

Assignment 1

Yuki Joyama (yj2803)

Problem 1

- (a) $X_2 \perp\!\!\!\perp X_3, X_7, X_8, X_{10}$
 $X_3 \perp\!\!\!\perp X_2 | X_1$
 $X_4 \perp\!\!\!\perp X_1, X_7, X_8, X_{10} | X_2, X_3$
 $X_5 \perp\!\!\!\perp X_1, X_2, X_3, X_7, X_8, X_9, X_{10} | X_4$
 $X_6 \perp\!\!\!\perp X_1, X_2, X_3, X_4, X_7, X_8, X_9, X_{10} | X_5$
 $X_7 \perp\!\!\!\perp X_1, X_2, X_4, X_5, X_6 | X_3$
 $X_8 \perp\!\!\!\perp X_1, X_2, X_4, X_5, X_6 | X_3, X_7$
 $X_9 \perp\!\!\!\perp X_1, X_2, X_3, X_5, X_6, X_7 | X_4, X_8, X_{10}$
 $X_{10} \perp\!\!\!\perp X_1, X_2, X_3, X_4, X_5, X_6, X_7 | X_8$
- (b) $X_2 \perp_d X_9 | X_4$: False
 $X_7 \perp_d X_5 | \{X_3, X_8\}$: True
 $\{X_2, X_4\} \perp_d X_7 | \{X_6, X_9, X_{10}\}$: False

Problem 2

Problem 3

Agreements: $D \perp\!\!\!\perp \{A, C\} | B$

Disagreements: $A \perp\!\!\!\perp C | \phi(a), C \perp\!\!\!\perp \{A, D\} | B$ (b)

Problem 4

- (a) b, e
- (b) An unshielded collider in a DAG will violate the Markov equivalence with respect to the chain DAG. Taken this into account, we can consider two cases that will be Markov equivalent to the chain DAG:
1. Flip all the arrows to left
 2. For $X_i (i = 2, 3, \dots, p-1)$, flip all the arrows to left before X_i
- There are $p-2$ possible X_i s that can be the pivot of the arrows. Therefore, $p-2+1 = p-1$ DAGs are Markov equivalent to the chain DAG.

Problem 5

- (a)

```
## $paths
## [1] "C -> B -> A <- E -> F -> G -> H" "C -> B -> A <- E -> F -> H"
## [3] "C -> B <- G -> H" "C -> B <- G <- F -> H"
## [5] "C -> E -> A <- B <- G -> H" "C -> E -> A <- B <- G <- F -> H"
## [7] "C -> E -> F -> G -> H" "C -> E -> F -> H"
## [9] "C -> F -> G -> H" "C -> F -> H"
## [11] "C -> F <- E -> A <- B <- G -> H" "C -> H"
##
## $open
## [1] FALSE FALSE FALSE FALSE FALSE FALSE TRUE TRUE TRUE TRUE FALSE TRUE
```

(b)

E and G are not d-separated given A and B.

(c)

```
## A _||_ C | B, E
## A _||_ D | C, E
## A _||_ D | B, E
## A _||_ F | C, E, G
## A _||_ F | B, E
## A _||_ G | B, C, F
## A _||_ G | B, E
## A _||_ H | C, F, G
## A _||_ H | C, E, G
## A _||_ H | B, C, F
## A _||_ H | B, E
## B _||_ D | C, E
## B _||_ D | C, F
## B _||_ D | C, G
## B _||_ E | C, F
## B _||_ E | C, G
## B _||_ F | C, G
## B _||_ H | C, G
## C _||_ D
## C _||_ G | F
## D _||_ F | C, E
## D _||_ G | F
## D _||_ G | C, E
## D _||_ H | C, F
## D _||_ H | C, E
## E _||_ G | F
## E _||_ H | C, F
```

