Lecture 5 - The Expenditure Function, with an Application to Food Stamps

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1 The Expenditure Function

So far, we've analyzed problems where income was held constant and prices changed. This gave us the Indirect Utility Function. Now, we want to analyze problems where utility is held constant and expenditures change. This gives us the Expenditure Function.

These two problems are closely related—in fact, they are '*duals*.' As you saw on Pset #1, most economic problems have a *dual problem*, which means an inverse problem. For example, the dual of choosing output in order to maximize profits is minimizing costs at a given output level; cost minimization is the dual of profit maximization. Similarly, the dual of maximizing utility subject to a budget constraint is the problem of minimizing expenditures subject to a utility constraint. Minimizing costs subject to keeping utility from falling below a utility constraint is the dual of maximizing utility subject to not exceeding a budget constraint.

1.1 Setup of expenditure function

Consumer's primal problem: maximize utility subject to a budget constraint. Consumer's dual problem: minimizing expenditure subject to a utility constraint (i.e. a level of utility the consumer must achieve). The dual problem yields the "expenditure function," the minimum expenditure required to attain a given utility level.

1. Start with:

$$\begin{aligned} & \max U(x,y) \\ s.t.\ p_x x + p_y y \quad \leq & \quad I \end{aligned}$$

2. Solve for x^*, $y^* \Rightarrow u^* = U(x^*,\, y^*)$ given p_x, p_y, I.
   $V = V\left(p_x, p_y, I\right)$

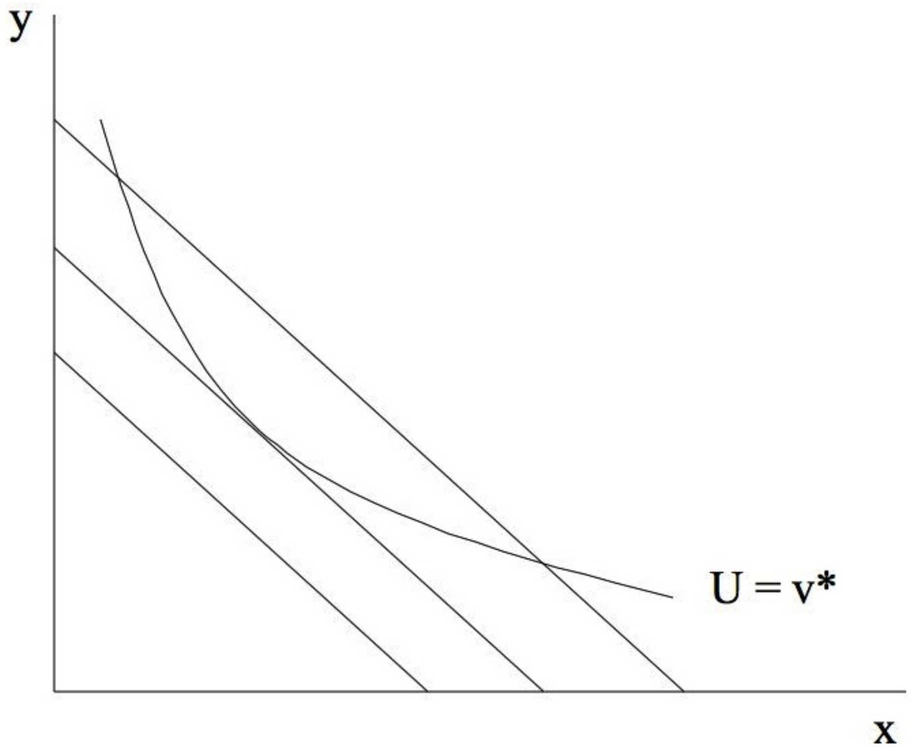
   V is the indirect utility function, and its solution is equal to u^*

3. Now solve the following problem:

$$\begin{aligned} & \min p_x x + p_y y \\ s.t. U(x,y) \quad \geq & \quad v^* \end{aligned}$$

   which gives $E = p_x x^* + p_y y^*$ for $U(x^*,\, y^*) = u^*$.
   $E = E\left(p_x, p_y, V^*\right)$
   E is the indirect utility function and its solution is equal to $p_x x^* + p_y y^*$

1.2 Graphical representation of the dual problem



- The dual problem consists of choosing the lowest budget set tangent to a given indifference curve. Example:

$$\begin{aligned} \min E &= p_x x + p_y y \\ s.t.\ x^{.5} y^{.5} &\geq U_p \end{aligned}$$

where U_p comes from the primal problem.

$$L = p_x x + p_y y + \lambda \left(U_p - x^{.5} y^{.5} \right)$$

$$\begin{aligned} \frac{\partial L}{\partial x} &= p_x - \lambda .5 x^{-.5} y^{.5} = 0 \\ \frac{\partial L}{\partial y} &= p_y - \lambda .5 x^{.5} y^{-.5} = 0 \\ \frac{\partial L}{\partial \lambda} &= U_p - x^{.5} y^{.5} = 0 \end{aligned}$$

- The first two of these equations simplify to:

$$x = \frac{p_y y}{p_x}$$

We substitute into the constraint $U_p = x^{.5}y^{.5}$ to get

$$\begin{aligned} U_p &= \left(\frac{p_y y}{p_x}\right)^{.5} y^{.5} \\ x^* &= \left(\frac{p_y}{p_x}\right)^{.5} U_p,\ y^* = \left(\frac{p_x}{p_y}\right)^{.5} U_p \\ E^* &= p_x\left(\frac{p_y}{p_x}\right)^{.5} U_p + p_y\left(\frac{p_x}{p_y}\right)^{.5} U_p \\ &= 2p_x^{.5}p_y^{.5}U_p \end{aligned}$$

1.3 Expenditure function: What is it good for?

The expenditure function is an essential tool for making consumer theory operational for public policy analysis. Using the expenditure function, we can 'monetize' otherwise incommensurate trade-offs to evaluate costs and benefits. The need for this type of calculation arises frequently in policy analysis and is the basis for most cost-benefit analyses.

As we have stressed earlier the semester, we don't know that 'utils' are. This presents a problem if we want to determine *how much* harm or benefit a certain policy imposes on an individual. The expenditure function gives us a convenient way to potentially circumvent this problem. Using the expenditure function, we can figure out *how much money* a consumer would have to be compensated (which could be a positive or negative number) to leave her equally well off after a policy is implemented as she was initially. So, the expenditure function permits us to calculate a 'money metric.'

Example: In 2007, the U.S. federal government enacted a policy that is slowly phasing out the sale of high wattage incandescent light bulbs. As these bulbs fail and supplies dwindle, consumers will eventually have to replace their high wattage incandescents with either lower wattage incandescent bulbs, compact fluorescent bulbs (CFLs), or Light Emitting Diodes (LEDs). All have disadvantages: lower wattage bulbs are dimmer; CFLs produce harsh, unnatural light, and take time to reach full intensity after they are powered up (especially in the cold); LEDs are expensive, and the light is often a bit harsh as well.

