# Homework5

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## Problem 1

a) The following table shows the descriptive statistics for all variables of interest in 50 States.

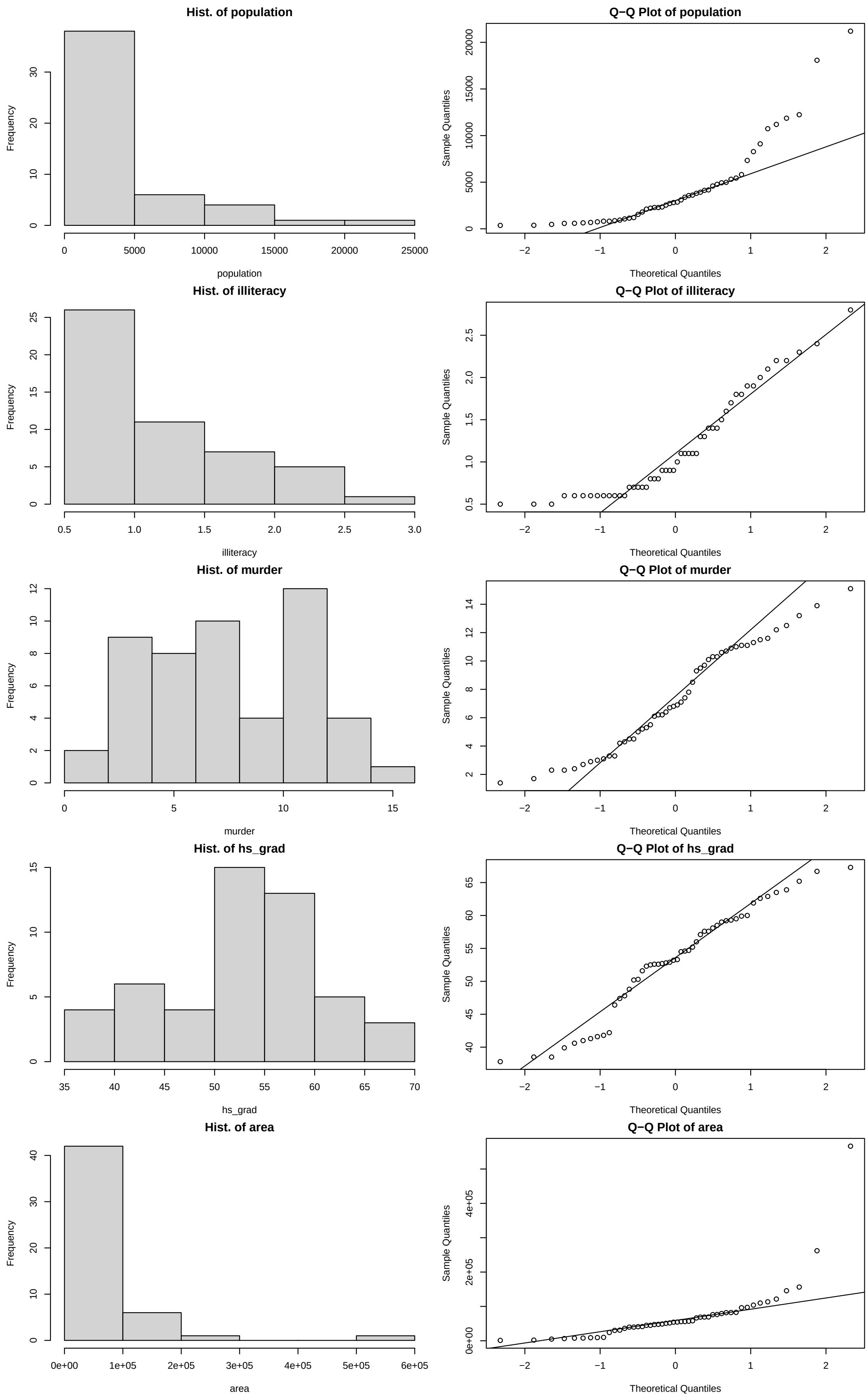
| **Characteristic** | **N = 50**[1] |
|---|---|
| Population | 4,246.4 / 2,838.5 (4,464.5) |
| Income per capita | 4,435.8 / 4,519.0 (614.5) |
| Illiteracy (%) | 1.2 / 1.0 (0.6) |
| Life Expectancy (years) | 70.9 / 70.7 (1.3) |
| Murder rate (per 100,000) | 7.4 / 6.9 (3.7) |
| High graduates (%) | 53.1 / 53.3 (8.1) |
| Number of days below freezing | 104.5 / 114.5 (52.0) |
| Land area (mile ^2) | 70,735.9 / 54,277.0 (85,327.3) |