With the option “type = all.pairs”:

```
## A _||_ C | B, E
## A _||_ C | B, D, E
## A _||_ C | B, E, F
## A _||_ C | B, D, E, F
## A _||_ C | B, E, G
```

```

## A _||_ C | B, D, E, G
## A _||_ C | B, E, F, G
## A _||_ C | B, D, E, F, G
## A _||_ C | B, E, H
## A _||_ C | B, D, E, H
## A _||_ C | B, E, F, H
## A _||_ C | B, D, E, F, H
## A _||_ C | B, E, G, H
## A _||_ C | B, D, E, G, H
## A _||_ C | B, E, F, G, H
## A _||_ C | B, D, E, F, G, H
## A _||_ D | B, E
## A _||_ D | C, E
## A _||_ D | B, C, E
## A _||_ D | B, E, F
## A _||_ D | C, E, F
## A _||_ D | B, C, E, F
## A _||_ D | B, E, G
## A _||_ D | C, E, G
## A _||_ D | B, C, E, G
## A _||_ D | B, E, F, G
## A _||_ D | C, E, F, G
## A _||_ D | B, C, E, F, G
## A _||_ D | B, E, H
## A _||_ D | C, E, H
## A _||_ D | B, C, E, H
## A _||_ D | B, E, F, H
## A _||_ D | C, E, F, H
## A _||_ D | B, C, E, F, H
## A _||_ D | B, E, G, H
## A _||_ D | C, E, G, H
## A _||_ D | B, C, E, G, H
## A _||_ D | B, E, F, G, H
## A _||_ D | C, E, F, G, H
## A _||_ D | B, C, E, F, G, H
## A _||_ F | B, E
## A _||_ F | B, C, E
## A _||_ F | B, D, E
## A _||_ F | B, C, D, E
## A _||_ F | B, E, G
## A _||_ F | C, E, G
## A _||_ F | B, C, E, G
## A _||_ F | B, D, E, G
## A _||_ F | C, D, E, G
## A _||_ F | B, C, D, E, G
## A _||_ F | B, E, H
## A _||_ F | B, C, E, H
## A _||_ F | B, D, E, H
## A _||_ F | B, C, D, E, H
## A _||_ F | B, E, G, H
## A _||_ F | C, E, G, H
## A _||_ F | B, C, E, G, H
## A _||_ F | B, D, E, G, H
## A _||_ F | C, D, E, G, H

```

```

## A _||_ F | B, C, D, E, G, H
## A _||_ G | B, E
## A _||_ G | B, C, E
## A _||_ G | B, D, E
## A _||_ G | B, C, D, E
## A _||_ G | B, C, F
## A _||_ G | B, C, D, F
## A _||_ G | B, E, F
## A _||_ G | B, C, E, F
## A _||_ G | B, D, E, F
## A _||_ G | B, C, D, E, F
## A _||_ G | B, E, H
## A _||_ G | B, C, E, H
## A _||_ G | B, D, E, H
## A _||_ G | B, C, D, E, H
## A _||_ G | B, C, F, H
## A _||_ G | B, C, D, F, H
## A _||_ G | B, E, F, H
## A _||_ G | B, C, E, F, H
## A _||_ G | B, D, E, F, H
## A _||_ G | B, C, D, E, F, H
## A _||_ H | B, E
## A _||_ H | B, C, E
## A _||_ H | B, D, E
## A _||_ H | B, C, D, E
## A _||_ H | B, C, F
## A _||_ H | B, C, D, F
## A _||_ H | B, E, F
## A _||_ H | B, C, E, F
## A _||_ H | B, D, E, F
## A _||_ H | B, C, D, E, F
## A _||_ H | B, E, G
## A _||_ H | C, E, G
## A _||_ H | B, C, E, G
## A _||_ H | B, D, E, G
## A _||_ H | C, D, E, G
## A _||_ H | B, C, D, E, G
## A _||_ H | C, F, G
## A _||_ H | B, C, F, G
## A _||_ H | C, D, F, G
## A _||_ H | B, C, D, F, G
## A _||_ H | B, E, F, G
## A _||_ H | C, E, F, G
## A _||_ H | B, C, E, F, G
## A _||_ H | B, D, E, F, G
## A _||_ H | C, D, E, F, G
## A _||_ H | B, C, D, E, F, G
## B _||_ D | C, E
## B _||_ D | A, C, E
## B _||_ D | C, F
## B _||_ D | C, E, F
## B _||_ D | A, C, E, F
## B _||_ D | C, G
## B _||_ D | C, E, G

```

```

## B _||_ D | A, C, E, G
## B _||_ D | C, F, G
## B _||_ D | C, E, F, G
## B _||_ D | A, C, E, F, G
## B _||_ D | C, E, H
## B _||_ D | A, C, E, H
## B _||_ D | C, F, H
## B _||_ D | C, E, F, H
## B _||_ D | A, C, E, F, H
## B _||_ D | C, G, H
## B _||_ D | C, E, G, H
## B _||_ D | A, C, E, G, H
## B _||_ D | C, F, G, H
## B _||_ D | C, E, F, G, H
## B _||_ D | A, C, E, F, G, H
## B _||_ E | C, F
## B _||_ E | C, D, F
## B _||_ E | C, G
## B _||_ E | C, D, G
## B _||_ E | C, F, G
## B _||_ E | C, D, F, G
## B _||_ E | C, F, H
## B _||_ E | C, D, F, H
## B _||_ E | C, G, H
## B _||_ E | C, D, G, H
## B _||_ E | C, F, G, H
## B _||_ E | C, D, F, G, H
## B _||_ F | C, G
## B _||_ F | C, D, G
## B _||_ F | C, E, G
## B _||_ F | A, C, E, G
## B _||_ F | C, D, E, G
## B _||_ F | A, C, D, E, G
## B _||_ F | C, G, H
## B _||_ F | C, D, G, H
## B _||_ F | C, E, G, H
## B _||_ F | A, C, E, G, H
## B _||_ F | C, D, E, G, H
## B _||_ F | A, C, D, E, G, H
## B _||_ H | C, G
## B _||_ H | C, D, G
## B _||_ H | C, E, G
## B _||_ H | A, C, E, G
## B _||_ H | C, D, E, G
## B _||_ H | A, C, D, E, G
## B _||_ H | C, F, G
## B _||_ H | A, C, F, G
## B _||_ H | C, D, F, G
## B _||_ H | A, C, D, F, G
## B _||_ H | C, E, F, G
## B _||_ H | A, C, E, F, G
## B _||_ H | C, D, E, F, G
## B _||_ H | A, C, D, E, F, G
## C _||_ D