Although this policy is designed to reduce aggregate electricity consumption and pollution, the policy will leave some consumers worse off (despite their slightly lower utility bills). How much would consumers need to be compensated to make them indifferent to the policy change? This calculation depends on the expenditure function. Let's say that consumer utility prior to the ban is given by $\bar{U}$ and expenditures by:

$$E_{pre} = E\left(p_{IN}, p_{CFL}, p_{LED}, p_a, \bar{U}\right),$$

where p_{IN} is the price of an incandescent lightbulb, p_{CFL} and p_{LED} are the prices of CLFs and LEDs respectively, and p_a is the price of all other goods. Notice that we do *not* assume that these three types of bulbs differ only according to price (i.e., that they are three 'quantities' of the same good). Because these lights have different characteristics, consumers will have different preferences over them.

To attain the same level of utility after the ban on high wattage incandescents, consumers would need this much income to be indifferent:

$$E_{post} = E\left(p_{IN} = \infty, p_{CFL}, p_{LED}, p_a, \bar{U}\right).$$

The difference $E_{post} - E_{pre}$ is the amount of money that we would need to compensate homeowners to leave them indifferent between the world with and without high wattage incandescent bulbs.

Of course, we don't usually know the expenditure function, so this isn't as easy to apply in practice as it is in theory. But it turns out that if we have an estimate of the compensated elasticity of demand for a good, this is often enough to make a rough calculation. You'll see this in the Schanzenbach (2011) study.

One very crude estimate we could make at the outset is this. Let $x^*_{pre} = (x^*_{IN}, x^*_{CFL}, x^*_{LED}, x^*_a)$ equal the consumer's original *chosen* bundle (because this bundle is chosen, we are justified in treating it as their optimal choice), with $E_{pre} = x^*_{IN} \times p_{IN} + x^*_{CFL} \times p_{CFL} + x^*_{LED} \times p_{LED} + x^*_a \times p_a$. In the post period, we have $p'_{IN} = \infty$, meaning incandescent bulbs are no longer available. So we could calculate how much more income the consumer would need to substitute from incandescent to the cheapest alternative light source:

$$\Delta E' = E'_{post} + E_{pre} \simeq x^*_{IN} \times \min\{p_{CFL}, p_{LED}\} + x^*_{CFL} \times p_{CFL} + x^*_{LED} \times p_{LED} + x^*_a \times p_a - E_{pre},$$

where I've assumed for simplicity that prices other than p_{IN} are unchanged. Assuming that $\min\{p_{CFL}, p_{LED}\} > P_{IN}$, which implies that $\Delta E' > 0$. In this case, $\Delta E'$ could either underestimate or overestimate the amount of compensation needed to leave the consumer indifferent. It would tend to be an *underestimate* of the necessary compensation where the consumer prefers incandescent to CFL or LED light. In that case, simply providing sufficient resources to purchase CFL or LED bulbs in place of incandescent bulbs *might* leave the consumer worse off than she was initially. On the other hand, since $\Delta E' > 0$, the new budget would allow the consumer purchase more of x_a (all other goods) than under the original budget set. It's possible that the consumer would strictly prefer a bundle that included more x_a and fewer bulbs than in the original bundle, x^*_{pre}. Thus, $\Delta E'$ is a crude estimate because we do not know if over or underestimates the correct value.

In certain cases, we can say more. Take a case where $\{p_{CFL}, p_{LED}\} < P_{IN}$. This is a plausible possibility since CFLs are heavily subsidized (one can often get CFLs for free from the local utility company). As long as the consumer was originally buying some positive number of incandescent bulbs, $x^*_{IN} > 0$ (that is, even with $p_{IN} > p_{CFL} = 0$, she still chose to buy at least some incandescent bulbs) our crude calculation above would imply that $\Delta E' < 0$, meaning that the consumer now needs *less* income to achieve the same utility. Mechanically, she will be buying cheaper CFLs rather than incandescent bulbs from now on, so she needs less money to purchase the same number of bulbs. Clearly, by non-satiation, $\Delta E'$ strictly *underestimates* the compensation required to leave the consumer indifferent; the consumer is always worse off when the budget set is reduced since all *goods* have positive marginal utility. Another way to see this intuitively: the new hypothetical budget set, E'_{post} excludes the originally chosen bundle x^*_{pre} without allowing the consumer to purchase any previously unavailable bundle—that is, the new budget set is a strict subset of the old budget set. Since the original bundle was the consumer's chosen point in the original budget set, it must at least weakly dominate all other points in that set. Excluding that bundle from the budget set without introducing new choices must therefore make the consumer at least weakly worse off.[1]

1.4 Compensation and Over-Compensation

Let's say we were considering a policy that raised prices for some consumers, perhaps by raising the cost of gasoline. Policymakers might be legitimately concerned that this policy change would adversely affect low income consumers. To offset this effect, they might provide cash compensation to offset their loss. How large should this transfer be?

A typical policy response would be to set compensation equal to the full amount of the price increase multiplied by the consumer's initial expenditure on gasoline. Let C equal the compensation amount, with

$$C = \Delta P_g \times Q_{g,0}.$$

Here, $\Delta P_g = P_{g,1} - P_{g,0}$ is the policy-induced price change and $Q_{g,0}$ is the quantity that the consumer was purchasing initially (i.e., at time $t = 0$).

Is C the right amount of compensation—this is, neither too much or too little? If you knew the consumer's utility function (a tall order, of course), you could calculate the exact answer as

$$C^* = E\left(P_{g1}, P_a, V\left(P_{g0}, P_a, I_0\right)\right) - E\left(P_{g0}, P_a, V\left(P_{g0}, P_a, I_0\right)\right),$$

where I_0 is the consumer's initial budget, P_a are the prices of all other goods (assumed constant over time), and $V(\cdot)$ is the indirect utility function. You would then directly compare $C^* \lesseqqgtr C$ to see if C is above or below the exact compensation required. Absent knowledge of each consumer's utility function, can we say anything more?

The answer is yes. A bit of thought should convince you that it must be the case that

$$C \geq C^*.$$

The simple compensation scheme $C = \Delta P \times Q_0$ *always* weakly overestimates the actual compensation required. Why? As a starting point, note that C *must* be an upper bound on C^*. Clearly, if we compensate the consumer the full amount of money required to buy her initial bundle, she must be at least as well off as before; she can have the original bundle *or* she can choose many others that were not initially affordable. (These other bundles would have less gasoline but more of other goods

[1] Technically, I'm using a different form of argument here called Revealed Preference, which was developed by Nobel Laureate Paul Samuelson as a weaker alternative (i.e., requiring fewer and weaker assumptions) to the axiomatic utility theory that we are using in class. I will not have time to teach the theory of Revealed Preference this semester, but the idea is quite intuitive and can serve as a useful shortcut.

else. You should demonstrate to yourself that some previously infeasible bundles are now feasible with prices P_{g1}, P_a and income $I_0 + C$.)

