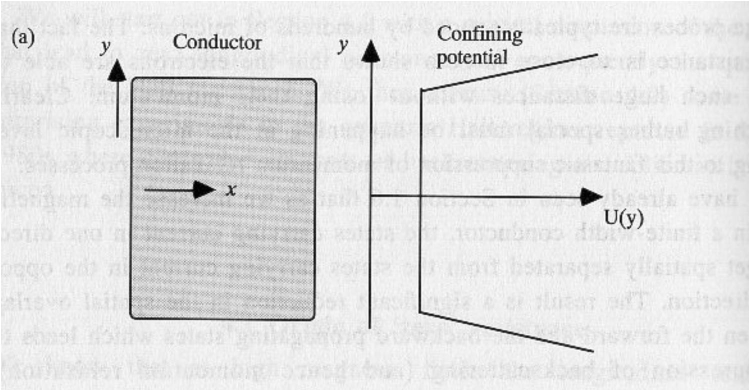
Quantum mechanical:
wace function is squeezed in y-direction and extended along the edges

- Effect of finite sample size: edge states

confining potential at edges of sample → U(y)

dispersion relation in lowest order perturbation theory:

$$E_{N,k_x} = (N+\frac{1}{2})\hbar w_c + \langle \Psi_{N,k_x} | U(y) | \Psi_{N,k_x} \rangle$$



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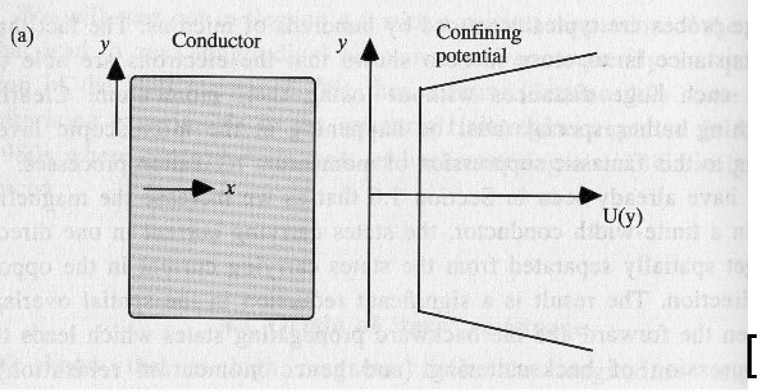

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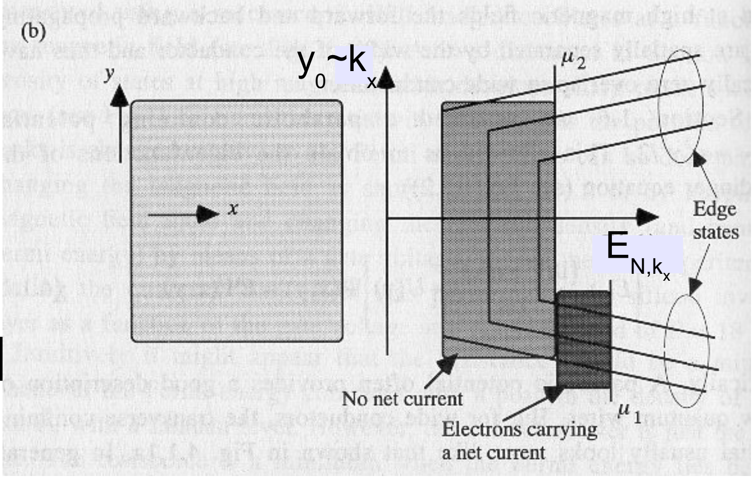

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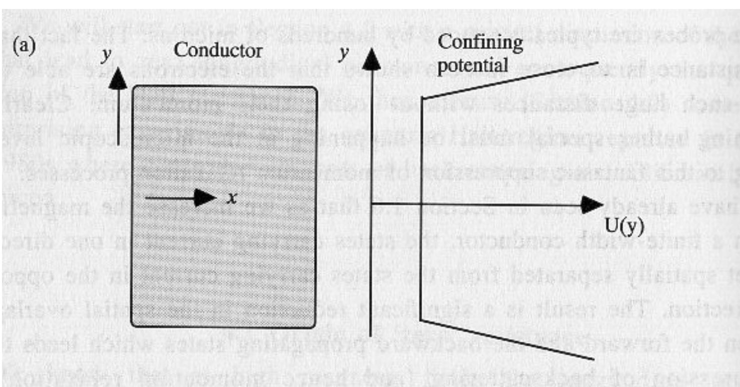

