Uncertainty and Unemployment Revisited: The Consequences of Financial and Labor Contracting Frictions*

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Abstract

I build a novel search model to study how uncertainty shocks to firm-level productivity affect unemployment through the financial channel of incomplete labor contracts. The model's core is a labor contracting friction that implies wage insensitivity to firm-level idiosyncratic shocks. Hence, wage bills are debt-like commitments by firms to workers, which firms are less likely to take on when high uncertainty raises firm default risks. Therefore, when uncertainty is high, firms hire fewer workers, and unemployment increases. Quantitatively, I find that, on top of aggregate productivity shocks, uncertainty shocks generate 26% of the increases in unemployment during past recessions. The effect of uncertainty shocks diminishes greatly if the financial or labor contracting friction is absent. I then use the model to analyze the impact of labor market policies under elevated uncertainty. Increasing unemployment benefits, such as in the U.S., makes it more expensive and risky for firms to pay wages, thus amplifying the recession. Subsidizing firms' wage bills, such as in Germany, performs better by insuring firms from idiosyncratic risk, but it still causes inefficiency since the misallocation losses of labor hoarding outweigh the gains of providing insurance.

Keywords: search and matching, financial frictions, incomplete labor contracts, uncertainty, volatility, firm heterogeneity, business cycles, labor market policies.
JEL Codes: E24, E32, E44, D53, D83, J08.

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1 Introduction

Unemployment increased a lot during recessions, and so did the uncertainty faced by firms. How much of the observed increase in unemployment was due to the elevated uncertainty of firm-level productivity? In this paper, I address this question by constructing and parameterizing a novel search model that centers on firm financial and labor contracting frictions. I find that, because of these frictions, elevated uncertainty generates 26% of the increases in unemployment during past recessions.

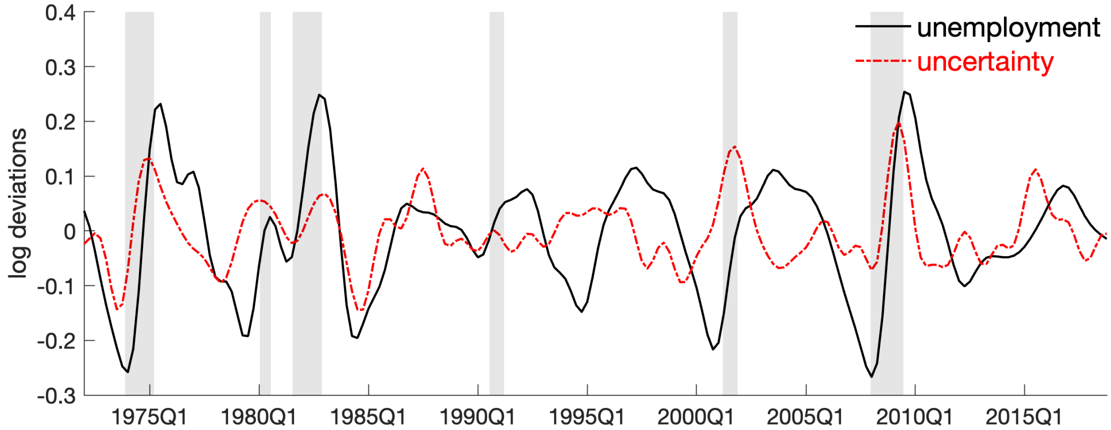
The model's core is a labor contracting friction that implies wages are insensitive to firm-level idiosyncratic shocks within the intertemporal firm-worker labor contracts. Wages are otherwise flexible. Particularly, this labor market friction does not require wages to be sticky: they can adjust fully in response to workers' outside opportunities. Instead, the key is that wages do not respond to firm-level idiosyncratic shocks. The restriction is consistent with existing empirical evidence showing that the pass-through from firms' idiosyncratic transitory shocks to workers' earnings is insignificant (Guiso, Pistaferri and Schivardi, 2005; Rute Cardoso and Portela, 2009). I also prove that this contracting friction can be micro-founded intuitively by assuming that firms have private information about their shocks.

This labor market friction affects unemployment by interacting with financial market incompleteness. In my model, the financial friction means firms face default risk. And when firms hire workers, it means that firms need to take on the commitment to pay workers wages. Since the labor contracting friction implies wages cannot adjust in response to firm-specific shocks, wage bills are isomorphic to debt. Therefore, when firm-level idiosyncratic risk increases, firms are averse to taking on these debt-like wage commitments associated with hiring. In this way, when uncertainty is high, firms hire fewer workers, and unemployment increases. I refer to this mechanism as the financial channel of incomplete labor contracts.