But we can say more? Again, yes. If the consumer has standard indifference curves that are bowed towards the origin (diminishing MRS), a change in the price of gasoline will cause the consumer to partially substitute towards other goods. This substitution partly blunts the effect of the price increase as the consumer re-optimizes her bundle given the new prices. If we raise the price of gasoline but hold the consumer's utility at its initial level, her optimally chosen bundle will rotate along the original indifference curve to a new location where the new price ratio is tangent to the initial indifference curve. This new bundle will cost more at the new prices than the original bundle at the old prices (unless there exists a perfect substitute for gasoline available at the original price[2]). But this new bundle (which holds utility constant) will cost strictly less than $I_0 + C$. The difference between the cost of the new bundle at the new prices and the cost of the old bundle at the old prices (both lying on the initial indifference curve) is equal to C^*. I leave it as an exercise for you to demonstrate graphically why it must be the case that $C \geq C^*$.

1.5 Relation between Expenditure function and Indirect Utility function

How do the solutions to the Dual and Primal problems compare?

- Examining the relationship between the expenditure and indirect utility functions:

$$\begin{aligned}
V(p_x, p_y, I_0) &= U_0 \\
E(p_x, p_y, U_0) &= I_0 \\
V(p_x, p_y, E(p_x, p_y, U_0)) &= U_0 \\
E(p_x, p_y, V(p_x, p_y, I_0)) &= I_0
\end{aligned}$$

- The Expenditure function and Indirect Utility function are *inverses* one of the other.

- Let's verify this in the example we saw above. Recall that the primal problem gave us factor demands x_p^*, y_p^* as a function of prices and income (not utility).

- The dual problem gave us expenditures (budget requirement) as a function of utility and prices.

$$x_p^* = \frac{I}{2p_x},\ y_p^* = \frac{I}{2p_y},\ U^* = \left(\frac{I}{2p_x}\right)^{.5}\left(\frac{I}{2p_y}\right)^{.5}$$

Now plug these into expenditure function:

$$E^* = 2U_p p_x^{.5} p_y^{.5} = 2\left(\frac{I}{2p_x}\right)^{.5}\left(\frac{I}{2p_y}\right)^{.5} p_x^{.5} p_y^{.5} = I$$

[2]Consider an extreme case where gasoline and kerosene are *perfect* substitutes and have identical initial prices. In that case, a rise in the price of gasoline would have no effect on consumer welfare since consumers would simply switch to kerosene. If consumers received compensation C along with the price change, they would be strictly better off than before the price change.

Finally notice that the multipliers are such that the multiplier in the dual problem is the inverse of the multiplier in the primal problem.

$$\lambda_P = \frac{U_x}{p_x} = \frac{U_y}{p_y}$$
$$\lambda_D = \frac{p_x}{U_x} = \frac{p_y}{U_y}$$

2 Demand Functions

Now, let's use the Indirect Utility function and the Expenditure function to get Demand functions. Up to now, we have been solving for:

- Utility as a function of prices and budget
- Expenditure as a function of prices and utility

Implicitly we have already found demand schedules—a demand schedule is immediately implied by an individual utility function. For any utility function, we can solve for the quantity demanded of each good as a function of its price, holding the price of all other goods constant *and* holding *either* income *or* utility constant.

2.1 Uncompensated ('Marshallian') demand—Holding *income* constant

- In our previous example where:

  $$U(x, y) = x^{.5}y^{.5}$$

  we derived:

  $$x(p_x, p_y, I) = 0.5\frac{I}{p_x}$$
  $$y(p_x, p_y, I) = 0.5\frac{I}{p_y}$$

- In general we will write these demand functions (for individuals) as:

$$\begin{aligned} x_1^* &= d_1(p_1, p_2, ..., p_n, I) \\ x_2^* &= d_2(p_1, p_2, ..., p_n, I) \\ &... \\ x_n^* &= d_n(p_1, p_2, ..., p_n, I) \end{aligned}$$

- We call this "Marshallian" demand after Alfred Marshall (who first drew demand curves). You are also welcome to call it uncompensated demand.

2.2 Compensated ('Hicksian') demand—Holding *utility* constant

- Similarly we derived that:

$$
\begin{aligned}
x(p_x, p_y, U) &= \left(\frac{p_y}{p_x}\right)^{.5} U_p \\
y(p_x, p_y, U) &= \left(\frac{p_x}{p_y}\right)^{.5} U_p
\end{aligned}
$$

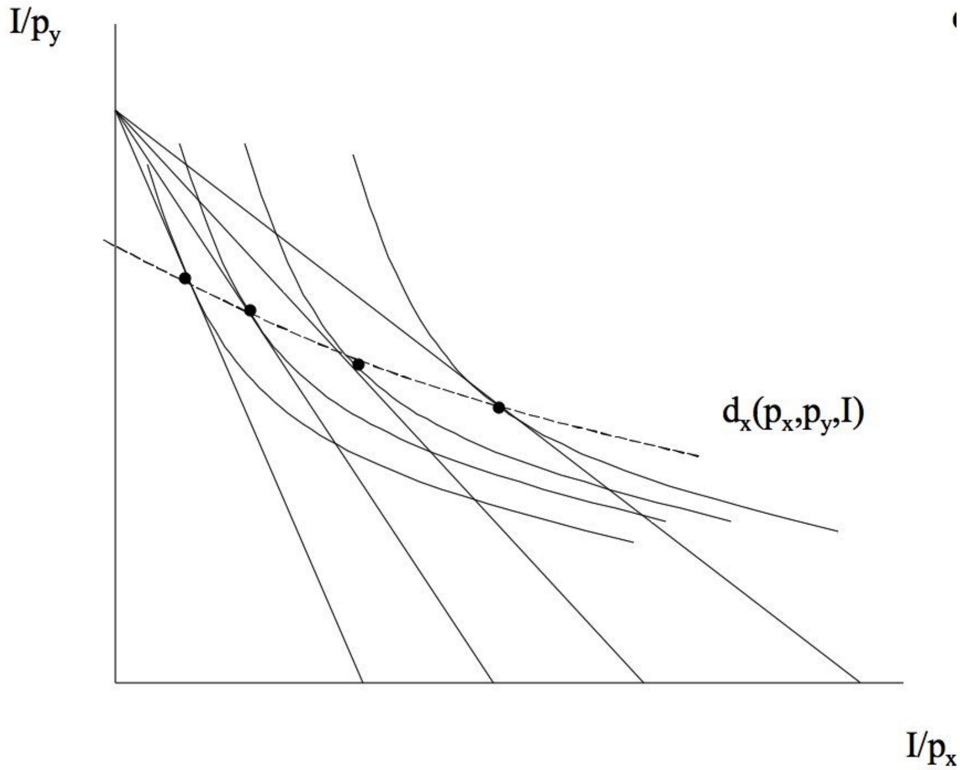
- In general we will write these demand functions (for individual) as:

$$
\begin{aligned}
x^*_{1,c} &= h_1(p_1, p_2, ..., p_n, U) \\
x^*_{2,c} &= h_2(p_1, p_2, ..., p_n, U) \\
&\ldots \\
x^*_{n,c} &= h_n(p_1, p_2, ..., p_n, U)
\end{aligned}
$$

- This is called "Hicksian" or compensated demand after John Hicks. This demand function takes *utility as an argument, not income*. This turns out to be an important distinction.