[1]Mean / Median (SD)

b)

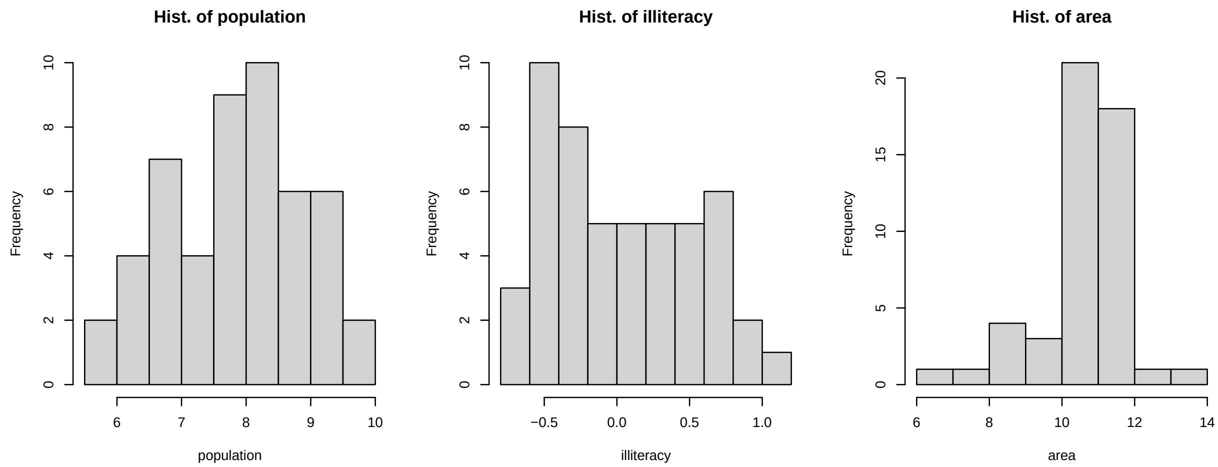
| var | statistic | p.value |
|---|---|---|
| population | 0.770 | <0.001 |
| income | 0.977 | 0.43 |
| illiteracy | 0.883 | <0.001 |
| life_exp | 0.977 | 0.442 |
| murder | 0.953 | 0.047 |
| hs_grad | 0.953 | 0.046 |
| frost | 0.955 | 0.053 |
| area | 0.572 | <0.001 |

The results of Shapiro-Wilk test indicates that variable `population`, `illiteracy`, `murder`, `hs_grad`, and `area` is not normally distributed.
The histogram and Q-Q plots for these variables are as follows:

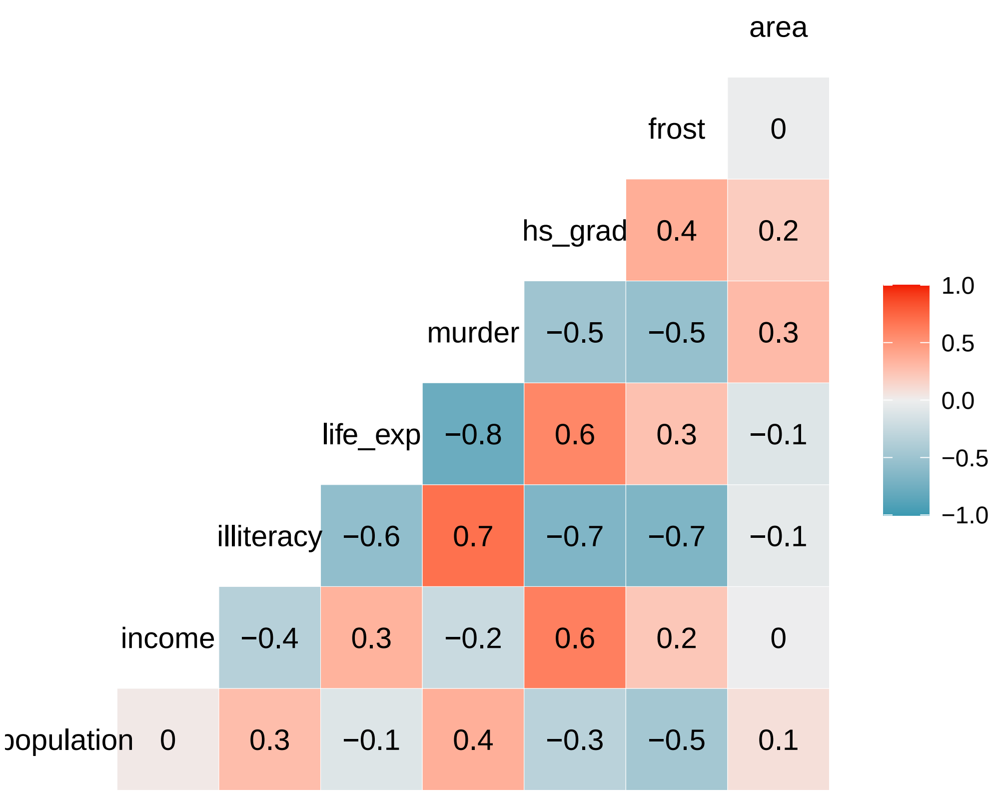

Hist. of population
Q−Q Plot of population
Hist. of illiteracy
Q−Q Plot of illiteracy
Hist. of murder
Q−Q Plot of murder
Hist. of hs_grad
Q−Q Plot of hs_grad
Hist. of area
Q−Q Plot of area
Frequency
Sample Quantiles
Theoretical Quantiles
population
illiteracy
murder
hs_grad
area

Given the shape of the histograms, I will log-transform `population`, `illiteracy`, and `area`.

Now, let's check these histograms.




I will use the data set including the log-transformed variables for the later analysis. Let's check the correlation between each variable and linear regression model including all variables.




```
##
## Call:
## lm(formula = life_exp ~ ., data = df_val)
##
```

```
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.44702 -0.42901  0.04546  0.50742  1.68911
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  6.799e+01  1.798e+00  37.809  < 2e-16 ***
## population   2.537e-01  1.311e-01   1.936   0.0597 .
## income      -4.417e-06  2.475e-04  -0.018   0.9858
## illiteracy   1.883e-01  4.204e-01   0.448   0.6565
## murder      -3.114e-01  4.659e-02  -6.684 4.12e-08 ***
## hs_grad      5.482e-02  2.552e-02   2.148   0.0375 *
## frost       -4.669e-03  3.173e-03  -1.471   0.1487
## area         7.314e-02  1.102e-01   0.663   0.5107
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7335 on 42 degrees of freedom
## Multiple R-squared:  0.7441, Adjusted R-squared:  0.7014
## F-statistic: 17.45 on 7 and 42 DF,  p-value: 1.368e-10
```

