

Uncertainty and Unemployment Revisited: The Consequences of Financial and Labor Contracting Frictions*

Yajie Wang[†]
University of Rochester

August 27, 2022

click [here](#) for the most recent version

Abstract

I build a novel model to study how uncertainty shocks to firm-level productivity affect unemployment through the financial channel of incomplete labor contracts. In my model, labor contracting frictions imply wage insensitivity to firms' idiosyncratic shocks. Hence, wage bills are debt-like commitments by firms to workers, which firms are less likely to take on when high uncertainty raises firm default risks. Therefore, when uncertainty is high, firms hire fewer workers, and unemployment increases. Quantitatively, I find that uncertainty shocks, together with aggregate productivity shocks, explain 90% of the increase in unemployment during the Great Recession. The model's quantitative performance deteriorates greatly if either the financial channel or uncertainty shocks are absent. Given the model's quantitative success, I use it to analyze the impact of the United States and German labor market policies that expanded a lot in recent recessions. The U.S. policy raises unemployment benefits, making it more expensive for firms to pay wages, amplifying the recession. Germany subsidizes firms' wage bills to keep workers employed, which outperforms the U.S. policy but still yields a negative impact since its misallocation losses outweigh its gains from insuring firms.

Keywords: search and matching, financial frictions, incomplete labor contracts, uncertainty, volatility, firm heterogeneity, business cycles, labor market policies.

JEL Codes: E24, E32, E44, D53, D83, J08.

*I am greatly indebted to Yan Bai and Narayana Kocherlakota for their invaluable advice and encouragement. I also thank George Alessandria, Jonas Arias, David Autor, Martin Beraja, Mark Bills, Gaston Chaumont, Pablo D'Erasmus, Lukasz Drozd, Burcu Eyigungor, Sasha Indarte, Philipp Kircher, Zheng Liu, Igor Livshits, Ryan Michaels, Makoto Nakajima, Christina Patterson, Edouard Schaal, Bryan Stuart, and participants in seminars and conferences at the University of Rochester, the Federal Reserve Bank of Philadelphia, Midwest Macro (Logan), North America Summer Meeting (Miami), Asian Meeting of the Econometric Society (Tokyo), and Young Economist Symposium (Yale) for their helpful comments and discussions.

[†]Email: yajie.wang@rochester.edu; Address: Department of Economics, 116 Harkness Hall, Rochester, NY 14620.

1 Introduction

What accounts for the increase in unemployment during recessions? My approach to answering this question is motivated by the recent work that finds the crucial role of uncertainty¹ shocks in driving business cycles.² Particularly, a key reason for uncertainty shocks to cause recessions is firm financial frictions (Alfaro, Bloom and Lin, 2019; Arellano, Bai and Kehoe, 2019; Gilchrist, Sim and Zakrajšek, 2014). Therefore, I build a novel model to study how uncertainty shocks to firm-level productivity affect unemployment through the financial channel of incomplete labor contracts, where the labor market incompleteness enables financial frictions to affect unemployment.

The key driving force in my model is the risk associated with hiring workers. That is, hiring is a risky investment because employment decisions are made before firms know the productivity. So, wage bills are debt-like commitments by firms to workers, which firms are less likely to take on when high uncertainty raises firm default risks due to financial frictions. Therefore, when uncertainty is high, firms are exposed to higher default risks, so they hire fewer workers, and unemployment increases. My quantitative general equilibrium model shows that this mechanism can generate 90% of the increase in unemployment during the Great Recession with uncertainty shocks and aggregate productivity shocks.

My model is built on Schaal (2017), who makes an important contribution by introducing uncertainty shocks into a directed search model that allows endogenous separations, entry and exit, and on-the-job search. However, he finds that the search model alone cannot generate a sufficient increase in unemployment during the Great Recession, even though uncertainty increased a lot, particularly during that period. My model can greatly improve the search framework's performance in explaining unemployment by considering two extra margins, firm financial frictions and labor contracting frictions.

As for financial frictions, I assume that firms can only borrow through state-uncontingent debt, and default is costly because it leads to liquidation. The price of debt reflects the firm's default probability and the post-default recovery from the firm's value. I also model the agency friction that managers can divert firms' funds for their private interests, which constrains the firms' incentives to save, so the default risk will not be offset by a large stock of savings.

On the other hand, the labor contracting frictions are based on two assumptions. First, I assume that labor contracts are incomplete in the sense that firms' promised lifetime utilities to workers only respond to aggregate shocks that affect workers' outside options. Second, I assume that firms have commitments to labor contracts, but workers do not. As a result, I can prove wage

¹ By "uncertainty", I mean volatility, i.e., the conditional (cross-sectional) variance of firm-level productivity.

² Papers that point out the crucial role of uncertainty shocks over business cycles include Basu and Bundick (2017), Bloom et al. (2018), Christiano, Motto and Rostagno (2014), Fajgelbaum, Schaal and Taschereau-Dumouchel (2017), and Fernández-Villaverde et al. (2011).

insensitivity. That is, although wages can respond flexibly to aggregate shocks that affect workers' outside options, wages cannot adjust in response to firm-level idiosyncratic shocks. Intuitively, this is plausible because it is hard to contract on firm-specific shocks. In my micro-founded model of asymmetric information, because workers do not know the shocks to firms, they will not accept wage cuts when firms use the idiosyncratic shocks as requests. Plus, empirical evidence based on employer-employee matched data also documents that there is no significant pass-through from firms' idiosyncratic transitory shocks to workers' earnings ([Guiso, Pistaferri and Schivardi, 2005](#); [Rute Cardoso and Portela, 2009](#)). My approach here to model labor contracting frictions is, as far as I know, new to the literature.

Together and only together, firm financial frictions and labor contracting frictions can generate realistic fluctuations in unemployment. Specifically, as typical search models, I assume that firms hire workers before knowing productivity, so employment is like a risky investment in the sense that firms are uncertain about its returns. Then an increase in idiosyncratic firm risk causes more exposure to default risk due to financial frictions. And hiring is riskier because incomplete labor contracts also prevent firms from hedging against idiosyncratic shocks. Therefore, firms are less likely to hire workers when the uncertainty of firm-level productivity increases. In this way, aggregate increases in uncertainty translate into higher unemployment. Notice that neither firm financial frictions nor labor contracting frictions are effective individually. Suppose either debt contracts or labor contracts are complete. Firms can use the state-contingent instruments to hedge against shocks fully and eliminate the idiosyncratic risk. In other words, it is the labor contract incompleteness that enables firm financial frictions to affect unemployment.

This financial channel of incomplete labor contracts greatly helps to understand the unemployment fluctuations over business cycles. In my model, uncertainty shocks together with aggregate productivity shocks can explain 90% of unemployment volatility in the data. Furthermore, to understand the U.S. past recessions from the 70s to the Great Recession, I apply the particle filter to my model and estimate the historical aggregate productivity shocks and uncertainty shocks. Then I feed the estimated structural shocks to the model to predict unemployment. The result shows that my model can generate similar patterns of unemployment variations to past recessions. Notably, it explains 90% of the increase in unemployment during the Great Recession.

My financial channel of incomplete labor contracts is the key to this quantitative success. Without it, the model behaves like a canonical search model, whose quantitative performance in explaining unemployment deteriorates greatly. This is consistent with the finding by [Schaal \(2017\)](#) that the canonical search framework alone is insufficient to explain the rise in unemployment during the Great Recession, even if uncertainty shocks are considered. The reason is that the model without the financial channel underestimates the effect of uncertainty shocks so much that it cannot generate a realistic increase in unemployment. Additionally, my model also shows that, compared with other recessions, financial frictions contribute more to the Great Recession because

it has the largest increase in uncertainty, which interacts with financial frictions and amplifies the rise in unemployment.

I also find that my financial channel of incomplete labor contracts requires interaction with uncertainty shocks in particular. Specifically, the recalibrated model with only aggregate productivity shocks displays little amplification from the financial channel. The reason is similar to [Shimer's \(2005\)](#) unemployment volatility puzzle. Although a negative aggregate productivity shock can also push firms into difficult financial conditions, equilibrium wages fall a lot according to the free entry condition, which greatly absorbs the negative impact of the aggregate productivity shock. Nevertheless, this offsetting effect of equilibrium wages is much weaker for uncertainty shocks. Since the production function is decreasing returns to scale, profits are convex with respect to productivity. The uncertainty shock spreads the productivity distribution and pushes the firm's value higher, i.e., the Oi-Hartman-Abel effect of increasing volatility ([Oi \(1961\)](#), [Hartman \(1972\)](#), and [Abel \(1983\)](#)). As a result, wages do not decrease much to satisfy the free entry condition. And my financial channel of wage commitments is still effective. Therefore, unlike a typical search model focusing on aggregate productivity shocks, my model suggests that uncertainty shocks are crucial to understanding unemployment dynamics.

Lastly, given its quantitative success, I use the model to analyze the impact of American and German labor market policies that expanded a lot during the Great Recession and the recent Covid recession. The U.S. policy increased unemployment benefits during Covid. My model's impulse response function suggests that a 1% increase in unemployment benefits when uncertainty is high raises unemployment by an additional 5% in response to a 5% uncertainty shock. Higher unemployment benefits push wages higher, make employment even riskier for firms, and thus amplify the negative impact of uncertainty shocks. The average welfare loss due to this policy is 3.9 basis points.

On the other hand, Germany has a different social security system of subsidizing firms to pay wages and keep workers employed. I revise my model to incorporate this policy so firms can let part of the workforce idle when uncertainty is high. The government will pay 84% of these idle workers' wages, and firms will pay the rest. I choose this replacement rate such that it also costs 5 basis points of output as the U.S. policy. My model suggests that this policy of wage subsidies insures firms and thus stabilizes unemployment fluctuations. Nevertheless, aggregate output decreases because wage subsidies encourage labor hoarding in relatively low-productivity firms, preventing efficient labor reallocation. Because the losses due to misallocation outweigh the gains from insuring firms, welfare is lower. On average, the German labor market policy causes a 2.4 basis points welfare loss, which still outperforms the U.S. policy.

In contrast, when the two policies are evaluated by the counterfactual model without financial frictions, their negative impacts are greatly underestimated, particularly for the U.S. policy. The welfare loss induced by the U.S. policy decreases from 3.9 to only 0.2 basis points, and that by

the German policy decreases from 2.4 to only 0.4 basis points. Consequently, the model without financial frictions misleadingly suggests that the U.S. policy is better. Therefore, I conclude that ignoring financial frictions biases the evaluation of labor market policies.

Related Literature. My paper contributes to three strands of literature. First, my paper builds on the literature that finds the crucial role of uncertainty shocks in driving business cycles.³ This finding motivates [Schaal \(2017\)](#) to introduce uncertainty shocks into a canonical search framework to understand the fluctuations in unemployment. He finds that the canonical search model cannot generate a realistic increase in unemployment during the Great Recession. My work builds on [Schaal \(2017\)](#) by adding firm financial frictions and labor contracting frictions. Adding financial frictions is motivated by the finding that financial frictions are a core reason for uncertainty shocks to cause recessions ([Alfaro, Bloom and Lin, 2019](#); [Arellano, Bai and Kehoe, 2019](#); [Christiano, Motto and Rostagno, 2014](#); [Gilchrist, Sim and Zakrajšek, 2014](#)). And adding labor contracting frictions enables financial frictions to affect unemployment. The two sources of frictions interact with each other and generate the financial channel of incomplete labor contracts. My quantitative results show that incorporating this channel substantially improves the search model’s performance in explaining unemployment fluctuations.

Second, my work complements a large and still growing literature using wage rigidity with respect to aggregate productivity shocks to explain unemployment volatility ([Bils, Chang and Kim, 2022](#); [Fukui, 2020](#); [Gertler and Trigari, 2009](#); [Hall, 2005](#); [Hall and Milgrom, 2008](#); [Menzio and Moen, 2010](#); [Rudanko, 2019](#); [Schoefer, 2021](#); [Shimer, 2004](#)). Distinguished from wage rigidity to aggregate productivity shocks, I allow wages to respond flexibly to aggregate shocks that affect workers’ outside options, while the novel driving force in my model is wage insensitivity to firms’ idiosyncratic shocks. To justify this setup, I provide a micro-founded theory of asymmetric information. The idea follows [Hall and Lazear \(1984\)](#) and [Lemieux, MacLeod and Parent \(2012\)](#), who use information frictions to justify wage rigidity with respect to aggregate productivity shocks. The same logic applies here to firms’ idiosyncratic shocks, probably more plausible because firm-specific productivity is harder for workers to know than aggregate labor market conditions. Besides, empirical evidence based on employer-employee matched data also supports my approach by showing that the pass-through from firms’ transitory idiosyncratic shocks to workers’ earnings is very little ([Guiso, Pistaferri and Schivardi, 2005](#); [Rute Cardoso and Portela, 2009](#)). With this novel ingredient, my model complements the wage rigidity literature by providing an alternative mechanism—the financial channel of wage insensitivity to firms’ idiosyncratic shocks. And my

³Uncertainty in my paper specifically refers to micro-level volatility, i.e., the volatility of firm-level idiosyncratic productivity documented by [Bloom et al. \(2018\)](#). As for the macro-level volatility of aggregate productivity shocks, [Leduc and Liu \(2016\)](#), [Freund and Rendahl \(2020\)](#), and [Cacciatore and Ravenna \(2021\)](#) use homogeneous-agent models to study the effect of macro-level volatility on unemployment through nominal rigidity or wage rigidity. [Den Haan, Freund and Rendahl \(2021\)](#) emphasize the real option channel of macro-level volatility due to search frictions. Due to the computational burden, my paper does not include macro-level volatility, supported by [Schaal \(2017\)](#), who finds that the size and impact of macro-level volatility shocks are small.

quantitative result indicates this mechanism’s success in explaining unemployment fluctuations over business cycles.

Third, there is a growing literature adding firm financial frictions to search models ([Blanco and Navarro, 2016](#); [Boz, Durdu and Li, 2015](#); [Christiano, Trabandt and Walentin, 2011](#); [Chugh, 2013](#); [Eckstein, Setty and Weiss, 2019](#); [Garin, 2015](#); [Mumtaz and Zanetti, 2016](#); [Petrosky-Nadeau, 2014](#); [Schoefer, 2021](#); [Sepahsalari, 2016](#); [Wasmer and Weil, 2004](#); [Zanetti, 2019](#)). The novelty of my model is incorporating labor contracting frictions with respect to firms’ idiosyncratic shocks to interact with financial frictions. My approach to modeling firm financial frictions follows the literature on risky debt with endogenous default decisions ([Arellano, Bai and Kehoe, 2019](#); [Khan and Thomas, 2013](#); [Ottonello and Winberry, 2020](#)). Modeling financial frictions in a search model with decreasing returns-to-scale production functions is technically challenging. The difficulty stems from the dimensionality curse due to the firm’s continuum of historical-dependent labor contracts. One of my theoretical contributions is uniquely determining wage payments within labor contracts. Given the expression of wages, labor contracts are no longer a necessary part of the state variables, resolving the dimensionality curse. This approach to tractability is, as far as I know, novel to the literature.

Layout. The paper proceeds as follows. I first set up the model in [Section 2](#). [Section 3](#) calibrates the model and presents the quantitative results, including the event study for U.S. past recessions and labor market policy experiments. Lastly, [section 4](#) concludes.

2 Model

To study the impact of aggregate shocks on unemployment, I build a directed search and matching model. The equilibrium is block recursive to provide tractability, following [Menzio and Shi \(2010, 2011\)](#), [Kaas and Kircher \(2015\)](#), and [Schaal \(2017\)](#).⁴ The model also features financial frictions with firm default risks, following [Arellano, Bai and Kehoe \(2019\)](#).

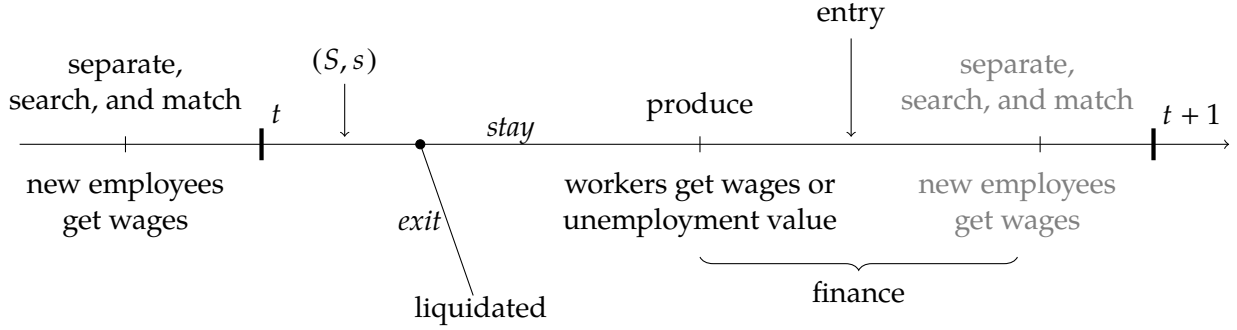
2.1 Environment and Timing

There are four types of agents in the economy: workers, firms, managers, and international financial intermediaries. Workers are infinitely lived and risk-neutral. They have the same productivity. The total mass of workers is normalized to one unit. Firms are also risk-neutral. They hire workers to produce homogeneous goods and finance by borrowing from the financial intermediaries.

Firms’ idiosyncratic productivity is drawn from the Markov process $\pi_z(z'|z, \sigma)$, where σ is

⁴ As for other search models with multi-worker firms, [Acemoglu and Hawkins \(2014\)](#) and [Elsby and Michaels \(2013\)](#) introduce Nash bargaining into random search. Because my paper focuses on business cycles, I leverage directed search with block recursivity to solve the problem globally out of the steady-state.

Figure 1: Timing



the aggregate volatility shock to the economy. Higher volatility implies a more widely spread distribution of tomorrow's idiosyncratic productivity shocks, so firms are more likely to draw a low idiosyncratic productivity. The other aggregate shock in the economy is the aggregate productivity shock A . I use S to summarize the two aggregate shocks (A, σ) . Firms also face an i.i.d. random operating cost shock ϵ , which follows a normal distribution $\Phi_\epsilon \equiv \mathcal{N}(\mu_\epsilon, \sigma_\epsilon^2)$. I use s to denote the two firm-specific shocks (z, ϵ) .

I assume that job search is directed. Each labor submarket is indexed by a promised utility x , which is the lifetime utility firms promise to workers hired from this submarket. The submarket tightness θ is the ratio of vacancies to the number of workers looking for jobs in each submarket. I use $p(\theta)$ to denote the job-finding rate of workers and $q(\theta)$ to denote the vacancy-filling rate of firms. The relation between x and θ will be determined by the free entry condition in equilibrium.

Following the implicit contract literature, I assume that firms are committed to labor contracts while workers are not. So, workers can leave the firm whenever their outside option is better. One justification for this approach is that firms care about their reputations more than individual workers. I denote the recursive-from labor contract as $C = \{w, \tau, W'(S', s'), d(S', s')\}$, where w is the current wage payment, τ is the layoff probability, $W'(S', s')$ is the next-period employment value promised by the firm, and $d(S', s')$ is the indicator for the firm's exit decision.

Figure 1 shows the timing. At the end of the preceding period, firms and workers interacted in the labor market to separate, search, and match. They draw up labor contracts in this stage. And newly employed workers receive wages. At the beginning of the current period, all shocks (S, s) realize. Then firms decide to exit or not. If a firm exits, it defaults on all its debts, including labor contracts, and its operations are liquidated. Otherwise, firms produce based on the number of employees as determined at the end of the last period. At the same time, employed workers receive wages according to continuing labor contracts. Unemployed workers also obtain unemployment benefits in this stage. Next, potential new firms can pay an entry cost to enter, after which both new entrants and incumbent firms participate simultaneously in the labor market. Firms borrow from international financial intermediaries to finance the expenditure during the process.

2.2 Worker's Problem

There are two types of workers in the economy: unemployed and employed workers. I abstract from the participation margin.

