Homework 2

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Problem 1

p, the probability of having at least one dental checkup during a two-year period, is 0.73.

(a) Let X be the probability of x out of 56 random individuals to have at least one dental checkup. X~Bin(56, 0.73)

$P(X = 40) = f(x) = \binom{n}{x}p^x(1-p)^{n-x} = \binom{56}{40}(0.73)^{40}(1-0.73)^{56-40} = 0.11$

Therefore, the probability that exactly 40 of these individuals will have at least one dental check up is **11.33%**

(b) The probability that at least 40 of these individuals will have at least one dental checkup can be denoted as: $P(X >= 40) = 1 - P(X < 40) = 1 - P(X <= 39)$

$P(X <= 39) = P(X = 0) + P(X = 1) + ... + P(X = 39) = 0.33$
Therefore, $P(X >= 40) = 1 - P(X <= 39) = 0.67$
$\rightarrow$ **66.79%**

(c)

$np = 56 \times 0.73 = 40.88 > 10$ and $nq = 56 \times 0.73 = 15.12 > 10$
Thus we can approximate X to the normal distribution X~N(40.88, 3.32) where P(X = 40) becomes $P(X = 40 - \frac{1}{2})$ and P(X >= 40) becomes $P(X >= 40 - \frac{1}{2})$

When X~N(40.88, 3.32),
$P(X = 39.5) =$ **0.11** $P(X >= 39.5) =$ **0.66**

These are similar to the results of (a) and (b).

(d) The expected value of X~Bin(56, 0.73) is:
$\mu = E(X) = np =$ **40.88**

(e) The variance of X~Bin(56, 0.73) is:
$\sigma^2 = var(X) = np(1-p) = 11.04$
Therefore, the standard deviation is **3.32**

Problem 2

Suppose the number of tornadoes in the U.S. follows a Poisson distribution with parameter $\lambda = 6$ tornadoes per year.
Let X denote the number of tornadoes in the U.S. per year. X~Poi(6)

(a)

The probability of having fewer than 3 tornadoes in the U.S. next year is
$P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$
$P(X = x) = f(x) = \frac{\lambda^x e^{-\lambda}}{x!}$ so,

$P(X = 0) = \frac{6^0 e^{-6}}{0!} = 0$
$P(X = 1) = \frac{6^1 e^{-6}}{1!} = 0.01$
$P(X = 2) = \frac{6^2 e^{-6}}{2!} = 0.04$

Therefore, $P(X < 3) = \mathbf{0.06}$

(b)

The probability of having exactly 3 tornadoes in the U.S. next year is
$P(X = 3) = \frac{6^3 e^{-6}}{3!} = \mathbf{0.09}$

(c)

The probability of having more than 3 tornadoes in the U.S. next year is
$P(X > 3) = 1 - P(X <= 3) = 1 - (P(X < 3) + P(X = 3)) = \mathbf{0.85}$

Problem 3