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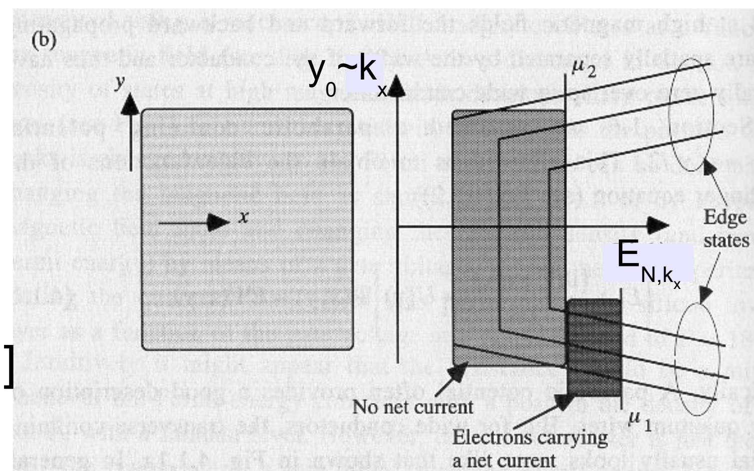
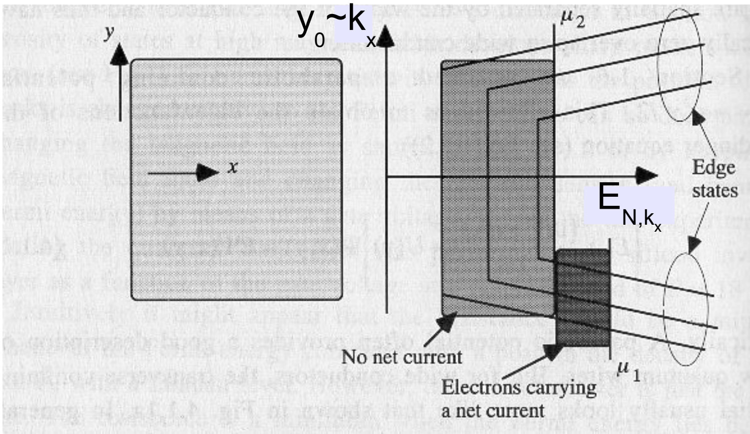


for $E_F \neq E_N$ → one edge state / edge channel per occupied (bulk) Landau Level !!!

- Effect of finite sample size: edge states

Properties of edge states / edge channels:

- only edge states contribute to a net current (for applied voltage)



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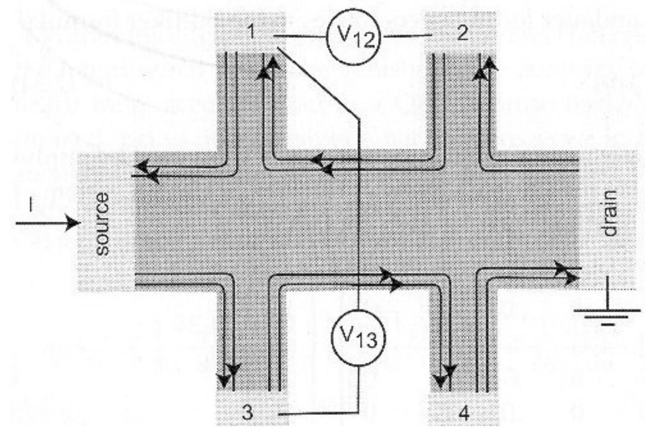

[4]

- edge states at two opposite sample edges carry currents in opposite directions

$$v_{N,k_x}=\frac{1}{\hbar}\frac{\partial E_{N,k_x}}{\partial k_x}=\frac{1}{\hbar}\frac{\partial U(y_0(k_x))}{\partial k_x}=\frac{1}{\hbar}\frac{\partial U(y_0(k_x))}{\partial y_0}\frac{\partial y_0}{\partial k_x}=\frac{c}{eB}\frac{\partial U(y_0)}{\partial y_0}$$

differs in sign for opposite edges

$$E_{N,k_x}=(N+\frac{1}{2})\hbar w_c+\langle\Psi_{N,k_x}|U(y)|\Psi_{N,k_x}\rangle \approx (N+\frac{1}{2})\hbar w_c+U(y_0(k_x)) \quad \text{with} \quad y_0=\frac{\hbar c}{eB}k_x$$

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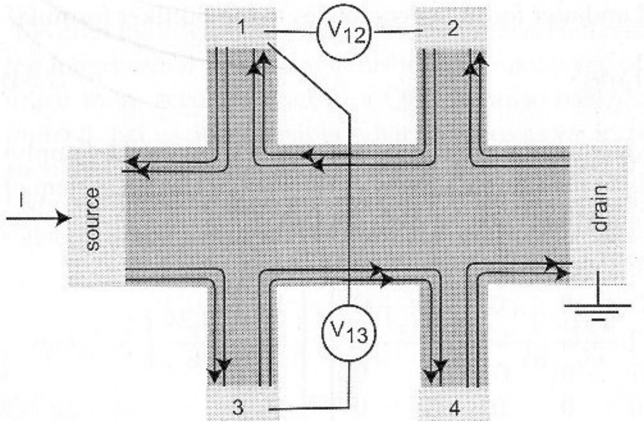

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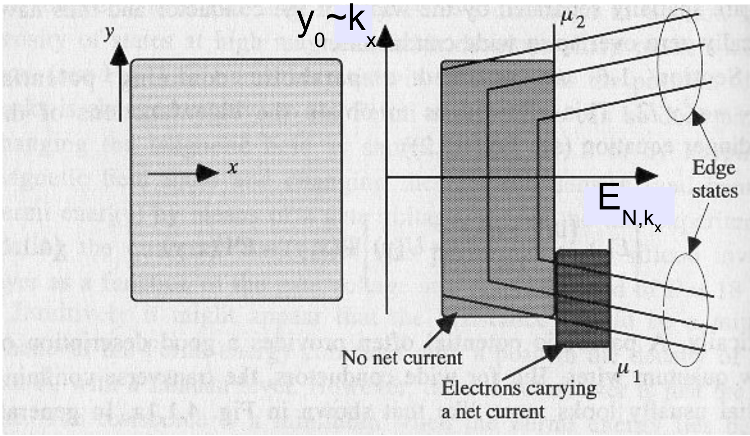

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- no scattering with bulk states as there are no allowed states to be scattered in