My model is built on Schaal (2017). He in turn extends the directed search framework in Menzio and Shi (2010) to have multi-worker firms and decreasing returns to scale production technology, which enables within-firm endogenous hirings, separations, and on-the-job search. My model allows for both aggregate productivity shocks and uncertainty shocks, as in Schaal (2017). Despite two aggregates shocks and firm heterogeneity, the model keeps its tractability because directed search provides block recursivity (Kaas and Kircher, 2015; Menzio and Shi, 2011; Schaal, 2017).

Then, I extend the model by incorporating the labor contracting friction, along with a more standard firm financing friction. The latter assumes firms can only borrow through state-uncontingent debt, and default is costly because it leads to liquidation. The price of debt reflects the firm's default probability and the post-default recovery from the firm's value. I also model the agency friction that managers can divert firms' funds for their private interests, which constrains the

Figure 1: Unemployment and Uncertainty



Notes: This graph shows the quarterly time series of unemployment and uncertainty. The solid black presents unemployment from the Current Population Survey (CPS). The dot-dash red line depicts the uncertainty measured by the interquartile range of sales growth rates across firms, where the firm sales growth rates are residualized from firm effects and industry-quarter fixed effects. The data source of firms' sales is Compustat. The series of unemployment and uncertainty are detrended by a band-pass filter to focus on fluctuations between 6 and 32 quarters. The shaded bars are the U.S. recessions from NBER.

firms' incentives to save, so the default risk will not be offset by a large stock of savings. The financial contracting friction interacts with the labor contracting friction to generate risk. Neither contracting friction is effective individually. If labor contracts are complete, firms can use them as state-contingent financial instruments to replace the state-uncontingent debt. If the financial market is complete, how wages are paid within labor contracts is inconsequential because it is the present value of wages that determines the incentives of hiring and firing.[1] In sum, the labor contracting and financial frictions are effective only when they are together.

I use the model for three quantitative analyses. First, I show that the model accounts for 90% of unemployment volatility in the data with both aggregate productivity shocks and uncertainty shocks. The model is calibrated by matching the business cycle statistics of GDP and the interquartile range (IQR) of firm sales growth rates, labor market flows, and financial market moments. I document that adding uncertainty shocks to aggregate productivity shocks generates 22% of the unemployment volatility. Additionally, I find that, without modeling contracting frictions, the unemployment volatility explained by considering uncertainty shocks decreases to only 12%.

Second, I use the model to explain the increases in unemployment during the U.S. past recessions. As Figure 1 shows, in recessions, uncertainty — as measured by the IQR of firm sales growth rates — rose greatly, and so did unemployment. Therefore, I use my model to quantify how much of the observed increases in unemployment are due to elevated uncertainty. I first apply the particle filter to my model and estimate the historical aggregate productivity shocks and uncertainty

[1] See Pissarides (2009) for similar neutrality of incumbent workers' wage rigidity with respect to aggregate shocks.

shocks using the data of GDP and the IQR of firm sales growth.[2] Then, I let the model predict unemployment by feeding in the estimated structural shocks. I find that considering uncertainty shocks, on top of aggregate productivity shocks, generates 26% of the increase in unemployment during the past recessions on average. Particularly, this number increases to 40% for the Great Recession. Uncertainty is especially important for understanding the Great Recession because this recession has a large increase in firm-level idiosyncratic risk but only a mild decrease in aggregate productivity.

The key to the impact of uncertainty shocks is the interaction between financial and labor contracting frictions. Without them, the model collapses to a canonical search model, where the increase in unemployment explained by uncertainty shocks decreases from 24% to only 7%. Moreover, this model is particularly incapable of explaining the increase in unemployment during the Great Recession, consistent with the finding in Schaal (2017). The reason is that uncertainty shock is the key driving force of the Great Recession, and the model without contracting frictions greatly underestimates the impact of elevated uncertainty.

