Homework 1

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Problem 1

a) Qualitative, ordinal
b) Qualitative, binary
c) Qualitative, ordinal
d) Quantitative, continuous
e) Quantitative, discrete

Problem 2

The depression scores for 14 individuals with a recent bike crash history (I will define this as group A):
45, 39, 25, 47, 49, 5, 70, 99, 74, 37, 99, 35, 8, 59

a) Descriptive summaries

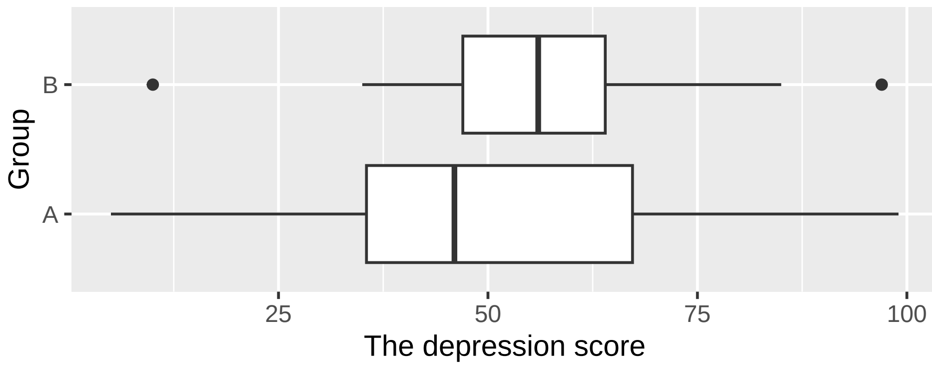
Mean: $\sum_{i=1}^{14} \frac{x_i}{14} = 49.36$
Median: $(45 + 47)/2 = 46$
Range: $99 - 5 = 94$
SD: $\sqrt{\frac{1}{14-1} \sum_{i=1}^{14} (x_i - \overline{x})^2} = 28.85$

b) Box plot of the depression scores of group A

The box plot and histogram (plot not shown) indicate right-skewed, unimodal distribution.

The depression scores for 13 individuals with a recent car crash history (I will define this as group B):
67, 50, 85, 43, 64, 35, 47, 97, 58, 58, 10, 56, 50

a) Side-by-side box plot of the depression scores stratified by type of accident



b) Based on the box plot and histogram (plot not shown), group A appears to have right-skewed, unimodal distribution while group B has left-skewed, unimodal distribution. This is also supported by the fact mean > median in group A and mean < median in group B.

c) Judging from the box plot, group A appears to have a lower typical depression score.

Problem 3

Tossing one fair 12-sided die.

a) event A: an even number appears
Let $\Omega = \{1, 2, \ldots, 12\}$
Let A $= \{2, 4, 6, 8, 10, 12\}$, $A \subset \Omega$
P(A) $= \frac{6}{12} = \frac{1}{2}$

b) event B: number 10 appears Let $\Omega = \{1, 2, \ldots, 12\}$
Let B $= \{10\}$, $A \subset \Omega$
P(B) $= \frac{1}{12}$

c) $P(B \cup A) = P(A) + P(B) - P(B \cap A)$
$P(B \cap A)$ is the probability of number 10 appears (which is also an even number), so $P(B \cap A) = \frac{1}{12}$
Therefore, $P(B \cup A) = P(A) + P(B) - P(B \cap A) = \frac{1}{2} + \frac{1}{12} - \frac{1}{12} = \frac{1}{2}$

d) If event A and event B are independent, $P(A \cap B)$ should be equal to $P(A)P(B)$.
Now, given $P(A)P(B) = \frac{1}{2} \times \frac{1}{12} = \frac{1}{24} \neq P(A \cap B)$, we can say that these two events are not independent.

Problem 4

Let A be having dementia and B be having positive CT scan findings.
5% of 75+ year-old women have dementia so $P(A) = 0.05$
Among women with dementia, 80% have positive CT scan findings so $P(B|A) = \frac{P(B \cap A)}{P(A)} = 0.8$
Given $P(A) = 0.05$, $P(A \cap B) = 0.04$
10% showed a positive CT scan findings among women who don't have dementia, and this gives $P(B|\bar{A}) = \frac{P(B \cap \bar{A})}{P(\bar{A})} = 0.1$
Given $P(\bar{A}) = 1 - 0.05 = 0.95$, $P(B \cap \bar{A}) = 0.095$

Now, $P(B) = P(A \cap B) + P(\bar{A} \cap B) = 0.04 + 0.095 = 0.135$
Therefore, the probability of a woman with a positive CT scan findings to have dementia is $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.04}{0.135} = 0.30$

The answer to this question would be 30%.