Unemployed Worker's Problem. An unemployed worker receives unemployment benefits \bar{u} in the current period and chooses a submarket x_u to job search to maximize their unemployment value. The matching probability $p(\theta(S, x_u))$ depends on the aggregate shocks and the promised utility of the submarket. Therefore, the unemployment value is:

$$U(S) = \max_{x_u} \bar{u} + p(\theta(S, x_u))x_u + (1 - p(\theta(S, x_u)))\beta \mathbb{E} U(S'). \quad (1)$$

Employed Worker's Problem. The value of employment depends on the contingent labor contract $C = \{w, \tau, W'(S', s'), d(S', s')\}$. The worker receives his wage w in the current period and can simultaneously search for other jobs as well. I use x to denote his choice of on-the-job search submarket. If he successfully gets a new job, he receives x as lifetime utility. Notice that the job finding rate $p(\theta(S, x))$ is discounted by the relative on-the-job search efficiency λ , which matches the job-to-job transition rate.

In the next period, if the worker is laid off or the firm exits, he will be unemployed and receive the unemployment value $U(S')$. Otherwise, he can still work for the firm and receive the promised utility $W'(S', s')$. Notice that I assume firms are fully committed to labor contracts, but workers are not. Therefore, for promised utilities lower than the unemployment value, the worker will voluntarily leave the job and become unemployed. The following equation formalizes the value of employment:

$$\begin{aligned} W(S, s, C) = & \max_x w + \lambda p(\theta(S, x))x \\ & + (1 - \lambda p(\theta(S, x)))\beta \mathbb{E} \left\{ [\tau + (1 - \tau)(\pi_d + (1 - \pi_d)d(S', s'))]U(S') \right. \\ & \left. + (1 - \tau)(1 - \pi_d)(1 - d(S', s')) \max\{W'(S', s'), U(S')\} \right\}. \end{aligned} \quad (2)$$

where π_d is the exogenous exit rate of firms.

2.3 Firm's Problem

Firms maximize their present values, namely, the discounted cumulative sum of equity payouts.

A firm's states include realized aggregate shocks $S \in \mathcal{S}$, realized firm-specific shocks $s \in \mathcal{s}$, the number of employees n , and the set of promised utilities to its employees $\{W(S, s; i)\}_{i \in [0, n]}$, where

i is the index of incumbent employees within the firm.

Firms optimize over the current equity payout Δ , next-period debt b' , next-period employment n' , the number of workers to hire n_h , the submarket x_h in which to search, and next-period exit decisions $d(S', s')$. I assume that a firm only posts vacancies in one submarket each period. Firms also choose the current-period wages of incumbent workers $w(i)$, the layoff probability $\tau(i)$, the wages of newly hired workers $w_h(i')$, and the set of next-period lifetime utilities $\{W(S', s'; i')\}_{S' \in S', s' \in s'; i' \in [0, n']}$, subject to the participation constraint (8) and the promise-keeping constraint (9). I use $w(i)$ for incumbent employee i 's wage, \bar{w}_m for the manager's wage⁵, and $w_h(i')$ for the wage of a newly hired employee i' .

Equations (3) to (11) summarize the firm's problem starting from the production stage:

$$J(S, s, b, n, \{W(S, s; i)\}_{i \in [0, n]}) = \max_{\substack{\Delta, b', n', n_h, x_h, d(S', s'), \\ \{w(i), \tau(i)\}_{i \in [0, n]}, \\ \{w_h(i')\}_{i' \in (n' - n_h, n']}, \\ \{W'(S', s'; i'), \bar{W}(i')\}_{S' \in S', s' \in s'; i' \in [0, n']}} \Delta \quad (3)$$

$$+ \beta(1 - \pi_d) \mathbb{E}_{S', s' | S, s} \left\{ (1 - d(S', s')) J(S', s', b', n', \{W(S', s'; i')\}_{S' \in S', s' \in s'; i' \in [0, n']}) \right\}$$

$$\text{s.t. } \Delta = Azn^\alpha - \int_0^n w(i) di - \bar{w}_m - \epsilon - b - c \frac{n_h}{q(\theta(S, x_h))} - \int_{n' - n_h}^{n'} w_h(i') di' + Q(S, z, b', n') b' \geq 0, \quad (4)$$

$$n' = \int_0^n (1 - \tau(i))(1 - \lambda p(\theta(S, x^*(S)))) di + n_h, \quad (5)$$

$$x^*(S) = \arg \max_x p(\theta(S, x)) [x - \beta \mathbb{E} U(S')], \quad (6)$$

$$W'(S', s'; i') = U(S') + \bar{W}(i'), \quad (7)$$

$$\bar{W}(i') \geq 0, \quad (8)$$

$$W(S, s, C) \geq \begin{cases} W(S, s, i) & \text{for } i \in [0, n], \\ x_h & \text{for newly hired employees,} \end{cases} \quad (9)$$

$$i'(i) = \int_0^i (1 - \tau(j))(1 - \lambda p(\theta(S, x^*(S)))) dj, \forall i \in [0, n], \quad (10)$$

$$Q(S, z, b', n') b' - n_h \frac{c}{q(\theta(S, x_h))} - \int_{n' - n_h}^{n'} w_h(i') di' \geq M(S, z, n) - F_m(S, z), \quad (11)$$

$$\text{where } F_m(S, z) = \left[\frac{\bar{w}_m + (1 - \lambda) \frac{\beta}{1 - \beta} \bar{w}_m}{(1 - \Phi(A \xi \mathbb{E}[A' z' n'^\alpha - \int_0^{n'} w(i') di' - \bar{w}_m - \epsilon'])) \zeta \mathbb{E} z'} \right]^{\frac{1}{\alpha}} \bar{u}.$$

The firm chooses its equity payouts Δ for the current period. I assume that firms are subject to

⁵ One manager per firm.

the non-negative equity payout constraint in equation (4). I adopt this assumption so that firms cannot always raise cash through equity issuance, which ensures financial frictions are effective in my model. Equity payouts Δ equal output Azn^α minus the wage payments to incumbent employees $\int_0^n w(i)di$, minus the manager's wage \bar{w}_m , minus the stochastic operating cost ϵ , minus debt b , minus vacancy posting costs $c \frac{n_h}{q(\theta(S, x_h))}$, minus wage payments to newly hired workers $\int_{n'-n_h}^{n'} w_h(i')di'$, and plus borrowings $Q(S, z, b', n')b'$. I assume that output is decreasing returns to scale with respect to the number of employees by letting α be smaller than one. This assumption helps generate meaningful firm sizes, essential to capturing firms' downsizing behaviors when subject to high volatility. The parameter c is the posting cost per vacancy. To hire n_h new workers, the firm needs to post $\frac{n_h}{q(\theta(S, x_h))}$ vacancies, where q denotes the vacancy-filling rate. The total vacancy posting cost is correspondingly $c \frac{n_h}{q(\theta(S, x_h))}$. The bond price Q is determined such that the international financial intermediaries break even, which will be defined later.

Equation (5) is the law of motion for employment. The firm's next-period number of employees is the sum of staying employees and new hires. Employees can separate from the firm for two reasons, on-the-job search and layoffs. Employees optimally choose an on-the-job search submarket to maximize their expected lifetime utility as in eq. (6). I use $x^*(S)$ to denote the optimal on-the-job search market. Then the probability for a worker to transit to another firm is $\lambda p(\theta(S, x^*(S)))$. If the worker does not find a new job, he faces a layoff probability $\tau(i)$. Therefore, eq. (5) means that the staying employees plus new hires sum to the next-period employment.

Eq. (7) assumes a specific contract form for the next-period promised utilities, which are comprised of two parts: the outside option of unemployment $U(S')$ and a utility markup chosen by the firm $\bar{W}(i')$. The promised utility markup can be contingent on workers, but it does not vary across states. The state-uncontingency of $\bar{W}(i')$ is crucial for effective financial frictions. Suppose firms' future promises to workers could be contingent on states. Labor contracts will then serve as a much better financial instrument than state-uncontingent bonds, which I conceive of as a counterfactual. Appendix D uses asymmetric information to provide a micro-founded model to justify this setup. The idea is based on Hall and Lazear (1984), who prove the optimality of predetermined wages when considering information frictions. Specifically, suppose workers do not have information about the firm's conditions. Firms can lie to pay less to workers. Because workers do not know whether the firm is truly facing a bad shock, they will not accept the wage cut. Section 2.10 discusses the assumption of a state-uncontingent promised utility markup in detail.

Recall that I assume firms are committed to labor contracts, but workers are not. Therefore, the participation constraint (8) shows that the firm should promise a non-negative utility markup to retain its workers. Otherwise, the worker would rather be unemployed. Furthermore, the promise-keeping constraint (9) requires the firm to adhere to its commitment that the worker's employment value is at least the promised lifetime utility. For incumbent worker $i \in [0, n]$, his promised utility

is $W(S, s, i)$, one of the firm's state variables. For a newly hired worker, his promised utility is x_h , according to the firm's choice of hiring submarket. Finally, eq. (10) formalizes the transition of the employee's index from i to i' .

The last constraint (11) reflects the agency frictions between shareholders and managers, following Jensen (1986) and Arellano, Bai and Kehoe (2019).⁶ This constraint dampens the firm's saving incentives so that financial frictions are effective in the model. Otherwise, firms will build up a large cash buffer and never default.

The micro-foundation of the agency frictions is as follows. I assume that there is a pool of potential managers from which each firm can hire one manager to operate the firm. The total mass of managers is much smaller than of workers, so I abstract from managers when calculating unemployment. Each manager can also be self-employed and produce \bar{w}_m units of goods. The market for managers is competitive, so a manager's wage is also \bar{w}_m .

Each period consists of a day and night. During the day, managers are monitored by the firm's shareholders, so managers adopt the firm's optimal policies. The manager uses borrowing $Q(S, z, b', n')b'$ and sales to pay dividends, wages of incumbent workers, his own wage, the operating cost, and debt. Search happens overnight, and the manager is supposed to use the remaining resources to pay vacancy posting costs and the wages of new workers. However, what happens during the night cannot be observed by shareholders until the next day. Therefore, the manager can propose an alternative production plan to the financial intermediary to borrow as much as possible at night. To convince the financial intermediary of the new plan (\bar{b}', \bar{n}') , the manager needs to provide proof by posting vacancies to have \bar{n}' workers in the next period if hiring is necessary. The manager thus needs to pay vacancy posting costs and wages for newly hired workers for the alternative proposal. In sum, to maximize available funds, the manager will come up with a proposal to achieve maximum possible borrowing net of hiring costs:

$$M(S, z, n) = \max_{\substack{b', n', n_h, x_h, d(S', s'), \\ \{\tau(i)\}_{i \in [0, n]}, \{w_h(i')\}_{i' \in (n' - n_h, n']}, \\ \{W'(S', s'; i'), \bar{W}(i')\}_{S' \in S', s' \in s'; i' \in [0, n']}}} Q(S, z, b', n')b' - n_h \frac{c}{q(\theta(S, x_h))} - \int_{n' - n_h}^{n'} w_h(i') di' \quad (12)$$

$$\text{s.t. (5), (7), (8), and (9).} \quad (13)$$

Given the maximum net borrowing $M(S, z, n)$, the remaining credit available for the manager is the maximum net borrowing minus the previous borrowing plus the originally planned but unused money for search, i.e., $M(S, z, n) - Q(S, z, b', n')b' + n_h \frac{c}{q(\theta(S, x_h))} + \int_{n' - n_h}^{n'} w_h(i') di'$.

The manager wants to use the remaining resources to hire workers to produce for his own

⁶ Workers own firms in the model, so they are shareholders. I do not explicitly model equity payouts in the worker's problem because workers are risk-neutral and the free entry condition implies that the firm's net present value is zero.

project in the next period. After the next-period production occurs, shareholders learn what has occurred. The extra workers will be laid-off and search for jobs. Because the manager only needs to hire workers for the next-period production, the outside value of unemployment benefits \bar{u} is the lowest wage for the manager to retain workers to produce. The manager will use the rest of the funds to hire as many workers as possible. The number of workers n_s is determined by

$$n_s = \frac{M(S, z, n) - Q(S, z, b', n')b' + n_h \frac{c}{q(\theta(S, x_h))} + \int_{n'-n_h}^{n'} w_h(i') di'}{\bar{u}}. \quad (14)$$

The manager takes advantage of the firm's productivity for his sided project, so the output is

$$\zeta z' n_s^\alpha,$$

where ζ indicates the profitability of the manager's own project.

I also assume that there is an auditing technology to detect a manager's intention to deviate at night. The effectiveness of the auditing technology, ξA , is based on a measure of auditing quality, ξ , proportional to aggregate productivity. The incentive and available resources to use the auditing technology are approximated by the firm's expected income $\mathbb{E}[A' z' n'^\alpha - \int_0^{n'} w(i') di' - \bar{w}_m - \epsilon]$. The more the firm expects to earn, the more it can and should pay for the auditing technology. I assume that the probability of the manager being caught is Gaussian and determined by the amount of auditing:

$$\Phi\left(\xi A \mathbb{E}[A' z' n'^\alpha - \int_0^{n'} w(i') di' - \bar{w}_m - \epsilon]\right). \quad (15)$$

I model the auditing technology to match the correlation between credit spreads and aggregate output, so the financial effect of aggregate productivity shocks is consistent with the empirical covariance. Otherwise, a positive aggregate productivity shock will cause counterfactually higher credit spreads because firms would have higher income and borrow substantially to avoid the managerial deviations. With the auditing technology, firms do not need to borrow that much when aggregate productivity is high, so the credit spreads decrease, as in the data.

Suppose the manager deviates from the firm's optimal policies and works on his side project. In that case, shareholders will find out and fire the manager the next day. Assume the manager faces probability λ of becoming self-employed (else returning to the manager market), which approximates the punishment for deviation. Therefore, to avoid manager deviations, the firm should not operate with significant unused credit so the manager cannot hire many workers and the side project is not attractive. To do so, the firm should satisfy the following constraint such

that the manager prefers to be honest:

$$\left(1 - \Phi\left(\xi A \mathbb{E}[A' z' n'^\alpha - \int_0^{n'} w(i') di' - \bar{w}_m - \epsilon]\right)\right) \mathbb{E}_t \beta \zeta A_{t+1} z_{t+1} n_s^\alpha + \lambda \mathbb{E}_t \sum_{j=2}^{\infty} \beta^j \bar{w}_m \leq \mathbb{E}_t \sum_{j=1}^{\infty} \beta^j \bar{w}_m,$$

which delivers the agency friction constraint (11) by plugging in equation (14).

The agency friction constraint (11) incentivizes firms to borrow in the spirit of Jensen (1986). Without agency frictions, firms have strong incentives to save and grow out of financial frictions. There are other options to reduce firms' savings, such as using a lower discount factor. But the model requires an unrealistically low discount factor to match observed leverage since firms are very opposed to liquidation. For other ways to make firms borrow in the presence of financial frictions, Quadrini (2011) provides one summary.

2.4 Wages

The state of the firm's problem (3) is an infinite-dimensional object because of the set of promised utilities. This section shows how to simplify the firm's problem by deriving wages and default decisions.

First, the promise-keeping constraint (9) always binds. Otherwise, firms could lower wages and earn more. Moreover, Proposition 1 shows that the participation constraint (8) also binds.

Proposition 1 *The participation constraint (8) binds, i.e., $\bar{W}(i') = 0$, for any worker i' .*

Appendix C provides the proof. Here, I explain the intuition. The participation constraint (8) requires that the promised utility markup should be non-negative. Suppose there exists a strictly positive promised utility markup $\bar{W}(i') > 0$. Then the firm can have a relatively low current wage according to the binding promise-keeping constraint (9). Namely, a positive promised utility markup can be understood as borrowing from the employee by backloading wages. However, borrowing from employees is more costly than borrowing from lenders through bonds. The reason is that lenders only discount the firm's default probability, but employees also discount the layoff risks. Therefore, firms never want to backload wages and promise a positive utility markup.

Given the binding promise-keeping constraint (9) and the participation constraint (8), I am able to determine wages. The binding participation constraint (8) implies that the promised utilities always equal the unemployment value U . From the worker's problem (1) and (2) and the binding promise-keeping constraint (9), an incumbent worker's wage is

$$\begin{aligned} w(S) &= U(S) - \lambda \max_x p(\theta(S, x)) [x - \beta \mathbb{E} U(S')] - \beta \mathbb{E} U(S') \\ &= \bar{u} + (1 - \lambda) \max_x p(\theta(S, x)) [x - \beta \mathbb{E} U(S')]. \end{aligned} \tag{16}$$

That is, an incumbent worker's wage equals the outside payoff of being unemployed minus gains from on-the-job search.

Similarly, a newly hired worker's wage equals

$$w_h(S) = x_h - \beta \mathbb{E} U(S'). \quad (17)$$

The uniquely determined wages in (16) and (17) are crucial for solving the problem quantitatively. Given the wages, the infinite-dimensional distribution of promised utilities not informative as a state variable, and the firm's problem can be simplified by removing the implicit contract constraints, (7), (8), and (9).

In terms of employment, the following Lemma 2.1 shows that while the model pins down the firm's total layoffs, the individual worker's layoff probability is undetermined.

Lemma 2.1 *The firm's total layoffs $\int_0^n \tau(i) di$ is uniquely determined, but the individual probability of layoff $\tau(i)$ is not.*

Proof For each optimal policy, eq. (5) determines the firm's total layoffs

$$\int_0^n \tau(i) di = n - \frac{n' - n_h}{1 - \lambda p(\theta(S, x^*(S)))}. \quad (18)$$

As long as total layoffs are constant, any perturbation of individual layoff probabilities $\{\tau(i)\}_{i \in [0, n]}$ does not affect the firm's value. \square

The key reason Lemma 2.1 holds is homogeneous workers, so the distribution of layoff probabilities is irrelevant. Therefore, I will focus on the symmetric decision rule that all employees face the same layoff probability throughout the rest of this paper.

2.5 Firm's Default Decision and Cash on Hand

To further reduce the number of dimensions, I next explore the firm's default decision and rewrite the firm's problem using cash on hand as a state variable.

