Assignment 1

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Problem 1

(a) $X_2 \perp\!\!\!\perp X_3, X_7, X_8, X_{10}$
$X_3 \perp\!\!\!\perp X_2 | X_1$
$X_4 \perp\!\!\!\perp X_1, X_7, X_8, X_{10} | X_2, X_3$
$X_5 \perp\!\!\!\perp X_1, X_2, X_3, X_7, X_8, X_9, X_{10} | X_4$
$X_6 \perp\!\!\!\perp X_1, X_2, X_3, X_4, X_7, X_8, X_9, X_{10} | X_5$
$X_7 \perp\!\!\!\perp X_1, X_2, X_4, X_5, X_6 | X_3$
$X_8 \perp\!\!\!\perp X_1, X_2, X_4, X_5, X_6 | X_3, X_7$
$X_9 \perp\!\!\!\perp X_1, X_2, X_3, X_5, X_6, X_7 | X_4, X_8, X_{10}$
$X_{10} \perp\!\!\!\perp X_1, X_2, X_3, X_4, X_5, X_6, X_7 | X_8$

(b) $X_2 \perp_d X_9 | X_4$: False
$X_7 \perp_d X_5 | \{X_3, X_8\}$: True
$\{X_2, X_4\} \perp_d X_7 | \{X_6, X_9, X_{10}\}$: False

Problem 2

Problem 3

Agreements: $D \perp\!\!\!\perp \{A, C\} | B$
Disagreements: $A \perp\!\!\!\perp C | \phi$ (a), $C \perp\!\!\!\perp \{A, D\} | B$ (b)

Problem 4

(a) b, e

(b) An unshielded collider in a DAG will violate the Markov equivalence with respect to the chain DAG. Taken this into account, we can consider two cases that will be Markov equivalent to the chain DAG:
1.Flip all the arrows to left
2.For $X_i (i = 2, 3, ..., p-1)$, flip all the arrows to left before X_i
There are p-2 possible X_is that can be the pivot of the arrows. Therefore, $p-2+1 = p-1$ DAGs are Markov equivalent to the chain DAG.

Problem 5

(a)

```
## $paths
## [1] "C -> B -> A <- E -> F -> G -> H" "C -> B <- G -> H"
## [3] "C -> E -> A <- B <- G -> H"      "C -> E -> F -> G -> H"
## [5] "C -> F -> G -> H"                "C -> F <- E -> A <- B <- G -> H"
## [7] "C -> H"
##
## $open
## [1] FALSE FALSE FALSE  TRUE  TRUE FALSE  TRUE
```

(b)

```
## E and G are not d-separated given A and B.
```

(c)

```
## A _||_ C | B, E
## A _||_ D | C, E
## A _||_ D | B, E
## A _||_ F | C, E, G
## A _||_ F | B, E
## A _||_ G | B, C, F
## A _||_ G | B, E
## A _||_ H | C, G
## A _||_ H | B, C, F
## A _||_ H | B, E
## B _||_ D | C, E
## B _||_ D | C, F
## B _||_ D | C, G
## B _||_ E | C, F
## B _||_ E | C, G
## B _||_ F | C, G
## B _||_ H | C, G
## C _||_ D
## C _||_ G | F
## D _||_ F | C, E
## D _||_ G | F
## D _||_ G | C, E
## D _||_ H | C, G
## D _||_ H | C, F
## D _||_ H | C, E
## E _||_ G | F
## E _||_ H | C, G
## E _||_ H | C, F
## F _||_ H | C, G
```

