Homework 1

Yuki Joyama

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Problem 1

a) Qualitative, ordinal
b) Qualitative, binary
c) Qualitative, ordinal
d) Quantitative, continuous
e) Quantitative, discrete

Problem 2

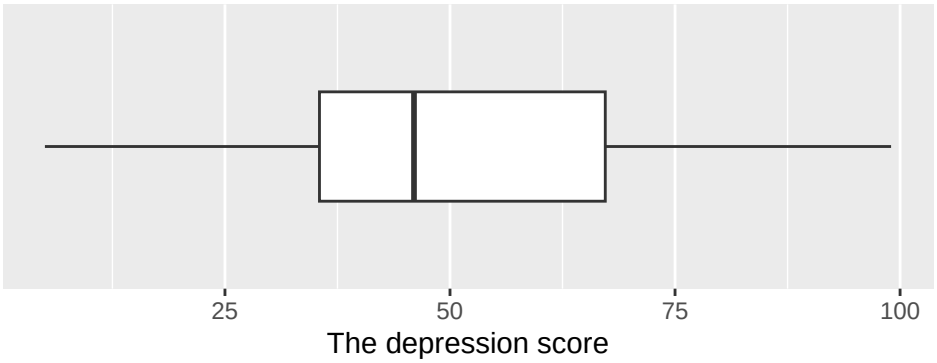
The depression scores for 14 individuals with a recent bike crash history (I will define this as group A): 45, 39, 25, 47, 49, 5, 70, 99, 74, 37, 99, 35, 8, 59

a) Descriptive summaries

Mean: $\sum_{i=1}^{14} \frac{x_i}{14} = 49.36$
Median: $(45 + 47)/2 = 46$
Range: $99 - 5 = 94$
SD: $\sqrt{\frac{1}{14-1}\sum_{i=1}^{14}(x_i - \overline{x})^2} = 28.85$

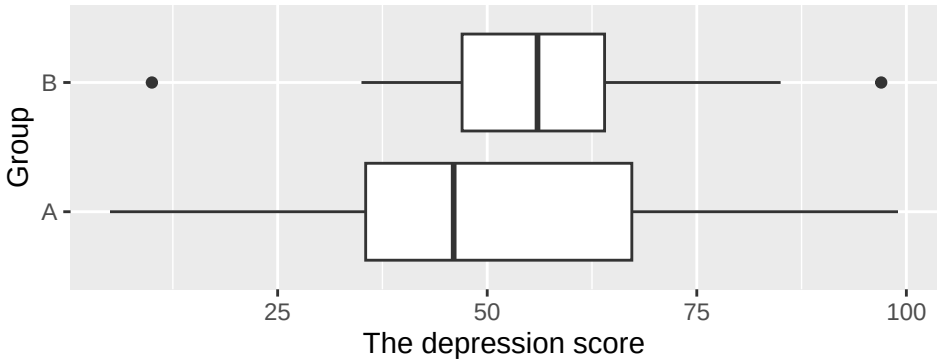
b) Box plot of the depression scores of group A

The box plot shows right-skewed, unimodal distribution.



The depression scores for 13 individuals with a recent car crash history (I will define this as group B): 67, 50, 85, 43, 64, 35, 47, 97, 58, 58, 10, 56, 50

a) Side-by-side box plot of the depression scores stratified by type of accident



b) Based on the box plot, group A appears to have right-skewed, unimodal distribution while group B has left-skewed, unimodal distribution. This is also supported by the fact mean > median in group A and mean < median in group B.

c) Judging from the box plot, group A appears to have a lower typical depression score.

Problem 3

Tossing one fair 12-sided die.

a) event A: an even number appears
Let $\Omega = \{1, 2, \ldots, 12\}$
Let A = $\{2, 4, 6, 8, 10, 12\}$, $A \subset \Omega$
P(A) = $\frac{6}{12} = \frac{1}{2}$

b) event B: number 10 appears Let $\Omega = \{1, 2, \ldots, 12\}$
Let B = $\{10\}$, $A \subset \Omega$
P(B) = $\frac{1}{12}$

c) $P(B \cup A) = P(A) + P(B) - P(B \cap A)$
$P(B \cap A)$ is the probability of number 10 appears (which is also an even number), so $P(B \cap A) = \frac{1}{12}$
Therefore, $P(B \cup A) = P(A) + P(B) - P(B \cap A) = \frac{1}{2} + \frac{1}{12} - \frac{1}{12} = \frac{1}{2}$

d) If event A and event B are independent, $P(A \cap B)$ should be equal to $P(A)P(B)$.
Now, given $P(A)P(B) = \frac{1}{2} \times \frac{1}{12} = \frac{1}{24} \neq P(A \cap B)$, we can say that these two events are not independent.

Problem 4

5% of women above age of 75 have dementia. Among women (75+ years old) with dementia, 80% have positive findings on their CT scan. Among women (75+ years old) who don't have dementia, 10% will have a positive CT scan findings. A randomly-selected woman (75+ years old) had a positive CT scan findings. What is the probability that she actually has dementia?