• Effect of finite sample size: edge states

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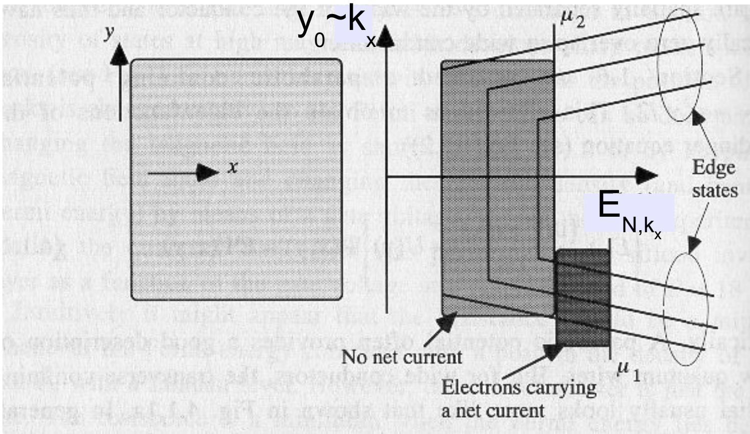

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differs in sign for opposite edges

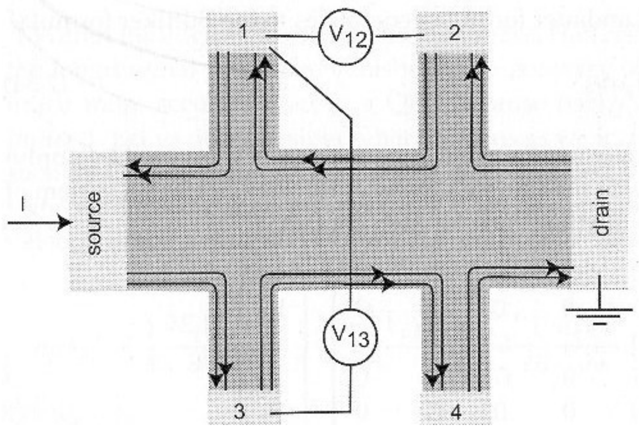
- backscattering over the sample width is exponentially suppressed (states localized at the edges have no overlapp)

- no scattering with bulk states as there are no allowed states to be scattered in

- scattering between different channels is suppressed due to the energy gap of $\hbar w_c$ between the channels

• Effect of finite sample size: edge states

Properties of edge states / edge channels:



[4]

→ no scattering / no momentum relaxation

→ 1 dimensional ballistic transport in edge channels for $E_F \neq E_N$ even though impurities are present

→ „supercurrent“ in IQHE

Outline

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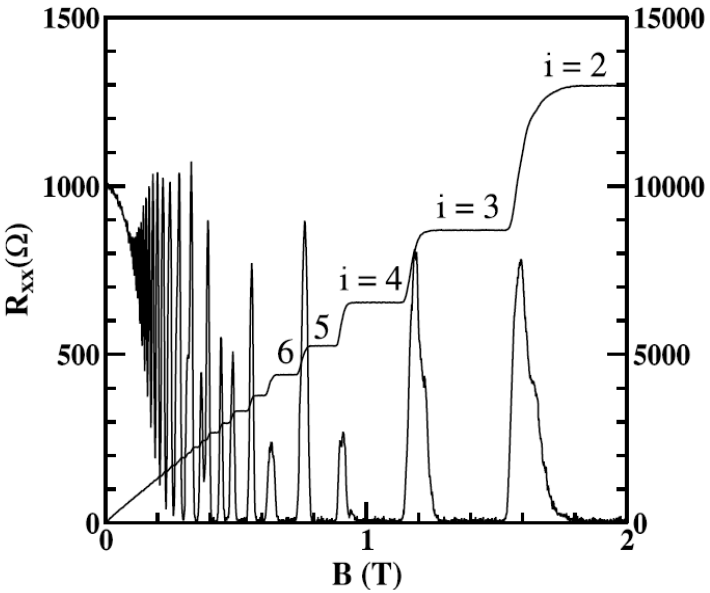
IV Integer Quantum Hall Effect - explanations

IV Integer Quantum Hall Effect – explanations

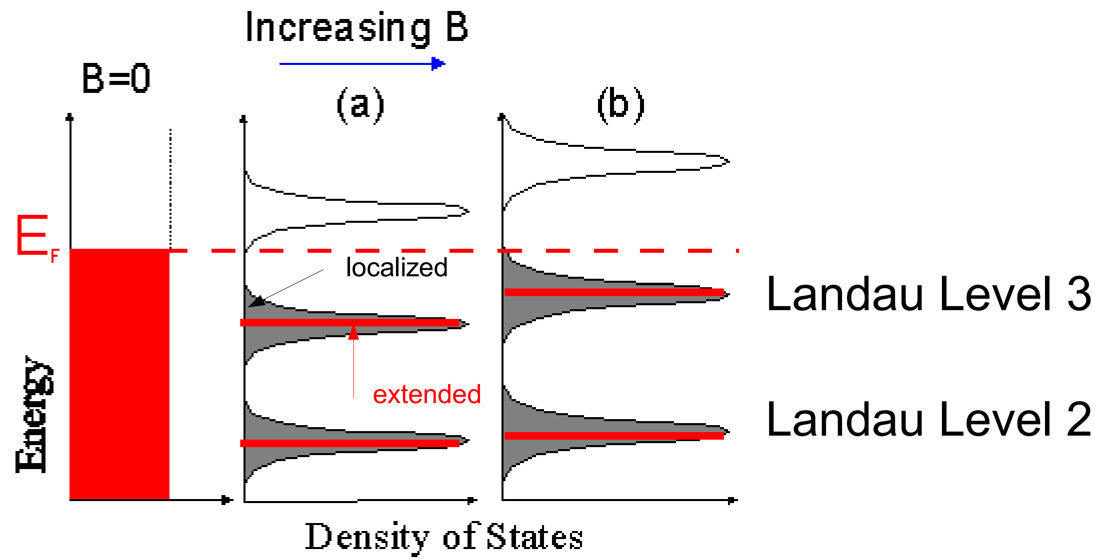
1) Longitudinal resistance R_x

- vanishes at plateau regions of R_{Hall}
- jumps about 13 orders of magnitude when R_{Hall} changes

