of bounded on [a,b] is integrable. inf EU(IP)3 sup {LILP)3 (Criterion of integrability)

(D YE>Q F partition P of [a, b] s.t. U(Lp)-L(Lp) (E 

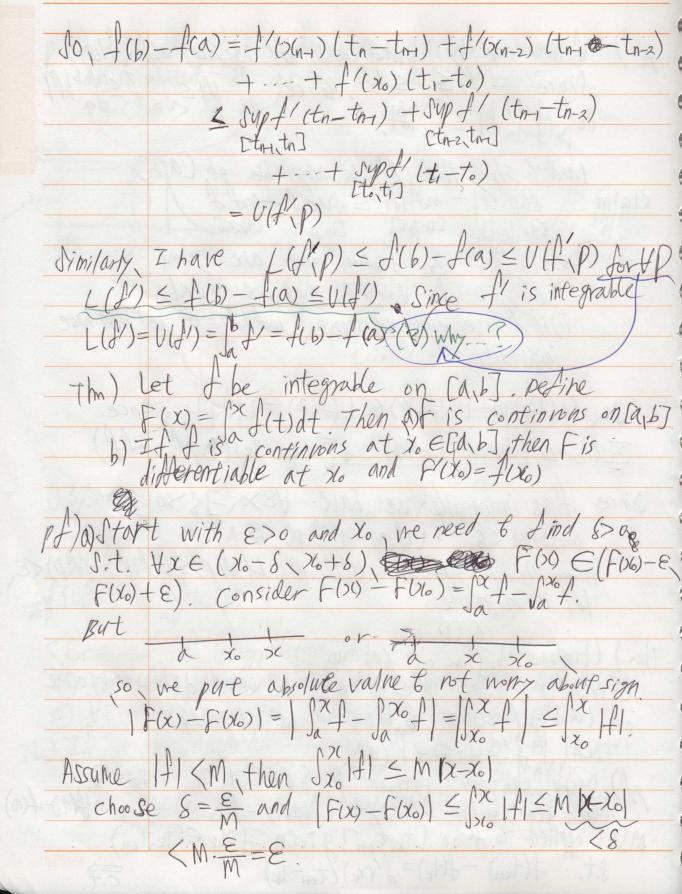
Pf) It we already showed that If is integrable, then

Since -If \( \leq \frac{1}{2} \leq \leq \frac{1}{2} \rightarrow \frac{1} Noted & Show that If is integrable on (a,b].

claim Sup If - inf If & Sup f -inf &

Ctr. trail Ctr. trail Ctr. trail Ctr. trail 4) if f >0 for 4 values, they are equal ii) if f < 0 for & values, they are equal. ii) if I have different signs, int 181=0, and will have So, (((f), p)-L((f), p) ≤ U(f, p) -L(f, p), Since E (tren-tre) (Sup/f/-in/f/) < 12 (tren-tre) (supf-inff) Since f is integrable we have  $f \in (0, -1) = 0$  st.

mesh(p)  $(8 \rightarrow V|f,p) - L(f,p) \in (0, -1) = V(f,p) - L(f,p) = V(f,p) - L(f,p) = V(f,p) - L(f,p) = V(f,p) = V(f,p)$ If is integrable Thm) (Fundamental Thin of (alculus I) Given f is continuous on [a,b], and differentiable on (ab). Assume & that I' is integrable. Then (b) = f(b) - f(a) Italy with a partition  $P = \{a = t_0 < t_1 < - - < t_n = b\}$   $f(b) - f(a) = (f(b) - f(t_m)) + (f(t_m) - f(t_{n-2})) + \cdots + (f(t_l) - f(n))$ MV. Tapplied to f on  $Ct_k, t_{k+1} \supset f(x_k)$   $f(t_k) = f'(x_k) (t_{k+1} - t_k)$ S.t.  $f(t_{k+1}) - f(t_k) = f'(x_k) (t_{k+1} - t_k)$  C.f



b) Assume that I is continuous at >6. F'(xo) = lim for-f(xo) which is egual to f(xo) It's sufficient to show (im (FUU-FIXE) - f(XE)) = 0  $\frac{F(x) - F(x_0)}{x - x_0} = \int_{-x_0}^{x} f(t) dt$ Hence  $(x_0) = \frac{1}{x_0} \int_{x_0}^{x_0} f(x_0)$ Hence  $(x_0) = \frac{1}{x_0} \int_{x_0}^{x_0} f(x_0)$ = 1 (f(t)-f(xo))dt

= 1 (f(t)-f(xo))dt

Small when x is near xo,

since f is continuous at xo,

Given &>0,75>0 S.t. Whenever |t-xo|(8, |f(t)-f(xo))(8, |f(t)-f(xo))| So, F(x)-F(6) - f(x6) ≤ x+x60 | x | f(+x) - f(€x6) | dt Triangular  $\langle tx \rightarrow 6 | \int_{X_0}^{X_0} \mathcal{E} = \mathcal{E} , \text{ when } | x \rightarrow 26 | \langle 8 |$ by def &F'(Xo) &ists and eguml & fixe)