c) Automatic procedures In this section, I will use backward and forward procedures.

```
## Start:  AIC=-23.71
## life_exp ~ population + income + illiteracy + murder + hs_grad +
##     frost + area
##
##              Df Sum of Sq    RSS     AIC
## - income      1    0.0002 22.596 -25.712
## - illiteracy  1    0.1079 22.704 -25.475
## - area        1    0.2368 22.833 -25.192
## <none>                    22.596 -23.713
## - frost       1    1.1645 23.760 -23.200
## - population  1    2.0155 24.611 -21.441
## - hs_grad     1    2.4822 25.078 -20.502
## - murder      1   24.0347 46.631  10.512
##
## Step:  AIC=-25.71
## life_exp ~ population + illiteracy + murder + hs_grad + frost +
##     area
##
##              Df Sum of Sq    RSS      AIC
## - illiteracy  1    0.1095 22.705 -27.4708
## - area        1    0.2616 22.858 -27.1370
## <none>                    22.596 -25.7125
## - frost       1    1.2628 23.859 -24.9936
## - population  1    2.3859 24.982 -22.6937
## - hs_grad     1    4.4112 27.007 -18.7959
## - murder      1   24.4834 47.079   8.9907
##
## Step:  AIC=-27.47
## life_exp ~ population + murder + hs_grad + frost + area
##
##              Df Sum of Sq    RSS     AIC
```

```
## - area        1    0.2157 22.921 -28.998
## <none>                    22.705 -27.471
## - population  1    2.2792 24.985 -24.688
## - frost       1    2.3760 25.082 -24.495
## - hs_grad     1    4.9491 27.655 -19.612
## - murder      1   29.2296 51.935  11.899
##
## Step:  AIC=-29
## life_exp ~ population + murder + hs_grad + frost
##
##              Df Sum of Sq    RSS     AIC
## <none>                    22.921 -28.998
## - frost       1     2.214 25.135 -26.387
## - population  1     2.450 25.372 -25.920
## - hs_grad     1     6.959 29.881 -17.741
## - murder      1    34.109 57.031  14.578
```

```
##
## Call:
## lm(formula = life_exp ~ population + murder + hs_grad + frost,
##     data = df_val)
##
## Coefficients:
## (Intercept)   population       murder      hs_grad        frost
##   68.720810     0.246836    -0.290016     0.054550    -0.005174
```

```
## Start:  AIC=30.44
## life_exp ~ 1
##
##              Df Sum of Sq    RSS     AIC
## + murder      1    53.838 34.461 -14.609
## + hs_grad     1    29.931 58.368  11.737
## + illiteracy  1    28.688 59.611  12.791
## + income      1    10.223 78.076  26.283
## + frost       1     6.064 82.235  28.878
## <none>                    88.299  30.435
## + population  1     1.054 87.245  31.835
## + area        1     1.042 87.257  31.842
##
## Step:  AIC=-14.61
## life_exp ~ murder
##
##              Df Sum of Sq    RSS     AIC
## + hs_grad     1    4.6910 29.770 -19.925
## + frost       1    3.1346 31.327 -17.378
## + population  1    2.9854 31.476 -17.140
## + income      1    2.4047 32.057 -16.226
## + area        1    1.4583 33.003 -14.771
## <none>                    34.461 -14.609
## + illiteracy  1    0.1292 34.332 -12.797
##
## Step:  AIC=-19.93
## life_exp ~ murder + hs_grad
##
```

```
##              Df Sum of Sq    RSS     AIC
## + population  1    4.6350 25.135 -26.387
## + frost       1    4.3987 25.372 -25.920
## <none>                    29.770 -19.925
## + illiteracy  1    0.8366 28.934 -19.351
## + area        1    0.1236 29.647 -18.134
## + income      1    0.1022 29.668 -18.097
##
## Step:  AIC=-26.39
## life_exp ~ murder + hs_grad + population
##
##              Df Sum of Sq    RSS     AIC
## + frost       1   2.21416 22.921 -28.998
## + illiteracy  1   1.05998 24.075 -26.542
## <none>                    25.135 -26.387
## + income      1   0.11819 25.017 -24.623
## + area        1   0.05387 25.081 -24.495
##
## Step:  AIC=-29
## life_exp ~ murder + hs_grad + population + frost
##
##              Df Sum of Sq    RSS     AIC
## <none>                    22.921 -28.998
## + area        1  0.215741 22.706 -27.471
## + illiteracy  1  0.063655 22.858 -27.137
## + income      1  0.010673 22.911 -27.021
```

```
##
## Call:
## lm(formula = life_exp ~ murder + hs_grad + population + frost,
##     data = df_val)
##
## Coefficients:
## (Intercept)       murder      hs_grad   population        frost
##   68.720810    -0.290016     0.054550     0.246836    -0.005174
```