Notice that the outside value is zero when a firm exits, so a firm defaults and exits when it cannot satisfy the non-negative equity payout constraint (4). Define cash on hand X as:

$$X = Azn^\alpha - n[\bar{u} + (1 - \lambda)\mu(S)] - \bar{w}_m - \epsilon - b, \quad (19)$$

where $\mu(S) \equiv \max_x p(\theta(S, x))[x - \beta \mathbb{E} U(S')]$. Then, a firm defaults if and only if:

$$X + M(S, z, n) < 0, \quad (20)$$

where $M(S, z, n)$ is maximum net borrowing as defined in equation (12). Therefore, a firm's default decision can be summarized by the operating cost cutoff $\bar{\epsilon}(S, z, b, n)$, defined as:

$$\bar{w}_m + \bar{\epsilon}(S, z, b, n) \equiv Azn^\alpha - \int_0^n w(i)di - b + M(S, z, n). \quad (21)$$

So, the firm defaults when the operating cost is higher than the cutoff $\bar{\epsilon}(S, z, b, n)$ such that the firm cannot satisfy the non-negative equity payout constraint, i.e.,

$$d(S, s, b, n) = \begin{cases} 0, & \text{if } \epsilon \leq \bar{\epsilon}(S, z, b, n), \\ 1, & \text{if } \epsilon > \bar{\epsilon}(S, z, b, n). \end{cases} \quad (22)$$

Plugging in the default cutoff (21) and wages (16) and (17), I rewrite the firm's problem (3) using cash on hand X as a state variable:

$$V(S, z, X, n) = \max_{\substack{\Delta, b', n', \\ \tau, n_h, x_h}} \Delta + \beta(1 - \pi_d) \mathbb{E}_{S', z' | S, z} \int_{-\infty}^{\bar{\epsilon}(S', z', b', n')} V(S', z', X', n') d\Phi(\epsilon') \quad (23)$$

$$\text{s.t. (6),} \quad (24)$$

$$n' = (1 - \tau)(1 - \lambda p(\theta(S, x^*(S))))n + n_h, \quad (25)$$

$$\Delta = X + Q(S, z, b', n')b' - n_h \frac{c}{q(\theta(S, x_h))} - n_h[x_h - \beta \mathbb{E} U(S')] \geq 0, \quad (26)$$

$$X' = A'z'n^\alpha - n'[\bar{u} + (1 - \lambda)\mu(S')] - \bar{w}_m - \epsilon' - b', \quad (27)$$

$$\bar{w}_m + \bar{\epsilon}(S', z', b', n') = A'z'n'^\alpha - n'[\bar{u} + (1 - \lambda)\mu(S')] - b' + M(S', z', n'), \quad (28)$$

$$Q(S, z, b', n')b' - n_h \frac{c}{q(\theta(S, x_h))} - n_h[x_h - \beta \mathbb{E} U(S')] \geq M(S, z, n) - F_m(S, z). \quad (29)$$

The non-negative equity payout constraint (26) reveals that firms' decisions depend on cash on hand X . When cash on hand is too low, the firm defaults because it cannot fully pay wages and debts. On the other hand, when cash on hand is sufficiently high, the firm is not constrained by (26). In this case, the firm solves the following relaxed problem:

$$\hat{V}(S, z, X, n) = \max_{\substack{b', n', \\ \tau, n_h, x_h}} X + Q(S, z, b', n')b' - n_h \frac{c}{q(\theta(S, x_h))} - n_h[x_h - \beta \mathbb{E} U(S')] \quad (30)$$

$$+ \beta(1 - \pi_d) \mathbb{E}_{S', z' | S, z} \int_{-\infty}^{\bar{\epsilon}(S', z', b', n')} V(S', z', X', n') d\Phi(\epsilon')$$

$$\text{s.t. (6), (25), (27), (28), and (29).} \quad (31)$$

For the relaxed problem, cash on hand does not affect the firm's choices. Let $\hat{b}(S, z, n)$, $\hat{n}(S, z, n)$,

$\hat{\tau}(S, z, n)$, $\hat{n}_h(S, z, n)$, and $\hat{x}_h(S, z, n)$ denote the optimal policies for the relaxed problem. The following Lemma 2.2 characterizes firms' decisions with respect to cash on hand.

Lemma 2.2 (*Decision Cutoffs*): *If $X < -M(S, z, n)$, the firm cannot satisfy the nonnegative external equity payout condition and has to default. If $X \geq \hat{x}(S, z, n) \equiv -\{Q(S, z, \hat{b}, \hat{n})\hat{b} - \hat{n}_h \frac{c}{q(\theta(\hat{S}, \hat{x}_h))} - \hat{n}_h[\hat{x}_h - \beta \mathbb{E} U(S')]\}$, the firm solves the relaxed problem (30), and the level of cash on hand does not affect the optimal decisions.*

Proof If the firm's cash on hand X is less than $-M(S, z, n)$, even though the firm borrows as much as possible, it cannot make nonnegative external equity payouts. So, the firm defaults and exits. If the firm's cash on hand X is more than $\hat{x}(v, z, n)$, then $(\hat{b}, \hat{n}, \hat{\tau}, \hat{n}_h, \hat{x}_h)$ is also the solution to the firm's problem (23), because constraint (26) holds automatically. In this case, cash on hand does not affect any constraints, and the optimal decisions do not depend on cash on hand. \square

Lemma 2.2 provides the method to solve the firm's problem by level of cash on hand. This partitioning method has been used by Khan and Thomas (2013), Arellano, Bai and Kehoe (2019), and Ottonello and Winberry (2020).

2.6 Bond Pricing

I assume that the economy's financial market is small compared with the rest of the world, so the risk-free interest rate in the international financial market is exogenous. This assumption ensures the block recursivity and thus computational tractability.

International financial intermediaries supply one-period bonds to firms. They are risk-neutral and competitive. The opportunity cost of lending is the risk-free interest rate r in the world financial market, equal to $1/\beta - 1$. Financial intermediaries break even when lending to firms. If the firm defaults, financial intermediaries can recover a value proportional to the firm's expected income $A'z'n'^\alpha - w(S)n' - \bar{m} - \mu_\epsilon$. This recovery function indicates that lenders can recover more when the firm has a higher value. Lian and Ma (2021) shows that this earnings-based lending prevails in the U.S.

Let η denote the recovery rate. And the break-even bond price $Q(S, z, b', n')$ is:

$$Q(S, z, b', n') = \beta \mathbb{E}_{S', z' | S, z} \left\{ (1 - \pi_d) \Phi(\bar{\epsilon}(S', z', b', n')) + [1 - (1 - \pi_d) \Phi(\bar{\epsilon}(S', z', b', n'))] \min \left\{ \eta \frac{A'z'n'^\alpha - w(S)n' - \bar{w}_m - \mu_\epsilon}{b'}, 1 \right\} \right\}. \quad (32)$$

2.7 Firm Entry

Potential new firms pay a fixed cost k_e to enter. New entrants' productivity will be drawn from the stationary distribution of idiosyncratic productivity $g_z(\cdot)$. New entrants do not produce in the entry period but hire workers as do incumbent firms. New firms start with zero debt and no labor. Then the new entrant's problem is:

$$J_e(S, z) = \max_{n_h, x_h} -n_h \frac{c}{q(\theta(S, x_h))} - n_h [x_h - \beta \mathbb{E} U(S')] \quad (33)$$

$$+ \beta(1 - \pi_d) \mathbb{E}_{S', z' | S, z} \int_{-\infty}^{\tilde{e}(S', z', b_0, n_h)} V(S', z', X', n_h) d\Phi(\epsilon'), \quad (34)$$

$$\text{s.t. } b_0 = 0, \text{ (27), and (28).} \quad (35)$$

I use n_e , x_e , and d_e to denote the new entrant's optimal policies.

Notice that both incumbent firms and new entrants only post vacancies in the markets with the lowest hiring cost. Define the minimum hiring cost per worker as

$$\kappa(S) \equiv \min_{x_h} [x_h + \frac{c}{q(\theta(S, x_h))}]. \quad (36)$$

In equilibrium, only submarkets with the lowest hiring cost are active. Given the equilibrium hiring cost $\kappa(S)$, the mapping from the market's promised utility x to the market intensity θ is

$$\theta(S, x) = \begin{cases} q^{-1}\left(\frac{c}{\kappa(S) - x}\right), & \text{if } x \leq \kappa(S) - c, \\ 0, & \text{if } x \geq \kappa(S) - c. \end{cases} \quad (37)$$

Notice that the upper bound of the vacancy filling probability q is one. When the submarket's promised utility x is higher than $\kappa - c$, no firm posts vacancies there because the vacancy filling probability cannot be greater than one to compensate for the hiring cost. In this case, the market is inactive, and the market intensity is zero.

The value of $\kappa(S)$ is determined by the free entry condition, which requires that the entry cost equals the expected entry value:

$$k_e = \sum_z J_e(S, z) g_z(z), \forall S. \quad (38)$$

Therefore, the free entry condition closes the model by pinning down the hiring cost $\kappa(S)$ for all aggregate states S .

2.8 Equilibrium

This section defines the block recursive equilibrium of the model.

Definition 2.1 Let s^f summarize the firm's state variables (S, z, X, n) . The block recursive equilibrium consists of the policy and value functions of unemployed workers $\{x_u(S), U(S)\}$; of employed workers $\{x(S, s, C), W(S, s, C)\}$; of incumbent firms $\{\Delta(s^f), b'(s^f), n'(s^f), \tau(s^f), n_h(s^f), x_h(s^f), w(S), w_h(S)\}$; of new firms $\{n_e(S), x_e(S), J_e(S)\}$; the hiring cost per worker $\kappa(S)$; the labor market tightness function $\theta(S, x; \kappa(S))$; and bond price schedules $Q(S, z, b', n')$ such that

1. Given the bond price schedules, the hiring cost, and the labor market tightness, the policy and value functions of unemployed workers, employed workers, incumbent firms, and entering firms solve their respective problems (1), (2), (16), (17), (23), and (33).
2. The bond price schedule satisfies (32).
3. The hiring cost per worker and the labor market tightness function satisfy (36) and (37).
4. The free entry condition (38) holds.

2.9 Aggregate Transitions

Let $Y(z, X, n)$ denote the mass of firms with states (z, X, n) , which is the sum of incumbent firms and new entrants which do not default. The law of motion of the firm distribution is:

$$\begin{aligned}
 & Y'(z', X', n') \\
 &= \sum_{z, X, n, \epsilon'} (1 - \pi_d)(1 - d(S', s'; S, z, X, n)) \mathbb{1}\{X'(S', s'; S, z, X, n) = X'\} \phi(\epsilon') \pi_z(z'|z, \sigma) \mathbb{1}\{n'(S, z, X, n) = n'\} Y(z, X, n) \\
 &+ m_e(S, Y) \sum_{z, \epsilon'} (1 - \pi_d)(1 - d_e(S', s'; S, z)) \mathbb{1}\{X'_e(S', s'; S, z) = X'\} \phi(\epsilon') \pi_z(z'|z, \sigma) \mathbb{1}\{n_e(S) = n'\} g_z(z).
 \end{aligned} \tag{39}$$

Following [Arellano, Bai and Kehoe \(2019\)](#), I assume that defaulting firms can still produce as long as their sales can cover the minimum wage required to retain workers for production. As discussed in Section 2.3, the necessary wage equals the unemployment benefits. These firms' employees will be laid off after production occurs and search for new jobs in the labor market, so they are not counted in unemployment in the current period. This setup relieves the concern that varying default rates mechanically drive the fluctuations of output and unemployment. Let $d^p(S', s'; S, z, X, n)$ be the production indicator, which equals one if and only if the firm does not default or satisfies $A'z'n'^\alpha > \bar{u}n'$ and zero otherwise. I use $Y^p(z, n)$ to denote the distribution

of producing firms, which thus evolves per:

$$\begin{aligned} \Upsilon^p(z', n') &= \sum_{z, X, n, \epsilon'} (1 - \pi_d)(1 - d^p(S', s'; S, z, X, n)) \pi_z(z'|z, \sigma) \mathbb{1}\{n'(S, z, X, n) = n'\} \Upsilon(z, X, n) \\ &\quad + m_e(S, \Upsilon) \sum_{z, \epsilon'} (1 - \pi_d)(1 - d_e^p(S', s'; S, z)) \pi_z(z'|z, \sigma) \mathbb{1}\{n_e(S) = n'\} g_z(z). \end{aligned} \quad (40)$$

The mass of entrants $m_e(S, \Upsilon)$ is determined such that total jobs found by workers equals the total jobs created by incumbent firms and new entrants:⁷

$$JF_{\text{workers}}(S, \Upsilon) = JC_{\text{incumbents}}(S, \Upsilon) + m_e(S, \Upsilon) JC_{\text{entrants}}(S, \Upsilon), \quad (41)$$

where

$$JF_{\text{workers}}(S, \Upsilon) = p(\theta(S, x_u^*(S))) \left(1 - \sum_{z, X, n} n \Upsilon(z, X, n) \right) + \sum_{z, X, n} \lambda p(\theta(S, x^*(S))) n \Upsilon(z, X, n), \quad (42)$$

$$JC_{\text{incumbents}}(S, \Upsilon) = \sum_{z, X, n} n_h(S, z, X, n) \Upsilon(z, X, n), \quad (43)$$

$$JC_{\text{entrants}}(S, \Upsilon) = \sum_z g_z(z) n_e(S, z). \quad (44)$$

Aggregate output is the sum of all firms' output:

$$Y = \sum_{z, n} A z n^\alpha \Upsilon^p(z, n), \quad (45)$$

and the unemployment rate u is the share of workers who do not produce:

$$u = 1 - \sum_{z, n} n \Upsilon^p(z, n). \quad (46)$$

⁷ Over the business cycle, jobs created by incumbent firms, $JC_{\text{incumbents}}$, can occasionally be larger than jobs found by workers, JF_{workers} . If the total mass of workers is restricted to one, then entry will be negative and not well-defined. To deal with this issue, I assume that when incumbent firms hire more workers than find jobs, the entry m_e is zero, and the mass of workers increases such that equation (41) holds. Then I normalize the economy so that the mass of workers is one unit again. This setup can be understood as an increase in labor force participation. Simulation shows that the average annual population growth rate is less than 0.5%, implying that the potential problem of negative entry is small. Another way to solve this problem is to assign different entry costs for different aggregate states. See [Kaas and Kircher \(2015\)](#) for this treatment.

2.10 Discussions of the Assumptions

My model’s key assumption and driving force is the incompleteness of financial instruments and labor contracts. This section discusses and justifies this assumption.

First, I assume that firms can only borrow through state-uncontingent debt. Beyond the inherent plausibility given the widespread real-world existence of state-uncontingent debt, much related literature uses this assumption to model firms’ default risks ([Arellano, Bai and Kehoe, 2019](#); [Khan and Thomas, 2013](#); [Ottonello and Winberry, 2020](#)). I also assume that firms can only have non-negative equity payouts. This assumption ensures that firms do not have deep pockets by issuing large amounts of equity, which in turn guarantees that financial frictions are meaningful in my model. Empirically, according to the calculations in [Schoefer \(2021\)](#) using U.S. Flow of Funds data, the average equity raised by U.S. firms from 1951Q4 to 2019Q4 is negative because of share repurchases. Therefore, on average equity issuance is not a primary financing tool for U.S. firms. Admittedly, a more realistic model could have equity issuance, but the recalibration will indicate sizable equity issuance costs. I conjecture that my model’s implications would thus not change much. Therefore, I follow the literature to exclude equity financing for simplicity.

Second, novel to the search literature, I assume that labor contracts do not insure firms from idiosyncratic shocks. Specifically, the promised utility is assumed to have a state-uncontingent utility markup over the unemployment value. The state-uncontingency makes wages debt-like commitments that firms are less likely to take on when volatility is high. This mechanism helps my model explain unemployment better than [Schaal \(2017\)](#).

The assumption of state-uncontingency distinguishes my framework from the textbook Diamond–Mortensen–Pissarides search models and a subsequent group of models assuming wage rigidity. As [Shimer \(2005\)](#) points out, typical search models cannot generate the observed unemployment volatility because wage movements largely cancel out the employment impacts of aggregate productivity shocks in equilibrium. One of the solutions to Shimer’s puzzle is wage rigidity ([Hall, 2005](#)). However, whether wage rigidity is a proper assumption to explain unemployment is still an open question ([Kudlyak, 2014](#); [Pissarides, 2009](#); [Rudanko, 2009](#)). To clarify, instead of making the debatable assumption of wage rigidity, I allow wages to vary with the outside value of unemployment. That is, wages can still decrease to offset the effects of aggregate shocks as in textbook search model. The difference is that I assume labor contracts do not allow firms to hedge against idiosyncratic risks. My quantitative results show that the micro-level risk caused by wage commitments is the key to explaining unemployment dynamics over business cycles. On the other hand, suppose the labor contract is state-contingent. Then it will be a much better financial instrument than state-uncontingent debt. I conceive this implication as counterfactual.

To justify the state-uncontingency of labor contracts, Appendix [D](#) provides a micro-founded theory of information frictions. The idea is based on [Hall and Lazear \(1984\)](#). When workers do not

clearly observe firm shocks, firms have the incentive to lie to workers to pay lower wages. Workers know this, so they do not accept wage cuts when the firm declares a bad shock. Therefore, the incentive-compatible lifetime promised utility features a state-uncontingent utility markup over the worker's outside value.

The state-uncontingency of labor contracts can be tested empirically. One implication or potential criticism is that it will cause inefficient separations. However, [Jäger, Schoefer and Zweimüller \(2019\)](#) show that separations are indeed inefficient in the data. Using a quasi-experimental repeal of an unemployment insurance extension in Austria, they do not find that surviving jobs display higher resilience. This implies that separated jobs are not simply the ones with the lowest surplus. In line with their conclusion of inefficient separations, my assumption of state-uncontingent labor contracts can also have similar implications.

With both types of incompleteness, my financial channel of wage commitments can improve a search model's performance in explaining unemployment. Moreover, there has been empirical evidence in the existing literature to support the interaction between finance and labor. [Favilukis, Lin and Zhao \(2020\)](#) find that wage growth and labor shares are the primary factors predicting credit spreads. [Donangelo et al. \(2019\)](#) use Census data and find that firms with higher labor shares are more sensitive to shocks. [Schoefer \(2021\)](#) shows that industries with higher labor shares are associated with more procyclical cash flows.

3 Quantitative Analysis

In this section, I first parametrize the model by matching moments. Then I explain the mechanism and show the critical connection between financial frictions and volatility shocks and unemployment volatility. Next, I apply the model to U.S. business cycles to see to what degree the model can explain unemployment dynamics during recessions. Finally, I conduct policy experiments to investigate the labor market policies of the U.S. and Germany during the recent Covid recession.

I use grid search to solve the problem numerically. The global solution is computationally tractable because of block recursivity.

3.1 Parameterization

There are four shocks (A, σ, z, ϵ) in the economy. The logs of aggregate productivity and volatility both follow AR(1) processes:

$$\log A_{t+1} = \rho_A \log A_t + \sigma_A \sqrt{1 - \rho_A^2} \epsilon_t^A, \quad (47)$$

$$\log \sigma_{t+1} = (1 - \rho_\sigma) \log \bar{\sigma} + \rho_\sigma \log \sigma_t + \sigma_\sigma \sqrt{1 - \rho_\sigma^2} \epsilon_t^\sigma, \quad (48)$$

where the innovations ϵ_t^A and ϵ_t^σ follow the standard normal distribution. I follow [Schaal \(2017\)](#) and allow ϵ_t^A and ϵ_t^σ to be correlated with the correlation coefficient $\rho_{A\sigma}$.

Firm j 's idiosyncratic productivity also follows an AR(1) process:

$$\log z_{jt+1} = \rho_z \log z_{jt} + \sigma_t \sqrt{1 - \rho_z^2} \epsilon_{jt}^z, \quad (49)$$

where ϵ_{jt}^z follows the standard normal distribution, and the volatility shock σ_t controls the standard deviations of the innovation.

The i.i.d. operating cost shock ϵ 's distribution, $\Phi(\cdot)$, is normally distributed with mean μ_ϵ and standard deviation σ_ϵ .

I follow [Menzio and Shi \(2010\)](#) and [Schaal \(2017\)](#) in using the following job finding probability function:

$$p(\theta) = \theta(1 + \theta^\gamma)^{-1/\gamma}. \quad (50)$$

Accordingly, the vacancy-filling rate $q(\theta)$ is $p(\theta)/\theta$.

I calibrate the parameters as closely as possible to [Schaal \(2017\)](#) for comparison. Table 1 shows the parameter values. The parameters in Panel A are exogenously assigned, following the literature. The quarterly discount factor β equals 0.988, corresponding to a 5% annual risk-free interest rate. The labor coefficient α is set as 0.66 to be consistent with the wage share. I follow [Khan and Thomas \(2008\)](#) to set the persistence of idiosyncratic productivity ρ_z as 0.95.

The remaining parameters in Panel B are calibrated by matching moments using U.S. data. Table 2 shows the moments in the data and the model. Because of the model's non-linearity, all parameters influence all moments jointly. However, each moment is primarily affected by certain parameters, and I organize them into four groups accordingly. The first two groups of parameters are related to aggregate shocks and the labor market, which are calibrated according to [Schaal \(2017\)](#). The other two groups of parameters, associated with the financial market and firm exit, are added upon [Schaal's \(2017\)](#) calibration for my financial channel.

The first set of parameters controls the AR(1) processes of aggregate shocks. For the aggregate productivity parameters (ρ_A, σ_A) , I use the autocorrelation and standard deviation of output as target moments. The data moments are calculated by [Schaal \(2017\)](#) using real GDP from the Bureau of Economic Analysis. He detrends the time series of output by an HP-filter with a parameter of 1,600 to obtain the log deviations.