```

```

## C _||_ G | F
## C _||_ G | D, F
## C _||_ G | E, F
## C _||_ G | D, E, F
## D _||_ F | C, E
## D _||_ F | A, C, E
## D _||_ F | B, C, E
## D _||_ F | A, B, C, E
## D _||_ F | C, E, G
## D _||_ F | A, C, E, G
## D _||_ F | B, C, E, G
## D _||_ F | A, B, C, E, G
## D _||_ F | C, E, H
## D _||_ F | A, C, E, H
## D _||_ F | B, C, E, H
## D _||_ F | A, B, C, E, H
## D _||_ F | C, E, G, H
## D _||_ F | A, C, E, G, H
## D _||_ F | B, C, E, G, H
## D _||_ F | A, B, C, E, G, H
## D _||_ G | C, E
## D _||_ G | A, C, E
## D _||_ G | B, C, E
## D _||_ G | A, B, C, E
## D _||_ G | F
## D _||_ G | C, F
## D _||_ G | B, C, F
## D _||_ G | A, B, C, F
## D _||_ G | E, F
## D _||_ G | C, E, F
## D _||_ G | A, C, E, F
## D _||_ G | B, C, E, F
## D _||_ G | A, B, C, E, F
## D _||_ G | C, E, H
## D _||_ G | A, C, E, H
## D _||_ G | B, C, E, H
## D _||_ G | A, B, C, E, H
## D _||_ G | C, F, H
## D _||_ G | B, C, F, H
## D _||_ G | A, B, C, F, H
## D _||_ G | C, E, F, H
## D _||_ G | A, C, E, F, H
## D _||_ G | B, C, E, F, H
## D _||_ G | A, B, C, E, F, H
## D _||_ H | C, E
## D _||_ H | A, C, E
## D _||_ H | B, C, E
## D _||_ H | A, B, C, E
## D _||_ H | C, F
## D _||_ H | B, C, F
## D _||_ H | A, B, C, F
## D _||_ H | C, E, F
## D _||_ H | A, C, E, F
## D _||_ H | B, C, E, F

```

```

## D _||_ H | A, B, C, E, F
## D _||_ H | C, E, G
## D _||_ H | A, C, E, G
## D _||_ H | B, C, E, G
## D _||_ H | A, B, C, E, G
## D _||_ H | C, F, G
## D _||_ H | A, C, F, G
## D _||_ H | B, C, F, G
## D _||_ H | A, B, C, F, G
## D _||_ H | C, E, F, G
## D _||_ H | A, C, E, F, G
## D _||_ H | B, C, E, F, G
## D _||_ H | A, B, C, E, F, G
## E _||_ G | F
## E _||_ G | C, F
## E _||_ G | B, C, F
## E _||_ G | A, B, C, F
## E _||_ G | D, F
## E _||_ G | C, D, F
## E _||_ G | B, C, D, F
## E _||_ G | A, B, C, D, F
## E _||_ G | C, F, H
## E _||_ G | B, C, F, H
## E _||_ G | A, B, C, F, H
## E _||_ G | C, D, F, H
## E _||_ G | B, C, D, F, H
## E _||_ G | A, B, C, D, F, H
## E _||_ H | C, F
## E _||_ H | B, C, F
## E _||_ H | A, B, C, F
## E _||_ H | C, D, F
## E _||_ H | B, C, D, F
## E _||_ H | A, B, C, D, F
## E _||_ H | C, F, G
## E _||_ H | A, C, F, G
## E _||_ H | B, C, F, G
## E _||_ H | A, B, C, F, G
## E _||_ H | C, D, F, G
## E _||_ H | A, C, D, F, G
## E _||_ H | B, C, D, F, G
## E _||_ H | A, B, C, D, F, G

```

According to the documentation, the default for this function is type = “missing.edge”. This returns a list of conditional independencies with minimal testable implication per missing edge while type = “all.pairs” returns all implied conditional independencies between two variables. This is why the first one is shorter than the second one.