The both procedures generated the same model (variables included in the final model: `murder`, `hs_grad`, `population`, `frost`). There does not appear to be a close call, as the elimination/addition of each variable consistently decreases the AIC value and indicates a better model fit. Therefore, I would keep all the variables suggested by the procedure.
Intuitively, we could assume that there is an association between `illiteracy` and `HS graduation rate`. My subset does not include both, so instead of checking for multicollinearity, let's examine correlation.

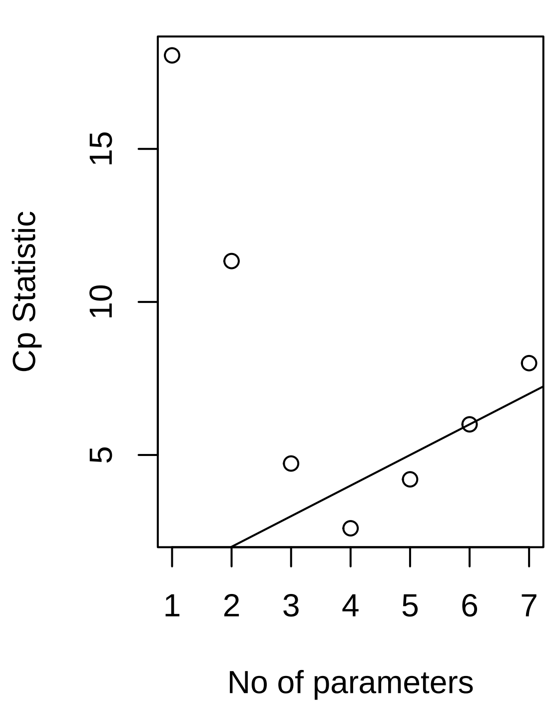
```
## [1] -0.6688091
```

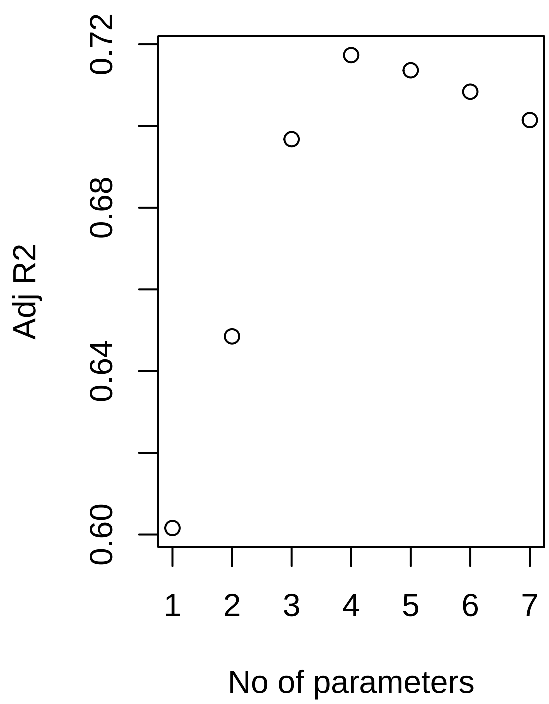
There seems to be a moderate negative association between the two variables.

