8.511 Problem Set 7

Yijun Jiang

October 29, 2015

1 Graphene in a Magnetic Field

1.1 Part (a)

Letting $|H - EI| = 0$, we have

$$E^2 = (\hbar v)^2 (k_x - ik_y)(k_x + ik_y) = (\hbar v|\mathbf{k}|)^2$$

Therefore, we recover the linear spectrum

$$E^{\pm}_{\mathbf{k}} = \pm \hbar v |\mathbf{k}|$$

1.2 Part (b)

$$\begin{aligned}
\pi_x &= -i\partial_x + \frac{e}{\hbar c} A_x = -i\partial_x \\
\pi_y &= -i\partial_y + \frac{e}{\hbar c} A_y = -i\partial_y + \frac{eB}{\hbar c} x
\end{aligned}$$

Therefore, the Schrödinger equation is

$$\hbar v \begin{pmatrix} 0 & -i\partial_x - \partial_y - i\frac{eB}{\hbar c}x \\ -i\partial_x + \partial_y + i\frac{eB}{\hbar c}x & 0 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = E \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

Acting by H on both sides, we have

$$(\hbar v)^2 \begin{pmatrix} (-i\partial_x - \partial_y - i\frac{eB}{\hbar c}x)(-i\partial_x + \partial_y + i\frac{eB}{\hbar c}x) & 0 \\ 0 & (-i\partial_x + \partial_y + i\frac{eB}{\hbar c}x)(-i\partial_x - \partial_y - i\frac{eB}{\hbar c}x) \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = E^2 \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

which simplifies to

$$(\hbar v)^2 \begin{pmatrix} -\partial_{xx} + (i\partial_y - \frac{eB}{\hbar c}x)^2 + \frac{eB}{\hbar c}[\partial_x, x] & 0 \\ 0 & -\partial_{xx} + (i\partial_y - \frac{eB}{\hbar c}x)^2 - \frac{eB}{\hbar c}[\partial_x, x] \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = E^2 \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

Since $[\partial_x, x] = 1$, the equation for ϕ_2 is thus

$$(\hbar v)^2 \left(-\partial_{xx} + \left(i\partial_y - \frac{eB}{\hbar c}x \right)^2 - \frac{eB}{\hbar c} \right) \phi_2 = E^2 \phi_2$$

1.3 Part (c)

The y dependence only enters by a phase $e^{ik_y y}$. Replacing $\partial_y \to ik_y$,

$$(\hbar v)^2 \left(-\partial_{xx} + \left(k_y + \frac{eB}{\hbar c} x \right)^2 - \frac{eB}{\hbar c} \right) f(x) = E^2 f(x)$$

This is just a harmonic oscillator equation,

$$\left(-\frac{\hbar^2}{2m}\partial_{xx} + \frac{1}{2}m\omega^2 \left(x + \frac{\hbar c}{eB} k_y \right)^2 \right) f(x) = E' f(x)$$

where

$$\omega = \frac{eB}{mc}$$
$$E' = \frac{E^2}{2mv^2} + \frac{1}{2}\hbar\omega$$

The eigenvalues are

$$E'_n = \left(n + \frac{1}{2} \right) \hbar\omega \qquad (n = 0, 1, 2 \cdots)$$

which means

$$E_n^2 = 2nmv^2\hbar\omega = \frac{2n\hbar eBv^2}{c} \qquad (n = 0, 1, 2 \cdots)$$

So we get the Landau levels

$$E_n^{\pm} = \pm v \sqrt{\frac{2e\hbar}{c} Bn} \qquad (n = 0, 1, 2 \cdots)$$

Plugging in numerical values,

$$E_1^+ \approx 9.18 \times 10^{-2} \text{ eV} \sim 10^3 \text{ K}$$

This gap is much larger than the Landau level spacing for a free 2DEG under the same magnetic field,

$$\Delta E_{free} = \frac{\hbar eB}{mc} \approx 1.16 \times 10^{-3} \text{ eV} \sim 13.4 \text{ K}$$

1.4 Part (d)

1.5 Part (e)

2 Shubnikov-de Haas Oscillation

2.1 Part (a)

2.2 Part (b)