Homework5

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Problem 1

a) The following table shows the descriptive statistics for all variables of interest in 50 States.

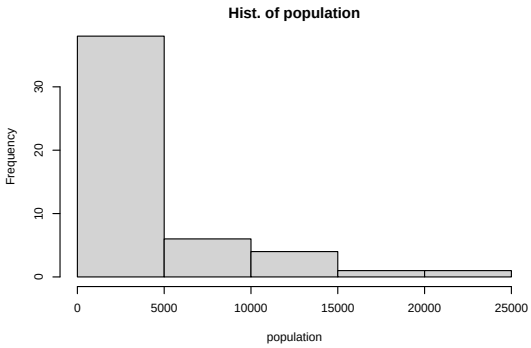
Characteristic	**N = 50**[1]
Population	4,246.4 / 2,838.5 (4,464.5)
Income per capita	4,435.8 / 4,519.0 (614.5)
Illiteracy (%)	1.2 / 1.0 (0.6)
Life Expectancy (years)	70.9 / 70.7 (1.3)
Murder rate (per 100,000)	7.4 / 6.9 (3.7)
High graduates (%)	53.1 / 53.3 (8.1)
Number of days below freezing	104.5 / 114.5 (52.0)
Land area (mile ^2)	70,735.9 / 54,277.0 (85,327.3)

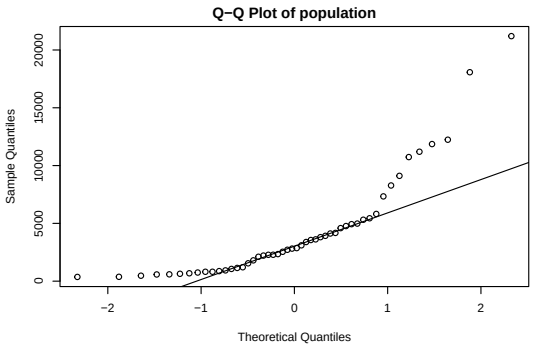
[1]Mean / Median (SD)

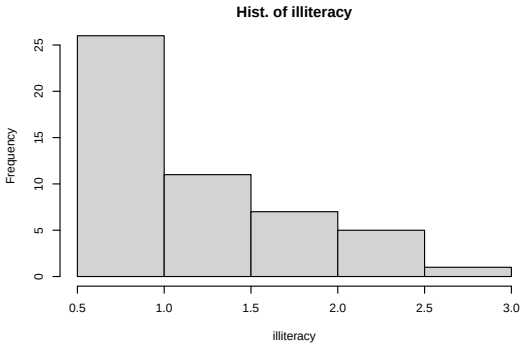
b)

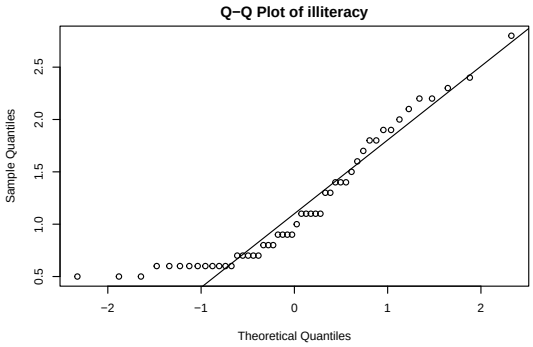
var	statistic	p.value
population	0.770	<0.001
income	0.977	0.43
illiteracy	0.883	<0.001
life_exp	0.977	0.442
murder	0.953	0.047
hs_grad	0.953	0.046
frost	0.955	0.053
area	0.572	<0.001