2.3 Graphical derivation of demand curves

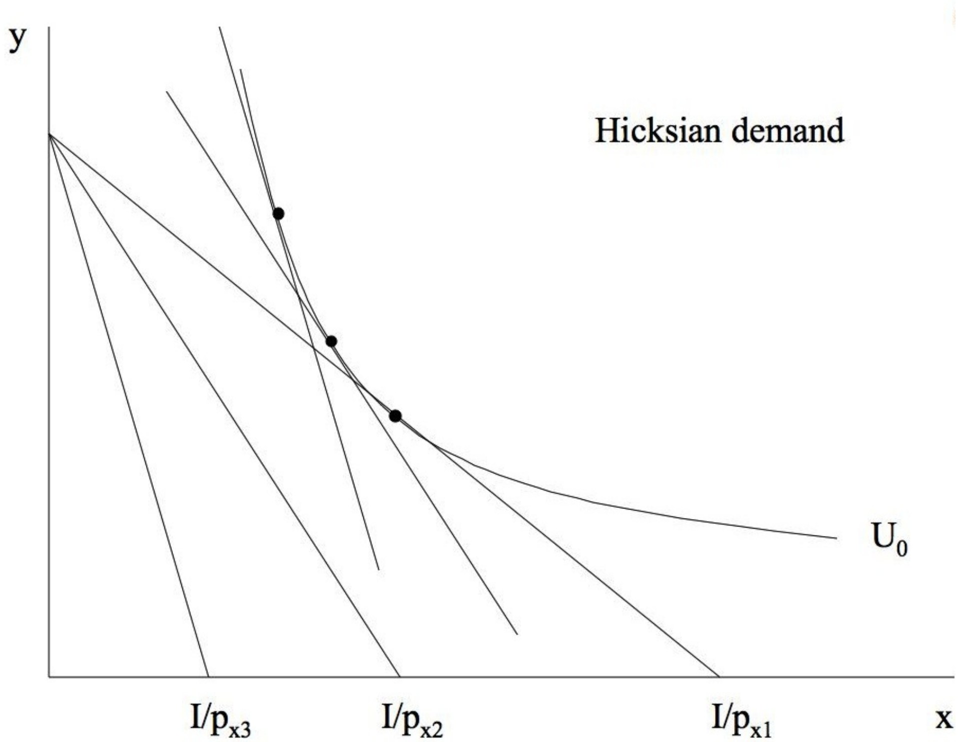
- A demand curve for x as a function of p_x



- So a demand function is a set of tangency points between indifference curves and budget set holding I and p_y (all other prices) constant.

- What type of demand curve is depicted above? *Marshallian* ('uncompensated') $d_x(p_x, p_y, I)$. Utility is not held constant, but income is.

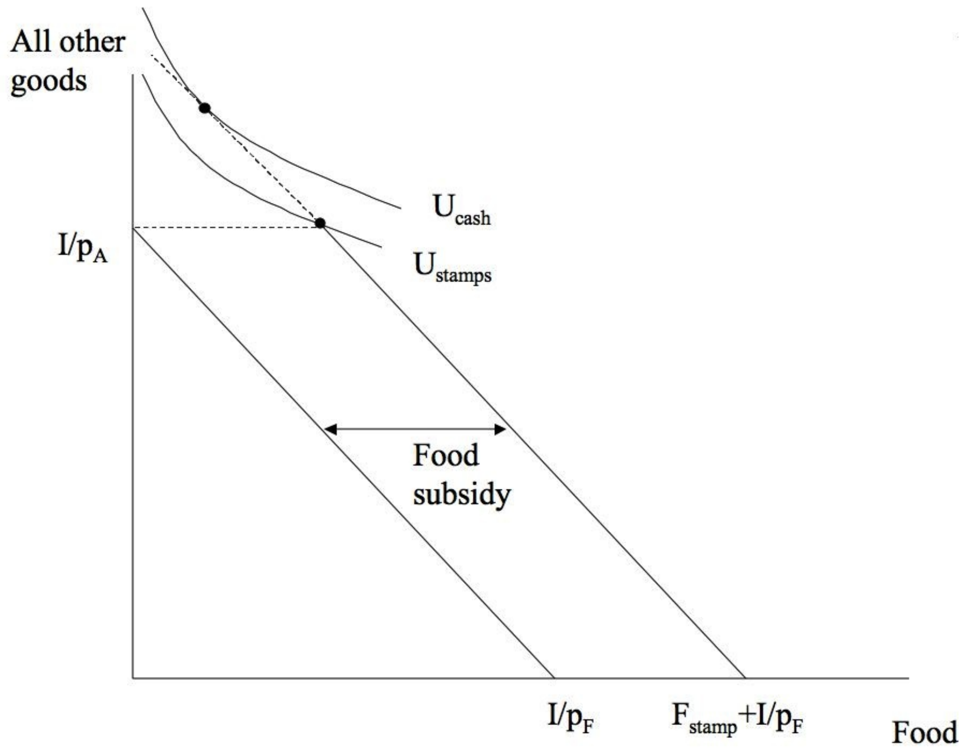
- Below is a Hicksian ('compensated') demand curve $h_x(p_x, p_y, I)$. Here, utility is held constant, which means that the budget set must rotate as prices change to keep the consumer on the same indifference curve.



- Now, we have the tools to analyze the Food Stamp program.

3 In-Kind Transfers: An application of consumer theory

- As noted in a prior lecture, there are numerous in-kind (versus cash) transfers made by government to citizens:
  - Food stamps.
  - Public housing.
  - Child care assistance.
  - Medical care.
  - Schooling.
- These transfers have two economic effects:
  - They shift the budget set outward so that the consumer can potentially buy more of the subsidized good *and* all other goods.
  - They place a kink in the budget set at the subsidy level of the in-kind good–thus they "force" consumption of at least the threshold level of the in-kind good. (see figure below)



- What are the consequences of this type of transfer for consumer welfare?
  - For a consumer who consumes on non-kinked section of new budget set?
  - For a consumer who consumers at the kink?
  - Can this transfer make a consumer worse off?