With the option "type = all.pairs":

```
## A _||_ C | B, E
## A _||_ C | B, D, E
## A _||_ C | B, E, F
## A _||_ C | B, D, E, F
## A _||_ C | B, E, G
```

```
## A _||_ C | B, D, E, G
## A _||_ C | B, E, F, G
## A _||_ C | B, D, E, F, G
## A _||_ C | B, E, H
## A _||_ C | B, D, E, H
## A _||_ C | B, E, F, H
## A _||_ C | B, D, E, F, H
## A _||_ C | B, E, G, H
## A _||_ C | B, D, E, G, H
## A _||_ C | B, E, F, G, H
## A _||_ C | B, D, E, F, G, H
## A _||_ D | B, E
## A _||_ D | C, E
## A _||_ D | B, C, E
## A _||_ D | B, E, F
## A _||_ D | C, E, F
## A _||_ D | B, C, E, F
## A _||_ D | B, E, G
## A _||_ D | C, E, G
## A _||_ D | B, C, E, G
## A _||_ D | B, E, F, G
## A _||_ D | C, E, F, G
## A _||_ D | B, C, E, F, G
## A _||_ D | B, E, H
## A _||_ D | C, E, H
## A _||_ D | B, C, E, H
## A _||_ D | B, E, F, H
## A _||_ D | C, E, F, H
## A _||_ D | B, C, E, F, H
## A _||_ D | B, E, G, H
## A _||_ D | C, E, G, H
## A _||_ D | B, C, E, G, H
## A _||_ D | B, E, F, G, H
## A _||_ D | C, E, F, G, H
## A _||_ D | B, C, E, F, G, H
## A _||_ F | B, E
## A _||_ F | B, C, E
## A _||_ F | B, D, E
## A _||_ F | B, C, D, E
## A _||_ F | B, E, G
## A _||_ F | C, E, G
## A _||_ F | B, C, E, G
## A _||_ F | B, D, E, G
## A _||_ F | C, D, E, G
## A _||_ F | B, C, D, E, G
## A _||_ F | B, E, H
## A _||_ F | B, C, E, H
## A _||_ F | B, D, E, H
## A _||_ F | B, C, D, E, H
## A _||_ F | B, E, G, H
## A _||_ F | C, E, G, H
## A _||_ F | B, C, E, G, H
## A _||_ F | B, D, E, G, H
## A _||_ F | C, D, E, G, H
```

```
## A _||_ F | B, C, D, E, G, H
## A _||_ G | B, E
## A _||_ G | B, C, E
## A _||_ G | B, D, E
## A _||_ G | B, C, D, E
## A _||_ G | B, C, F
## A _||_ G | B, C, D, F
## A _||_ G | B, E, F
## A _||_ G | B, C, E, F
## A _||_ G | B, D, E, F
## A _||_ G | B, C, D, E, F
## A _||_ G | B, E, H
## A _||_ G | B, C, E, H
## A _||_ G | B, D, E, H
## A _||_ G | B, C, D, E, H
## A _||_ G | B, C, F, H
## A _||_ G | B, C, D, F, H
## A _||_ G | B, E, F, H
## A _||_ G | B, C, E, F, H
## A _||_ G | B, D, E, F, H
## A _||_ G | B, C, D, E, F, H
## A _||_ H | B, E
## A _||_ H | B, C, E
## A _||_ H | B, D, E
## A _||_ H | B, C, D, E
## A _||_ H | B, C, F
## A _||_ H | B, C, D, F
## A _||_ H | B, E, F
## A _||_ H | B, C, E, F
## A _||_ H | B, D, E, F
## A _||_ H | B, C, D, E, F
## A _||_ H | C, G
## A _||_ H | B, C, G
## A _||_ H | C, D, G
## A _||_ H | B, C, D, G
## A _||_ H | B, E, G
## A _||_ H | C, E, G
## A _||_ H | B, C, E, G
## A _||_ H | B, D, E, G
## A _||_ H | C, D, E, G
## A _||_ H | B, C, D, E, G
## A _||_ H | C, F, G
## A _||_ H | B, C, F, G
## A _||_ H | C, D, F, G
## A _||_ H | B, C, D, F, G
## A _||_ H | B, E, F, G
## A _||_ H | C, E, F, G
## A _||_ H | B, C, E, F, G
## A _||_ H | B, D, E, F, G
## A _||_ H | C, D, E, F, G
## A _||_ H | B, C, D, E, F, G
## B _||_ D | C, E
## B _||_ D | A, C, E
## B _||_ D | C, F
```

```
## B _||_ D | C, E, F
## B _||_ D | A, C, E, F
## B _||_ D | C, G
## B _||_ D | C, E, G
## B _||_ D | A, C, E, G
## B _||_ D | C, F, G
## B _||_ D | C, E, F, G
## B _||_ D | A, C, E, F, G
## B _||_ D | C, E, H
## B _||_ D | A, C, E, H
## B _||_ D | C, F, H
## B _||_ D | C, E, F, H
## B _||_ D | A, C, E, F, H
## B _||_ D | C, G, H
## B _||_ D | C, E, G, H
## B _||_ D | A, C, E, G, H
## B _||_ D | C, F, G, H
## B _||_ D | C, E, F, G, H
## B _||_ D | A, C, E, F, G, H
## B _||_ E | C, F
## B _||_ E | C, D, F
## B _||_ E | C, G
## B _||_ E | C, D, G
## B _||_ E | C, F, G
## B _||_ E | C, D, F, G
## B _||_ E | C, F, H
## B _||_ E | C, D, F, H
## B _||_ E | C, G, H
## B _||_ E | C, D, G, H
## B _||_ E | C, F, G, H
## B _||_ E | C, D, F, G, H
## B _||_ F | C, G
## B _||_ F | C, D, G
## B _||_ F | C, E, G
## B _||_ F | A, C, E, G
## B _||_ F | C, D, E, G
## B _||_ F | A, C, D, E, G
## B _||_ F | C, G, H
## B _||_ F | C, D, G, H
## B _||_ F | C, E, G, H
## B _||_ F | A, C, E, G, H
## B _||_ F | C, D, E, G, H
## B _||_ F | A, C, D, E, G, H
## B _||_ H | C, G
## B _||_ H | A, C, G
## B _||_ H | C, D, G
## B _||_ H | A, C, D, G
## B _||_ H | C, E, G
## B _||_ H | A, C, E, G
## B _||_ H | C, D, E, G
## B _||_ H | A, C, D, E, G
## B _||_ H | C, F, G
## B _||_ H | A, C, F, G
## B _||_ H | C, D, F, G
```