d) Criterion-based procedures

```
## Subset selection object
## Call: regsubsets.formula(life_exp ~ ., data = df_val)
## 7 Variables  (and intercept)
```

```
##            Forced in Forced out
## population     FALSE      FALSE
## income         FALSE      FALSE
## illiteracy     FALSE      FALSE
## murder         FALSE      FALSE
## hs_grad        FALSE      FALSE
## frost          FALSE      FALSE
## area           FALSE      FALSE
## 1 subsets of each size up to 7
## Selection Algorithm: exhaustive
##          population income illiteracy murder hs_grad frost area
## 1  ( 1 ) " "        " "    " "        "*"    " "     " "   " "
## 2  ( 1 ) " "        " "    " "        "*"    "*"     " "   " "
## 3  ( 1 ) "*"        " "    " "        "*"    "*"     " "   " "
## 4  ( 1 ) "*"        " "    " "        "*"    "*"     "*"   " "
## 5  ( 1 ) "*"        " "    " "        "*"    "*"     "*"   "*"
## 6  ( 1 ) "*"        " "    "*"        "*"    "*"     "*"   "*"
## 7  ( 1 ) "*"        "*"    "*"        "*"    "*"     "*"   "*"
```




The Mallow's Cp criterion and Adjusted $R^2$ suggests that the model with four parameters (`population`, `murder`, `hs_grad`, `frost`) is the best fit.