3.1 The U.S. Food Stamp Program: Some facts

- The Food Stamps program is now called SNAP, which stands for Supplemental Nutrition Assistance Program
  - Administered by the U.S. Department of Agriculture, which is not normally considered a welfare agency.
  - In 2013, the program served 47.6 million participants (15.1 percent of the U.S. population!) in 23.0 million households.
  - Distributed an average of \$133 per month per participant, and \$275 per household.
  - Thus, the average annual benefit per recipient was about \$1,600, and per household was \$3,300
  - Benefit costs for 2013 were \$76.1 billion (that's about 40% more than 2007!)
  - Administrative costs typically add about 5 percent to the total (\$3.8 billion in 2014)
- Why use in-kind transfers instead of cash transfers?

3.2 Advantages of in-kind transfers

1. Guarantee nutrition?
2. Prevent use of cash on drugs, alcohol cigarettes$\Longleftrightarrow$Paternalism: invalid preferences of individuals
3. What is "valid" use of public money? Food versus recreation.
4. Political necessity (for public support).

3.3 Disadvantages of in-kind transfers

1. Restrict/distort choice.
2. Administration/enforcement costs. It's estimated that *half* of the cost of food stamp administration is fraud prevention.
3. Who is made better off by enforcing restrictions on food stamp recipients' choices? (Does it pass a Pareto test?)
4. Creation of underground market for trade in stamps ("shadow market"). Creation of criminals.

3.4 The value of food stamps: Some policy questions

Key policy questions to consider in comparing cash to in-kind food stamp transfers?

1. Are recipients "distorted?" That is, do they indeed spend more on food than they otherwise would if food stamps were given in cash?
2. Does cash versus in-kind have any effect on nutrition?
3. How costly are cash versus in-kind programs to administer?
4. What share of food stamps are trafficked? And at what price?

4 The value of food stamps: the Schanzenbach study

- This study analyzes a pair of food-stamp experiments in San Diego and Alabama implemented in the early 1990s by the U.S. Department of Agriculture.
- "Cash out" experiments: Food stamp benefits paid in cash to a random subset of recipients instead of food stamp coupons.
- Idea: Compare food and other expenditures among households receiving stamps and equivalent households randomly assigned cash instead.

- Notice: There is no pre-period (i.e., baseline data), so this is not a "difference-in-difference" comparison (unlike Card and Krueger). Is that a problem? Not necessarily. If the randomization is valid and we have a reasonably sized sample, we can be fairly confident that the counterfactual outcomes for the treatment and control groups should be comparable. In that case, we can compare outcomes without a baseline during the experiment to assess what food consumption would counterfactually have been for the treated group had they not been "cashed out."

- Concretely, let $Z = 1$ denote cash and $Z = 0$ denote stamps. Let Y_0 equal food expenditures if beneficiaries are assigned to stamps and Y_1 equal food expenditures if beneficiaries are assigned to cash. If the randomization is valid, $E(Y_1|Z = 1) = E(Y_1|Z = 0)$ and $E(Y_0|Z = 1) = E(Y_0|Z = 0)$, which implies that $E(Y_1 - Y_0|Z = 1) = E(Y_1|Z = 1) - E(Y_0|Z = 0)$. Hence, the contrast in food expenditures in the treatment ($Z = 1$) and control ($Z = 0$) groups gives the causal effect of cash versus stamps on food expenditures.

4.1 'Distorted' versus 'non-distorted' households

- Would we expect all or even most households to be worse off with food stamps than cash? Certainly not. Low income families spend a considerable share of their income on food–-about 30 percent of the household budget in the Schanzenbach sample, which is twice what average Americans spend. (This is logical. Food is a necessity. Relative to average households, poorer households spend a larger share of their incomes on necessities and a smaller share on luxuries.)

- And food stamps are not that generous: \$111 - \$370 per month for households of 1 - 4 children at the time of Schanzenbach's study. This is less than one-third of the typical low-income household budget.

- How can we measure which households are "distorted?" In an experiment with pre- and post-treatment data, this would be straightforward. We could use these data to identify households that decreased their food consumption after they were 'cashed out.' These households were, in all likelihood, distorted by the program. (We would of course still want to include the control group in this comparison to difference out the "time effect.") In the current study, identifying distorted households is more challenging because there are no baseline data, so no pre-post comparison is possible.

- Schanzenbach's primary approach is to label a household distorted if its monthly food spending is less than its food stamp amount. For the cash-recipient households, this poses no problem. Cash recipients that spend less than the transfer amount on food would likely have been 'distorted' by receipt of stamps had they not been in the cash-out group (that is, they would either have to have spent more on food than desired or simply not have used all of their stamps). By this definition, **20 percent** of cash-out households *would have been* distorted had they been assigned to the stamp group.

- Assuming that the randomization of households to stamp and cash groups was effective, we would expect approximately the same proportion of *stamp* households to be distorted. Of course, we don't know which households these are. Conditional on receiving stamps, it's likely that they will buy at least that amount of food rather than throw the stamps away. Hence, we cannot observe that they are spending more than they 'want to' on food.

- Schanzenbach's workaround to this problem is to rank households according to the ratio of food expenditures to stamp amount and then classify the *lowest* 20 percent as potentially distorted—that is, households that might have preferred to spend less. (The paper uses two other approaches to classification, neither perfect. All three generate similar results. In theory, one could identify the constrained households as those that spent *exactly* their food stamp allocations on food. In practice, it's hard to measure food expenditure that precisely. Moreover, if a household spent approximately $1 more or $1 less than their ford stamp allocation, would you really be confident that they were undistorted?

- See Tables 2a and 2b. Among the 20 percent of stamp households that are categorized as distorted, their excess food consumption is estimated to average 27 percent of their stamp allocation.

- Using the difference in food spending among 'distorted' households that do and do not receive the cash grant, Schanzenbach calculates a rough measure of the "distorted share" of food stamp benefits. This is simply the difference between what they would have spent on food and what they did spend on food divided by what they would have spent:

$$DS = \frac{F_s - F_c}{F_c},$$

where c stands for cash transfer and s is for stamps.

- Note the counter-factual assumption here, made plausible by the experimental design: the stamp households would have spent the same on food as the cash households except for the stamp restrictions.