The financial channel of incomplete labor contracts operates mainly through uncertainty shocks rather than aggregate productivity shocks. Suppose I introduce the financial and labor contracting frictions to the search model with only aggregate productivity shocks. The model only generates an additional 10% of the increase in unemployment during the Great Recession. However, this number increases to 27% when the model also has uncertainty shocks. Contracting frictions are not effective given aggregate productivity shocks because equilibrium wages decrease a lot in response to the aggregate productivity shock that is common to all firms, the same as in Shimer (2005). On the contrary, wages are stable when it is an uncertainty shock. The reason is that an uncertainty shock is also a dispersion shock, which spreads the distribution of firm-level productivity, maintaining firm values as well as the wage level.[3]

Lastly, given the importance of elevated uncertainty in affecting unemployment, I assess the impacts of labor market policies that target high-uncertainty periods. First, I analyze the policy of raising unemployment benefits, as what is implemented by the U.S. during the Covid recession. Although this policy is used to help unemployed workers, my model shows that higher unemployment benefits push wages higher, making hiring riskier for firms, so unemployment increases even more. Another labor market policy that expanded a lot during recessions is subsidizing firms to pay wages, as Germany did during the Great Recession and the Covid Recession. My model suggests that wage subsidies can insure firms against idiosyncratic shocks, thus weakening the negative impact of high uncertainty, which outperforms the policy of raising unemployment benefits. However, wage subsidies also encourage labor hoarding and thus cause misallocation.

[2] A particle filter is a Monte Carlo Bayesian estimator for the posterior distribution of structural shocks and allows non-linear systems. It is like a Kalman filter but can be applied to my non-linear model.

[3] This positive impact of increasing volatility on firm values is called the Oi-Hartman-Abel effect (Oi (1961), Hartman (1972), and Abel (1983)).

My quantitative analysis shows the misallocation losses outweigh the gains from providing insurance. Therefore, this policy also hurts efficiency. Notice that the micro-level frictions are the key to evaluating the policies. Suppose contracting frictions are ignored in the analysis. Then the misspecified model will greatly underestimate the efficiency losses induced by the two policies. In particular, it will misleadingly suggest that increasing unemployment benefits is better than subsiding wage payments.

Related Literature. My paper contributes to three strands of literature. First, I build on the studies that find the crucial role of uncertainty shocks in driving business cycles (Basu and Bundick, 2017; Bloom et al., 2018; Christiano, Motto and Rostagno, 2014; Fajgelbaum, Schaal and Taschereau-Dumouchel, 2017; Fernández-Villaverde et al., 2011).[4] Particularly, Schaal (2017) introduces uncertainty shocks into a canonical search framework with aggregate productivity shocks. He finds that although uncertainty shocks can increase unemployment through the reallocation of labor across firms, the magnitude is far from sufficient to explain the increase in unemployment during the Great Recession. By contrast, in Arellano, Bai and Kehoe's (2019) financial friction model, elevated uncertainty generates a sizable negative impact comparable to the Great Recession. Alfaro, Bloom and Lin (2019), Christiano, Motto and Rostagno (2014), and Gilchrist, Sim and Zakrajšek (2014) also find that financial frictions are a key reason for uncertainty shocks to cause recessions. But they all focus on the Walrasian competitive equilibrium of a spot labor market, and so their analysis abstracts from unemployment. In my paper, I model both the search friction in Schaal (2017) and the firm financial friction in Arellano, Bai and Kehoe (2019). My model's novelty is considering a labor contracting friction that implies wage insensitivity to firm-level idiosyncratic shocks, as supported by the empirical evidence in the literature. Theoretically, this labor market incompleteness is the key to having meaningful firm financial friction under the search framework, where the firm-worker employment relationships are not restricted to be static.

Second, my work complements the literature that uses wage stickiness to generate unemployment volatility (Bils, Chang and Kim, 2022; Fukui, 2020; Gertler and Trigari, 2009; Hall, 2005; Hall and Milgrom, 2008; Menzio and Moen, 2010; Rudanko, 2019; Schoefer, 2021; Shimer, 2004). I contribute to this literature by proposing an alternative mechanism—the financial channel of incomplete labor contracts. The labor market incompleteness in my model does not require wage stickiness. Instead, it requires wage insensitivity to firms' idiosyncratic shocks, which I find important in understanding unemployment volatility. This labor contracting friction is micro-founded by modeling that firms have private information about their shocks. It is also supported by the direct empirical evidence that the pass-through from firms' transitory idiosyncratic shocks to workers'

[4] Uncertainty in my paper specifically refers to micro-level volatility, i.e., the volatility of firm-level idiosyncratic productivity documented by Bloom et al. (2018). For studies on macro-level volatility of aggregate productivity shocks, see, e.g., Leduc and Liu (2016), Freund and Rendahl (2020), Cacciatore and Ravenna (2021), and Den Haan, Freund and Rendahl (2021). My paper does not include macro-level volatility due to the computational burden. But I conjecture it will not change the result much because I do not assume sticky wages and, according to Schaal (2017), the size and impact of macro-level volatility are small.

earnings is very little (Guiso, Pistaferri and Schivardi, 2005; Rute Cardoso and Portela, 2009). Plus, indirect evidence includes that firms with higher labor shares are more sensitive to shocks (Donangelo et al., 2019), and wage growth and labor shares are the primary factors predicting credit spreads (Favilukis, Lin and Zhao, 2020).