(d) The summary of the simulated data (N=10000) is shown below.

##	A	B	C	D
##	Min. : -3.689914	Min. : -3.66834	Min. : -3.715549	Min. : -3.397235
##	1st Qu.: -0.684691	1st Qu.: -0.66291	1st Qu.: -0.678795	1st Qu.: -0.682320
##	Median : -0.011717	Median : 0.02416	Median : -0.010833	Median : -0.004992

```
## Mean      :-0.005188   Mean      : 0.01402   Mean      :-0.002295   Mean      :-0.014104
## 3rd Qu.: 0.664292   3rd Qu.: 0.68337   3rd Qu.: 0.674964   3rd Qu.: 0.656542
## Max.      : 3.359318   Max.      : 3.78853   Max.      : 3.388888   Max.      : 3.638291
##           E           F           G
## Min.      :-4.148542   Min.      :-3.733913   Min.      :-3.489782
## 1st Qu.: -0.672298   1st Qu.: -0.672419   1st Qu.: -0.671242
## Median    : 0.023623   Median    : 0.002315   Median    : 0.016195
## Mean      : 0.004979   Mean      : 0.001992   Mean      : 0.005013
## 3rd Qu.: 0.670245   3rd Qu.: 0.681429   3rd Qu.: 0.684476
## Max.      : 4.160425   Max.      : 3.991859   Max.      : 3.415378
##           H
## Min.      :-3.482478
## 1st Qu.: -0.655090
## Median    : 0.005633
## Mean      : 0.006265
## 3rd Qu.: 0.660801
## Max.      : 3.735608
```

Markov blanket for vertex B are:

```
## [1] "C" "G" "A" "E"
```

Let's check the linear regression of $B \sim Mb(B, \mathcal{G}) + \text{remaining covariates}$ using this simulated data.

```
##
## Call:
## lm(formula = B ~ A + C + D + E + F + G + H, data = sim)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.5674 -0.5001  0.0082  0.5023  3.0091
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.0127456  0.0074923   1.701   0.0889 .
## A             0.3360154  0.0079161  42.447 < 2e-16 ***
## C            -0.2107823  0.0081714 -25.795 < 2e-16 ***
## D             0.0008288  0.0078329   0.106   0.9157
## E             0.0404660  0.0083848   4.826 1.41e-06 ***
## F             0.0028081  0.0104751   0.268   0.7887
## G             0.4678821  0.0107319  43.597 < 2e-16 ***
## H            -0.0003218  0.0093214  -0.035   0.9725
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7491 on 9992 degrees of freedom
## Multiple R-squared:  0.4435, Adjusted R-squared:  0.4431
## F-statistic: 1138 on 7 and 9992 DF,  p-value: < 2.2e-16
```

The coefficients for variables outside of the Markov blanket (D, F, H) have p-values greater than 0.05, indicating their independence from B . Meanwhile, the coefficients for variables within the Markov blanket have p-values less than 0.05, confirming that the Markov blanket property holds for vertex B .

Code

```
# problem 5
library(dagitty)

# construct fig 5 DAG
g <- dagitty('dag {
  D [pos="0,0"]
  E [pos="1,0"]
  C [pos="1,-1"]
  A [pos="2,0"]
  B [pos="3,0"]
  F [pos="4,0"]
  G [pos="4,-1"]
  H [pos="5,-1"]

  D -> E -> A <- B <- G -> H
  C -> E -> F -> G
  C -> H
  C -> B
  C -> F -> H
}')

# a: path from C to H
paths(g, "C", "H")

# b: d-separation between E and G given A and B
if(dseparated(g, "E", "G", c("A", "B"))){
  message("E", " and ", "G", " are d-separated given A and B.")
} else {
  message("E", " and ", "G", " are not d-separated given A and B.")
}

# c: list the conditional independencies relationships implied by the model
impliedConditionalIndependencies(g)
impliedConditionalIndependencies(g, type = "all.pairs")

set.seed(2024)
# d: simulate data from this DAG, which associates the DAG with a linear structural equation model
# path coefficient (-0.7, 0.7), sample size = 10000
sim <- simulateSEM(
  g,
  b.default = NULL,
  b.lower = -0.7,
  b.upper = 0.7,
  N = 10000
)

summary(sim)

markovBlanket(g, 'B')

# construct a linear model
```

```
lm_b = lm(B ~ A + C + D + E + F + G + H, sim)
summary(lm_b)
```