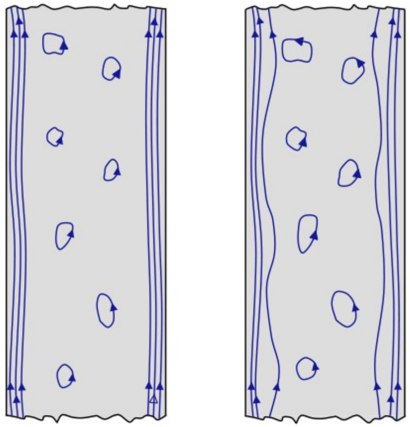


[5]

IV Integer Quantum Hall Effect – explanations

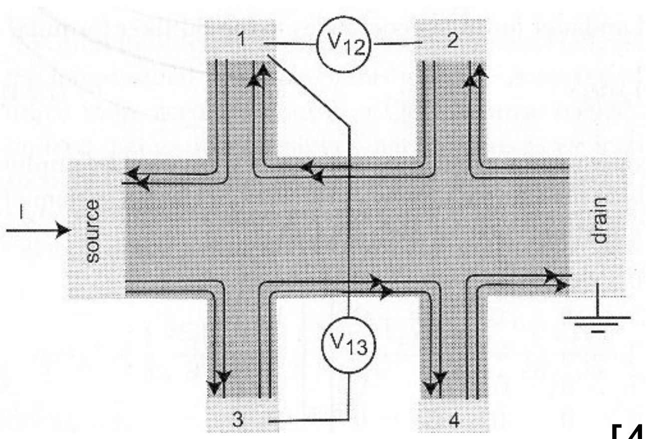


[6]



→ 1 dimensional ballistic transport for $E_F \neq E_N$ in 3 channels [2]

IV Integer Quantum Hall Effect – explanations



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→ $\mu_{source} = \mu_3 = \mu_4$

→ $\mu_{drain} = \mu_2 = \mu_1$

→ $V_x = V_{12} = (\mu_2 - \mu_1)/e = 0 = V_{34}$

→ $\boxed{R_x = V_x/I_x = 0}$

IV Integer Quantum Hall Effect – explanations

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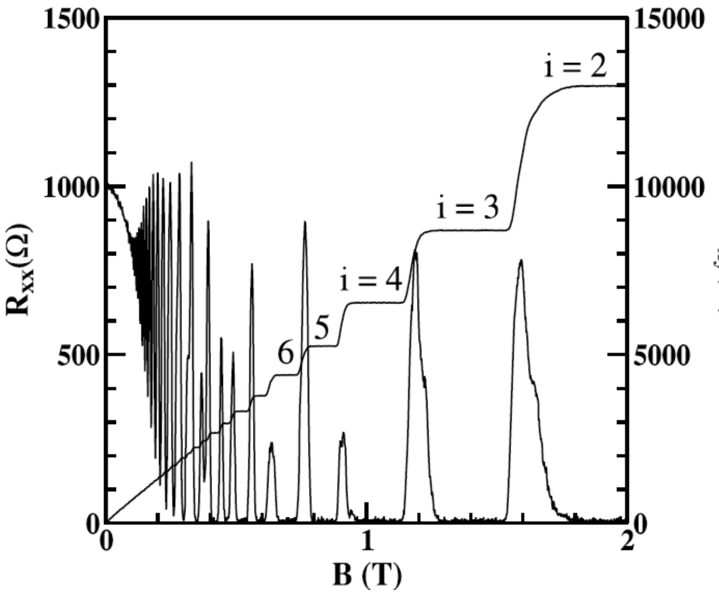