To calibrate the process of volatility shocks, I follow [Bloom et al. \(2018\)](#) to use the interquartile range of sales growth rates across firms (IQR) to reflect the degree of volatility in the economy.

Table 1: Parameter Values

Parameters	Notation	Value	Source/Matched Moment
Panel A: Assigned Parameters			
Discount factor	β	0.988	5% annual interest rate
Decreasing returns to scale coefficient	α	0.66	Labor share
Persistence of productivity	ρ_z	0.95	Khan and Thomas (2008)
Panel B: Parameters from Moment Matching			
Aggregate shocks			
Persistence of aggregate productivity	ρ_A	0.920	Autocorrelation of output
SD of aggregate productivity	σ_A	0.024	SD of output
Mean of volatility	$\bar{\sigma}$	0.248	Mean of IQR
Persistence of volatility	ρ_σ	0.880	Autocorrelation of IQR
SD of volatility	σ_σ	0.092	SD of IQR
Correlation between ϵ_t^A and ϵ_t^σ	$\rho_{A\sigma}$	-0.020	Correlation (output, IQR)
Labor market			
Unemployment benefits	\bar{u}	0.142	EU rate
Vacancy posting cost	c	0.001	UE rate
Relative on-the-job search efficiency	λ	0.100	EE rate
Matching function elasticity	γ	1.600	ϵ_{UE}/θ
Entry cost	k_e	15.21	Entry/Total job creation
Mean operating cost	$\bar{w}_m + \mu_\epsilon$	0.001	Average establishment size
Financial market			
SD of production costs	σ_ϵ	0.080	Mean credit spread
Agency friction	$\tilde{\zeta}$	2.400	Median leverage
Auditing quality	ξ	1.780	Correlation (output, spreads)
Recovery rate	η	2.410	Correlation (IQR, spreads)
Exit			
Exogenous exit rate	π_d	0.021	Annual exit rate

Notes: Panel A shows parameters exogenously assigned. Panel B shows parameters calibrated to match the targeted data moments in Table 2.

I obtain the sales of firms from CRSP-Compustat, which provides balance sheet data of publicly listed firms. I use the Consumer Price Index for All Urban Consumers (CPI) to deflate sales. To avoid the composition of firms influencing the IQR, I follow [Bloom et al. \(2018\)](#) and use only firms with at least 100 quarters of observations. I also drop firms in the finance and public administration sector. I also follow [Davis and Haltiwanger \(1992\)](#) by measuring the sales growth rate at quarter t as $(y_t - y_{t-4})/((y_t + y_{t-4})/2)$, so growth rates are less affected by extreme values of sales. Next, because firms may respond heterogeneously to shocks in different industries, the IQR of the original sales growth rates may reflect not only the underlying volatility shocks but also heterogeneous responses. Therefore, I follow [Bloom et al. \(2018\)](#) and [Schaal \(2017\)](#) and measure volatility controlling for firms' permanent heterogeneity and industry heterogeneity over business cycles. Specifically, I project firms' sales growth on firm-level fixed effects and industry-quarter

Table 2: Matched Moments

		Benchmark Model		No Financial Frictions	
Moments	Data	$A + \sigma$	A	$A + \sigma$	A
Aggregate shocks					
Autocorrelation of output	0.839	0.868	0.877	0.840	0.870
SD of output	0.016	0.015	0.015	0.019	0.017
Mean of IQR	0.171	0.169	0.160	0.167	0.165
Autocorrelation of IQR	0.647	0.611	-	0.614	-
SD of IQR	0.013	0.011	-	0.012	-
Correlation (output, IQR)	-0.351	-0.305	-	-0.318	-
Labor market					
UE rate	0.834	0.814	0.817	0.845	0.824
EU rate	0.076	0.083	0.080	0.073	0.074
EE rate	0.085	0.081	0.082	0.085	0.082
$\epsilon_{UE/\theta}$	0.720	0.717	0.707	0.705	0.714
Average establishment size	15.6	15.4	15.3	15.0	15.6
Entry/Total job creation	0.21	0.18	0.18	0.17	0.17
Financial market					
Mean credit spread (%)	1.09	0.96	0.97	-	-
Median leverage (%)	26	21	21	-	-
Correlation (output, spreads)	-0.549	-0.503	-	-	-
Correlation (IQR, spreads)	0.462	0.448	-	-	-
Exit					
Annual exit rate (%)	8.9	9.0	9.2	9.0	9.1

Notes: This table shows the targeted data moments and moments matched by the benchmark model and the model without financial frictions. $A + \sigma$ means the model has both aggregate productivity shocks and volatility shocks, and A means the model only has aggregate productivity shocks. Table 6 reports the recalibrated parameters of the four models.

fixed effects to obtain residuals of sales growth⁸ and use these residuals as my measure of volatility to construct the IQR. Given the time series of IQR, I compute its mean, detrend the time series with an HP-filter, and compute the autocorrelation and standard deviations to serve as targets for the volatility shock parameters ($\mu_\sigma, \rho_\sigma, \sigma_\sigma$). I also use the correlation between output and IQR to pin down the correlation between aggregate productivity shocks and volatility shocks $\rho_{A\sigma}$. The output data is quarterly real GDP per capita from the Bureau of Economic Analysis, retrieved from FRED. It is detrended by the HP-filter with 1,600 as the parameters to obtain the log deviations.

The second group of parameters is related to the labor market. The unemployment utility \bar{u} is the opportunity cost of working, affecting wages and thus firms' firing decision; the vacancy posting cost c primarily affects firms' hiring decisions; and the relative on-the-job search efficiency λ influences the probability of job-to-job transitions. I calibrate these three parameters using

⁸ Firm industry is based on the Standard Industrial Classification (SIC) at the 3-digit level.

the transition probability from employment to unemployment (EU), the transition probability from unemployment to employment (UE), and the transition probability from employment to employment (EE). The data moments for EU, UE, and EE are the quarterly versions of the monthly ones in [Schaal \(2017\)](#), who obtains the monthly EU and UE rates from [Shimer \(2005\)](#) and the EE rate from [Nagypál \(2007\)](#). The matching function elasticity γ is calibrated to match the elasticity of UE rates to the labor market intensity θ , which [Schaal \(2017\)](#) obtains from [Shimer \(2005\)](#). The entry cost k_e is calibrated to match the share of jobs created by entrants, which is calculated by [Schaal \(2017\)](#) using Business Employment Dynamics (BED). The mean operating cost affects firms' exit decisions and thus can be pinned down by the average establishment size, measured by [Schaal \(2017\)](#) using the 2002 Economic Census. Notice that the mean operating cost μ_e and the manager's wage \bar{w}_m symmetrically influence firms' cash on hand, so I calibrate $\mu_e + \bar{w}_m$ using the average establishment size.

Parameters in the third group deal with the financial market. First, I use the average credit spread to calibrate the standard deviation of the operating cost, σ_e . The credit spread is the difference between the yield on Baa and Aaa corporate bonds. The data source is Moody's, retrieved from FRED, Federal Reserve Bank of St. Louis. Correspondingly, the credit spread in the model is the annualized difference between the actual borrowing cost and the risk-free interest rate:

$$\frac{1}{Q(v, z, b', n')} - \frac{1}{\beta}. \quad (51)$$

Because the agency friction constraint incentivizes firms to borrow, I use the median leverage of firms to calibrate the agency friction parameter $\tilde{\zeta} \equiv \zeta / (\bar{w}_m + (1 - \lambda) \frac{\beta}{1 - \beta} \bar{w}_m)$. Leverage is the ratio of the firm's total debt to its annualized sales. The data moment of median leverage is from [Arellano, Bai and Kehoe \(2019\)](#). Next, I use the correlation between output and credit spreads to parameterize the auditing technology ξ , and I use the correlation between IQR and credit spreads for the recovery rate η . Targeting the two correlations anchors the effects of aggregate productivity shocks and volatility shocks through the financial channel.

The last parameter is the exogenous exit rate π_d , which helps generate exits beyond defaults. I use the annual exit rate calculated from Business Dynamics Statistics (BDS) to calibrate π_d .

3.2 Differences from the Calibration of [Schaal \(2017\)](#)

My parametrization is based on [Schaal \(2017\)](#) when estimating parameters related to aggregate shocks and the labor market. I follow his calibration closely except for the following three differences.

First, [Schaal \(2017\)](#) uses a monthly frequency, while my model's frequency is quarterly. I choose the quarterly frequency to accommodate the data moments related to the financial market. As

is common in the finance literature, leverage should be one of the target moments, defined as a firm's debt over annualized sales. In a quarterly model, annualized sales in the denominator equal four times the quarterly sales. However, suppose the model is monthly. Annualized sales in the denominator will be 12 times the monthly sales. Therefore, when targeting the same median leverage in the data, the monthly model implies the firm's debt is much higher than its per-period sales, and the default risks will be counterfactually high. Thus, I follow the finance literature and use a quarterly model.

Second, [Schaal \(2017\)](#) uses 0.85 as the decreasing returns to scale coefficient α , and I use 0.66. Neither of us explicitly models capital, while [Schaal \(2017\)](#) chooses 0.85 to approximate the total decreasing returns. But he also points out that the results are unaffected when targeting a labor share of 0.66. Because my mechanism is about wage commitments, I choose to target the wage share so that the size of firm commitments is consistent with the data. If I used 0.85 as the decreasing returns to scale coefficient, wage commitments would be larger, increasing the risk to firms and generating counterfactually high credit spreads.

Third, to calibrate the volatility shock process, [Schaal \(2017\)](#) uses the interquartile range (IQR) of innovations to idiosyncratic productivity calculated by [Bloom et al. \(2018\)](#). Instead, I follow both [Bloom et al. \(2018\)](#) and [Arellano, Bai and Kehoe \(2019\)](#) and use the IQR of firms' sales growth rates. I make this deviation because targeting the IQR of innovations to idiosyncratic productivity leads to a counterfactually high sales volatility. Specifically, the IQR of sales growth in the model will be more than five times the data. Because sales volatility determines firm default probability, the counterfactually high volatility of sales leads to counterfactually large default rates and extremely high credit spreads. To keep the magnitude of financial effects reasonable, I use the IQR of firms' sales growth rates as in [Arellano, Bai and Kehoe \(2019\)](#), who also model volatility shocks and financial frictions simultaneously. The main difference between using the IQR of firms' sales growth rates and the IQR of idiosyncratic productivity innovations is the level of volatility $\bar{\sigma}$.⁹ But, they have very similar business cycle behaviors in terms of innovations to volatility, i.e., ϵ_t^σ . In particular, Figure 7 compares the log deviations of estimated aggregate productivity shocks and volatility shocks of the model without financial frictions with [Schaal \(2017\)](#), showing that the two volatility shocks have similar variations over business cycles.

As a validation of this calibration choice, Table 3 shows that my counterfactual model without financial frictions has very similar business cycle statistics to [Schaal \(2017\)](#). Further, Figure 5 displays the changes in unemployment during recessions, and the model without financial frictions also yields very similar patterns to [Schaal \(2017\)](#). Therefore, our divergence in calibration is not the main reason for our different quantitative performances.

⁹ One concern about the idiosyncratic productivity measured by [Bloom et al. \(2018\)](#) is that they use revenue TFP, which can reflect firm pricing power instead of productivity ([Bils, Klenow and Ruane, 2021](#); [Hsieh and Klenow, 2009](#)).

Table 3: Business Cycle Statistics

	Y	Y/L	U	V	Hirings	Quits	Layoffs	Wages
Panel A: Data								
Std Dev.	0.016	0.012	0.121	0.138	0.058	0.102	0.059	0.008
cor(Y,x)	1	0.590	-0.859	0.720	0.677	0.720	-0.462	0.555
Panel B: Benchmark Model								
<i>Both A and σ Shocks</i>								
Std Dev.	0.015	0.013	0.106	0.097	0.048	0.029	0.111	0.011
cor(Y,x)	1	0.910	-0.500	0.774	0.140	0.884	-0.202	0.876
<i>Only A Shock</i>								
Std Dev.	0.015	0.011	0.079	0.081	0.019	0.028	0.053	0.010
cor(Y,x)	1	0.988	-0.901	0.904	0.010	0.964	-0.853	0.980
Panel C: Model Without Financial Frictions								
<i>Both A and σ Shocks</i>								
Std Dev.	0.019	0.016	0.088	0.084	0.038	0.026	0.069	0.014
cor(Y,x)	1	0.986	-0.781	0.826	0.051	0.919	-0.569	0.964
<i>Only A Shock</i>								
Std Dev.	0.017	0.014	0.069	0.064	0.023	0.023	0.047	0.012
cor(Y,x)	1	0.994	-0.876	0.906	-0.024	0.975	-0.787	0.985

Notes: Panel A shows the business cycle moments in the data. Panels B and C report moments of 3,000-quarter simulations of the benchmark model and the model without financial frictions, with and without volatility shocks. Both the data and the model simulations are log-detrended by the HP filter with smoothing parameter 1600. To be consistent with [Schaal \(2017\)](#), Y denotes output, Y/L is output per worker, U represents unemployment, and V is vacancies.

3.3 Business Cycle Statistics

To assess how well my model can explain business cycles, I report simulated business cycle statistics in Table 3. To compute the moments, I simulate the model for 3,000 quarters and use the log deviations from an HP-filter trend with a smoothing parameter of 1,600. Beyond the benchmark model, I consider three alternative models for comparison. All models are recalibrated by matching the same moments. Table 2 contains the calibration results and Table 6 reports the recalibrated parameters.

Since all models are calibrated, they have similar predictions for output and labor productivity in the first two columns, and I will focus on their differences in terms of unemployment volatility in brief. The next section will investigate the mechanism in greater detail.

Benchmark Model With Both Shocks. The standard deviation of unemployment is 0.121 in the data (Panel A), and my benchmark model with both aggregate productivity shocks and volatility shocks can generate a standard deviation of 0.106 (Panel B), which is 10 percent higher than the 0.090 in [Schaal \(2017\)](#). Next, I use three alternative models to explain the roles of both financial frictions and volatility shocks in my model's quantitative performance.

Benchmark Model With Only Aggregate Productivity Shocks. The second part of Panel B calibrates the same model but keeps only the aggregate productivity shocks, i.e., the volatility of firms' idiosyncratic productivity is no longer time-varying. Now the model only generates a standard deviation of unemployment of 0.079. This finding implies that volatility shocks are necessary for the financial channel's effectiveness. Distinguished from aggregate productivity shocks, the offsetting effect of equilibrium wages is much smaller for volatility shocks. High volatility spreads the distribution of firms' idiosyncratic productivity, and high productivity firms may still benefit from elevated volatility, which props up wages. This is called as the Oi-Hartman-Abel effect (Oi (1961), Hartman (1972), Abel (1983)) in the volatility literature. Therefore, though wages still decline when volatility is high, the decline is much smaller than when it is a negative aggregate productivity shock.

Model Without Financial Frictions and With Both Shocks. The first part of Panel C shows the results when the model does not have financial frictions but has both shocks. It generates a 0.088 standard deviation of unemployment, in line with the 0.090 in Schaal (2017). Comparing it with the 0.106 of the benchmark model reveals the important role of financial frictions in the model. Without financial frictions, my model is similar to Schaal (2017). Adding the financial channel amplifies the effect of aggregate shocks on unemployment and generates more unemployment volatility.

Model Without Financial Frictions and With Only Aggregate Productivity Shocks. I show the model statistics without financial frictions and with only the aggregate productivity shock in the second part of Panel C. It generates a standard deviation of unemployment of 0.069, consistent with the 0.067 in the corresponding model of Schaal (2017). This number is similar to the benchmark model with only the aggregate productivity shock, and confirms the complementarity between volatility shocks and the financial channel of wage commitments.

3.4 Inspecting the Mechanism

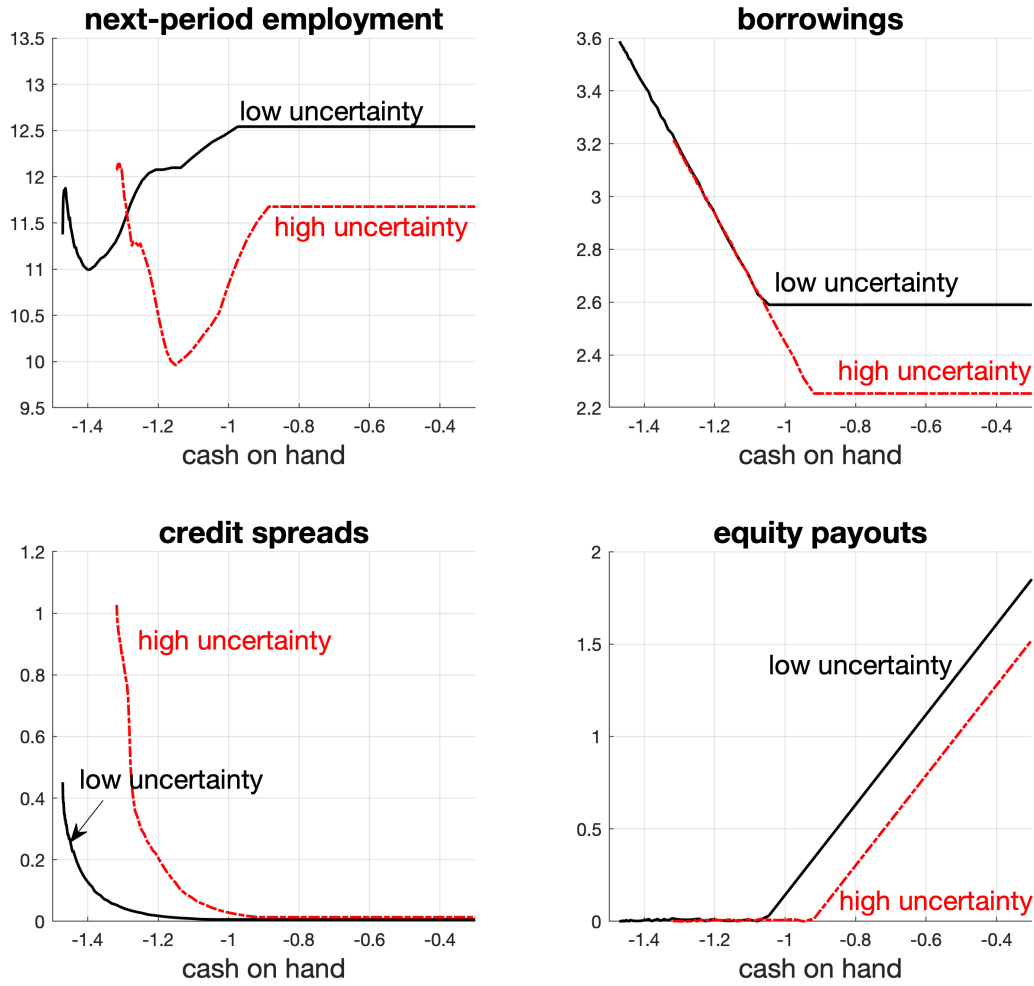
In this section, I explain the mechanism in my model. First, I show how the interaction between the financial channel of wage commitments and volatility shocks works. Next, I illustrate the importance of the financial channel by comparing it with the recalibrated model without financial frictions. Finally, I explain why the mechanism needs to interact with volatility shocks.

3.4.1 The Financial Channel of Wage Commitments

I use the firm's decision rules to explain how high volatility leads firms to cut employment due to financial concerns over wage payments.

Figure 2 shows how firms' decisions depend on cash on hand X and the level of volatility. I

Figure 2: Decision Rules with Different Volatilities



Notes: The four panels plot a median firm's decision rules for next-period employment, borrowing, credit spread, and equity payouts with respect to cash on hand. The solid black lines are for the low volatility state, and the dash-dot red lines are for the high volatility state. Aggregate productivity is set at the high level. "High" or "low" means one unconditional standard deviation above or below the mean. The firm's idiosyncratic productivity and current employment are fixed at their median levels.

vary the cash on hand on the horizontal axis and fix the firm's idiosyncratic productivity and employment at their median levels. The decision rules are next-period employment n' , borrowing Qb' , credit spread $1/Q - 1/\beta$, and equity payouts Δ . The solid black lines are for the low volatility state, one unconditional standard deviation below the mean volatility. The dash-dot red lines are for the high volatility state, one unconditional standard deviation higher than the mean. The decision rules are in line with the patterns in [Arellano, Bai and Kehoe \(2019\)](#) because both models have similar financial channels.

