<Math 104>

01.16 Tue

objects ⇒ real # functions designed over R

↳ will talk about completeness axioms

$\sqrt{2}$ is not rational $\neq \frac{p}{q}$

↳ not converge if we only consider $\mathbb{Q}$.

Some series such as $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\cdots$ converges

Some series such as $1+\frac{1}{2}+\frac{1}{3}+\cdots$ not converge.

* $f: R \to R$ ⇒ check continuous, differentiable, integrable.

Fundamental Theorem of Calculus

* Sequence of functions

⇒ ex) $f_1, f_2, \cdots, f_n \xrightarrow{\text{converge}} f$

↳ pointwise & uniform

↳ Find the existence of f.

Ch1

$\mathbb{N}$ ⇒ Natural number $= \{1, 2, 3, \cdots\}$

Set does not have to be infinite.

↳ ex) $S = \{a, 2\}$

$S = \{x \in \mathbb{N} \mid x = y^2 \text{ for some } y \in \mathbb{N}\} = \{1, 4, 9, 16, \cdots\}$

$= \{x \mid x \in \mathbb{N} \text{ and } x = y^2 \text{ for some } y \in \mathbb{N}\}$

☆ Peano axioms ⇒ ① For each $n \in N$, there is a unique sucessor called $n+1$.

② There is no number n such that 1 is the successor of n.

☆ If S is a subset of N s.t. a) $1 \in S$ and b) if $n \in S$, then $n+1 \in S$, then $S = N$.

⟨Mathematical induction⟩

Suppose you have $P_1, P_2, \cdots$

Problem here is how to show all P_n's are true?

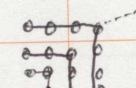
Steps ⇒ 1) check P_1

2) Assume that P_n is true, and check that P_{n+1} is also true.

Then, all P_n are true.

ex) Show that for each $n \in N$,

$$1+3+5+\cdots+(2n+1) = (n+1)^2 \quad (*)$$

Sol 1) 

Sol 2) P_1 : $(*)$ when $n = 1$

Assume P_n is true where $(*)$ $n = n$, and we want to show that P_{n+1} is also true.

So, assume $1+3+5+\cdots+(2n+1) = (n+1)^2$ and prove

$$\underbrace{1+3+\cdots+(2n+1)}+(2(n+1)+1) = ((n+1)+1)^2$$

By assumption, $(n+1)^2+(2n+3) = (n+2)^2$

So, check, $n^2+2n+1+2n+3 = n^2+4n+4$.

$\therefore$ Therefore, by induction, P_n is true for all $n \in \mathbb{N}$.

ex) Show that for every $n \in \mathbb{N}$, $|\sin nx| \leq n|\sin x|$ for every $x \in \mathbb{R}$.

sol) $P_1 \Rightarrow |\sin 1 \cdot x| \leq |\sin x|$ for all $x \in \mathbb{R}$

$P_2 \Rightarrow |\sin 2x| \leq 2|\sin x|$ for all $x \in \mathbb{R}$.

$\vdots$

P_1 is obviously true.

Assume that P_n is true, we need to show P_{n+1} is also true.

Assume $|\sin nx| \leq n|\sin x|$ for $\forall x \in \mathbb{R}$, need to show $|\sin (n+1)x| \leq (n+1)|\sin x|$ $\forall x \in \mathbb{R}$.

$$
\begin{aligned}
|\sin(nx+x)| &= |\sin nx \cdot \cos x + \cos nx \cdot \sin x| \quad \text{--- trig identity} \\
&\leq |\sin nx \cdot \cos x| + |\cos nx \cdot \sin x| \quad \text{--- triangular equality} \\
&= |\sin nx||\cos x| + |\sin x||\cos nx| \quad \text{---- //} \\
&\leq |\sin nx| + |\sin x| \\
&\leq n|\sin x| + |\sin x| = (n+1)|\sin x|
\end{aligned}
$$

$\therefore$ proved.