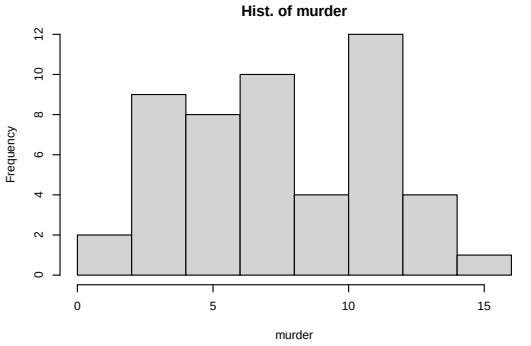
The results of Shapiro-Wilk test indicates that variable `population`, `illiteracy`, `murder`, `hs_grad`, and `area` is not normally distributed.
The histogram and Q-Q plots for these variables are as follows:

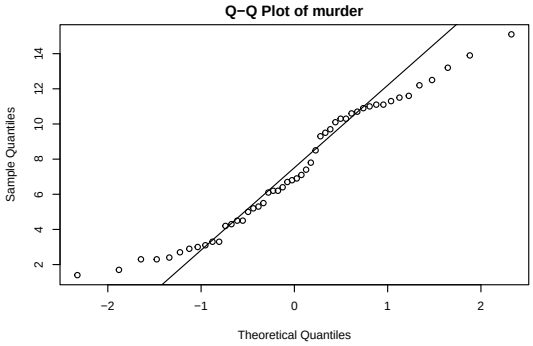

Hist. of population
Frequency
0
10
20
30
0
5000
10000
15000
20000
25000
population

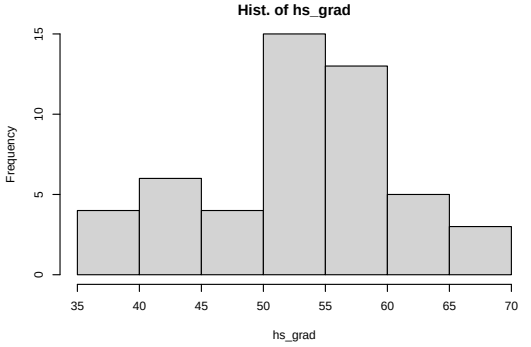

Q−Q Plot of population
Sample Quantiles
0
5000
10000
15000
20000
−2
−1
0
1
2
Theoretical Quantiles

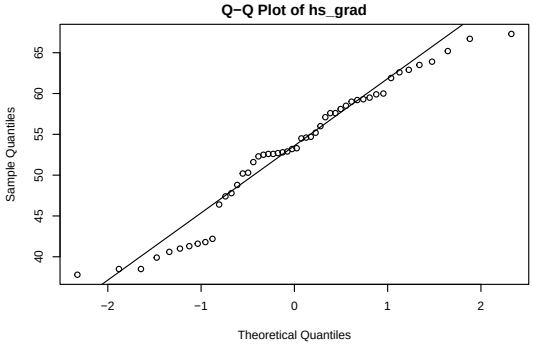

Hist. of illiteracy
Frequency
0
5
10
15
20
25
0.5
1.0
1.5
2.0
2.5
3.0
illiteracy

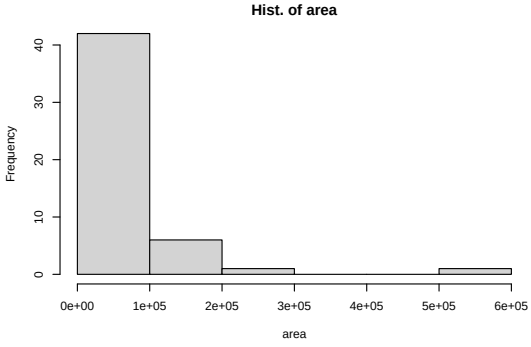

Q−Q Plot of illiteracy
Sample Quantiles
0.5
1.0
1.5
2.0
2.5
−2
−1
0
1
2
Theoretical Quantiles