First, the relations between the decision rules and cash on hand are consistent with Lemma 2.2. When cash on hand is higher than a cutoff, firms' employment, borrowing, and credit spreads

no longer depend on cash on hand. The equity payouts increase with cash on hand one for one in this case. When below the cutoff, the equity payout is zero because the non-negative equity payout constraint binds.¹⁰ As cash on hand decreases, firms need to borrow more to satisfy the non-negative equity payout constraint. Credit spreads subsequently rise. Employment for the next period decreases as cash on hand decreases because firms face higher default risks and cut employment to avoid defaulting on wage payments next period. But when cash on hand is very low, firms hire slightly more workers. The reason is the increased default probability. Conditional on survival, firms have higher expected productivity. Thus, firms decide to take on more risk and hire more workers.

Second, firms' decisions depend on the level of volatility. Higher volatility implies greater default risks because of the larger probability of drawing low productivity, resulting in higher credit spreads. Therefore, firms are tentative about borrowing and equity payouts. For a similar reason, firms also decrease the number of employees in high volatility states. Intuitively, firms make employment decisions, i.e., search, matching, and separation, in the labor market before knowing tomorrow's productivity. The committed wage payments are like debts in this sense, which firms have to pay or default in the next period. If the firm draws low productivity next period, wages can be too high compared to sales, and firms may default and be liquidated. Because of these concerns over meeting payroll, firms are precautionary in making commitments during high volatility periods. Low demand leads to higher unemployment. This is the key mechanism in my model.

3.4.2 Role of Financial Frictions

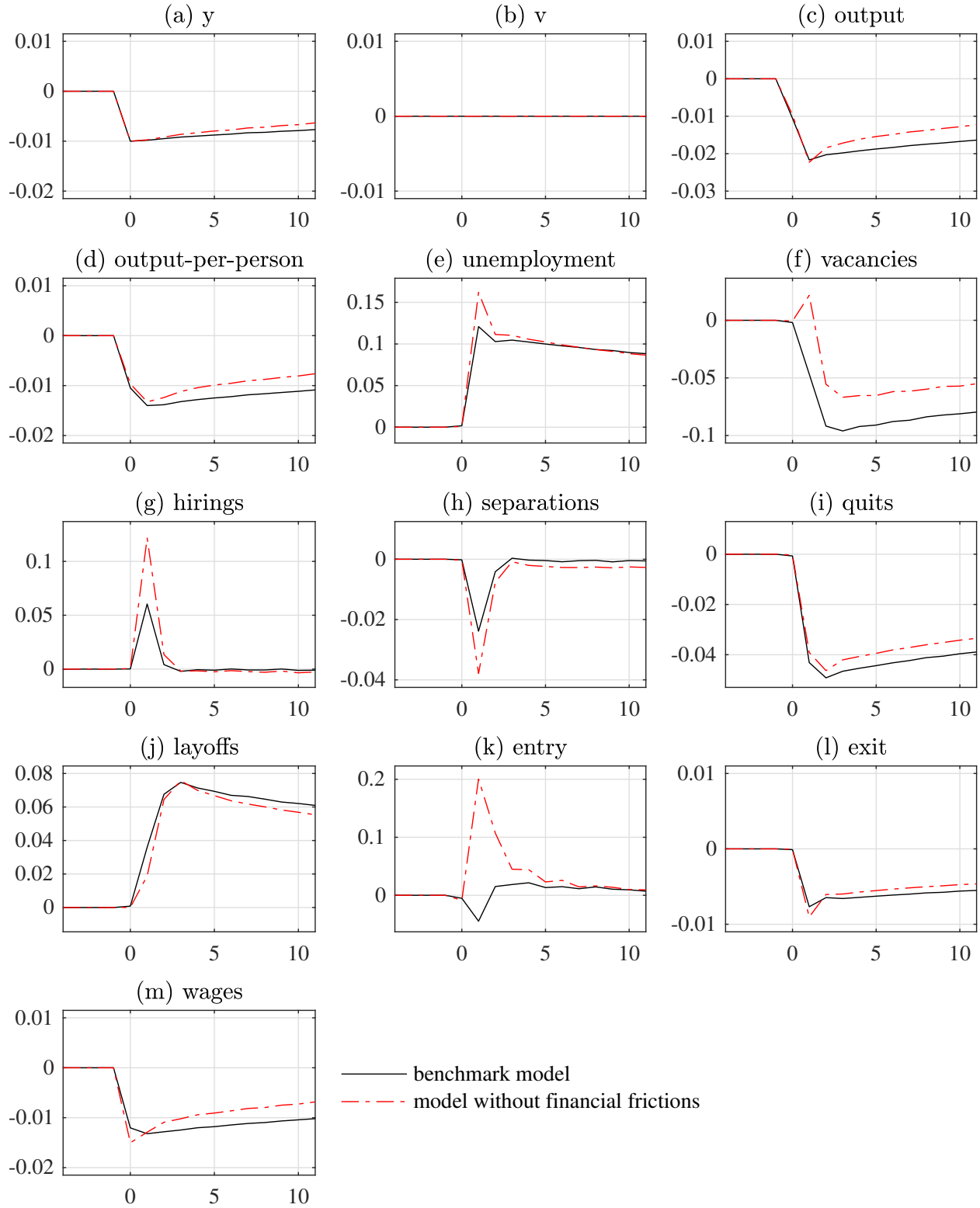
The model contains four sources of financial frictions. First, firms only have access to state-uncontingent debt to finance operations. Otherwise, firms would be able to allocate assets across states, repay less in bad states, and never default. Second, firms can default on their debts, and the debt prices reflect firms' default probabilities. Third, negative external equity payouts are not allowed, i.e., firms cannot use equity issuance to get rid of financial constraint. Fourth, firms are subject to agency frictions, so they cannot build up a savings buffer to grow out of default risks.

To understand the role of financial frictions in the model, I shut down all the financial incompleteness in a reference model. In this case, the financial market is complete with state-contingent assets and perfect enforcement of debts. Firms are not subject to the non-negative equity payout constraint. The monitor is perfect, so there is no agency problem. This model without financial frictions resembles [Schaal \(2017\)](#), who studies the effect of volatility shocks on unemployment under search and matching frictions.

I recalibrate the model to match the targeted moments except for the financial variables. Columns

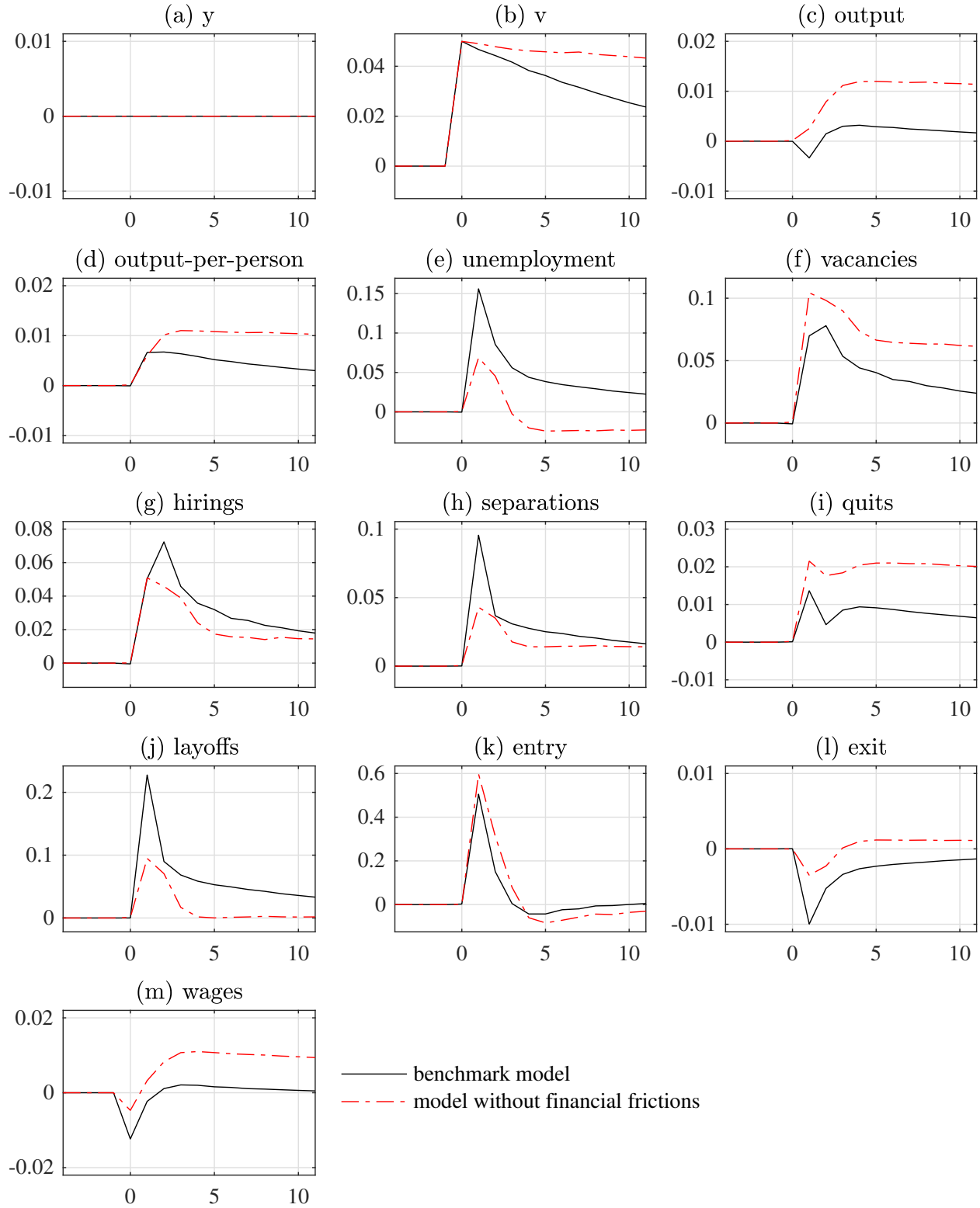
¹⁰ The slight difference between zero is due to computational errors of grid search.

Figure 3: Aggregate Impulse Responses to a 1% Negative Aggregate Productivity Shock



Notes: The panels are impulse responses to a 1% transitory negative aggregate productivity shock at quarter 0. The impulse responses are the average of 4,000 simulated paths, presented as log deviations from the mean. Solid black lines are the benchmark results. Dash-dot red lines are for the model without financial frictions. I use [Schaal's \(2017\)](#) code when plotting.

Figure 4: Aggregate Impulse Responses to a 5% Positive Volatility Shock



Notes: The panels are impulse responses to a 5% positive volatility shock at quarter 0. The impulse responses are the average of 4,000 simulated paths, presented as log deviations from the mean. Solid black lines are the benchmark results. Dash-dot red lines are for the model without financial frictions. I use [Schaal's \(2017\)](#) code when plotting.

"No Financial Frictions, $A + \sigma$ " of Table 2 and Table 6 show the results of the moment matching and the values of parameters, respectively. Because there are no financial variables to pin down the standard deviation of operating costs, I use the same σ_ϵ as in the benchmark. Financial parameters, including the agency friction $\tilde{\zeta}$, auditing quality ξ , and recovery rate η , are not applicable in this case.

The recalibrated parameters in Table 6 suggest that the standard deviations of both shocks need to increase to match the variations of aggregate output and IQR in the data, in particular the standard deviation of aggregate productivity shocks. The need for larger standard deviations of shocks for the model without financial frictions suggests that financial frictions amplify the effect of shocks.

I use the impulse responses in Figures 3 and 4 to illustrate how financial frictions affect aggregate outcomes.

A Negative Aggregate Productivity Shock. Figure 3 plots the impulse responses to a negative aggregate productivity shock. To draw the impulse responses, I simulate the economy's distribution 4,000 times with stochastic aggregate shocks. At quarter 0, I impose a 1% negative aggregate productivity shock. Then I let the economy evolve stochastically again. The impulse responses in Figure 3 are the average of the 4,000 simulated paths. The solid black lines are from the benchmark model, and the dash-dot red lines are without financial frictions.

For the benchmark model in solid black lines, a 1% negative aggregate productivity shock leads to a 2% decline in output and 10% higher unemployment.

The dash-dot red lines show the results when there are no financial frictions, showing that the changes in output and unemployment are similar to in the benchmark. This finding implies that the financial channel primarily operates through volatility shocks. The reason is that wages decline more with financial frictions (Panel (m)), which offsets the negative impact of lower aggregate productivity.

A Positive Volatility Shock. Figure 4 displays the impulse responses following a 5% positive volatility shock. The methodology to draw the impulse responses is the same, except I shock the simulations with a 5% positive volatility shock at quarter 0.

In the benchmark model, a 5% positive volatility shock lowers output slightly and raises unemployment by 15%, while the model without financial frictions generates an output boom and much less of an increase in unemployment. This result explains why Schaal (2017) finds it difficult for a search model to generate a sufficient increase in unemployment during the Great Recession. In his model, the effect of volatility on unemployment is mainly through the reallocation of workers across firms. My work builds on his by incorporating a financial channel of wage commitments into the model, which interacts with volatility shocks and improves the model's ability to explain

unemployment fluctuations.

3.4.3 Role of Volatility Shocks

Recall that Table 3 shows that with only the aggregate productivity shock, the standard deviation of unemployment is significantly smaller. In this section, I explain why the financial channel of wage commitments is particularly influential when interacting with volatility shocks instead of aggregate productivity shocks.

In the last panels of Figures 3 and 4, I show the impulse responses of wages. The decline of wages is much larger in response to a negative aggregate productivity shock than a volatility shock.¹¹ Because the lower wages offset the effect of the negative aggregate productivity shock, the financial channel of wage commitments does not amplify unemployment volatility much. The offsetting equilibrium effect of wages is also the reason for the unemployment volatility puzzle in Shimer (2005). He finds that the calibrated standard Diamond-Mortensen-Pissarides model generates less than 10% of the observed standard deviation of unemployment. In that model, because of the free entry condition, the decline in wages largely absorbs the effect of aggregate productivity shocks. Similarly, the free entry condition in my model also leads to a large decrease in wages to offset the impact of aggregate productivity shocks.

Nonetheless, the offsetting effect of wage dynamics is smaller for volatility shocks. The reason why wages do not decrease much is due to a property of volatility shocks. A positive volatility shock spreads the distribution of firm-level productivity. Since a firm's profit is convex in terms of its idiosyncratic productivity, a wider distribution delivers a higher expected profit. The volatility literature calls this property the Oi-Hartman-Abel effect (Oi (1961), Hartman (1972), Abel (1983)). The Oi-Hartman-Abel effect is stronger for firms with high productivity because firm productivity is persistent. These firms' high expected values means that wages do not need to decrease much to satisfy the free entry condition. Since the equilibrium wage is not low enough for a median firm to offset the higher risk of drawing low idiosyncratic productivity, it hires less. At the aggregate level, unemployment increases.

Therefore, the financial channel of wage commitment needs to interact with volatility shocks to improve the model's prediction of unemployment volatility significantly. This finding provides a new way to understand the fluctuations of unemployment. Unlike a typical search model with only aggregate productivity shocks and homogeneous firms, I argue that volatility shocks are crucial to understanding unemployment because firms have limited ability to hedge against idiosyncratic risks.

¹¹ Actually, both the wages of incumbent workers and newly hired workers increase when volatility increases. The average wage in Figure 4 decreases because the share of newly hired workers increases, who have lower wages.

3.5 Event Study for U.S. Recessions

The preceding section shows how my model can generate most of the standard deviation of unemployment in the data. In this section, I apply my model to U.S. recessions in particular.

To leverage the particle filter, I first obtain the mapping between a set of state variables and simulated observables in the spirit of [Krusell and Smith \(1998\)](#). This step approximates my infinite-dimensional heterogeneous agent model with a finite-dimensional non-linear state-space model, making it possible to estimate the historical aggregate shocks using the particle filter. The set of state variables include log aggregate productivity A and log volatility v up to five-quarter lags, their interactions, and their respective interactions with credit spreads. I project the aggregate variables on the state variables to obtain a mapping between observables and the underlying states. The aggregate variables of interest are aggregate output, the interquartile range of firm sales growth (IQR), and unemployment. The R^2 of the mapping between the state variables and output, IQR, and unemployment are 0.999998, 0.9997, and 0.99998, respectively. The high R^2 indicates that the state variables can forecast the observables well. That is, the state-space mapping successfully captures the relation between the underlying shocks and observables, which facilitates estimating the shocks from data in the next step.

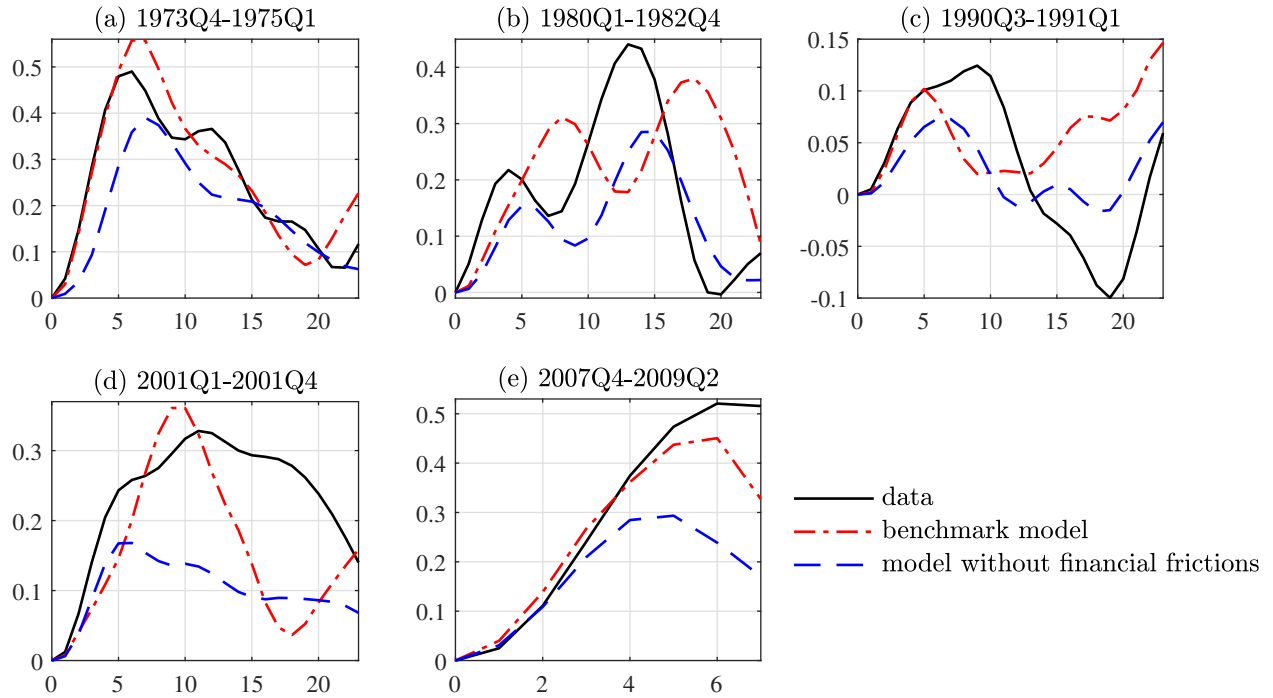
To estimate the underlying aggregate productivity shocks and volatility shocks over U.S. business cycles, I apply the particle filter to U.S. data, following the approach in [Bocola and Dovis \(2019\)](#). And as [Schaal \(2017\)](#), I use band-pass filtered log real GDP per capita and IQR to jointly smooth out the underlying shocks. This delivers the series of estimated shocks $\{\hat{\epsilon}_t^A, \hat{\epsilon}_t^\sigma\}$. Next, I let the model to predict unemployment by feeding the estimated shocks.