Third, there is a growing literature adding firm financial frictions to search models. Christiano, Trabandt and Walentin (2011), Chugh (2013), Garin (2015), Mumtaz and Zanetti (2016), Petrosky-Nadeau (2014), Sepahsalari (2016), Wasmer and Weil (2004), and Zanetti (2019) introduce intra-period collateral constraints into their search models. By contrast, my model allows intertemporal borrowing and lending, where the financial friction is modeled as risky debt and endogenous default probabilities, as in Arellano, Bai and Kehoe (2019), Khan and Thomas (2013), and Ottonello and Winberry (2020). This kind of financial frictions is more suitable for studying uncertainty shocks because the impact of uncertainty is intertemporal rather than within-period. Like me, Blanco and Navarro (2016) also model firm default risk in a search framework. However, wages in their model are pure internal transfers between the firm and its workers, so they can solve the problem by joint surplus maximization. My model differs from theirs in the labor contracting friction, which interacts with the financial friction and turns out to be the key to understanding uncertainty's impact on unemployment. My methodological contribution is to prove that wage payments are uniquely determined in this case, which keeps the tractability to solving the model.

Layout. The paper proceeds as follows. I first set up the model in Section 2. Section 3 calibrates the model and presents quantitative results, including the event study for U.S. past recessions and labor market policy experiments. Lastly, section 4 concludes.

2 Model

To study the impact of aggregate shocks on unemployment, I build a directed search and matching model. The equilibrium is block recursive to provide tractability, following Menzio and Shi (2010, 2011), Kaas and Kircher (2015), and Schaal (2017).[5] The model also features the financial friction with firm default risks, following Arellano, Bai and Kehoe (2019).

2.1 Environment and Timing

There are four types of agents in the economy: workers, firms, managers, and international financial intermediaries. Workers are infinitely lived and risk-neutral. They have the same productivity. The total mass of workers is normalized to one unit. Firms are also risk-neutral. They hire workers

[5] As for other search models with multi-worker firms, Acemoglu and Hawkins (2014) and Elsby and Michaels (2013) introduce Nash bargaining into random search. Because my paper focuses on business cycles, I leverage directed search with block recursivity to solve the problem globally out of the steady-state.

to produce homogeneous goods and finance by borrowing from the financial intermediaries.

Firms' idiosyncratic productivity is drawn from the Markov process $\pi_z(z'|z,\sigma)$, where σ is time-varying uncertainty of firm-level productivity. Higher uncertainty implies a more widely spread distribution of tomorrow's idiosyncratic productivity shocks, so firms are more likely to draw a low idiosyncratic productivity. The other aggregate shock in the economy is the aggregate productivity shock A. I use S to summarize the two aggregate shocks (A,σ). Firms also face an i.i.d. random operating cost shock ϵ, which follows a normal distribution $\Phi_\epsilon \equiv \mathcal{N}(\mu_\epsilon, \sigma_\epsilon^2)$. I use s to denote the two firm-specific shocks (z,ϵ).

I assume that job search is directed. Each labor submarket is indexed by a promised utility x, which is the lifetime utility firms promise to workers hired from this submarket. The submarket tightness θ is the ratio of vacancies to the number of workers looking for jobs in each submarket. I use $p(\theta)$ to denote the job-finding rate of workers and $q(\theta)$ to denote the vacancy-filling rate of firms. The relation between x and θ will be determined by the free entry condition in equilibrium.

Following the implicit contract literature, I assume that firms are committed to labor contracts while workers are not. So, workers can leave the firm whenever their outside option is better. One justification for this approach is that firms care about their reputations more than individual workers. I denote the recursive-from labor contract as $C = \{w,\ \tau,\ W'(S',s'),\ d(S',s')\}$, where w is the current wage payment, τ is the layoff probability, $W'(S',s')$ is the next-period employment value promised by the firm, and $d(S',s')$ is the indicator for the firm's exit decision.