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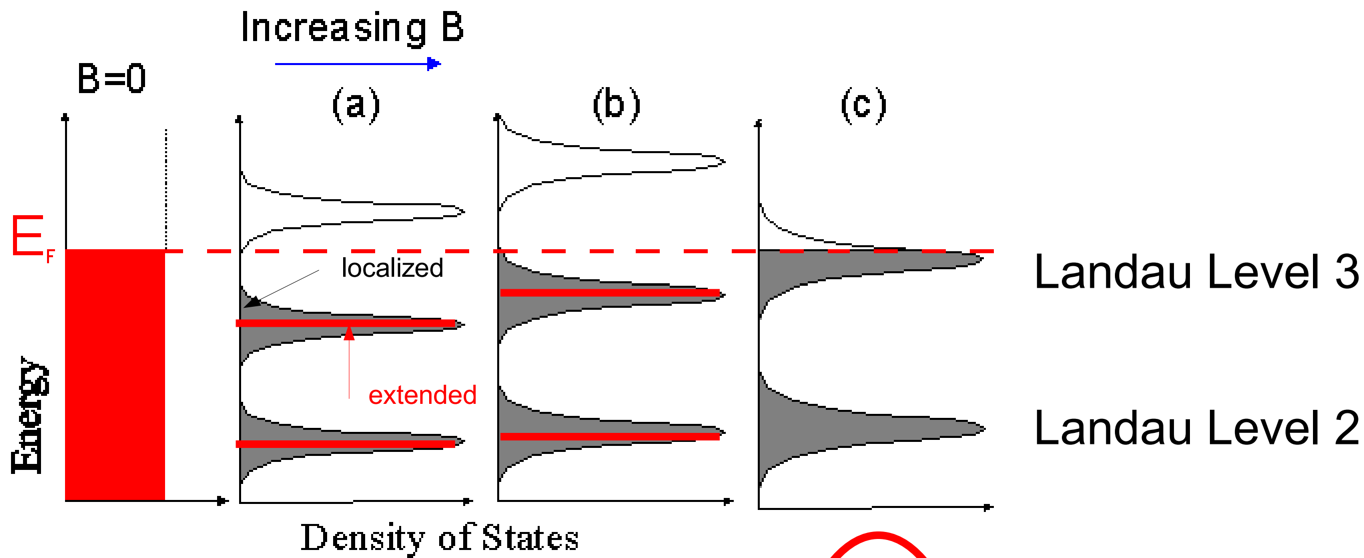
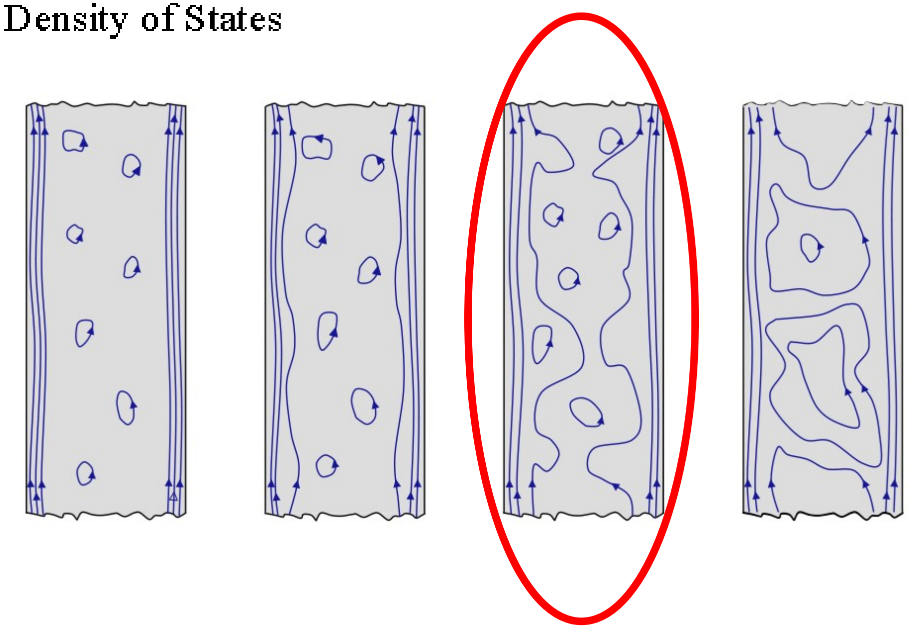
IV Integer Quantum Hall Effect – explanations



[6]

backscattering over whole sample is possible for $E_F = E_N$



[2]

IV Integer Quantum Hall Effect – explanations

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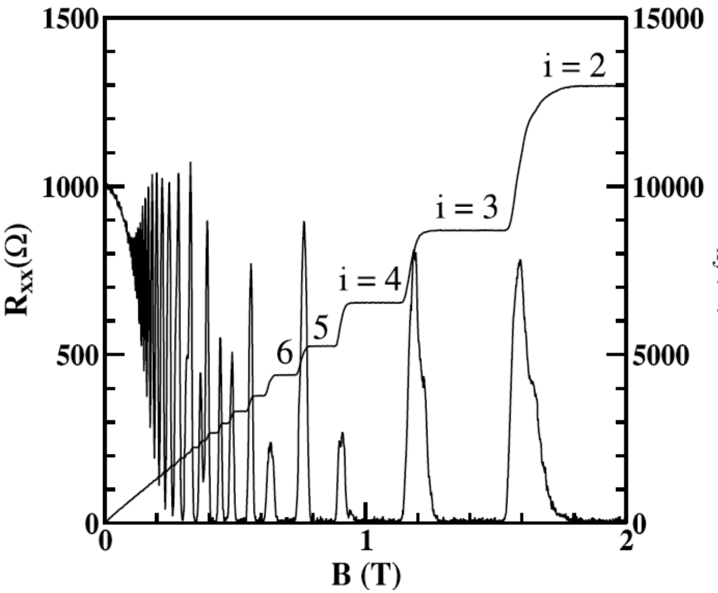
- vanishes at plateau regions of R_{Hall} for $E_F \neq E_N$
- jumps about 13 orders of magnitude when R_{Hall} changes for $E_F = E_N$



