14.03/003: Problem Set 2

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Due October 5, 2015 @ 5pm at E18-266 (or in class)

1 Short Questions

For each of the statements below, determine whether the statement is True, False, or Uncertain, and explain why. Answers without explanation will receive no credit. (Even if it is True, elaborate why it is so.)

1. Adam and Brian have different preferences both of which satisfy all of the 5 axioms introduced in class. At certain prices, incomes, and shapes of their utility functions, it can be that every good is normal to Adam while every good is inferior to Brian.

2. Peter and John's preferences satisfy 5 axioms. Peter consumes every good strictly more than John does. Then Peter attains higher utility than John.

3. A Giffen good must be an inferior good, but an inferior good need not be a Giffen good.

4. Utility functions which are increasing and convex in all goods (thus exhibiting a decreasing marginal rate of substitution) generate indifference curves that are downward sloping in every pair of goods (that is, they have a negative slope in x-y space) and are bowed towards the origin.

5. If a consumer chooses a consumption bundle for which the marginal rate of substitution between two goods does not equal the ratio of their prices, then the consumer is not choosing an optimal, utility-maximizing bundle.

2 Utility Maximization and Marshallian Demand Functions

A consumer has the following utility function:

$$U(x, y) = \frac{1}{4} \log x + \frac{3}{4} \log y.$$

The prices of goods x and y are p_x and p_y, and the consumer cannot buy more than her income allows, which is denoted by m.

1. Set up the consumer's constrained maximization problem. Solve for the optimal consumption bundle.

2. Compute the consumer's utility when she consumes her optimal bundle. Given this result, rewrite the utility maximization problem as a cost minimization problem.

3. Given the demand curve you derived, determine the uncompensated price effect, $\frac{\partial x^*}{\partial p_x}$, and the income effect, $\frac{\partial x^*}{\partial m}x^*$.

4. Using the Slutsky equation, find the substitution effect, $\frac{\partial h_x}{\partial p_x}$.

3 Indirect Utility and Expenditure Function

In an economy with two goods, food (x) and clothing (y), a consumer has the following indirect utility function

$$v(p_x, p_y, m) = \log m - \alpha \log p_x - (1-\alpha)p_y, \tag{1}$$

where p_x and p_y are prices of goods x and y, m her income, and $\alpha \in (0,1)$ a parameter.

1. Remember that

$$v(p_x, p_y, m) := \left[\max_{x,y} u(x,y) \quad \text{subject to} \quad p_x x + p_y y = m\right]. \tag{2}$$

   Write down the Lagrangian of the problem on the right hand side of (2). Then use the envelope theorem to obtain $\frac{\partial v}{\partial p_x}$, $\frac{\partial v}{\partial p_y}$, and $\frac{\partial v}{\partial m}$ in the general problem (2) and in our particular example (1). Use these results to obtain the Marshallian demands of this consumer.

2. Find the expenditure function.

3. With another application of the envelope theorem, derive the Hicksian demands from the expenditure function you have just obtained.

4. Suppose that you learn that this consumer initially spends $\frac{2}{3}$ of her income on purchases of food and $\frac{1}{3}$ of her income on clothing. What would be your best estimate of the parameter α in equation (1)? Use this value of α for the remaining questions.

5. Assume that the initial prices are $p_x = p_y = 1$, the initial income is $m = 200$, and the price of food increases to $p_x^{new} = 8$ after an agricultural shock. How much additional income would be necessary to bring this consumer back to her welfare level achieved under the initial prices?

4 Consumer Theory

For each of the following, state which (if any) of the 5 consumer theory axioms are violated. Be sure to explain your answer.

1. Sally wants a vacuum and is comparing 3 different makers. Dyson has power but creates fair amount of noise; Hoover is quiet but weigh a fair amount; Toshiba is light but only has fair power. The table below gives the summary, in which the question mark indicates that the corresponding characteristic is unknown.

	Power	Noise	Weight
Dyson	Good	Fair	?
Hoover	?	Good	Fair
Toshiba	Fair	?	Good

When she compares two makers, she only focuses on the comparable characteristics and strictly prefers the Good one over the Fair one.

2. Jess is about to decide which college to attend. She has her career path in mind either in computer science or in music. She has three offers: MIT, Berklee, and Harvard. MIT offers 8 classes of computer science and 1 class of music; Berklee offers 1 class of computer science and 8 classes of music; Harvard offers 5 classes of computer science and 5 classes of music. She prefers MIT and Berklee over Harvard, and Berklee over MIT.

3. Cindy loves cupcakes. She especially likes the whip cream part, so usually she prefers a cupcake with more whipped cream over ones with less whipped cream. However, when there are raisins in them, yikes, she wants to have as few raisins as possible. This means that however much whipped cream a cupcake has on top of raisins, if there is another cupcake with less (or no) raisins in it, she strictly prefers it.

5 The Deadweight Loss of Housing Vouchers

The US government has asked you to design an experiment to measure the deadweight loss of housing vouchers. You start by selecting a sample of participants who are all currently receiving \$100 in housing vouchers each month. You randomly assign them to two groups: group 1 continues to receive housing vouchers, while group 2 receives \$100 in cash each month instead. In group 1, 50% of the participants used all of their housing vouchers but spent no additional money on housing, while the other 50% spent an additional \$50/month on housing. In group 2, 50% spent \$80/month on housing and 50% spent \$150/month on housing.

1. Which of these consumers are constrained, and which ones are unconstrained?

2. A critic of the housing voucher program looks at your data and concludes that 10% of spending on vouchers is pure waste. Explain why the critic thinks so, and also why he is probably wrong.

3. Based on prior research, you believe that the compensated elasticity of demand for housing is 0.25 (i.e. demand for housing is rather inelastic).

   (a) Draw the compensated demand curve on a graph for the unconstrained consumers and for the constrained consumers. On the X-axis put the expenditure on housing in dollars; on the Y axis put the price of housing in terms of expenditures on all other goods. Identify the deadweight loss of the program graphically. [Hints: Plug in some exact numbers for the constrained consumers, and these numbers will help you in the next question. A demand curve with elasticity 0.25 is defined by the formula $H = \gamma P^{-0.25}$, where H

is the quantity of housing purchased. Note that γ must be the amount of housing such that the marginal housing "unit" is worth exactly \$1 of other expenditures.]

(b) Explain how you would estimate the deadweight loss of housing vouchers for each constrained consumer. (You can approximate the compensated demand curves by linear functions if you like.)

4. A politician proposes to eliminate the deadweight loss and save the government money by cashing out the housing voucher program. Under her proposal, \$100 in vouchers will be replaced by \$100 in cash, minus the average deadweight loss you calculated. Who benefits and loses from this policy change?

5. Did you need to use group 1 in the calculations in (3)?

6. Suppose you had found that 50% of group 2 recipients spent \$80/month on housing, while the other 50% spent \$130/month on housing. Would this observation make you concerned about your calculations in (3)? Why or why not?

7. You have also collected data on the black market value of housing vouchers.

   (a) Suppose that you found that the black market price was \$0.75 for each dollar of housing vouchers. Show clearly with a graph how this would change your estimate of the deadweight loss. (No calculations required.)

   (b) Suppose you found that the black market price was \$0.25 for each dollar of housing vouchers. Would this make you concerned about your approach to estimating the deadweight loss? Explain why or why not.