

Discrete Distributions: Binomial & Poisson

Binomial Distribution

Consider a sample of n infants with a 0.06 probability of developing asthma. Calculate the probability that exactly 5 infants develop asthma.

```
# n = 5  
dbinom(5, 20, 0.06)
```

```
[1] 0.004765603
```

```
# n = 100  
dbinom(5, 100, 0.06)
```

```
[1] 0.1639175
```

Now, calculate the probability that at least 10 infants have asthma from a sample of 20.

$$P(X \geq 10) = 1 - P(X \leq 9)$$

```
1 - pbinom(9, 20, 0.06)
```

```
[1] 6.383682e-08
```

Create a function to graph the binomial distribution with parameters n and p .

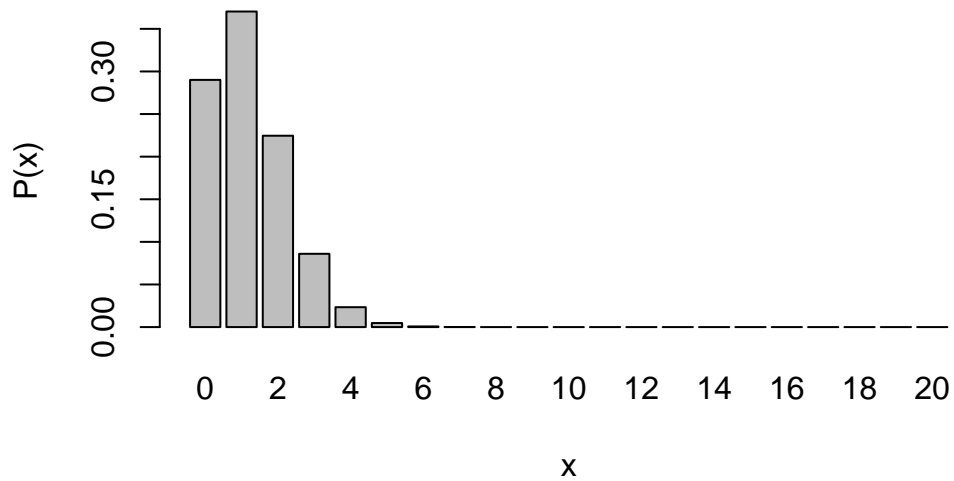
```
# define function
bin_graph = function(n, p) {

  x = dbinom(0:n, size = n, prob = p)
  barplot(x, names.arg = 0:n,
          main = paste0("n = ", n, ", p = ", p),
          ylab = "P(x)",
          xlab = "x")

}

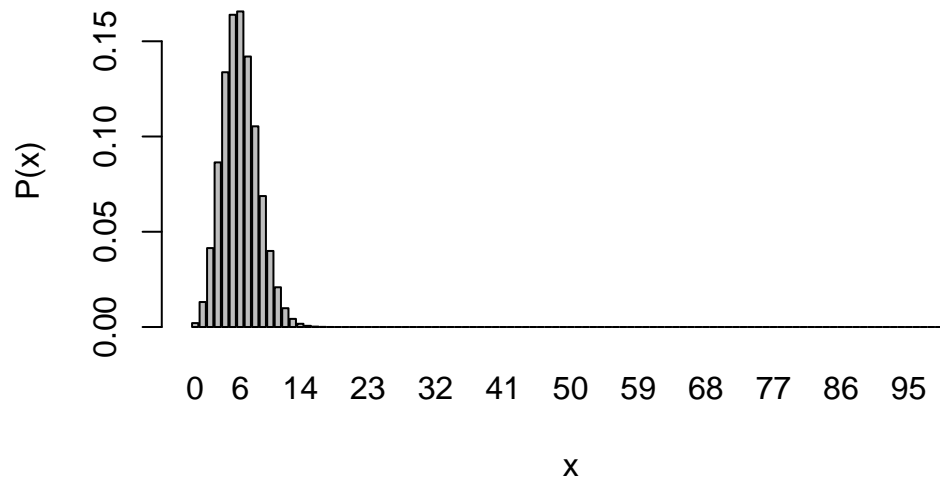
# call function
bin_graph(n = 20, p = 0.06)
```