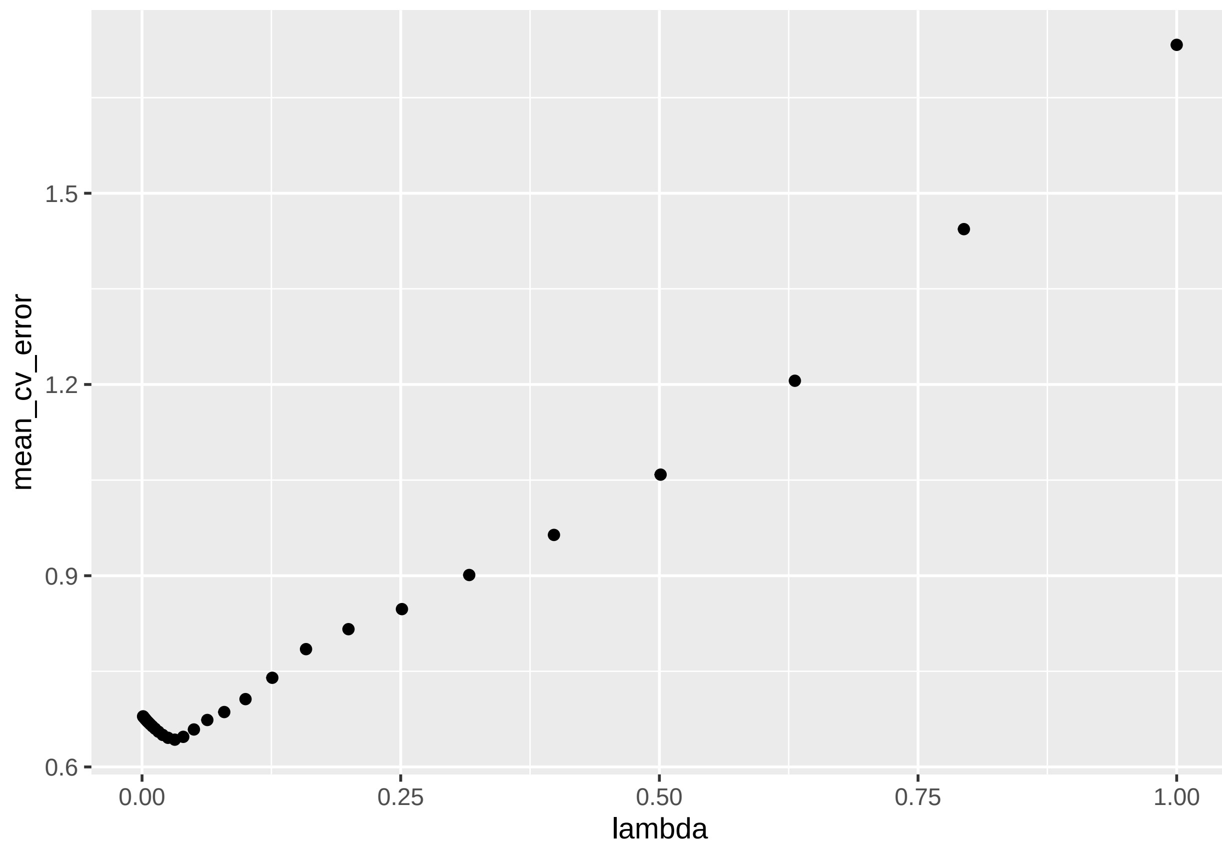
e) The LASSO method

```
## 8 x 1 sparse Matrix of class "dgCMatrix"
##                  s0
```

```
## (Intercept) 70.8786
## population   0.0000
## income       .
## illiteracy   .
## murder       .
## hs_grad      .
## frost        .
## area         .
```

```
## 8 x 1 sparse Matrix of class "dgCMatrix"
##                       s0
## (Intercept) 69.624719212
## population   0.132311050
## income       .
## illiteracy   .
## murder      -0.238481455
## hs_grad      0.040070763
## frost       -0.001484793
## area         .
```

```
##
## Call:  cv.glmnet(x = as.matrix(df_val[2:8]), y = df_val$life_exp, lambda = lambda_seq,      nfolds =
##
## Measure: Mean-Squared Error
##
##      Lambda Index Measure     SE Nonzero
## min 0.03162    16  0.6428 0.1960       5
## 1se 0.19953     8  0.8161 0.3026       2
```




```
## [1] 0.03162278
```

```
## 8 x 1 sparse Matrix of class "dgCMatrix"
##                       s0
## (Intercept) 68.904248047
## population   0.208096331
## income       .
## illiteracy   .
## murder      -0.277654060
## hs_grad      0.048555587
## frost       -0.004090184
## area         0.022049544
```