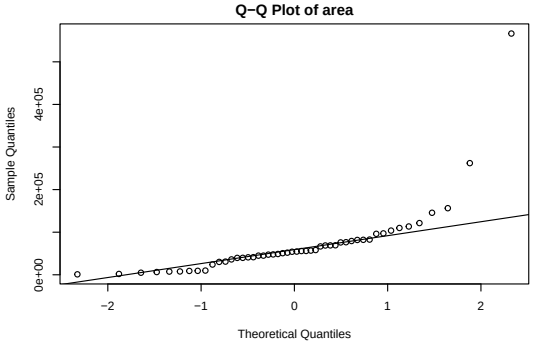

Hist. of murder
Frequency
0
2
4
6
8
10
12
0
5
10
15
murder


Q−Q Plot of murder
Sample Quantiles
2
4
6
8
10
12
14
−2
−1
0
1
2
Theoretical Quantiles


Hist. of hs_grad
Frequency
0
5
10
15
35
40
45
50
55
60
65
70
hs_grad

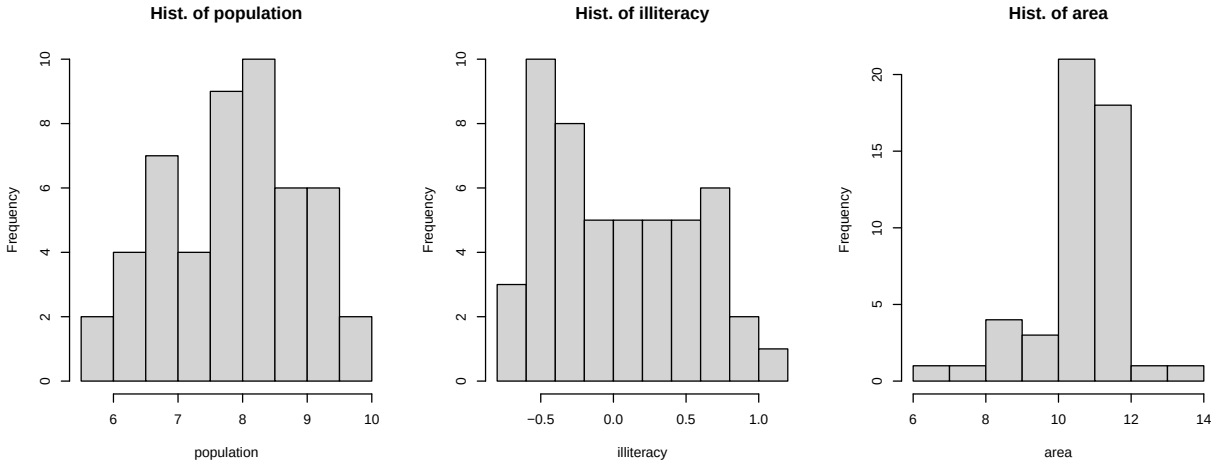

Q−Q Plot of hs_grad
Sample Quantiles
40
45
50
55
60
65
−2
−1
0
1
2
Theoretical Quantiles


Hist. of area
Frequency
0
10
20
30
40
0e+00
1e+05
2e+05
3e+05
4e+05
5e+05
6e+05
area