4.2 Estimating the welfare loss

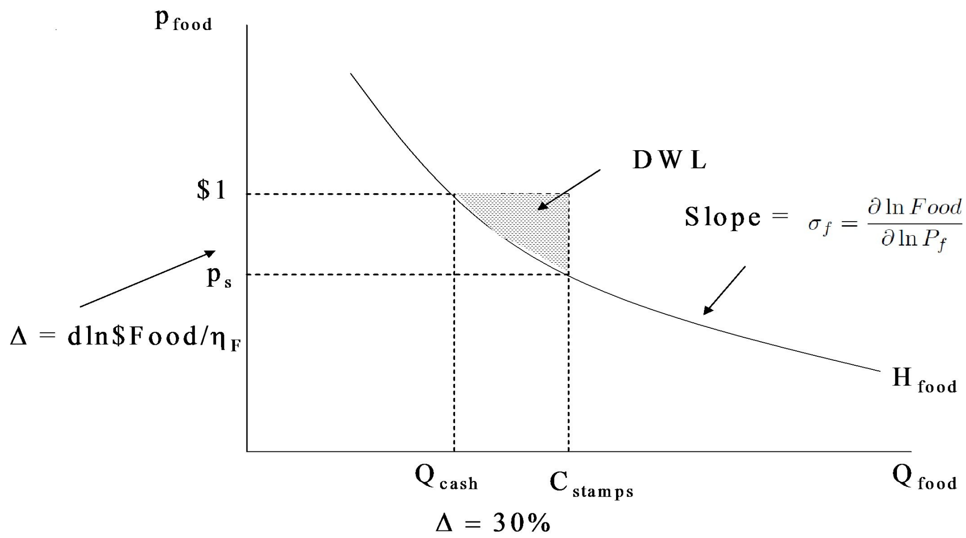
- Is the amount $F_s - F_c$ equal to the welfare loss for these households (HHs)? Of course not. These HHs still value the 'excess" food they consume by some positive amount, even if they would have preferred to spend that money on other goods. So the quantity $F_s - F_c$ strictly overestimates the size of the welfare loss.

- To calculate the DWL, we need to use some fundamental consumer theory. For this analysis, please refer to the diagram below that depicts the *compensated* (Hicksian) demand function for food. Why compensated demand? We want to answer the following question: holding consumer utility constant at its level achieved when the consumer is receiving food stamps, how much money could we give to consumers instead of stamps if we relaxed the constraint that they spend at least that amount on food? Because *utility* is held constant here, this question inherently involves the compensated demand function.

- Let's start by asking: What's the marginal utility of \$1 food for 'non-distorted' consumers relative to *all other goods*? At the utility maximizing optimum choice of food and all other goods for a non-distorted household, the marginal utility of \$1 spent on all other non-food goods must equal to the marginal utility of \$1 spent on food. So, a household that receives \$10 in cash gets the maximum value from that \$10. A household that is given \$10 in cash and told to spend it on food may get somewhat lower utility than if given cash. One way to see this is to note that for a non-distorted consumer choosing her optimum consumption bundle, the marginal utility of food and all other goods is equated with the price ratio. So, the next dollar spent on food has the same marginal utility value as the next dollar spent on all other goods:
$$\left.\frac{U_F}{U_A}\right|_{F^*,A^*} = \frac{P_F}{P_A}.$$
For a distorted household, however, we'd expect that
$$\left.\frac{U_{F'}}{U_{A'}}\right|_{F',A'} < \frac{P_F}{P_A}.$$
Given that the composite good "all" includes *all non-food goods*, it's also logical to assume that the consumer does not have strongly diminishing MRS in "all" relative to food. So, an extra dollar spent on "all" does not produce significantly less utility than the last dollar. But in the case of food, where the distorted consumer is up against the minimum stamp expenditure constraint, we would not expect this to be true; the marginal utility of food will be declining in additional consumption.

- How do we determine the marginal utility in dollar equivalent terms of this excess consumption?

- We know how much excess food the 'distorted' households are consuming: roughly 1/3rd more than they want to. If we knew the slope of the compensated demand function, we could figure out how much utility they were losing *in cash equivalent terms* from consuming excess food instead of spending the rest of their budget on other goods.



Hicksian Demand Curve for Food

- In this figure, the shaded area is the dead weight loss. This is the cost of excess food consumed minus the amount of "utility" (in dollar terms) they obtain from the excess food.

Aside: Is it valid to measure utility in dollar equivalent terms given that utility functions are ordinal?

- Since utility functions are only defined up to a monotone transformation, doesn't this mean the welfare loss calculations *in dollars* (a cardinal measure) are not uniquely defined for a given utility function? Actually, it does not.

- Consider the following thought experiment. Utility functions $U_1(\cdot)$ and $U_2(\cdot)$ are identical for consumer theory; $U_2(\cdot)$ is a monotone transformation of $U_1(\cdot)$. Hence, these two utility functions have identical preference rankings and choose the same bundles of goods for given income and prices. If we gave $U_1(\cdot)$ and $U_2(\cdot)$ each \$100 in cash, they would consume identical bundles to one another. Likewise, if we gave them \$100 in food stamps, they would consume identically.

- Imagine that $U_1(\cdot)$ and $U_2(\cdot)$ are distorted by food stamps so that they are forced to consume more food using \$100 in stamps than they would if given \$100 cash. How much additional cash (in addition to food stamps) would it take to make $U_1(\cdot)$ and $U_2(\cdot)$ indifferent between \$100 in cash versus \$100 in stamps plus additional cash?

- We don't know the numerical answer without an explicit functional form. But we do know that the answer must be the same for $U_1(\cdot)$ and $U_2(\cdot)$. Why? Both $U_1(\cdot)$ and $U_2(\cdot)$ would choose to buy the same bundles using the extra cash to get back on the original indifference curve associated with receiving \$100 in cash—and of course those bundles would cost the same since all consumers face the same prices.

- Hence, the DWL associated with food stamps (in dollars, not utils) is identical for both utility functions, despite the fact that the functions are not identical. If you want to demonstrate this to yourself, work an example with $U_1(X,Y) = X^{1/2}Y^{1/2}$ and $U_2(X,Y) = 1/2\ln X + 1/2\ln Y$.]

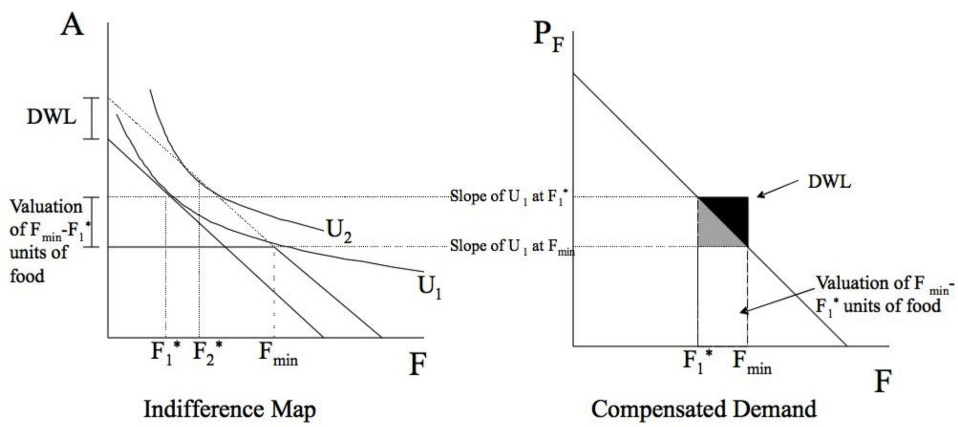
- How can we measure the DWL in dollar terms? The compensated demand curve allows us to calculate how much additional income the consumer would need to be indifferent between stamps + compensation versus cash exclusively (where the cash amount is equal to the face value of the stamps).