The lambda value that minimizes the test MSE turns out to be 0.0316228. The final model produced by the optimal lambda value does not include `income` and `illiteracy` because they were not influential enough.

f) Compare the subsets from c, d, and e

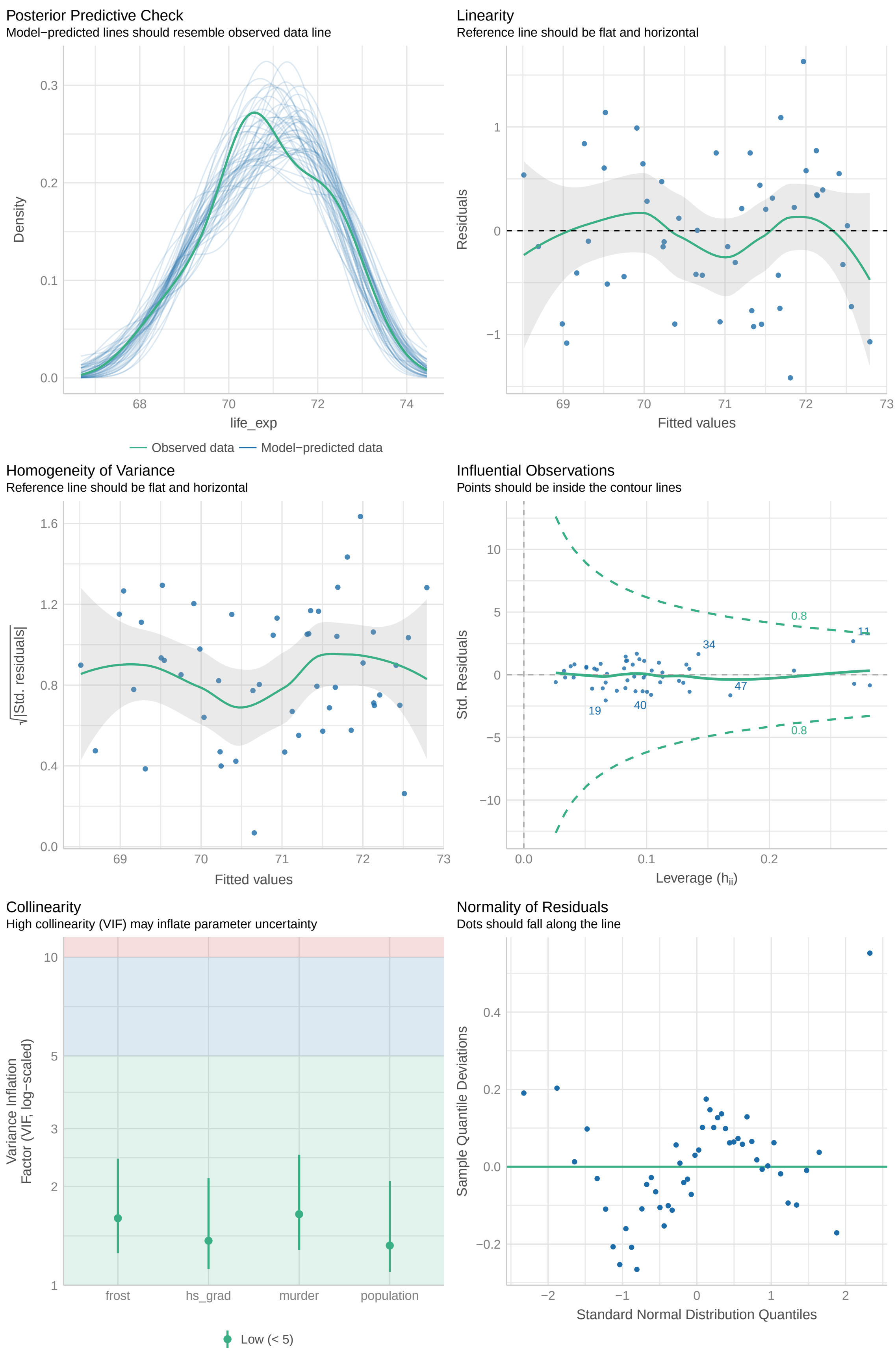
```
## Linear Regression
##
## 50 samples
##  4 predictor
##
## No pre-processing
## Resampling: Cross-Validated (10 fold)
```

```
## Summary of sample sizes: 46, 45, 46, 43, 46, 45, ...
## Resampling results:
##
##   RMSE      Rsquared   MAE
##   0.743866  0.7127871  0.6417565
##
## Tuning parameter 'intercept' was held constant at a value of TRUE
```

```
## glmnet
##
## 50 samples
##  5 predictor
##
## No pre-processing
## Resampling: Cross-Validated (10 fold)
## Summary of sample sizes: 46, 46, 46, 45, 44, 43, ...
## Resampling results:
##
##   RMSE       Rsquared   MAE
##   0.7471661  0.7437034  0.6434308
##
## Tuning parameter 'alpha' was held constant at a value of 1
## Tuning
##  parameter 'lambda' was held constant at a value of 0.03162278
```

The Root Mean Square Errors (RMSEs) from linear regression model (selected by c and d) and LASSO regression model (by e) indicate that the linear model has slightly better predictive ability in the testing data set. Thus, I will employ the linear regression model as my final model.

Let's check the model assumptions.


Posterior Predictive Check
Model−predicted lines should resemble observed data line
Density
life_exp
Observed data
Model−predicted data
Linearity
Reference line should be flat and horizontal
Residuals
Fitted values
Homogeneity of Variance
Reference line should be flat and horizontal
√|Std. residuals|
Fitted values
Influential Observations
Points should be inside the contour lines
Std. Residuals
Leverage (hii)
0.8
11
34
47
19
40
Collinearity
High collinearity (VIF) may inflate parameter uncertainty
Variance Inflation
Factor (VIF, log−scaled)
frost
hs_grad
murder
population
Low (< 5)
Normality of Residuals
Dots should fall along the line
Sample Quantile Deviations
Standard Normal Distribution Quantiles

g) Findings