n = 20, p = 0.06



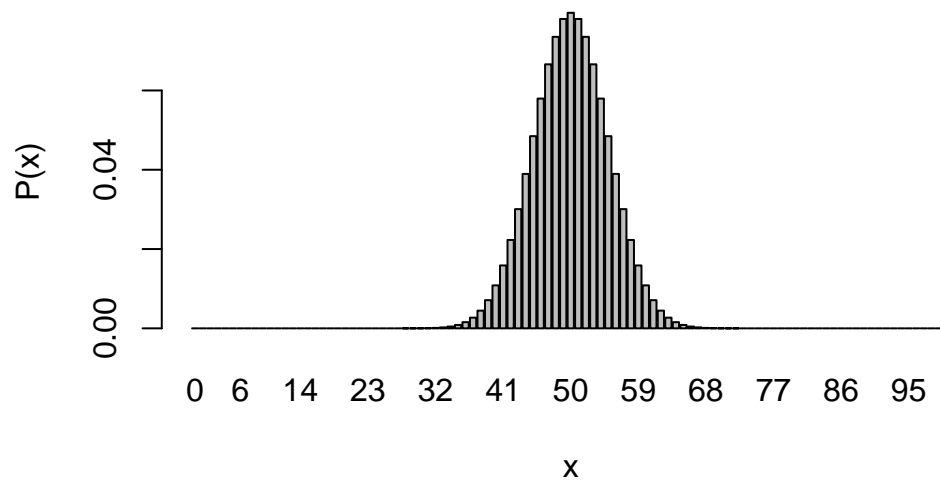
```
bin_graph(n = 100, p = 0.06)
```

n = 100, p = 0.06



```
bin_graph(n = 100, p = 0.5)
```

n = 100, p = 0.5



Poisson Distribution

Let λ be the rate (number of calls per hour).

Calculate the probability of having exactly 4 calls in the next hour for $\lambda = 10$.

```
dpois(4, 10)
```

```
[1] 0.01891664
```

Calculate the probability of receiving at most 8 calls in one hour.

```
ppois(8, 10)
```

```
[1] 0.3328197
```

What does the Poisson distribution look with with different lambdas? (Create a function to graph the Poisson distribution with parameter lambda.)