Figure 5 shows the peak-to-trough log deviations of unemployment in recessions since 1973. Panel A shows the results of the models with both aggregate productivity shocks, A and uncertainty shocks, σ . The black lines are the data and the dash-dotted red lines are the predictions of the benchmark model. It is clear that the benchmark model can generate a great share of the increase in unemployment during recessions. Notably, it can account for about 90% of the Great Recession.

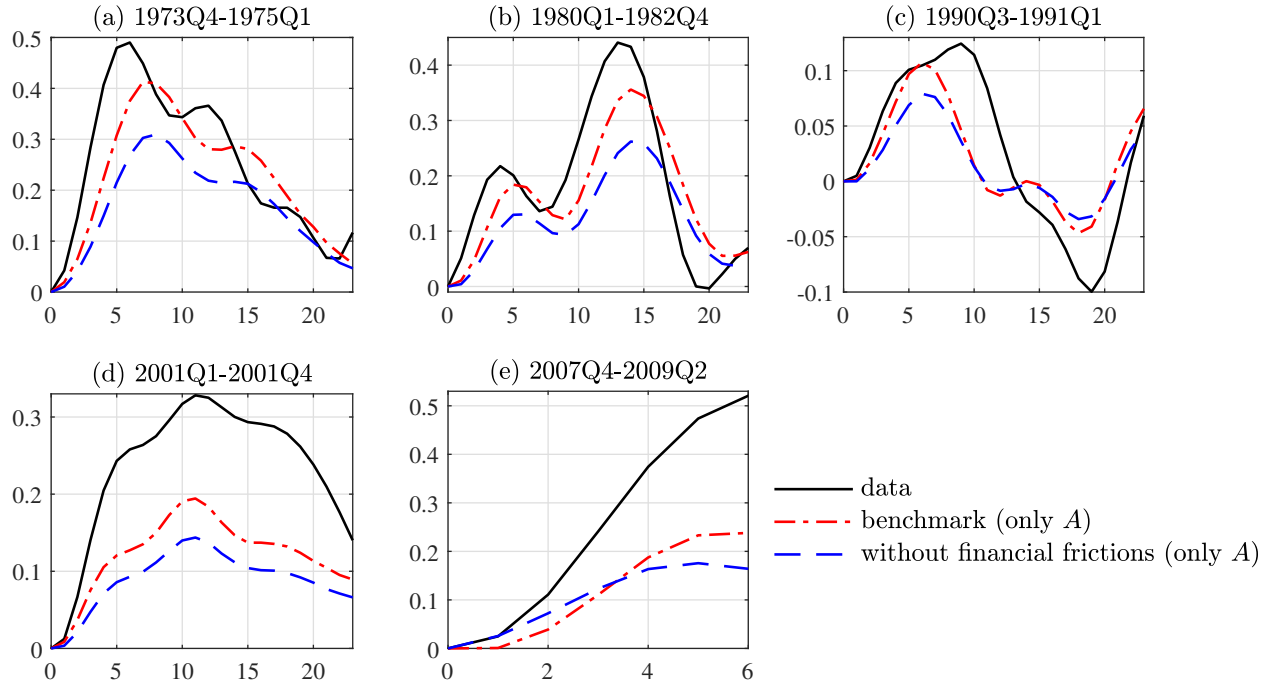
To understand the role of financial frictions, I also show the predictions of the counterfactual model without financial frictions using dashed blue lines. In line with the business cycle statistics in Table 3, the model without financial frictions generates less of an increase in unemployment during recessions. Moreover, the predictions for the early 2000s recession and the Great Recession particularly deteriorate. The reason is that the two recessions feature large increases in volatility measured as the interquartile firms' sales growth rates. At the same time, their aggregate output declines are smaller than the recession in the 70s and 80s. Therefore, the estimated volatility shocks play a bigger role than the aggregate productivity shocks during these two recessions. Given that the financial channel of wage commitments is particularly influential when interacting with volatility shocks, the model without financial frictions underestimates the impact of volatility

Figure 5: Unemployment Series With and Without Modeling Financial Frictions

Panel A: Models With Both Aggregate Productivity Shocks and Uncertainty Shocks



Panel B: Models With Only Aggregate Productivity Shocks



Notes: The panels show the model's predictions for unemployment during recessions. Models in Panel A have both aggregate productivity shocks, A , and uncertainty shocks, σ . I use the particle filter to jointly estimate the time series of aggregate productivity shocks and uncertainty shocks by matching output and the IQR of firm sales growth in the data. The data are detrended by a band-pass filter to focus on fluctuations between 6 and 32 quarters, following [Schaal \(2017\)](#). Models in Panel B have only aggregate productivity shocks. Given the estimated shocks, I show the model-predicted unemployment. The data on unemployment is the solid black lines. The unemployment fluctuations predicted by the benchmark model are the dash-dotted red lines, and model unemployment without financial frictions is the dashed blue line. Series are depicted in terms of log deviations from the peak preceding the recession. I use [Schaal's \(2017\)](#) code when plotting.

Table 4: Peak-To-Trough Changes During Recessions

	1973-1975	1980-1982	1990-1991	2001	2007-2009
Output					
Data	-0.082	-0.069	-0.021	-0.038	-0.048
Both A and σ shocks					
Benchmark model	-0.081	-0.068	-0.021	-0.036	-0.047
Model without financial frictions	-0.078	-0.067	-0.020	-0.037	-0.045
Only A shock					
Benchmark model	-0.082	-0.069	-0.021	-0.038	-0.048
Model without financial frictions	-0.081	-0.068	-0.021	-0.038	-0.047
Unemployment					
Data	0.490	0.441	0.124	0.328	0.521
Both A and σ shocks					
Benchmark model	0.577	0.380	0.147	0.361	0.451
Model without financial frictions	0.390	0.285	0.073	0.168	0.294
Only A shock					
Benchmark model	0.413	0.356	0.107	0.194	0.238
Model without financial frictions	0.309	0.262	0.079	0.144	0.176

Notes: The table shows the peak-to-trough changes in output and unemployment during recessions for both the data and models. I use the particle filter to jointly estimate the time series of aggregate productivity shocks and volatility shocks by matching output and the IQR of firm sales growth in the data. The data are detrended by a band-pass filter to focus on fluctuations between 6 and 32 quarters, following [Schaal \(2017\)](#). Given the estimated shocks, I show the peak-to-trough changes predicted by the model during recessions. Consistent with [Schaal \(2017\)](#), series are depicted in terms of log deviations from the peak preceding the recession.

shocks. Thus, it substantially underpredicts the increase in unemployment for the early 2000s recession and the Great Recession, while its predictions for the other recessions are less affected.

Panel B displays the results of the model with only aggregate productivity shocks. It shows that the model's performance in explaining recessions deteriorate in general. Furthermore, the difference between with and without financial frictions shrinks a lot, implying that the amplification effect of financial frictions on unemployment is much smaller for aggregate productivity. As you can see, as for aggregate productivity shocks alone, the amplification effect of financial frictions is very limited. The reason is similar to [Shimer's \(2005\)](#) puzzle. In the equilibrium, according to the free entry condition, wages decrease a lot to absorb the effect of negative aggregate productivity shocks, which are common to all firms. However, this offsetting effect of wages is much smaller for uncertainty shocks because high volatility spreads the distribution of firms' idiosyncratic productivity. And the probability of drawing high productivity also increases, which makes wages stay at a high level. Therefore, my financial channel of wage commitments is effective when interacting with uncertainty shocks.

Table 4 reports the peak-to-trough changes in output and unemployment during recessions. Because I estimate the underlying shocks by matching output, both the benchmark model and the

model without financial frictions have similar output declines as in the data. As for unemployment, considering financial frictions improves the model's ability to generate elevated unemployment in recessions. In particular, during the Great Recession, unemployment increased by 50%. The model without financial frictions can only explain about half of this rise, while the benchmark model can generate 90% of the increase in unemployment. Moreover, the results with only aggregate productivity shocks also deteriorate greatly, confirming the finding by [Schaal \(2017\)](#) that uncertainty shocks are crucial for understating the fluctuations in unemployment.

My result builds on [Schaal \(2017\)](#), who finds that a search model cannot replicate unemployment during the Great Recession. He also conjectures that adding financial frictions is a possible way to improve the model's performance. My model confirms that adding financial frictions does go a long way to make the model better account for the Great Recession.

3.6 Policy Implications

Given the quantitative performance of my model, I apply it to the labor market policies of the U.S. and Germany during the recent Covid recession. I also discuss how the measured effects of these policies are biased when financial frictions are not considered.

U.S. Policy. In the recent 2020 Covid-19 pandemic, the U.S. market uncertainty increased dramatically. Specifically, [Altig et al. \(2020\)](#) show that business executives are much more uncertain about their firms' future sales growth rates during the COVID-19 pandemic, according to the U.S. monthly panel Survey of Business Uncertainty (SBU) and the U.K. monthly Decision Maker Panel (DMP). At the same time, the U.S. government deployed economic support policies. One notable response was the U.S. Federal Pandemic Unemployment Compensation (FPUC) program, which increased unemployment benefits by an extra \$600 per week.

To figure out the aggregate impacts of this policy, I modify my model to incorporate it. Specifically, I model that the government increases unemployment benefits by 1% when volatility is high. Given the policy, I re-solve the model quantitatively. So, the policy is anticipated by the agents in the economy. For simplicity, I assume that the government collects tax revenue through a lump-sum tax, and this policy costs 4.81 basis points of output in the simulation.

Table 5 summarizes the impact of the U.S. policy on business cycles. Panel A displays the policy and its costs as a share of output. And Panel B compares the model-simulated moments of the benchmark model and the model with the U.S. policy. It is clear that raising unemployment benefits has negative impacts through the model's labor demand channel. In particular, output decreases by 0.41%, unemployment increases by 0.39 percentage points, the standard deviation of unemployment increases by 16%, and welfare decreases by 3.9 basis points. The reason is that the increased unemployment benefits distort the labor market by pushing wages higher by around

Table 5: The Impact of Labor Market Policies on Economic Performance

	Without Policy	U.S. Policy	German Policy
Panel A: Policies			
Increase in unemployment benefits	-	1%	-
The replacement rate of wage subsidies	-	-	84.4%
The ratio of policy costs to output	-	4.81‰	4.86‰
Panel B: Economic Performance			
<i>Benchmark Model</i>			
Mean of output	100	99.593	99.938
SD of output	0.015	0.015	0.015
Mean of unemployment (%)	5.823	6.210	5.804
SD of unemployment	0.106	0.123	0.104
Mean of average wages	100	100.061	100.014
SD of average wages	0.011	0.011	0.011
UE rate	0.814	0.799	0.814
EU rate	0.083	0.085	0.083
EE rate	0.081	0.080	0.081
Mean credit spread (%)	0.96	0.96	0.97
Median leverage (%)	21	21	21
Annual exit rate (%)	9.0	9.0	9.0
Welfare	100	99.961	99.976
<i>Model Without Financial Frictions</i>			
Mean of output	100	99.951	99.985
SD of output	0.019	0.019	0.019
Mean of unemployment	4.834	4.863	4.806
SD of unemployment	0.088	0.089	0.086
Mean of average wages	100	100.017	100.008
SD of average wages	0.014	0.014	0.014
UE rate	0.845	0.844	0.845
EU rate	0.073	0.073	0.072
EE rate	0.085	0.084	0.085
Annual exit rate (%)	9.0	9.0	9.0
Welfare	100	99.998	99.996

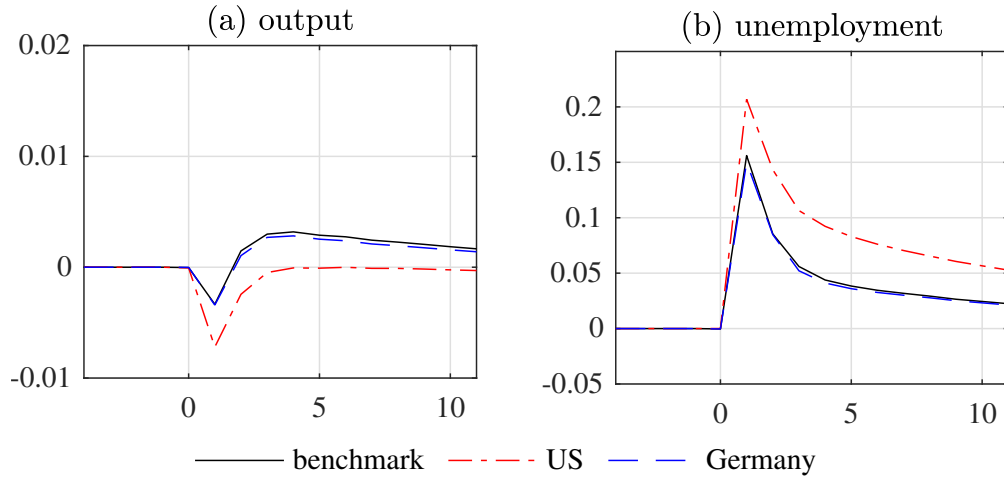
Notes: The table shows the model-simulated moments without and with the labor market policies. Panel A specifies the policies, and Panel B displays the moments of 3,000-quarter simulations of the benchmark model and the model without financial frictions. Given each policy, I re-solve the model. Therefore, policies are anticipated by the agents in the economy. The output, average wages, and welfare levels are normalized to 100 for the two models without policy. The standard deviations of output, unemployment, and average wages use the log deviations from the HP-filter with parameter 1,600 as before.

6.1 basis points in the simulation. Therefore, firms hire fewer workers, making it harder for the unemployed to find jobs. Specifically, the policy decreases the unemployment-to-employment transition rate by 1.5 percentage points. Overall, since the U.S. policy distorts the economy by increasing the marginal cost of the labor force, aggregate output and welfare decrease.

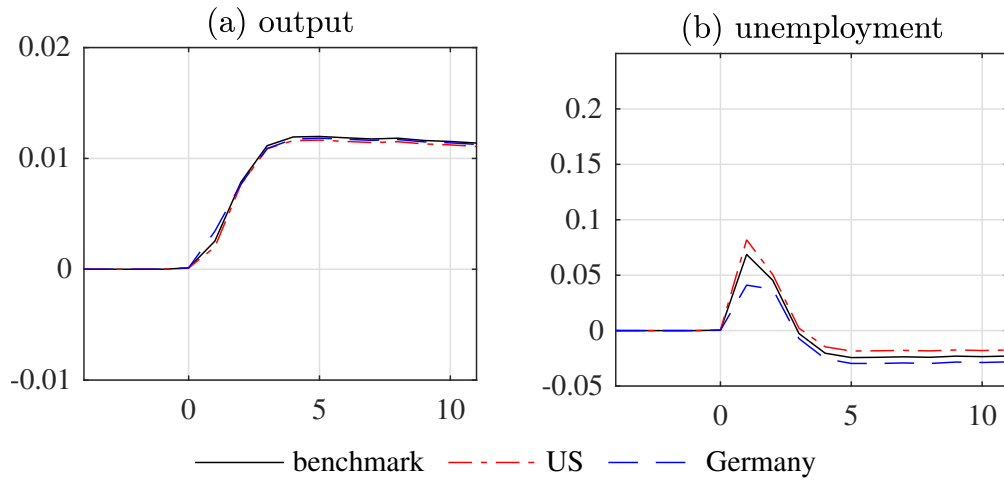
Next, Panel A of Figure 6 shows the impulse responses to a 5% positive volatility shock. The

Figure 6: Policy Implications of a Transitory Positive Volatility Shock

Panel A: Benchmark Model



Panel B: Model Without Financial Frictions



Notes: The panels are impulse responses to a 5% positive volatility shock at quarter 0. The impulse responses are the average of 4,000 simulated paths, presented as log deviations from the mean. Solid black lines are the benchmark results. Dash-dot red lines are for the model with enhanced U.S. unemployment benefits. Dashed blue lines are for the model with German wage subsidies. I use [Schaal's \(2017\)](#) code when plotting.

solid black lines are the results of the benchmark model as shown in Figure 4, and the dashed red lines are for the model with the U.S. policy, which raises unemployment benefits by 1% when the volatility shock hits. It turns out that this policy amplifies the recession by generating lower output and raising unemployment by an additional 5%. The reason is that increased unemployment value requires higher wages, which not only increases the cost of production but also strengthens the financial concern of wage payments. Therefore, the recession is deepened.

German Policy. Germany has a very different social security system from the U.S. As a part of

Germany's social insurance program, Kurzarbeit, the short-time work (STW) policy compensates part of workers' earnings losses when firms cut workers' hours. In other words, the government helps firms pay wages when there is a bad shock.¹² I model Germany's policy by allowing the firm to have an option to let part of its workforce idle when volatility is high. The government pays 84.4% of the idle workers' wages, and the firm pays the rest.¹³ This replacement rate is chosen to have the same share of government expenditure to output ratio as the U.S. Table 5 shows that this policy costs 4.86 basis points of output.

Table 5 also reports the model-simulated moments without and with the German policy. It turns out that this policy has its pros and cons. On the positive side, the unemployment rate decreases by 1.9 basis points, and its standard deviation over business cycles is 1.9 percent lower. In other words, the German labor market policy can lower and stabilize unemployment by providing state-contingent insurance to firms to help them pay wages and retain employees, weakening the financial channel of wage commitments. Plus, since the policy reduces separations, it can save the resources spent on search.

However, the policy's overall impact is negative—aggregate output decreases by 6.2 basis points, and welfare decreases by 2.4 basis points. The negative effect is due to the policy-induced distortions. The wage subsidies encourage labor hoarding, which misallocates the labor force to low-productivity firms that are supposed to separate from their employees. As a result, the labor market is less efficient, so aggregate output and welfare decrease.

The dash-dot blue lines in Figure 6 show the impulse responses to a transitory positive volatility shock under Germany's policy. The impulse responses almost do not change because the policy's distortive costs offset the stabilization benefits from insuring firms. But it can still be observed that output decreases slightly more and unemployment increases slightly less, consistent with the findings in Table 5.

Biased Policy Evaluation Without Consideration of Financial Frictions. I have already shown that financial frictions are crucial for my model's implications for business cycles. Another question is how important financial frictions are for policy evaluation. To answer this question, I apply the above two policy experiments to the recalibrated model without financial frictions.

The lower part of Table 5 reports the model's moments when financial frictions are absent, without and with the policies. As we can see, the negative impacts of both labor market policies diminish dramatically. The welfare loss induced by the U.S. policy decreases from 3.9 to 0.2 basis points. And the German policy-induced welfare loss decreases from 2.4 to 0.4 basis points. Panel B of Figure 6 also displays similar results with the impulse response to a 5% positive volatility shock. Without considering financial frictions, the U.S. policy generates a much smaller decrease in output

¹² Cooper, Meyer and Schott (2017) provide information on this system.

¹³ Although my model does not have an explicit component of working hours, it does not affect the results since each firm has a continuum of the workforce to be idle, equivalent to modeling hours cut.

and a much smaller increase in unemployment. The reason is that when the financial channel of wage commitments is absent, the policy's negative influence is weakened substantially. On the other hand, the German policy displays a stronger stabilization effect by generating slightly higher output and lower unemployment. In this case, the policy-induced distortion is much smaller because wage subsidies do not twist firms' liquidity incentives. Therefore, the policy's benefits outweighed its drawbacks, and the recession is dampened.

To sum up, the U.S. policy pays unemployed workers more during high volatility states, making it more expensive for firms to pay wages; Germany's policy provides wage subsidies when firms cannot afford to retain workers, which helps them keep workers when facing shocks. Both policies expanded aggressively during the Covid recession. My quantitative results show that the U.S. policy significantly amplifies recessions and the German policy has mild negative impacts. Both policies' negative effects are largely underestimated when the financial channel is omitted. It is worth noting that my model focuses on the labor demand mechanism. It does not include worker-side risk aversion or demand effects, which may provide additional benefits for the two policies through other channels.