- We know the following:
  - The shadow value of the marginal food items for un-distorted households in terms of other goods foregone is \$1.00 (per dollar of expenditure).
  - We know the difference in quantity of food consumed P_c. About 20-30% more.
  - We'd like to know the welfare loss for distorted households in cash equivalent terms.

- To get this, we need the Compensated Demand Elasticity for food, which is defined as

$$\sigma_F = \frac{\partial \ln Q_F}{\partial \ln P_F} = \frac{\partial Q_F}{\partial P_F} \times \frac{P_F}{Q_F}$$

- Reemphasizing: we are using the compensated demand curve because we want to determine out how to make the consumer equally well off under the cash-out and stamp conditions—that is, we are holding utility constant.



- So, plug in from some existing studies. These suggest that σ_F is in the range of -0.16 to -0.28. That is a 10 percent increase in food prices reduces demand by 2 to 3 percent. (How is σ_F known? Let's say that this is a difficult parameter to estimate, and that the existing estimates come from surveys on consumers' preferences and purchasing behavior—which is quite imperfect.) For concreteness, let's say that $\sigma_F = -0.2$.

- Notice from the from the definition of the elasticity :

$$\sigma_F = \frac{\partial \ln Q_F}{\partial \ln P_F} \Rightarrow \partial \ln P_F = \frac{\partial \ln Q_f}{\sigma_F}$$

  This identity says that we can solve for the change in consumers' marginal willingness to pay for food items as good consumption increases by extrapolating along the compensated demand curve. We know that at the unconstrained consumption bundle, the marginal willingness to pay for a dollar of food is \$1. As we increase food consumption beyond this point (in proportional terms), the marginal willingness to pay decreases with slope $1/\sigma_F$. We can use our estimate of this elasticity, along with our knowledge the excess food consumption induced by the SNAP program, to infer the DWL.

- Let's write the compensate demand function for food as follows:

$$F = kP_F^{-0.2},$$

where k is a positive constant. Why this function? Because it has a constant elasticity of -0.2. To see this, note that:

$$\frac{\partial F}{\partial P} = -0.2kP_F^{-1.2}$$

$$\frac{\partial F}{\partial P} \times \frac{P}{F} = -\frac{0.2kP_F^{-1.2} \times P_F}{kP_F^{.2}} = -0.2.$$

Rearranging this function, we can express the marginal willingness to pay for food as a function of the quantity consumed

$$P_F = \left(\frac{F}{k}\right)^{-5}$$

- Let's use this function to determine the area of the DWL. Following the figure above, at a price of $P_F = 1$, the consumer spends F^* on food. This implies that $F^* = k$. We know that distorted consumers consume one-third more food than desired, implying that for them, $F_{min} = \frac{4}{3}k$. How much would a consumer be willing to pay for this additional quantity of food? We can answer that question by integrating the (inverse) demand curve:

$$\int_k^{\frac{4}{3}k} \left(\frac{F}{k}\right)^{-5} dF = \Big|_k^{\frac{4}{3}k} - \frac{1}{4}k^5F^{-4} = \frac{1}{4}k\left[1 - \left(\frac{4}{3}\right)^{-4}\right] = 0.17k.$$

  This area corresponds to the gray shaded area in the figure above plus the rectangle below it.

- What would the consumer's valuation have been of the equivalent amount of cash? We can safely assume that a dollar is worth a dollar to the consumer! We know that the additional expenditure on food is equal to $\frac{1}{3}k$. Thus, the value of the equivalent amount of cash would be $\frac{1}{3}k$. This corresponds to the grey triangle, the rectangle below it, and the black triangle above it.

- The DWL (the area of the black triangle) is therefore equal to $0.33k - 0.17k = 0.16k$. In percentage terms, we can say that distorted consumers only receive $\frac{16}{33} = 48\%$ of the economic value of the extra food consumed beyond the desired quantity.

- So what is the total value of food stamps for constrained households that spend 30% more on food than they would if unconstrained? Let S equal the face value of stamps received. Distorted consumers value these stamps at approximately equal to $S \times (0.7 \times 1 + 0.3 \times 0.48) = 0.84 \times S$. For the first 70% of stamps received, there is no economic loss—even for distorted consumers. For the remaining 30%, the marginal loss is \$0.48 on the dollar. Thus, on average, constrained recipients obtain \$0.84 in economic value per dollar on their total food stamp expenditure.

- What does this calculation imply for the total welfare loss from the program? Food stamp distribution in 2013 was about \$76 billion. Approximately 20 percent of recipients were distorted. They consume about 30% more food than desired, and their welfare loss on the excess food averages \$0.48 on the dollar. Hence, $DWL \approx 76 \times 0.20 \times 0.30 \times 0.48 = \$2.19$ billion.

- Concretely, USDA could 'cash out' food stamps and simultaneously reduce benefits by $\sim$ \$2.2 billion without making recipients worse off. However, notice that it would only want to reduce benefits to 'distorted' consumers—there is no welfare loss to those who are infra-marginal. This would be administratively infeasible. Alternatively, it could cash out food stamps while keeping benefit levels unchanged and make recipients $\sim$ \$2.2 billion better off ($\sim$ \$46 per recipient).

- If cashing out food stamps also reduced administrative costs, this might produce additional savings.

- If cashing out food stamps also shut down the underground market (eliminating transfers to criminals), this would increase targeting efficiency—more of the benefits would go to recipients rather than the grocers willing to traffic in food stamps.

4.3 Nutrition

The exercise above analyzes how efficiently the food stamp program maximizes the *utility* of participants. In fact, this is not the program goal. The goal is to,

> "...safeguard the health and well-being of the nation's population by raising levels of nutrition among low-income households."

So, perhaps the relevant question is whether cash transfers do a better or worse job than food stamps. What would you predict? What is the greater "nutrition" problem facing most households—too few calories or too many?