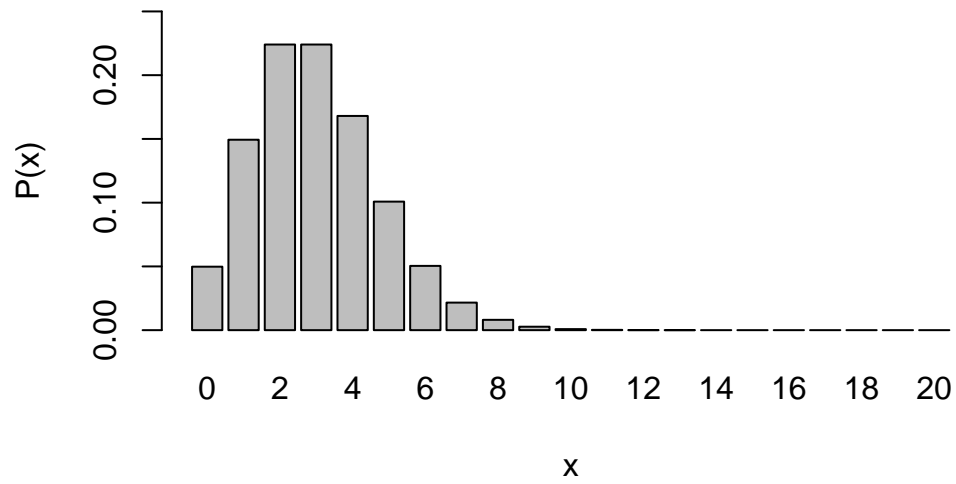
```
# define function
poi_graph = function(lam) {

  x = dpois(0:20, lambda = lam)
  barplot(x, names.arg = 0:20,
          ylim = c(0, 0.25),
          main = paste0("lambda = ", lam),
          ylab = "P(x)",
          xlab = "x")

}

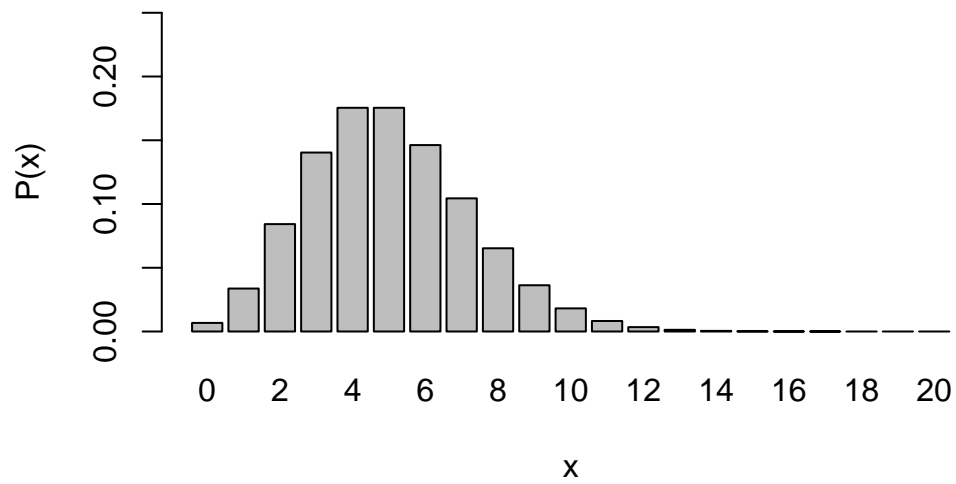
# call function
poi_graph(lam = 3)
```

lambda = 3



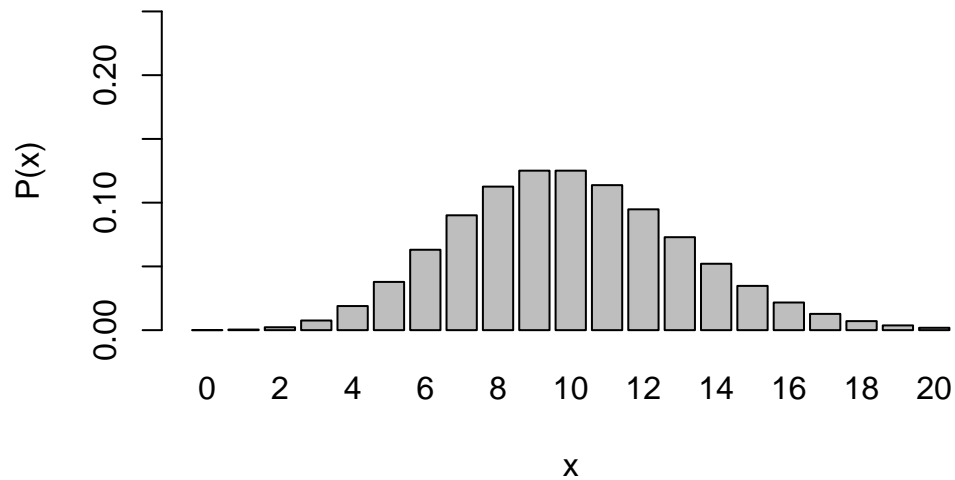
```
poi_graph(lam = 5)
```

lambda = 5



```
poi_graph(lam = 10)
```

lambda = 10



Exercise from Slide 26

Assume a rare birth defect occurs with probability 0.0001 and we collect a sample of 4,000 babies. Calculate the probability of having at least 10 babies with a birth defect.

Note: this probability can be computed using either the binomial or Poisson distribution.

```
# binomial  
1 - pbinom(9, 4000, 0.0001)
```

```
[1] 1.989076e-11
```

```
# Poisson  
1 - ppois(9, 0.4)
```

```
[1] 2.009826e-11
```

Both probabilities are approximately 0.