

14.03/14.003 Problem Set 1

Fall 2015

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Due Wednesday, 9/23/2015 at 5pm EST at E18-266 (or in class)

1 Implicit Function Theorem and Envelope Theorem

Consider the function:

$$y = f(x; a) = 4ax - 7x^2$$

where a is a parameter.

1. Find the critical point of this function. Denote it with $x^*(a)$.
2. Is this point a maximum, a minimum, or neither? (Show why)
3. For the rest of the problem, assume that $a > 0$. Using the critical point of the function $f(x; a)$ found in part 1, find $y^*(a)$, the maximized value of y as a function of a .
4. Find $\frac{dx^*}{da}$ and $\frac{dy^*}{da}$.
5. Now use the FOC from (1) and the implicit function theorem to find $\frac{dx^*}{da}$.
6. Use the envelope theorem to find $\frac{dy^*}{da}$. Why does this theorem allow you to simplify your calculations with respect to point 4.?

2 Minimum Wages and Employment

Assume that you have been asked to evaluate whether the labor market for hospital nurses (who are much more likely to be unionized than the average U.S. worker and could threaten to strike) is competitive or monopsonistic. The production function of nurse care depends only on labor input, and the output function looks as follows:

$$Y = -0.75L^2 + 15L$$

Here Y is the labor output (e.g. care quality) and L is the amount of labor used. The price of unit of care is $p = 3$. The labor supply (as a function of the wage offered) for nurses is as follows, where w is the wage in dollars per one unit of labor:

$$L = -8 + w$$

1. Find the marginal revenue product of labor curve (MRPL). Is curve downward sloping, and why?
2. Find the equilibrium wage and employment (w^C, L^C) that would exist if the labor market for nurses was competitive. [NOTE: a competitive firm takes the market wage as given, i.e. the firm assumes that the quantity of workers that it hires will have no effect on the price / wage of the next worker. (Obviously, the equilibrium market wage is such that it equates labor demand and supply)]. Demonstrate with a graph what the equilibrium looks like.
3. Now assume that you are in a Scandinavian country that has a national health care system where a single entity operates all hospitals, and is the only firm that hires nurses. Do you think that this entity takes the nurses' wages as given. In this scenario, find the marginal cost of labor curve (MCL). Is the MCL upward or downward sloping? Why is that? Compare the MCL curve to the nurses' supply of labor curve.
4. In a monopsonistic labor market with a single employer, what are the equilibrium wages and employment (w^M, L^M) that would equate market demand and supply for nurses? Illustrate this equilibrium also with a graph.
5. Compare the competitive and monopsonistic equilibria found above and explain why they differ. In the US health care system, which one would you argue to be the most likely scenario and why?
6. Due to successful lobbying by the nurses' union, the Congress has agreed to increase the minimum wage of nurses to $w_{min} = 16$. How will the new minimum wage affect the labor market equilibrium (wage and employment) in the competitive market? How about in the monopsonistic market? Here you will need to provide a mathematical answer, but you can additionally illustrate the new equilibria with graphs.
7. What if the government instead decides to raise the minimum wage of nurses to $w_{min} = 20$? Explain in some detail. [Note: If you were unable to derive the numerical values for the previous question you may give some intuition regarding how the results of the exercise depend on the level of the new minimum wage].

3 Minimum Wages and Employment in the US

There has been a vigorous debate about raising the current level of minimum wage in the US, and questions related to impacts it would have on businesses and workers especially in the service and fast-food industry. Please read the three related New York Times articles (posted on the Stellar site) and focus especially on what the experts are saying about the potential impacts of increased minimum wage. The economic experts generally take different sides of the minimum wage debate in the US.

1. Which model of the labor market from the ones discussed in class do you think most closely fit the empirical results on employment effects referred to by the various economic experts in these articles? Explain why. [Hint: do the results suggest labor markets for restaurant workers in the US are competitive, monopsonistic, or something else?] Which assumptions does the model you chose rely on?
2. Card and Krueger's classic study on the minimum wage took place during a recession. If they instead had looked at a change of minimum wage that took place immediately before an economic boom, how do you think that would have impacted the results? Illustrate with diagram how a simultaneous economic expansion in PA and NJ would have impacted the validity of their "natural experiment".
3. Card and Krueger measure the impact of the minimum wage immediately after its introduction, yet other economists distinguish between the short and long run impacts of minimum wage increases. Many imply that the long run impacts are likely to be larger (and more likely to be negative). List one or two specific economic reasons why the long run response would be larger, and elaborate on your reasoning. Consider e.g. that labor is not the only factor in fast food restaurants' production functions.
4. Consider a case where Texas is thinking about raising its state minimum wage and where workers are mobile across states (as well as countries). Assume that those Texan and Mexican workers who would most likely to be working in minimum wage jobs are perfect substitutes for each other in the labor market. In this scenario, how would the increased minimum wage affect labor supply (and immigration) between Texas and Mexico and the wages in each location. Explain both in words as well as using a graph.