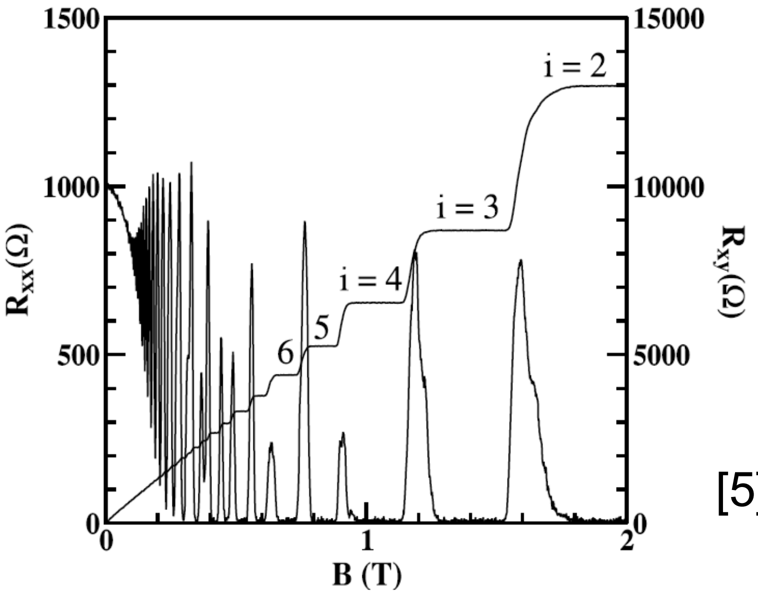
[5]

IV Integer Quantum Hall Effect – explanations

2) Hall resistance R_{Hall}

- shows plateau regions
- varies stepwise with step height given by

$$\boxed{R_{Hall} = \frac{h}{e^2} \cdot \frac{1}{N}} \quad N \in \mathbb{N}$$



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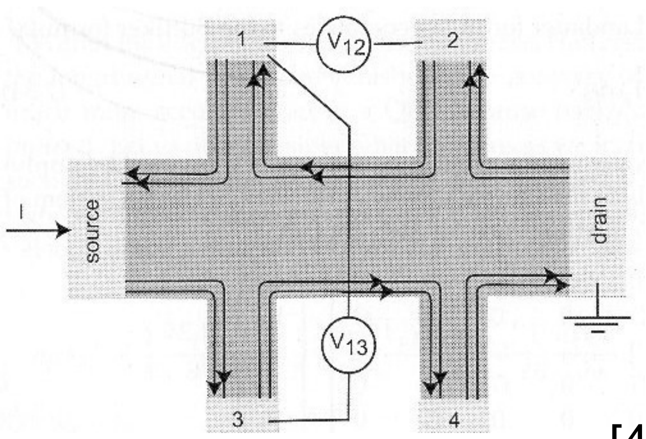

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→ 1 dimensional ballistic transport for $E_F \neq E_N$ in the channels

→ $\mu_{source} = \mu_3 = \mu_4$

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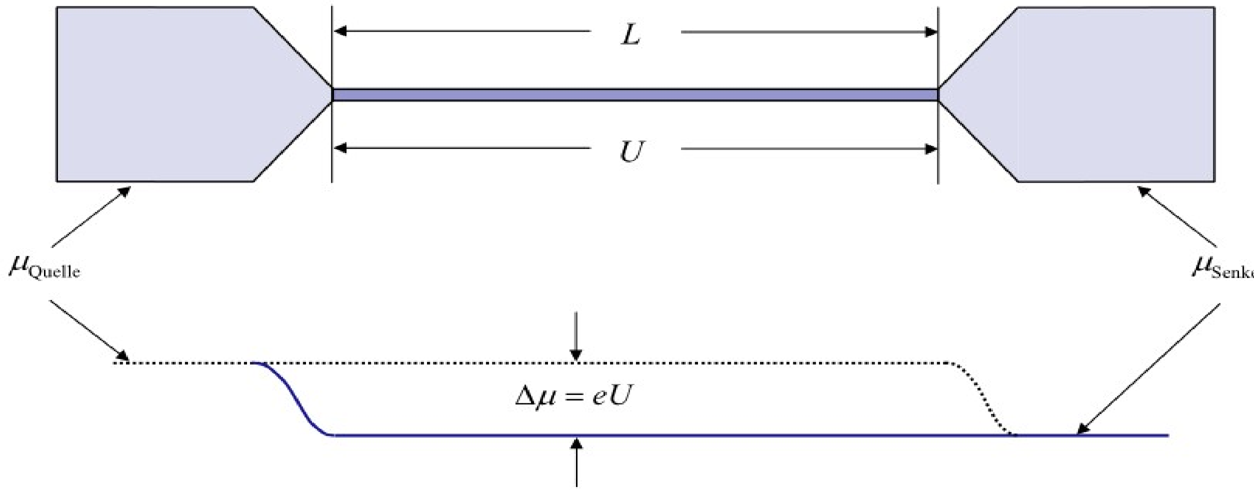
→ $V_{Hall} = V_{13} = (\mu_3 - \mu_1)/e = (\mu_{source} - \mu_{drain})/e$

IV Integer Quantum Hall Effect – explanations

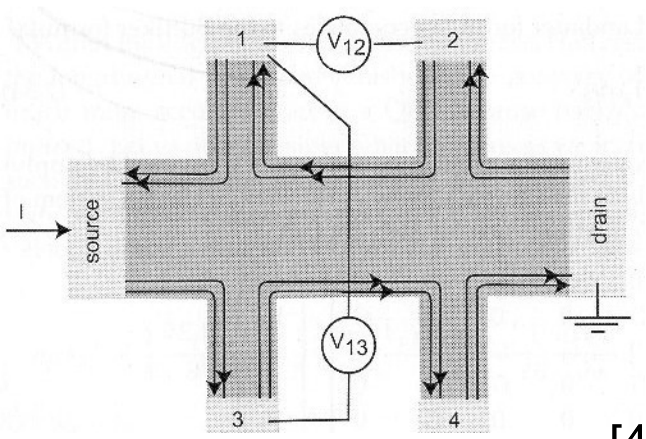


[2]