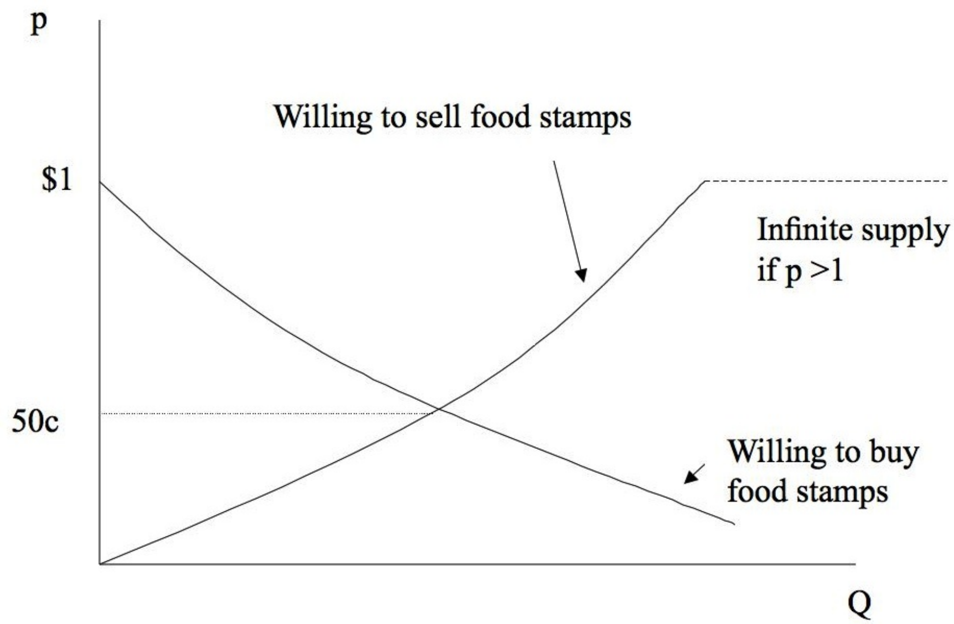
- **See Table 8C.** Largest expenditure reductions due to cash out are for:
  - Vegetables
  - Fruit
  - Meat
  - Legumes
  - Juice and soda
- No impact on alcohol consumption.
- No data on cigarette consumption.
- **See Table 9a.** In terms of Recommended Daily Allowances (RDA):
  - Reduction in calories from 126% to 119% of RDA.
  - On average, people still overeating, but less so.
  - On average, all RDAs still met.

- Looking at the share meeting the RDA:
  - 10 percentage point reduction in share eating 100% of RDA of calories.
  - 3 percentage point reduction in share eating 200% of RDA of calories.
  - Only other categories that appear affected are Iron and Calcium.
- So, the evidence on nutrition is not crystal clear but it's quite possible that reduction in calories outweighs reductions in other nutrients A lot of marginal money appears to go into soda and juice.
- **See Table 10a.** Some evidence that check recipients spend more on utilities (that's power, water and phone service, not 'utils!'). That's not necessarily bad.

4.4 The underground market for food stamps

One interesting wrinkle that these calculations do not incorporate is the underground market.

- Since some consumers value food at less than $1.00 at the margin while others do not, there are potential *gains from trade*. I can buy your food with my stamps, you can buy me something else.
- Moreover, grocers could in theory just give stamp holders cash (or sell them alcohol, cigarettes and other non-stamp goods) and redeem the stamps from the government at face value–though Electronic Benefits Transfer cards have made this harder.
- Why won't this market function efficiently such that stamps sell for $1.00 each?
  - This is fraud. Sellers could lose their stamp privileges, and buyers could be jailed.
  - Buyers will demand a 'risk premium.'
  - Consequently, sellers will not get full face value.
- It is likely that there will be a downward sloping demand curve of risk takers and an upward sloping supply curve of recipients who don't value food much at the margin. Theory predicts that the intersection of these curves will be below $1.00.



- But government will still pay $1.00 per stamp to the grocer. So, part of the food stamp money is a transfer to grocers who traffic in food stamps.
- What is the underground selling price? **See Table 6.**
- Food stamps sell for about $55 to $65 dollar per $100. This is a large transfer to stamp traffickers.
- The USDA estimated in 1996-1998 that about 3.5% of every dollar of food stamps was trafficked.
- Note additional costs of trafficking:
  - Enforcement costs of reducing trafficking
  - People in jails
- One further refinement. Imagine that there was no law enforcement in this market (and hence no risk premium) and so food stamps sold on the open market at face value ($100 in food stamps sold for $100). In this market, there would be no efficiency loss since it would function identically to a "cashed out" program. By contrast, the underground market in which food stamps sell at $65 per $100 face value has two flaws (not including criminal enforcement costs):
  1. There is a dead weight loss due to the fact that food stamp recipients will presumably continue to buy food until the marginal utility of consumption is well below a $1. This is the DWL we calculated above.

2. There is a loss in targeting efficiency. For, stamps that are sold on the black market, \$0.35 of every public dollar is a transfer to criminals (e.g., crooked grocers). Observe that this is *not* a dead weight loss—it is a *transfer*—since grocers value the \$0.35 gain at the margin like a cash transfer. But from the perspective of taxpayers, who ultimately bear the full costs of food stamps, this will be viewed as a waste of public resources.

- Some other "shadow" markets:
  - Human organ sales
  - Adopted children
  - Donor eggs for infertile couples
  - Prostitution
  - Recreational drugs
- General principle: When you prevent trades that people would otherwise engage in, market will attempt to undo this distortion through an underground market.
- The cost of enforcement to prevent this market from operating may be high:
  - Society must spend additional resources on monitoring, enforcement, imprisonment.
  - Some otherwise law-abiding citizens will engage in crime, go to jail, and perhaps commit other ancillary antisocial activities.
  - Examples: Consider U.S. experience with banning consumption of alcohol in the 1920s and 1930s ('prohibition'). Or consider the violence associated with the illicit drug trade. Open question: Would the world be more or less violent if cocaine were legalized?

5 Conclusions

1. Are Food Stamp recipients "distorted?" That is, do they indeed spend more on food than they otherwise would without food stamps?
   - Yes they are. About 23 cents is wasted on the dollar for stamps spent in excess of consumers' non-distorted consumption levels
2. Does cash versus in-kind have any effect on nutrition?
   - Cashing out does reduce caloric intake
   - Not clear it harms nutrition
3. What share of food stamps are trafficked, and at what price?

- About 3.5 percent of food stamps are trafficked illegally
- They sell at 50 to 60 percent at face value

4. How costly are cash versus in-kind programs to administer?

    - Cash versus EBT: EBT is about \$2.16 more expensive per person per month than sending checks. Nationally, that's about \$1.2 billion per year.
    - It is estimated that retailers also spend about \$1.04 billion per year to administer EBT.
    - Hence, EBT is very likely to reduce fraud, but there are substantial enforcement costs.

Other considerations:

- Food stamps have political support that direct transfer payments ("cash stamps") do not have; food stamps are *not* as likely to be viewed as a handout.
- Food stamps also have lobbying clout. The Farm lobby believes (or behaves as if its members believe) that food stamps are ultimately spent on farm products. Farmers view food stamps as a subsidy to them too, and they actively support the food stamp program. (As noted above, the food stamp program is run by the Department of Agriculture, a fact that would otherwise be hard to understand.) If food stamps were cashed out, lobbying support would likely decline.
- It thus appears plausible that cashing out the program would help recipients in the short run, harm them in the long run.
- Other considerations?