```
## B _||_ H | A, C, D, F, G
## B _||_ H | C, E, F, G
## B _||_ H | A, C, E, F, G
## B _||_ H | C, D, E, F, G
## B _||_ H | A, C, D, E, F, G
## C _||_ D
## C _||_ G | F
## C _||_ G | D, F
## C _||_ G | E, F
## C _||_ G | D, E, F
## D _||_ F | C, E
## D _||_ F | A, C, E
## D _||_ F | B, C, E
## D _||_ F | A, B, C, E
## D _||_ F | C, E, G
## D _||_ F | A, C, E, G
## D _||_ F | B, C, E, G
## D _||_ F | A, B, C, E, G
## D _||_ F | C, E, H
## D _||_ F | A, C, E, H
## D _||_ F | B, C, E, H
## D _||_ F | A, B, C, E, H
## D _||_ F | C, E, G, H
## D _||_ F | A, C, E, G, H
## D _||_ F | B, C, E, G, H
## D _||_ F | A, B, C, E, G, H
## D _||_ G | C, E
## D _||_ G | A, C, E
## D _||_ G | B, C, E
## D _||_ G | A, B, C, E
## D _||_ G | F
## D _||_ G | C, F
## D _||_ G | B, C, F
## D _||_ G | A, B, C, F
## D _||_ G | E, F
## D _||_ G | C, E, F
## D _||_ G | A, C, E, F
## D _||_ G | B, C, E, F
## D _||_ G | A, B, C, E, F
## D _||_ G | C, E, H
## D _||_ G | A, C, E, H
## D _||_ G | B, C, E, H
## D _||_ G | A, B, C, E, H
## D _||_ G | C, F, H
## D _||_ G | B, C, F, H
## D _||_ G | A, B, C, F, H
## D _||_ G | C, E, F, H
## D _||_ G | A, C, E, F, H
## D _||_ G | B, C, E, F, H
## D _||_ G | A, B, C, E, F, H
## D _||_ H | C, E
## D _||_ H | A, C, E
## D _||_ H | B, C, E
## D _||_ H | A, B, C, E
```

```
## D _||_ H | C, F
## D _||_ H | B, C, F
## D _||_ H | A, B, C, F
## D _||_ H | C, E, F
## D _||_ H | A, C, E, F
## D _||_ H | B, C, E, F
## D _||_ H | A, B, C, E, F
## D _||_ H | C, G
## D _||_ H | A, C, G
## D _||_ H | B, C, G
## D _||_ H | A, B, C, G
## D _||_ H | C, E, G
## D _||_ H | A, C, E, G
## D _||_ H | B, C, E, G
## D _||_ H | A, B, C, E, G
## D _||_ H | C, F, G
## D _||_ H | A, C, F, G
## D _||_ H | B, C, F, G
## D _||_ H | A, B, C, F, G
## D _||_ H | C, E, F, G
## D _||_ H | A, C, E, F, G
## D _||_ H | B, C, E, F, G
## D _||_ H | A, B, C, E, F, G
## E _||_ G | F
## E _||_ G | C, F
## E _||_ G | B, C, F
## E _||_ G | A, B, C, F
## E _||_ G | D, F
## E _||_ G | C, D, F
## E _||_ G | B, C, D, F
## E _||_ G | A, B, C, D, F
## E _||_ G | C, F, H
## E _||_ G | B, C, F, H
## E _||_ G | A, B, C, F, H
## E _||_ G | C, D, F, H
## E _||_ G | B, C, D, F, H
## E _||_ G | A, B, C, D, F, H
## E _||_ H | C, F
## E _||_ H | B, C, F
## E _||_ H | A, B, C, F
## E _||_ H | C, D, F
## E _||_ H | B, C, D, F
## E _||_ H | A, B, C, D, F
## E _||_ H | C, G
## E _||_ H | A, C, G
## E _||_ H | B, C, G
## E _||_ H | A, B, C, G
## E _||_ H | C, D, G
## E _||_ H | A, C, D, G
## E _||_ H | B, C, D, G
## E _||_ H | A, B, C, D, G
## E _||_ H | C, F, G
## E _||_ H | A, C, F, G
## E _||_ H | B, C, F, G
```

```
## E _||_ H | A, B, C, F, G
## E _||_ H | C, D, F, G
## E _||_ H | A, C, D, F, G
## E _||_ H | B, C, D, F, G
## E _||_ H | A, B, C, D, F, G
## F _||_ H | C, G
## F _||_ H | A, C, G
## F _||_ H | B, C, G
## F _||_ H | A, B, C, G
## F _||_ H | C, D, G
## F _||_ H | A, C, D, G
## F _||_ H | B, C, D, G
## F _||_ H | A, B, C, D, G
## F _||_ H | C, E, G
## F _||_ H | A, C, E, G
## F _||_ H | B, C, E, G
## F _||_ H | A, B, C, E, G
## F _||_ H | C, D, E, G
## F _||_ H | A, C, D, E, G
## F _||_ H | B, C, D, E, G
## F _||_ H | A, B, C, D, E, G
```