1 dimensional ballistic transport (current for one channel)

$$I = e \overbrace{\frac{1}{L}}^{\text{length of conductor}} \underbrace{\sum_k |v(k)|\left(f_r(k_r) - f_l(k_l)\right)}_{\textit{current summed over all L sites of 1 dimensional system}} = e\frac{1}{L}\frac{L}{2\pi}\int dk \underbrace{\frac{1}{\hbar}\left|\frac{\partial E}{\partial k}\right|}_{|v(k)|} \underbrace{\left(f_r(E+eV/2) - f_l(E-eU/2)\right)}_{\sim eV} = \frac{e^2}{h}V$$

IV Integer Quantum Hall Effect – explanations



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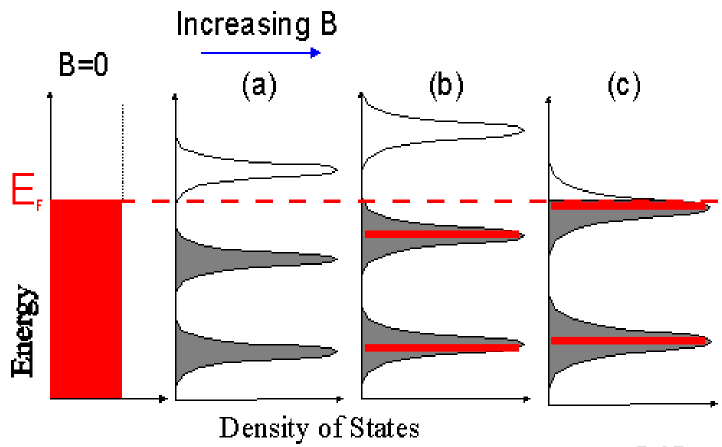
→ with $I = N\, e^2/h\; V_{Hall}$

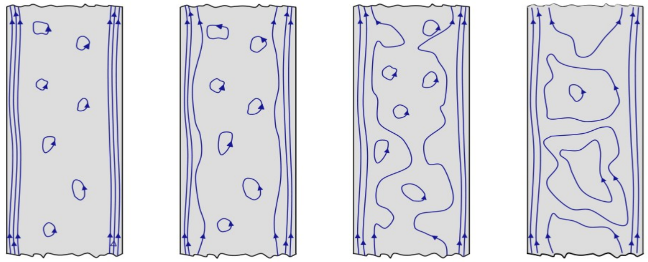
→ $$\boxed{R_{Hall} = \frac{V_{Hall}}{I} = \frac{h}{e^2}\frac{1}{N}}$$

IV Integer Quantum Hall Effect – explanations



[4]

- if B ↑ then N=const for $E_F \neq E_N$
  → plateau
- if B ↑ then N→ N-1 for $E_F = E_N$
  → R_{Hall} makes a step



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IV Integer Quantum Hall Effect – explanations

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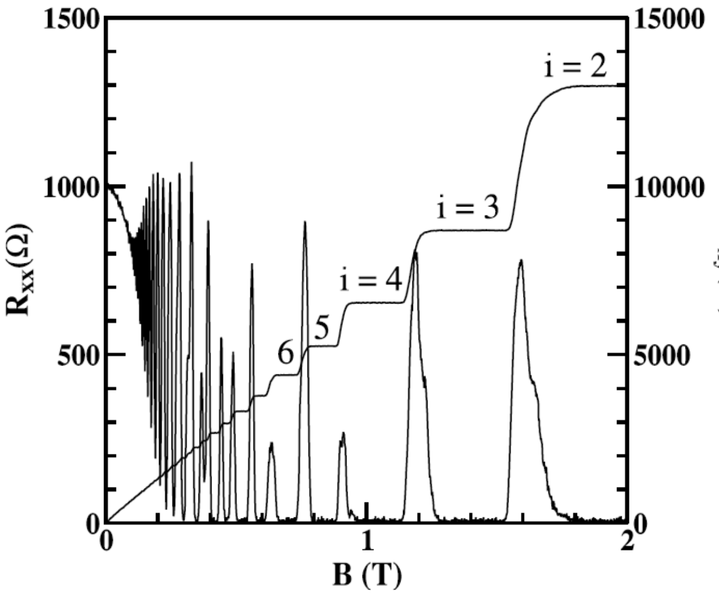


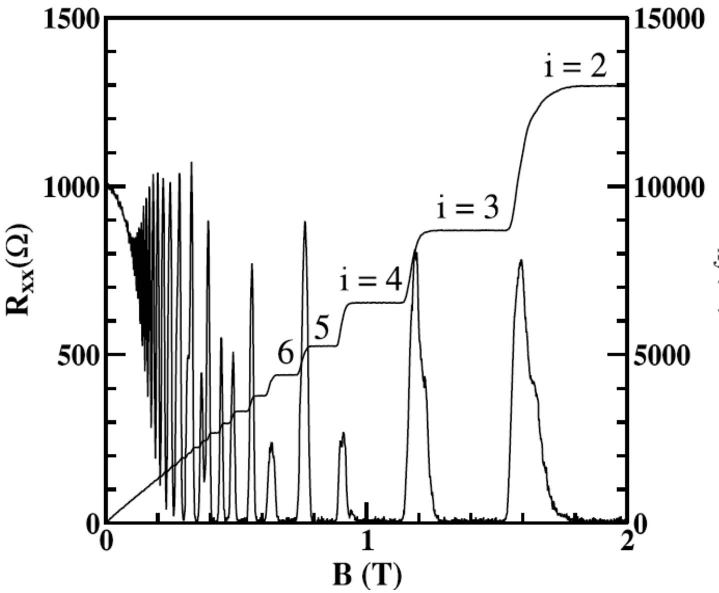
i=N

[5]