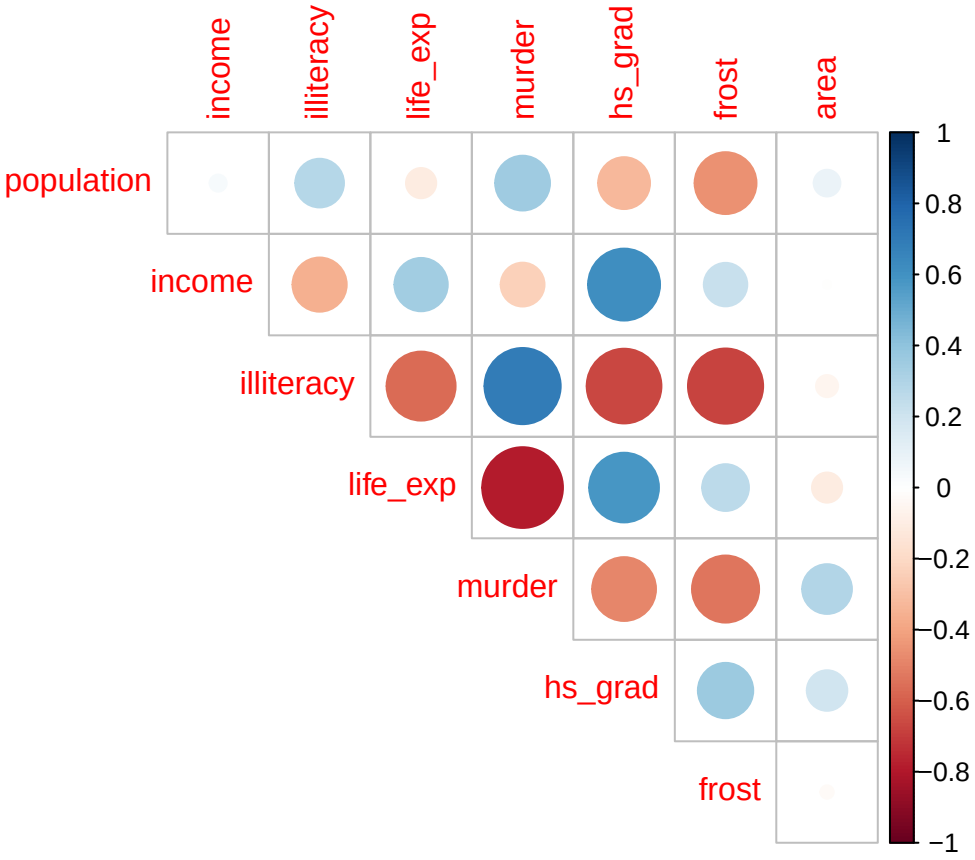

Q−Q Plot of area
Sample Quantiles
0e+00
2e+05
4e+05
−2
−1
0
1
2
Theoretical Quantiles

Given the shape of the histograms, I will log-transform `population`, `illiteracy`, and `area`.

Now, let's check these histograms.



I will use the data set including the log-transformed variables for the later analysis. Let's check the correlation between each variable and linear regression model including all variables.




```
##
## Call:
## lm(formula = life_exp ~ ., data = df_val)
##
```

```
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.44702 -0.42901  0.04546  0.50742  1.68911
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  6.799e+01  1.798e+00  37.809  < 2e-16 ***
## population   2.537e-01  1.311e-01   1.936   0.0597 .
## income      -4.417e-06  2.475e-04  -0.018   0.9858
## illiteracy   1.883e-01  4.204e-01   0.448   0.6565
## murder      -3.114e-01  4.659e-02  -6.684 4.12e-08 ***
## hs_grad      5.482e-02  2.552e-02   2.148   0.0375 *
## frost       -4.669e-03  3.173e-03  -1.471   0.1487
## area         7.314e-02  1.102e-01   0.663   0.5107
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7335 on 42 degrees of freedom
## Multiple R-squared:  0.7441, Adjusted R-squared:  0.7014
## F-statistic: 17.45 on 7 and 42 DF,  p-value: 1.368e-10
```

