Assignment2

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Problem 1

Given the local Markov property for UGs,
$X_i \perp\!\!\!\perp X\backslash Cl(X_i, \mathcal{G}) | Ne(X_i, \mathcal{G})$ and $X_i \nsim X_j$
$\Rightarrow X_i \perp\!\!\!\perp X_j \cup (X\backslash Cl(X_i, \mathcal{G}) \cup X_j) | Ne(X_i, \mathcal{G})$
$\Rightarrow X_i \perp\!\!\!\perp X_j | Ne(X_i, \mathcal{G}) \cup (X\backslash Cl(X_i, \mathcal{G}) \cup X_j)$ (by weak union)
$\Rightarrow X_i \perp\!\!\!\perp X_j | Ne(X_i, \mathcal{G}) \cup (X\backslash Ne(X_i, \mathcal{G}) \cup X_i \cup X_j)$
$\Rightarrow X_i \perp\!\!\!\perp X_j | X\backslash(X_i \cup X_j) \Rightarrow X_i \perp\!\!\!\perp X_j | X\backslash\{X_i, X_j\})$
This is the pairwise Markov property. Therefore, we can conclude that local Markov property $\Rightarrow$ pairwise Markov property in undirected graph $\mathcal{G}$.

Problem 2

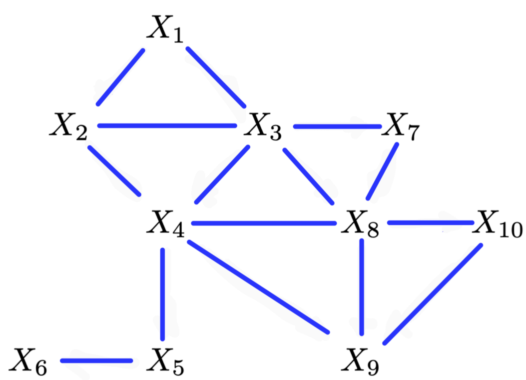

Figure 1: Moralized graph

(a) When factorizing by maximal clique, the joint distribution can be written as:
$p(x) = \frac{1}{Z}\phi_{123}(x_1, x_2, x_3)\phi_{234}(x_2, x_3, x_4)\phi_{378}(x_3, x_7, x_8)\phi_{348}(x_3, x_4, x_8)\phi_{489}(x_4, x_8, x_9)\phi_{234}(x_8, x_9, x_{10})$
$\phi_{45}(x_4, x_5)\phi_{56}(x_5, x_6)$
There are 8 factor potentials.

(b) When the scope has 3 variables, there are 3 parameters for each variable, 3 parameters for pairs, and 1 parameter for three-way interaction. When the scope has two variables, there are 2 parameters for single variable and 1 parameters for pairs. From (a), we have 6 scopes with 3 variables and 2 scopes with 2 variables. Therefore, there are $(3 + 3 + 1) \times 6 + (2 + 1) \times 2 = 48$ variables in total.

(c) There are 10 variables and each can take on either 0 or 1, so there are $2^{10} = 1024$ possible combinations. The joint distribution must sum to 1, and the last one will be determined automatically when all the other 9 variable are set. So $2^{10} - 1 = 1023$ parameters would be required.

(d) Factorization property in DAG $\mathcal{G}$:
$p(x_1, x_2, ..., x_{10}) = \prod_{i=1}^{10} p(x_i \mid Pa(X_i, \mathcal{G}))$
$=p(x_1)p(x_2|x_1)p(x_3|x_1)p(x_4|x_2, x_3)p(x_5|x_4)p(x_6|x_5)p(x_7|x_3)p(x_8|x_3, x_7)p(x_9|x_4, x_8, x_{10})p(x_{10}|x_8)$
When $x_i = 1$, the probability can be written as $p(x_i = 1|Pa(X_i, \mathcal{G}))$. The probability where $x_i = 0$ can be obtained by $1 - p(x_i = 1|Pa(X_i, \mathcal{G}))$ so we can focus on the combination of the parents. By the above factorization formula, the required number of parameters are $2^0 + 2^1 + 2^1 + 2^2 + 2^1 + 2^1 + 2^1 + 2^2 + 2^3 + 2^1 = 29$.