IV Integer Quantum Hall Effect – explanations

3) difference of plateau values
- of different samples and
- between different plateaus

is smaller than 10^{-10} times the quantized value



[5]

IV Integer Quantum Hall Effect – explanations

3) difference of plateau values
- of different samples and
- between different plateaus

is smaller than 10^{-10} times the quantized value

→ (nearly) complete suppression of momentum relaxation processes in the quantum Hall regime
→ truly ballistic conductor of incredibly high quality
→ quantization is extremely precise

→ accurate measurements possible
(e.g. due to independence of R_{Hall} of sample size

$$R_{Hall} = \frac{V_{Hall}}{I} = \frac{E_{Hall} \cdot W}{j \cdot W} = \frac{E_{Hall}}{j} = \rho_{Hall}$$

→ Hall resistance = Hall resistivity)

IV Integer Quantum Hall Effect – explanations

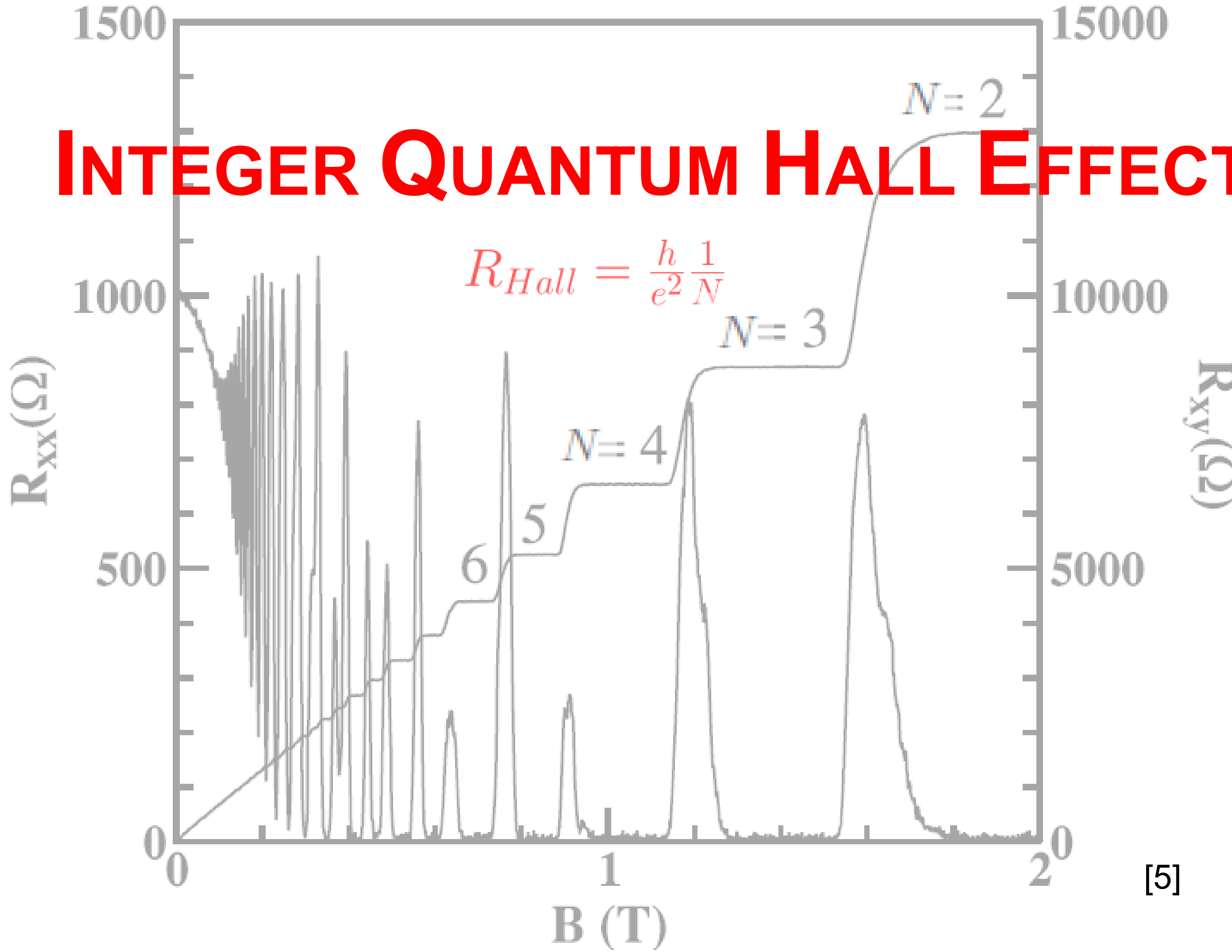
3) difference of plateau values

- of different samples and
- between different plateaus

is smaller than 10^{-10} times the quantized value

$\rightarrow$ $R_K = \frac{h}{e^2} = 25812.807\,\Omega$ is used as a resistance standard

Integer Quantum Hall Effect



[5]

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[6]]http://www.warwick.ac.uk/~phsbm/llqhe.gif

[7]]http://homepages.physik.uni-muenchen.de/~stefan.kehrein/teaching/MesoPhysikSoSe2011/MesoPhysikSoSe2011.html