4 Conclusion

This paper uses a novel model to study the financial effect of wage commitments on unemployment in the presence of volatility shocks. When volatility rises, firms are less likely to make wage commitments, so labor demand decreases and unemployment increases.

The model builds on [Schaal \(2017\)](#) by generating more unemployment volatility and a larger increase in unemployment during the Great Recession. This result is driven by the interaction between volatility shocks and the financial channel of wage commitment. The key is that firms have limited ability to hedge against the risks of idiosyncratic productivity variations.

With rich micro-foundations and realistic quantitative performance, my model has potential for wide application. As an example, I use this framework to investigate labor market policies. I find that raising unemployment benefits as the United States did during the Covid recession amplified the recession. On the other hand, a German approach of subsidizing firm wage bills has a mildly negative impact because of misallocation.

References

Abel, Andrew B. 1983. "Optimal investment under uncertainty." *The American Economic Review*, 73(1): 228–233.

- Acemoglu, Daron, and William B Hawkins.** 2014. "Search with multi-worker firms." *Theoretical Economics*, 9(3): 583–628.
- Alfaro, Iván, Nicholas Bloom, and Xiaoji Lin.** 2019. "The Finance Uncertainty Multiplier." Available at SSRN 3093871.
- Altig, Dave, Scott Baker, Jose Maria Barrero, Nicholas Bloom, Philip Bunn, Scarlet Chen, Steven J Davis, Julia Leather, Brent Meyer, Emil Mihaylov, et al.** 2020. "Economic uncertainty before and during the COVID-19 pandemic." *Journal of Public Economics*, 191: 104274.
- Arellano, Cristina, Yan Bai, and Patrick J Kehoe.** 2019. "Financial frictions and fluctuations in volatility." *Journal of Political Economy*, 127(5): 2049–2103.
- Basu, Susanto, and Brent Bundick.** 2017. "Uncertainty shocks in a model of effective demand." *Econometrica*, 85(3): 937–958.
- Bils, Mark, Peter J Klenow, and Cian Ruane.** 2021. "Misallocation or mismeasurement?" *Journal of Monetary Economics*, 124: S39–S56.
- Bils, Mark, Yongsung Chang, and Sun-Bin Kim.** 2022. "How sticky wages in existing jobs can affect hiring." *American Economic Journal: Macroeconomics*, 14(1): 1–37.
- Blanco, Julio A, and Gaston Navarro.** 2016. "The Unemployment Accelerator."
- Bloom, Nicholas, Max Floetotto, Nir Jaimovich, Itay Saporta-Eksten, and Stephen J Terry.** 2018. "Really uncertain business cycles." *Econometrica*, 86(3): 1031–1065.
- Bocola, Luigi, and Alessandro Dovis.** 2019. "Self-fulfilling debt crises: A quantitative analysis." *American Economic Review*, 109(12): 4343–77.
- Boz, Emine, C Bora Durdu, and Nan Li.** 2015. "Emerging market business cycles: The role of labor market frictions." *Journal of Money, Credit and Banking*, 47(1): 31–72.
- Cacciatore, Matteo, and Federico Ravenna.** 2021. "Uncertainty, wages and the business cycle." *The Economic Journal*, 131(639): 2797–2823.
- Carlsson, Mikael, Julián Messina, and Oskar Nordström Skans.** 2016. "Wage adjustment and productivity shocks." *The Economic Journal*, 126(595): 1739–1773.
- Christiano, Lawrence J, Mathias Trabandt, and Karl Walentin.** 2011. "Introducing financial frictions and unemployment into a small open economy model." *Journal of Economic Dynamics and Control*, 35(12): 1999–2041.
- Christiano, Lawrence J, Roberto Motto, and Massimo Rostagno.** 2014. "Risk shocks." *American Economic Review*, 104(1): 27–65.
- Chugh, Sanjay K.** 2013. "Costly external finance and labor market dynamics." *Journal of Economic Dynamics and Control*, 37(12): 2882–2912.
- Cooper, Russell, Moritz Meyer, and Immo Schott.** 2017. "The employment and output effects of short-time work in Germany." National Bureau of Economic Research.
- Davis, Steven J, and John Haltiwanger.** 1992. "Gross job creation, gross job destruction, and employment reallocation." *The Quarterly Journal of Economics*, 107(3): 819–863.
- Den Haan, Wouter J, Lukas B Freund, and Pontus Rendahl.** 2021. "Volatile hiring: uncertainty in

- search and matching models." *Journal of Monetary Economics*.
- Donangelo, Andres, Francois Gourio, Matthias Kehrig, and Miguel Palacios.** 2019. "The cross-section of labor leverage and equity returns." *Journal of Financial Economics*, 132(2): 497–518.
- Eckstein, Zvi, Ofer Setty, and David Weiss.** 2019. "Financial risk and unemployment." *International Economic Review*, 60(2): 475–516.
- Elsby, Michael WL, and Ryan Michaels.** 2013. "Marginal jobs, heterogeneous firms, and unemployment flows." *American Economic Journal: Macroeconomics*, 5(1): 1–48.
- Fajgelbaum, Pablo D, Edouard Schaal, and Mathieu Taschereau-Dumouchel.** 2017. "Uncertainty traps." *The Quarterly Journal of Economics*, 132(4): 1641–1692.
- Favilukis, Jack, Xiaoji Lin, and Xiaofei Zhao.** 2020. "The elephant in the room: the impact of labor obligations on credit markets." *American Economic Review*, 110(6): 1673–1712.
- Fernández-Villaverde, Jesús, Pablo Guerrón-Quintana, Juan F Rubio-Ramírez, and Martin Uribe.** 2011. "Risk matters: The real effects of volatility shocks." *American Economic Review*, 101(6): 2530–61.
- Freund, Lukas, and Pontus Rendahl.** 2020. "Unexpected effects: uncertainty, unemployment, and inflation."
- Fukui, Masao.** 2020. "A Theory of Wage Rigidity and Unemployment Fluctuations with On-the-Job Search." *Job Market Paper, Massachusetts Institute of Technology*.
- Garin, Julio.** 2015. "Borrowing constraints, collateral fluctuations, and the labor market." *Journal of Economic Dynamics and Control*, 57: 112–130.
- Gertler, Mark, and Antonella Trigari.** 2009. "Unemployment fluctuations with staggered Nash wage bargaining." *Journal of political Economy*, 117(1): 38–86.
- Gilchrist, Simon, Jae W Sim, and Egon Zakrajšek.** 2014. "Uncertainty, financial frictions, and investment dynamics." National Bureau of Economic Research.
- Guiso, Luigi, Luigi Pistaferri, and Fabiano Schivardi.** 2005. "Insurance within the firm." *Journal of Political Economy*, 113(5): 1054–1087.
- Hall, Robert E.** 2005. "Employment fluctuations with equilibrium wage stickiness." *American economic review*, 95(1): 50–65.
- Hall, Robert E, and Edward P Lazear.** 1984. "The excess sensitivity of layoffs and quits to demand." *Journal of Labor Economics*, 2(2): 233–257.
- Hall, Robert E, and Paul R Milgrom.** 2008. "The limited influence of unemployment on the wage bargain." *American economic review*, 98(4): 1653–74.
- Hartman, Richard.** 1972. "The effects of price and cost uncertainty on investment." *Journal of economic theory*, 5(2): 258–266.
- Hsieh, Chang-Tai, and Peter J Klenow.** 2009. "Misallocation and manufacturing TFP in China and India." *The Quarterly journal of economics*, 124(4): 1403–1448.
- Jäger, Simon, Benjamin Schoefer, and Josef Zweimüller.** 2019. "Marginal jobs and job surplus: a test of the efficiency of separations." National Bureau of Economic Research.

- Jensen, Michael C.** 1986. "Agency costs of free cash flow, corporate finance, and takeovers." *The American economic review*, 76(2): 323–329.
- Kaas, Leo, and Philipp Kircher.** 2015. "Efficient firm dynamics in a frictional labor market." *American Economic Review*, 105(10): 3030–60.
- Khan, Aubhik, and Julia K Thomas.** 2008. "Idiosyncratic shocks and the role of nonconvexities in plant and aggregate investment dynamics." *Econometrica*, 76(2): 395–436.
- Khan, Aubhik, and Julia K Thomas.** 2013. "Credit shocks and aggregate fluctuations in an economy with production heterogeneity." *Journal of Political Economy*, 121(6): 1055–1107.
- Krusell, Per, and Anthony A Smith, Jr.** 1998. "Income and wealth heterogeneity in the macroeconomy." *Journal of political Economy*, 106(5): 867–896.
- Kudlyak, Marianna.** 2014. "The cyclicalities of the user cost of labor." *Journal of Monetary Economics*, 68: 53–67.
- Leduc, Sylvain, and Zheng Liu.** 2016. "Uncertainty shocks are aggregate demand shocks." *Journal of Monetary Economics*, 82: 20–35.
- Lemieux, Thomas, W Bentley MacLeod, and Daniel Parent.** 2012. "Contract form, wage flexibility, and employment." *American Economic Review*, 102(3): 526–31.
- Lian, Chen, and Yueran Ma.** 2021. "Anatomy of corporate borrowing constraints." *The Quarterly Journal of Economics*, 136(1): 229–291.
- Menzio, Guido, and Espen R Moen.** 2010. "Worker replacement." *Journal of Monetary Economics*, 57(6): 623–636.
- Menzio, Guido, and Shouyong Shi.** 2010. "Block recursive equilibria for stochastic models of search on the job." *Journal of Economic Theory*, 145(4): 1453–1494.
- Menzio, Guido, and Shouyong Shi.** 2011. "Efficient search on the job and the business cycle." *Journal of Political Economy*, 119(3): 468–510.
- Mumtaz, Haroon, and Francesco Zanetti.** 2016. "The effect of labor and financial frictions on aggregate fluctuations." *Macroeconomic Dynamics*, 20(1): 313–341.
- Nagypál, Éva.** 2007. "Labor-market fluctuations and on-the-job search." *Available at SSRN 1088005*.
- Oi, Walter Y.** 1961. "The desirability of price instability under perfect competition." *Econometrica: journal of the Econometric Society*, 58–64.
- Ottonello, Pablo, and Thomas Winberry.** 2020. "Financial heterogeneity and the investment channel of monetary policy." *Econometrica*, 88(6): 2473–2502.
- Petrosky-Nadeau, Nicolas.** 2014. "Credit, vacancies and unemployment fluctuations." *Review of Economic Dynamics*, 17(2): 191–205.
- Pissarides, Christopher A.** 2009. "The unemployment volatility puzzle: Is wage stickiness the answer?" *Econometrica*, 77(5): 1339–1369.
- Quadrini, Vincenzo.** 2011. "Financial frictions in macroeconomic fluctuations." *FRB Richmond Economic Quarterly*, 97(3): 209–254.
- Rudanko, Leena.** 2009. "Labor market dynamics under long-term wage contracting." *Journal of*

- monetary Economics*, 56(2): 170–183.
- Rudanko, Leena.** 2019. “Firm wages in a frictional labor market.”
- Rute Cardoso, Ana, and Miguel Portela.** 2009. “Micro foundations for wage flexibility: wage insurance at the firm level.” *Scandinavian Journal of Economics*, 111(1): 29–50.
- Schaal, Edouard.** 2017. “Uncertainty and unemployment.” *Econometrica*, 85(6): 1675–1721.
- Schoefer, Benjamin.** 2021. “The financial channel of wage rigidity.” National Bureau of Economic Research.
- Sepahsalari, Alireza.** 2016. “Financial market imperfections and labour market outcomes.”
- Shimer, Robert.** 2004. “The consequences of rigid wages in search models.” *Journal of the European Economic Association*, 2(2-3): 469–479.
- Shimer, Robert.** 2005. “The cyclical behavior of equilibrium unemployment and vacancies.” *American economic review*, 95(1): 25–49.
- Wasmer, Etienne, and Philippe Weil.** 2004. “The macroeconomics of labor and credit market imperfections.” *American Economic Review*, 94(4): 944–963.
- Zanetti, Francesco.** 2019. “Financial shocks, job destruction shocks, and labor market fluctuations.” *Macroeconomic Dynamics*, 23(3): 1137–1165.

Appendix A Tables

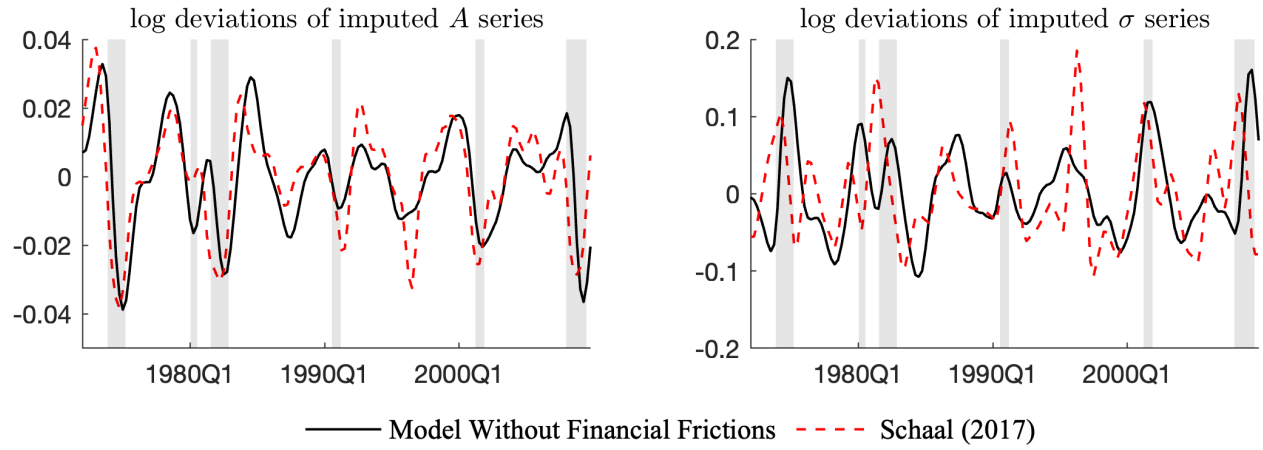
Table 6: Parameters of Reference Models

Parameters	Benchmark Model		No Financial Frictions	
	$A + \sigma$	A only	$A + \sigma$	A only
Aggregate shocks				
ρ_A	0.920	0.920	0.913	0.913
σ_A	0.024	0.028	0.041	0.035
$\bar{\sigma}$	0.248	0.250	0.313	0.280
ρ_σ	0.880	-	0.924	-
σ_σ	0.092	-	0.186	-
$\rho_{A\sigma}$	-0.020	-	-0.900	-
Labor market				
\bar{u}	0.142	0.142	0.141	0.141
c	0.001	0.001	0.001	0.002
λ	0.100	0.100	0.100	0.100
γ	1.600	1.600	1.600	1.600
k_e	15.21	14.87	11.19	11.47
$\bar{w}_m + \mu_\epsilon$	0.001	0.001	0.100	0.100
π_d	0.021	0.022	0.022	0.022
Financial market				
σ_ϵ	0.080	0.071	0.080	0.080
$\tilde{\zeta}$	2.400	2.400	-	-
ξ	1.780	1.780	-	-
η	2.410	2.410	-	-

Notes: The first column shows the calibrated parameters of the benchmark model. The other columns report the recalibrated parameters for reference models. $A + \sigma$ means the model has both aggregate productivity shocks and volatility shocks, and A means the model only has aggregate productivity shocks.

Appendix B Figures

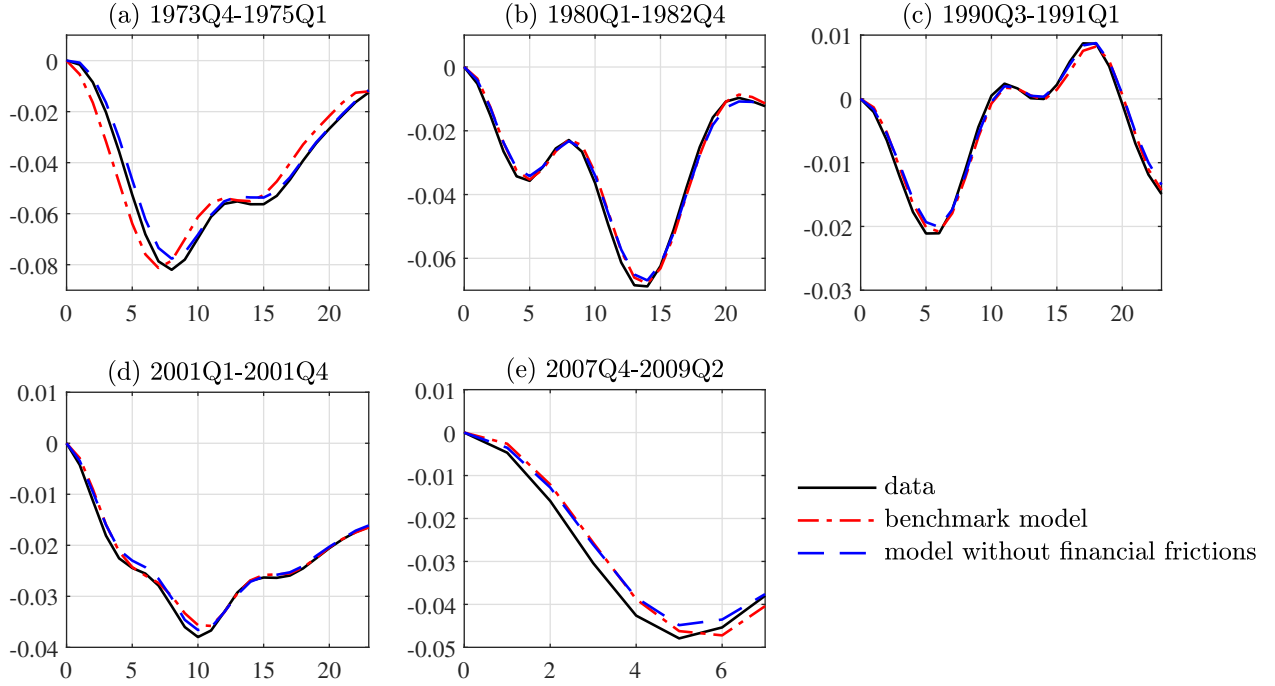
Figure 7: Comparison of Estimated Shocks with [Schaal \(2017\)](#)



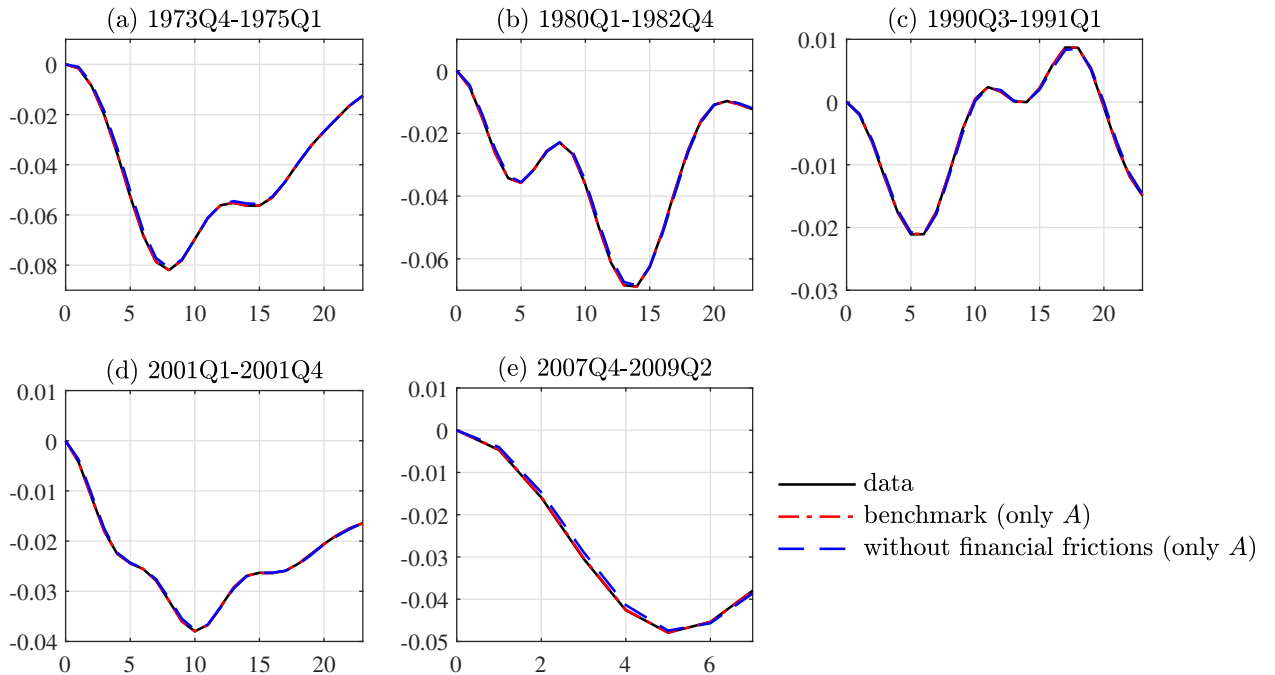
Notes: This figure compares the estimated aggregate productivity shocks and uncertainty shocks with [Schaal \(2017\)](#). I use the particle filter to estimate the shocks of the model without financial frictions by matching the data series of output and the IQR of firm sales growth. The black lines show the log deviations of aggregate productivity, A , and uncertainty, σ . The red dashed lines are the imputed shocks from [Schaal \(2017\)](#).

