Homework 3

Yuki Joyama

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Problem 1

Let n = 189, $\bar{X}$ (sample mean) = 129.81, s (sample standard deviation) = 30.58

a) Because we do not know the true population variance σ^2, 95% confidence interval of true mean weight can be obtained by the following:

$$\bar{X} - t_{n-1,1-\alpha/2}\frac{s}{\sqrt{n}}, \bar{X} + t_{n-1,1-\alpha/2}\frac{s}{\sqrt{n}}$$

Thus, 95% confidence interval is (125.43, 134.2)

b) We are 95% confident that the true mean weight of American women will be between (125.43, 134.2).

c) The statement is not valid because 171 pounds is not included in the 95% confidence interval.

Problem 2

a) Let s_1 and s_2 be the sample standard deviation from non-smoking group and smoking group accordingly. And let n_1 and n_2 be the number of observations in each group.
$\alpha = 0.05$
H_0: the variances in two groups are equal
H_1: the variances in two groups are not equal

The test statistic is
$F = \frac{s_1^2}{s_2^2} = 0.71 \sim F_{n_1-1,n_2-1}$

Now, $F_{n_1-1,n_2-1,1-\alpha/2} = 1.53 >$ F
Thus, we fail to reject the null hypothesis and accept H_0.

b) The answer from part a indicates that the variances between two groups are equal at 5% significance level. Therefore, I will perform two-sample independent t-test for equal variances.

c) $\alpha = 0.10$ Let μ_1 and μ_2 be the population mean of the two groups. ($\bar{X_1}, \bar{X_2}$: sample mean)
H_0: $\mu_1 = \mu_2$
H_1: $\mu_1 \neq \mu_2$

The pooled estimate of the standard deviation $s = \sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}} = 30.63$

The test statistic is
$t = \frac{\bar{X_1}-\bar{X_2}}{s\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = 0.6 \sim t_{n_1+n_2-2}$

$t_{n_1+n_2-2,1-\alpha/2} = 1.65 >$ t so we fail to reject the null hypothesis and accept H_0.
At 10% significance level, we can say that there is no statistically significant difference in the mother's weight at last menstrual period between smoking and non-smoking groups.

Problem 3

a) $\alpha = 0.01$
Let p be the proportion of pregnant American women suffer from hypertension. ($p_0 = 0.20$)
I will perform one-sample test for binomial proportion.

99% confidence interval for one population proportion can be calculated by:

$$\hat{p} - z_{1-\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z_{1-\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Thus, 99% CI is (0.02, 0.11)

Because 0.20 is not included in the 99% CI, we can conclude that we are 99% confident that our data does not support this claim.

b) $\alpha = 0.01$
H_0: $p = p_0$
H_1: $p < p_0$

Test statistic is $z = \frac{\hat{p}-p_0}{\sqrt{p_0(1-p_0)/n}}$ = -4.69 ~ N(0, 1)

z_α = -1.28 > z
Therefore, we reject the null hypothesis and accept H_1. We can conclude that at 10% significant level, the true proportion is indeed less than the claimed 20%. # Problem 4