Homework2

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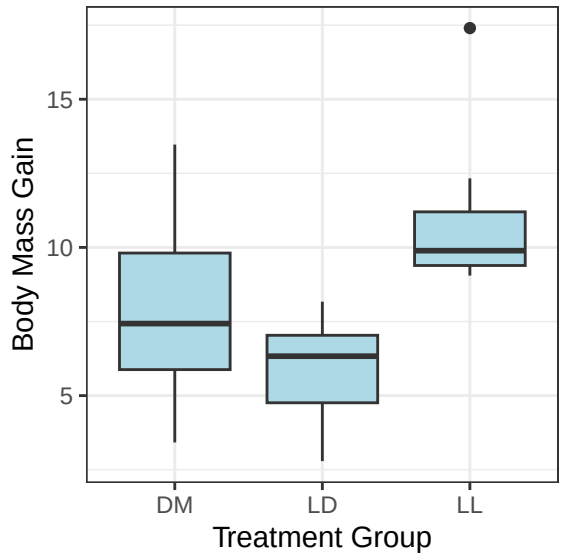
```
# libraries
library(tidyverse)
library(ggplot2)
library(perm)

# setup plot theme
theme_set(
  theme_bw() +
    theme(legend.position = "top")
  )

# import data
df = read_csv('./data/light.csv')
```

1. I will use boxplot to show the outcome by treatment group.

```
# boxplot
p1 = ggplot(df, aes(x = Light, y = BMGain)) +
  geom_boxplot(fill = "lightblue") +
  labs(x = "Treatment Group", y = "Body Mass Gain")
p1
```



2. Here I will subset the data to only consider LD (dark light) and LL (bright light) groups.

```r
# filter by these two groups
df2 = df |>
  filter(Light == 'LL' | Light == 'LD')
summary(df2)
```

```
##     Light               BMGain       Corticosterone       DayPct
##  Length:17          Min.   : 2.790   Min.   :  3.00   Min.   :21.85
##  Class :character   1st Qu.: 6.340   1st Qu.: 23.40   1st Qu.:40.50
##  Mode  :character   Median : 9.050   Median : 52.00   Median :61.45
##                     Mean   : 8.618   Mean   : 59.86   Mean   :57.49
##                     3rd Qu.: 9.890   3rd Qu.: 70.47   3rd Qu.:81.60
##                     Max.   :17.400   Max.   :191.22   Max.   :87.26
##   Consumption     GlucoseInt           GTT15           GTT120
##  Min.   :3.387   Length:17          Min.   :226.6   Min.   :118.3
##  1st Qu.:3.791   Class :character   1st Qu.:280.0   1st Qu.:153.7
##  Median :4.240   Mode  :character   Median :348.8   Median :227.3
##  Mean   :4.427                      Mean   :347.8   Mean   :251.8
##  3rd Qu.:4.873                      3rd Qu.:392.4   3rd Qu.:328.7
##  Max.   :7.177                      Max.   :500.0   Max.   :470.2
##     Activity
##  Min.   : 153
##  1st Qu.: 877
##  Median :1649
##  Mean   :2660
##  3rd Qu.:4482
##  Max.   :6702
```

3. I will redefine the variables using generic names as follows:

- LL group: $A = 1$
- LD group: $A = 0$
- BMGain: Y_{obs}

```r
# edit df2 accordingly
df2 = df2 |>
  mutate(A = ifelse(Light == "LL", 1, 0), # add variable A and input 1 for LL, 0 for LD group
         Y_obs = BMGain) |> # add new outcome column
  select(-Light, -BMGain)
```

To evaluate the causal effect of light at night on weight gain, I will need the following quantities:

```r
# define/calculate the quantities
N1 = sum(df2$A == 1)
N0 = sum(df2$A == 0)
N = N1 + N0
Yb_obs1 = df2 |>
  filter(A == 1) |>
  summarize(mean_Y_obs = mean(Y_obs)) |>
  pull(mean_Y_obs)
Yb_obs0 = df2 |>
  filter(A == 0) |>
```

```
  summarize(mean_Y_obs = mean(Y_obs)) |>
  pull(mean_Y_obs)
```

- Number of mice in LL group: $N_1 = 9$
- Number of mice in LD group: $N_0 = 8$
- Total number of mice in LL and LD group: $N = 17$
- Mean of the outcome variable for LL group: $\bar{Y}_1^{obs} = 11.01$
- Mean of the outcome variable for LD group: $\bar{Y}_0^{obs} = 5.93$

4.

```
# calculate t_obs
T_obs = Yb_obs1 - Yb_obs0
```

$T_{obs} = \bar{Y}_1^{\text{obs}} - \bar{Y}_0^{\text{obs}} = 5.08$

5. Under the completely randomized experiment where $N1$ and $N0$ are fixed, there are $\binom{N}{N_1} = 24310$ possibilities for A.

```
# enumerate them in a matrix
A = chooseMatrix(N, N1)
```

6. The sharp null hypothesis:
   $H_0 : Y_i^1 = Y_i^0 \quad$ for all i
   where Y_i^1 is the potential outcome for mouse i if they are assigned to $A = 1$, and Y_i^0 is the potential outcome for mouse i if they are assigned to $A = 0$.

```
# create df that has the group assignment based on the first row of matrix A
df3 = df2
df3$A = A[1,]

# calculate t under the first possibility of A, under the sharp null hypothesis
T_stat = mean(df3$Y_obs[df3$A == 1]) - mean(df3$Y_obs[df3$A == 0])
```

Under the sharp null hypothesis, the test statistic under the first row of matrix A is 1.55.