Figure 8: Output Series With and Without Modeling Financial Frictions

Panel A: Models With Both A And σ Shocks



Panel B: Models With Only A Shock



Notes: The panels show the model's predictions for output during recessions. Models in Panel A have both aggregate productivity shocks, A , and uncertainty shocks, σ . I use the particle filter to jointly estimate the time series of aggregate productivity shocks and uncertainty shocks by matching output and the IQR of firm sales growth in the data. The data are detrended by a band-pass filter to focus on fluctuations between 6 and 32 quarters, following [Schaal \(2017\)](#). Models in Panel B have only aggregate productivity shocks. Given the estimated shocks, I show the model-predicted output. The data on output is the solid black lines. The output fluctuations predicted by the benchmark model are the dash-dotted red lines. The output of the model without financial frictions is the dashed blue line. Series are depicted in terms of log deviations from the peak preceding the recession. I use [Schaal's \(2017\)](#) code when plotting.

Appendix C Proofs

Proposition 1 *The participation constraint binds, i.e., $\bar{W}(i') = 0$, for any worker i' .*

Proof I prove this proposition by contradiction. Suppose $\exists i'$ such that $\bar{W}(i') > 0$ in the optimal policy. The firm can construct an alternative policy with $\bar{W}(i') = 0$ and achieves a higher value. The main idea of the alternative policy is to frontload wages and borrow more.

Case 1. Suppose i' is an incumbent worker. Recall that the promise-keeping constraint always binds in the optimal policy. Construct an alternative policy by letting $\bar{W}(i')$ decrease to zero and increase the next-period wage of worker i' to satisfy tomorrow's binding promise-keeping constraint. Use δ to denote the decrease of tomorrow's wage bill $\int_0^{n'} w(i') di'$. At the same time, increase b' by δ so that next-period cash on hand is the same. Thus, the next-period default decisions are unchanged, and so is the expected value of the firm in the next period. The agency friction constraint (11) holds as well. On its left-hand side, borrowing increases, while the right-hand side decreases because of lower next-period wage bills.

Next, I prove that current equity payouts increase as follows. Because the future promised utility decreases, the worker's current wage should increase accordingly to satisfy today's promise-keeping constraint. To see how much the wage needs to increase, rewrite the worker's problem (2) as:

$$\begin{aligned} W(S, s, C) = \max_x w + \beta \mathbb{E} \Big[& [\tau + (1 - \tau)(\pi_d + (1 - \pi_d)d(S', s'))]U(S') \\ & + (1 - \tau)(1 - \pi_d)(1 - d(S', s'))[U(S') + \bar{W}(i')] \Big] \\ & + \lambda p(\theta(S, x)) \Bigg[x - \beta \mathbb{E} \Big[[\tau + (1 - \tau)(\pi_d + (1 - \pi_d)d(S', s'))]U(S') \\ & + (1 - \tau)(1 - \pi_d)(1 - d(S', s'))[U(S') + \bar{W}(i')] \Big] \Bigg]. \end{aligned} \quad (52)$$

Because the last term increases when $\bar{W}(i')$ decreases, the firm's current wage bill only needs to increase by less than $(1 - \tau)(1 - \pi_d)\beta \mathbb{E}[(1 - d(S', s'))]\delta$ to satisfy the same employment value W .¹⁴

Recall that b' increases by δ , and the default probability is unaffected, so the change in current equity payouts is:

$$\begin{aligned} \Delta^{\text{new}} - \Delta^{\text{old}} &\geq Q(S, z, b' + \delta, n)(b' + \delta) - Q(S, z, b', n)b' - (1 - \tau)(1 - \pi_d)\beta \mathbb{E}[(1 - d(S', s'))]\delta \\ &= \beta \mathbb{E}_{S', z' | S, z} \left\{ (1 - \pi_d)\Phi(\bar{\epsilon}(S', z', b', n')) \right\} (b' + \delta) - \beta \mathbb{E}_{S', z' | S, z} \left\{ (1 - \pi_d)\Phi(\bar{\epsilon}(S', z', b', n')) \right\} b' \\ &\quad + \beta \mathbb{E}_{S', z' | S, z} \left\{ [1 - (1 - \pi_d)\Phi(\bar{\epsilon}(S', z', b', n'))] \min \left\{ \eta \frac{A' z' n'^\alpha - (\int_0^{n'} w(i') di' - \delta) - \bar{w}_m - \mu_\epsilon}{b' + \delta}, 1 \right\} \right\} (b' + \delta) \end{aligned}$$

¹⁴ Notice that when the future promised utility decreases, the worker will do more on-the-job search, which allows the firm to lower the wage.

$$\begin{aligned}
& -\beta \mathbb{E}_{S',z'|S,z} \left\{ [1 - (1 - \pi_d)\Phi(\bar{\epsilon}(S',z',b',n'))] \min\left\{\eta \frac{A'z'n'^\alpha - \int_0^{n'} w(i')di' - \bar{w}_m - \mu_\epsilon}{b'}, 1\right\} b' \right. \\
& \quad \left. - (1 - \tau)(1 - \pi_d)\beta \mathbb{E}[(1 - d(S',s'))]\delta \right\} \\
& = \beta \mathbb{E}_{S',z'|S,z} \left\{ (1 - \pi_d)\Phi(\bar{\epsilon}(S',z',b',n')) \right\} \delta \\
& \quad + \beta \mathbb{E}_{S',z'|S,z} \left\{ [1 - (1 - \pi_d)\Phi(\bar{\epsilon}(S',z',b',n'))] \min\left\{\eta A'z'n'^\alpha - \left(\int_0^{n'} w(i')di' - \delta\right) - \bar{w}_m - \mu_\epsilon, b' + \delta\right\} \right. \\
& \quad \left. - \beta \mathbb{E}_{S',z'|S,z} \left\{ [1 - (1 - \pi_d)\Phi(\bar{\epsilon}(S',z',b',n'))] \min\left\{\eta A'z'n'^\alpha - \int_0^{n'} w(i')di' - \bar{w}_m - \mu_\epsilon, b'\right\} \right. \right. \\
& \quad \left. \left. - (1 - \tau)(1 - \pi_d)\beta \mathbb{E}[(1 - d(S',s'))]\delta \right\} \right\} \\
& \geq \beta \mathbb{E}_{S',z'|S,z} \left\{ (1 - \pi_d)\Phi(\bar{\epsilon}(S',z',b',n')) \right\} \delta - (1 - \tau)(1 - \pi_d)\beta \mathbb{E}[(1 - d(S',s'))]\delta \\
& \geq 0.
\end{aligned}$$

Case 2. Suppose i' is a newly hired worker. As before, construct an alternative policy by letting $\bar{W}(i') = 0$. Accordingly, increase the worker's wage in the next period and increase borrowing so that the firm's expected value does not change. The agency friction constraint holds as before. Next, lower the current wages of new hires $\int_{n'-n_h}^{n'} w_h(i')di'$ by less than $(1 - \tau)(1 - \pi_d)\beta \mathbb{E}[(1 - d(S',s'))]\delta$. Because borrowing increases by more than wages, current equity payouts increase.

Therefore, the alternative policy is feasible and better, so any policy with a loose participation constraint is not optimal. \square

Appendix D Micro-Foundations of Incomplete Labor Contracts

In this section, I use asymmetric information between the firm and its employees to show that the promised utility markup \bar{W} is state-uncontingent. The key assumption is that firms know the realized shocks immediately, but employees know the shocks later in the production stage. This information friction explains why labor contracts are not completely state-contingent, i.e., constraint (7). The logic follows Hall and Lazear (1984). They use asymmetric information to justify the optimality of pre-determined wages. We share the mechanism that firms can lie about the states, so the only incentive-compatible result is state-uncontingent promises. I will first set up the model with asymmetric information and then prove the optimality of state un-contingency.

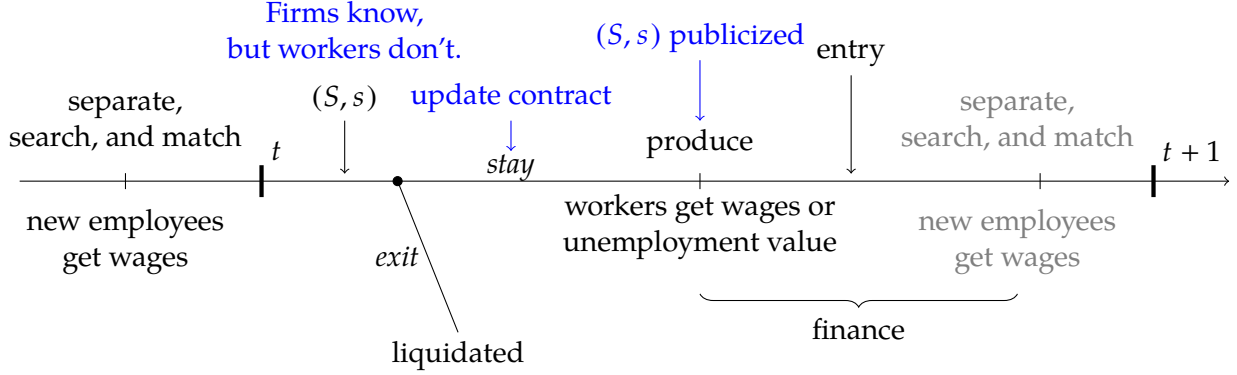
On top of the timeline in Figure 1, Figure 9 adds the timing for asymmetric information. When shocks (S, s) realize at the beginning of each period, firms know the shocks, but workers do not. If a worker leaves the firm now, he is unemployed and obtains the unemployment value in the current period. Given the shocks, firms choose to exit or stay. Staying firms declare their current shocks are \tilde{S} and \tilde{s} and update contracts. Notice that the declaration can differ from the true state since workers do not observe the information now. I allow the declarations to differ across the firm's employees. Given that the labor contract has been updated, the worker gets nothing in the current period if he leaves the firm now.¹⁵ At the production stage, the shocks (S, s) become public information. Workers receive wage payments according to the labor contract, which depends on the firm's declaration of the state (\tilde{S}, \tilde{s}) . At the end of the period, firms separate, search, and match.

The labor contract C contains $\{w, \tau, \bar{W}(S', s'), d(S', s')\}$. Notice I assume that the contract directly specifies the markup $\bar{W}(S', s')$ between the lifetime promised utility $W'(S', s')$ and the outside value of unemployment $U(S')$, which facilitates proving the markup's state-uncontingency.¹⁶

¹⁵ This assumption facilitates the proof because workers have no incentive to threaten to leave the firm (Proposition 2(i).)

¹⁶ In the traditional implicit contract literature, the labor contract specifies the lifetime utility. Instead, I assume that the contract specifies the promised utility markup in this paper, which is a weaker assumption in my context. Specifically, by asymmetric information, I will prove that the promised value in the contract is state-uncontingent. When the contract specifies the promised utility markup, it implies only the markup part is state-uncontingent, and the promised lifetime utility can still vary with aggregate states, similar to a standard Diamond-Mortensen-Pissarides search model. If the contract specifies the lifetime utility, then the whole lifetime utility is state-uncontingent, implying greater rigidity. This is a stronger assumption than is unnecessary for my model.

Figure 9: Timing With Asymmetric Information



Given the labor contract, the worker's employment value is:

$$\begin{aligned}
 W(S, s, C) = & \max_x w + \lambda p(\theta(S, x))x + (1 - \lambda p(\theta(S, x)))\tau\beta \mathbb{E}_{S'|S} U(S') \\
 & + (1 - \lambda p(\theta(S, x)))(1 - \tau)\beta \max \left\{ \underbrace{\mathbb{E}_{S'|S} U(S')}_{\text{leave before the contract is updated}}, \mathbb{E}_{S', s'|S, s} \{(\pi_d + (1 - \pi_d)d(S', s'))U(S')\} \right\} \\
 & + (1 - \pi_d)(1 - d(S', s')) \max \{ U(S') + \bar{W}(\tilde{S}^*, \tilde{s}^*), \underbrace{0 + \beta \mathbb{E}_{S''|S'} U(S'')}_{\text{leave after the contract is updated}} \} \}.
 \end{aligned} \tag{53}$$

As before, the worker receives the wage w at the production stage. The worker can conduct on-the-job search and leave the firm. If the worker stays but gets laid off, he will be unemployed in the next period and receive the unemployment value $U(S')$.

If the worker is not laid off, he can still leave the firm when the outside value is high enough. But the outside value depends on the timing of leaving the firm. If the worker leaves the firm before the contract is renewed, he is counted as unemployed and receives the unemployment value just like a laid-off worker. However, if he leaves the firm after the contract is renewed, he receives zero and gets the unemployment value one period later. This setup can be understood as the worker being ineligible to receive unemployment benefits after the labor relation renews, and drawing up contracts is time-consuming, so he does not have time to produce at home in the same period. Hence, the utility is zero in that period. This assumption will imply that workers have no incentive to threaten to quit when they find the firm lies (Proposition 2(i)), facilitating the proof.

If the labor relation persists, the worker will receive the lifetime utility $U(S') + \bar{W}(\tilde{S}^*, \tilde{s}^*)$. Notice that because of asymmetric information, the realized promised utility markup \bar{W} depends on the firm's declaration of shocks $(\tilde{S}^*, \tilde{s}^*)$. To clarify, $\{\bar{W}(S', s')\}$ in the labor contract is the set of utility markups for the next period. However, how much the worker can get in the next period depends on the firm's optimal declaration of shocks $(\tilde{S}^*, \tilde{s}^*; i)$.

A firm's states include realized aggregate shocks $S \in \mathcal{S}$, realized firm-specific shocks $s \in \mathcal{s}$, the number of employees n , and the set of promised utility markups to its employees $\{\bar{W}(S, s; i)\}_{S \in \mathcal{S}, s \in \mathcal{s}; i \in [0, n]}$, where i is the index of incumbent employees within the firm. In a slight abuse of notation, S and s inside $\bar{W}(\cdot, \cdot; i)$ refer to the possible shocks instead of the realized shocks.

Besides the choice variables in the original firm's problem (3), the firm now also chooses to declare the current shocks, $\tilde{S}(i)$ and $\tilde{s}(i)$, to each employee i . The following equations (54) to (60) summarize the firm's problem:

$$\begin{aligned} J(S, s, b, n, \{\bar{W}(S, s; i)\}_{S \in \mathcal{S}, s \in \mathcal{s}; i \in [0, n]}) = & \max_{\substack{\Delta, b', n', n_h, x_h, d(S', s') \\ \{\tilde{S}(i), \tilde{s}(i), w(i), \tau(i)\}_{i \in [0, n]}, \\ \{w_h(i')\}_{i' \in (n' - n_h, n']}, \\ \{\bar{W}(S', s'; i')\}_{S' \in \mathcal{S}', s' \in \mathcal{s}'; i' \in [0, n']}}} \Delta \end{aligned} \quad (54)$$

$$+ \beta(1 - \pi_d) \mathbb{E}_{S', s' | S, s} \left\{ (1 - d(S', s')) J(S', s', b', n', \{\bar{W}(S', s'; i')\}_{S' \in \mathcal{S}', s' \in \mathcal{s}'; i' \in [0, n']}) \right\}$$

$$\text{s.t. (4), (5), (6), (10), (11),} \quad (55)$$

$$\begin{aligned} W^E(i') \equiv \mathbb{E}_{S', s' | S, s} \{ & (\pi_d + (1 - \pi_d)d(S', s'))U(S') \\ & + (1 - \pi_d)(1 - d(S', s')) \max\{U(S') + \bar{W}(\tilde{S}^*, \tilde{s}^*; i'), 0 + \beta \mathbb{E}_{S'', s'' | S'} U(S'')\} \}, \end{aligned} \quad (56)$$

$$W^E(i') \geq \mathbb{E}_{S' | S} U(S'), \forall i' \in [0, n'], \quad (57)$$

$$\max_x w(i) + \lambda p(\theta(S, x))x + (1 - \lambda p(\theta(S, x)))\tau(i)\beta \mathbb{E}_{S' | S} U(S') \quad (58)$$

$$+ (1 - \lambda p(\theta(S, x)))(1 - \tau(i))\beta W^E(i') \geq U(S) + \bar{W}(\tilde{S}, \tilde{s}; i), \text{ for } i' \in [0, n' - n_h], \quad (59)$$

$$w_h(i') + \beta W^E(i') \geq x_h, \text{ for } i' \in (n' - n_h, n']. \quad (60)$$

Equations (56) to (60) describe the new implicit contract constraints in the presence of asymmetric information. First, equation (56) uses W^E to denote the worker's expected lifetime utility if he stays with the firm. Notice that W^E is also the last part of the employment value (53). Constraint (57) is the new participation constraint, meaning that the worker's expected utility is at least the expected unemployment value so that he will stay. Equation (58) is the new promise-keeping constraint for incumbent workers. This constraint requires the firm to commit to paying the employee at least the promised lifetime utility. The left-hand side is the incumbent worker's employment value, i.e., equation (53). The right-hand side is the promised lifetime utility, comprised of two parts—the unemployment value $U(S)$ and the promised utility markup $\bar{W}(\tilde{S}, \tilde{s}; i)$. Notice that $\tilde{S}(i)$ and $\tilde{s}(i)$ are the firm's declarations of shocks, two of the firm's choice variables. They can be different from the true shocks because firms know the realized shocks when renewing labor contracts, but workers do not. The declarations can be different across the firm's employees. Equation (60) is the new promise-keeping constraint for newly hired workers. Its left-hand side is the newly hired

worker's employment value. On the right-hand side, x_h is the submarket where the firm employs new workers, and x_h is also the promised lifetime utility of the vacancies posted in that submarket. Thus, equation (60) means the firm should guarantee that newly hired workers receive at least the lifetime utility promised by the offer.

The following Proposition 2 proves that the promised utility markup \bar{W} is state-uncontingent.

Proposition 2 *The labor relation between the firm and its employees has the following properties:*

- (i) *Workers do not leave the firm even if they find the firm lied.*
- (ii) *The promised utility markup \bar{W} is state-uncontingent.*

Proof As for point (i), recall that employees discover whether the firm lied about shocks in the production stage. If they leave the firm now, they get nothing today and start receiving the unemployment value in the next period. So, even if the firm gives the worker zero wages and fires them right after the production stage, the worker is willing to stay with the firm.

As for point (ii), because employees will not leave the firm regardless, according to point (i), lying about the shocks has no consequences for the firm. Thus, firms always declare the lowest employment surplus in $\{\bar{W}(S, s; i)\}_{S \in S, s \in s}$ to each employee i . Therefore, the incentive-compatible labor contract requires the promised utility markup \bar{W} to be state-uncontingent. \square