c) Automatic procedures In this section, I will use backward and forward procedures.

```
## Start:  AIC=-23.71
## life_exp ~ population + income + illiteracy + murder + hs_grad +
##     frost + area
##
##              Df Sum of Sq    RSS     AIC
## - income      1    0.0002 22.596 -25.712
## - illiteracy  1    0.1079 22.704 -25.475
## - area        1    0.2368 22.833 -25.192
## <none>                    22.596 -23.713
## - frost       1    1.1645 23.760 -23.200
## - population  1    2.0155 24.611 -21.441
## - hs_grad     1    2.4822 25.078 -20.502
## - murder      1   24.0347 46.631  10.512
##
## Step:  AIC=-25.71
## life_exp ~ population + illiteracy + murder + hs_grad + frost +
##     area
##
##              Df Sum of Sq    RSS      AIC
## - illiteracy  1    0.1095 22.705 -27.4708
## - area        1    0.2616 22.858 -27.1370
## <none>                    22.596 -25.7125
## - frost       1    1.2628 23.859 -24.9936
## - population  1    2.3859 24.982 -22.6937
## - hs_grad     1    4.4112 27.007 -18.7959
## - murder      1   24.4834 47.079   8.9907
##
## Step:  AIC=-27.47
## life_exp ~ population + murder + hs_grad + frost + area
##
##              Df Sum of Sq    RSS     AIC
```

```
## - area        1    0.2157 22.921 -28.998
## <none>                    22.705 -27.471
## - population  1    2.2792 24.985 -24.688
## - frost       1    2.3760 25.082 -24.495
## - hs_grad     1    4.9491 27.655 -19.612
## - murder      1   29.2296 51.935  11.899
##
## Step:  AIC=-29
## life_exp ~ population + murder + hs_grad + frost
##
##              Df Sum of Sq    RSS     AIC
## <none>                    22.921 -28.998
## - frost       1     2.214 25.135 -26.387
## - population  1     2.450 25.372 -25.920
## - hs_grad     1     6.959 29.881 -17.741
## - murder      1    34.109 57.031  14.578

##
## Call:
## lm(formula = life_exp ~ population + murder + hs_grad + frost,
##     data = df_val)
##
## Coefficients:
## (Intercept)   population       murder      hs_grad        frost
##   68.720810     0.246836    -0.290016     0.054550    -0.005174

## Start:  AIC=30.44
## life_exp ~ 1
##
##              Df Sum of Sq    RSS     AIC
## + murder      1    53.838 34.461 -14.609
## + hs_grad     1    29.931 58.368  11.737
## + illiteracy  1    28.688 59.611  12.791
## + income      1    10.223 78.076  26.283
## + frost       1     6.064 82.235  28.878
## <none>                    88.299  30.435
## + population  1     1.054 87.245  31.835
## + area        1     1.042 87.257  31.842
##
## Step:  AIC=-14.61
## life_exp ~ murder
##
##              Df Sum of Sq    RSS     AIC
## + hs_grad     1    4.6910 29.770 -19.925
## + frost       1    3.1346 31.327 -17.378
## + population  1    2.9854 31.476 -17.140
## + income      1    2.4047 32.057 -16.226
## + area        1    1.4583 33.003 -14.771
## <none>                    34.461 -14.609
## + illiteracy  1    0.1292 34.332 -12.797
##
## Step:  AIC=-19.93
## life_exp ~ murder + hs_grad
##
```

```
##              Df Sum of Sq    RSS     AIC
## + population  1    4.6350 25.135 -26.387
## + frost       1    4.3987 25.372 -25.920
## <none>                    29.770 -19.925
## + illiteracy  1    0.8366 28.934 -19.351
## + area        1    0.1236 29.647 -18.134
## + income      1    0.1022 29.668 -18.097
##
## Step:  AIC=-26.39
## life_exp ~ murder + hs_grad + population
##
##              Df Sum of Sq    RSS     AIC
## + frost       1   2.21416 22.921 -28.998
## + illiteracy  1   1.05998 24.075 -26.542
## <none>                    25.135 -26.387
## + income      1   0.11819 25.017 -24.623
## + area        1   0.05387 25.081 -24.495
##
## Step:  AIC=-29
## life_exp ~ murder + hs_grad + population + frost
##
##              Df Sum of Sq    RSS     AIC
## <none>                    22.921 -28.998
## + area        1  0.215741 22.706 -27.471
## + illiteracy  1  0.063655 22.858 -27.137
## + income      1  0.010673 22.911 -27.021


##
## Call:
## lm(formula = life_exp ~ murder + hs_grad + population + frost,
##     data = df_val)
##
## Coefficients:
## (Intercept)       murder      hs_grad   population        frost
##   68.720810    -0.290016     0.054550     0.246836    -0.005174
```

The both procedures generated the same model (variables included in the final model: `murder`, `hs_grad`, `population`, `frost`).

d) Criterion-based procedures

e) The LASSO method

f) Compare the subsets from c, d, and e

g) Findings