According to the documentation, the default for this function is type = "missing.edge". This returns a list of conditional independencies with minimal testable implication per missing edge while type = "all.pairs" returns all implied conditional independencies between two variables. This is why the first one is shorter than the second one.

(d)

Code

```
# problem 5
library(dagitty)

# construct fig 5 DAG
g <- dagitty('dag {
    D [pos="0,0"]
    E [pos="1,0"]
    C [pos="1,-1"]
    A [pos="2,0"]
    B [pos="3,0"]
    F [pos="4,0"]
    G [pos="4,-1"]
    H [pos="5,-1"]

    D -> E -> A <- B <- G -> H
    C -> E
    E -> F -> G
    C -> H
    C -> B
    C -> F
```

```
}')

# a: path from C to H
paths(g, "C", "H")

# b: d-separation between E and G given A and B
if(dseparated(g, "E", "G", c("A", "B"))){
  message("E"," and ", "G"," are d-separated given A and B.")
} else {
  message("E"," and ", "G", " are not d-separated given A and B.")
}

# c: list the conditional independencies relationships implied by the model
impliedConditionalIndependencies(g)
impliedConditionalIndependencies(g, type = "all.pairs")

# d: simulate data from this DAG, which associates the DAG with a linear structural equation model
# path coefficient (-0.7, 0.7), sample size = 10000
sim <- simulateSEM(
  g,
  b.default = NULL,
  b.lower = -0.7,
  b.upper = 0.7,
  N = 10000
)
```