4 Causal Inference: Gun Legislation in the U.S.

A very controversial and important policy question today relates to laws that control access to firearms. The big question is: Does gun control reduce violent crime? There is a lively debate on this issue both in the US and abroad, and international comparisons are often made by the commentators and the media. For example, gun control proponents note that the rate of gun violence is much lower in countries with fewer handguns, and therefore argue for stricter gun laws. Opponents argue, e.g., that the rate of violence is more related to other factors such as the availability of mental health care. Assessing whether greater restrictions on gun ownership would reduce violent crime requires the policy makers to know *the causal effect of such laws on gun violence* (and other violence).

How should the policy makers and/or researchers go about answering the question whether strict gun laws reduce violent crime? Let's introduce some mathematical notation to begin the process of answering it.

First, let X_{it} denote that individual i is either exposed to a very strict policy towards gun ownership ($X_{it} = 1$), OR towards a very lax policy towards gun ownership ($X_{it} = 0$) in year t .

Second, let Y_{it} denote the decision of individual i to commit a violent crime in year t [$Y_{it} = 1$] or not [$Y_{it} = 0$]. The potential outcomes under the two possible gun legislation scenarios are as follows:

Y_{0it} = the decision of individual i in year t if the gun legislation is very lax

Y_{1it} = the decision of individual i in year t if the gun legislation is very strict

Let us further define the expected effect of introducing a strict gun legislation as $T^* = E[Y_{1it} - Y_{0it} | X = 1]$. NOTE: we condition on $X = 1$ since we can only hope to learn about the causal effect of a “treatment” (here, the introduction of stricter gun laws) on those individuals who were subjected to the treatment.

1. Just hypothetically, what would be the ideal “policy experiment” that you would like to conduct to measure the impact of stricter gun laws on violent crime?
2. What is the fundamental problem of causal inference in this setting? If your aim is to answer the question: does a stricter gun legislation reduce violent crime - are you then interested in measuring $Y_{1it} - Y_{0it}$? Is it possible to measure this quantity?
3. In this setting, how would you write down the Average Treatment Effect (ATE) and the Average Treatment Effect on the Treated (ATT)? What is the conceptual difference between those two? Suppose that you are evaluating this question using the population of the US and the UK (i.e. your population of interest contains these two countries), and that the UK at some point decided to enact a much stricter gun legislation than before - and that you have data to estimate the effect of that policy change on the UK population. During the time when the gun legislation was changed in the UK there were no simultaneous legislation changes in the US. In your estimate for the UK the ATE or the ATT? Could you utilize the estimate for the UK to quantify the potential effect of stricter gun legislation in the US?
4. Assume that you have data for the rate of violent crime both in the UK and the US, and you know that the UK introduced additional restrictions on gun ownership in 1997. First, compare the rate of violent crime in the UK and the US in 1998 and calculate $\hat{T} = \hat{Y}_{UK,1998} - \hat{Y}_{US,1998}$, where \hat{Y} are the violent crime measures for the two countries in 1998.
 - (a) List all the assumptions that are required for \hat{T} to be an unbiased estimate of T^* (remember: unbiased means that $E[\hat{T}] = T^*$).
 - (b) Explain whether these assumptions are likely to be satisfied.
 - (c) If the assumptions are not satisfied, how will the estimate differ from T^* ? Specifically, would \hat{T} be an upward biased estimate of T^* (defined as $E[\hat{T}] > T^*$), a downward biased estimate (defined as $E[\hat{T}] < T^*$), or can the sign of the bias not be determined?
5. Assume next that you have data for the UK both in 1996 and 1998 (i.e. the year before and after the new legislation) and that you used these UK data to estimate the “causal effect of gun legislation on violent crimes” by comparing the pre-legislation and post-legislation rate of violent crime. This would give you $\hat{T} = \hat{Y}_{UK,1998} - \hat{Y}_{UK,1996}$

- (a) List all the assumptions that are required for \hat{T} to be an unbiased estimate of T^* .
 - (b) Explain whether these assumptions are likely to be satisfied.
 - (c) If the assumptions are not satisfied, would \hat{T} be an upward biased estimate of T^* (defined as $E[\hat{T}] > T^*$), a downward biased estimate (defined as $E[\hat{T}] < T^*$), or can the sign of the bias not be determined?
6. If you had the data from parts (4) and (5), how would you construct a Difference-in-Differences (DID) estimator for the causal effect of the introduction of the gun legislation on the violent crime rate in the UK? Under what assumptions will the DID estimator provide an unbiased estimate of T^* ? You should be as specific as possible about these assumptions and ideally use mathematical notation to accompany your answer.
 7. Assume that you had the data from the following graph to do your analysis, and you know that the UK gun legislation was changed in 1997. Given the graph, what do you think your DID estimator would say about the impact of gun legislation on violent crime. Why might that conclusion be misleading? What potential factors can you think of that would affect the comparability of the data across countries and over time, and how would your estimate be affected by that?

