Problem Set 1

This problem set is due at **11:59 pm** on **Wednesday, September 16th, 2015**. The exercises are **optional**. However, the problems are **mandatory**.

Exercise 1- 1: Do Exercise 3-2 in CLRS on page 61.

Exercise 1- 2: Do Exercise 3-4 in CLRS on page 62.

Exercise 1- 3: Do Exercise 4.3-2 in CLRS on page 87.

Exercise 1- 4: Do Exercise 4.3-8 in CLRS on page 87.

Using the master method in Section 4.5, you can show that the solution to the recurrence $T(n) = 4T(n/2) + n$ is $T(n) = \Theta(n^2)$. Show that a substitution proof with the assumption that $T(n) \leq cn^2$ fails. Then show how to subtract off a lower-order term to make a substitution proof work.

(Note: there was an error in CLRS before the third printing of the third edition where the recurrence was incorrectly specified as $T(n) = 4T(n/2) + n^2$.)

Exercise 1- 5: Do Exercise 4.4-1 in CLRS on page 92.

Exercise 1- 6: Do Exercise 4.4-2 in CLRS on page 92.

Exercise 1- 7: Read 15.2 in CLRS.

Exercise 1- 8: Read 15.3 in CLRS.

Problem 1- 1: 6.006 Review

(a) (Asymptotic Growth) Decide whether these statements are true or false for asymptotically non-negative functions f and g. You must justify all your answers to receive full credit by either giving a short proof (1-2 sentences) or exhibiting a counter-example.

i. Suppose $f(n) = \Theta(g(n))$, then $2^{f(n)} = \Theta(2^{g(n)})$

ii. For any constants $a, b > 0$, $af(n) + bg(n) = \Theta(\max(f(n), g(n)))$

iii. Suppose $f(n) = o(1)$, then $f(n)g(n) = o(1)$

iv. Rank the following functions by order of growth. In other words, find an arrangement $g_1, g_2, ..., g_{12}$ of the functions satisfying $g_1 = \Omega(g_2), g_2 = \Omega(g_3), ..., g_{11} = \Omega(g_{12})$. Partition your list into equivalence classes such that $f(n)$ and $g(n)$ are in the same class if and only if $f(n) = \Theta(g(n))$. In all terms, $\log$ functions are base 2.

$$\begin{array}{cccc} 2^n & n^3 & (\log n)^{\log n} & 100000^{100000000} \\ \log\log n & n\log n & n^{10} & 3^{\log^2 n} \\ 4^n & n! & n^{\log\log n} & \sum_{k=1}^{n}\log k \end{array}$$

(b) (Recurrences) Give asymptotic upper and lower bounds for $T(n)$ in each of the following recurrences. For all parts, assume that $T(n)$ is constant for $n \leq 2$.

i. $T(n) = 10T(n/3) + n^2$

ii. $T(n) = 9T(n/3) + n^2 \log n$

iii. $T(n) = T(\sqrt{n}) + \log n$

iv. $T(n) = T(n/4) + T(n/2) + n$

v. $T(n) = T(2n/3) + T(n/3) + n \log n$