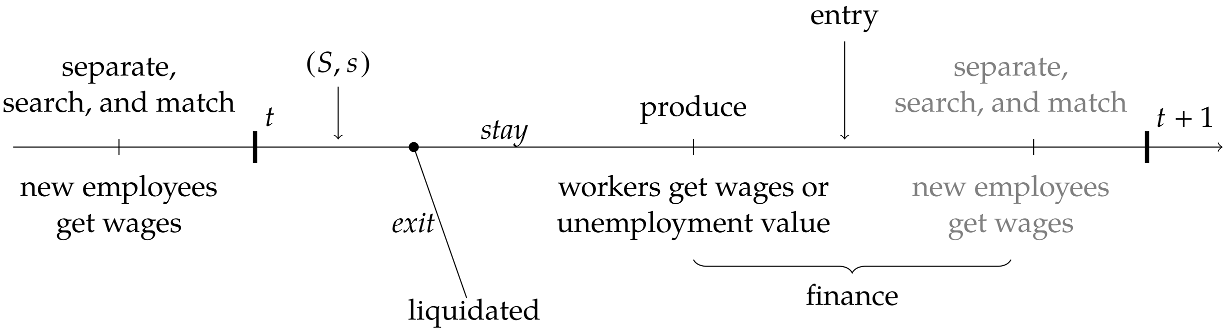
Figure 2 shows the timing. At the end of the preceding period, firms and workers interacted in the labor market to separate, search, and match. They draw up labor contracts in this stage. And newly employed workers receive wages. At the beginning of the current period, all shocks (S,s) realize. Then firms decide to exit or not. If a firm exits, it defaults on all its debts, including labor contracts, and its operations are liquidated. Otherwise, firms produce based on the number of employees as determined at the end of the last period. At the same time, employed workers receive wages according to continuing labor contracts. Unemployed workers also obtain unemployment benefits in this stage. Next, potential new firms can pay an entry cost to enter, after which both new entrants and incumbent firms participate simultaneously in the labor market. Firms borrow from international financial intermediaries to finance the expenditure during the process.

2.2 Worker's Problem

There are two types of workers in the economy: unemployed and employed workers. I abstract from the participation margin.

Unemployed Worker's Problem. An unemployed worker receives unemployment benefits $\bar{u}$ in the current period and chooses a submarket x_u to job search to maximize their unemployment

Figure 2: Timing



value. The matching probability $p(\theta(S, x_u))$ depends on the aggregate shocks and the promised utility of the submarket. Therefore, the unemployment value is:

$$\boldsymbol{U}(S) = \max_{x_u} \bar{u} + p(\theta(S, x_u))x_u + (1 - p(\theta(S, x_u)))\beta\, \mathbb{E}\, \boldsymbol{U}(S'). \tag{1}$$

Employed Worker's Problem. The value of employment depends on the contingent labor contract $C = \{w,\ \tau,\ W'(S', s'),\ d(S', s')\}$. The worker receives his wage w in the current period and can simultaneously search for other jobs as well. I use x to denote his choice of on-the-job search submarket. If he successfully gets a new job, he receives x as lifetime utility. Notice that the job finding rate $p(\theta(S, x))$ is discounted by the relative on-the-job search efficiency λ, which matches the job-to-job transition rate.

In the next period, if the worker is laid off or the firm exits, he will be unemployed and receive the unemployment value $\boldsymbol{U}(S')$. Otherwise, he can still work for the firm and receive the promised utility $W'(S', s')$. Notice that I assume firms are fully committed to labor contracts, but workers are not. Therefore, for promised utilities lower than the unemployment value, the worker will voluntarily leave the job and become unemployed. The following equation formalizes the value of employment:

$$\begin{aligned}\boldsymbol{W}(S, s, \boldsymbol{C}) = \max_{x}\ & w + \lambda p(\theta(S, x))x \\ & + (1 - \lambda p(\theta(S, x)))\beta\, \mathbb{E}\Big\{[\tau + (1-\tau)(\pi_d + (1-\pi_d)d(S', s'))]\boldsymbol{U}(S') \\ & + (1-\tau)(1-\pi_d)(1 - d(S', s'))\max\{W'(S', s'), \boldsymbol{U}(S')\}\Big\}.\end{aligned} \tag{2}$$

where π_d is the exogenous exit rate of firms.

2.3 Firm's Problem

Firms maximize their present values, namely, the discounted cumulative sum of equity payouts.

A firm's states include realized aggregate shocks $S \in \boldsymbol{S}$, realized firm-specific shocks $s \in \boldsymbol{s}$, the number of employees n, and the set of promised utilities to its employees $\{W(S,s;i)\}_{i\in[0,n]}$, where i is the index of incumbent employees within the firm.

Firms optimize over the current equity payout Δ, next-period debt b', next-period employment n', the number of workers to hire n_h, the submarket x_h in which to search, and next-period exit decisions $d(S',s')$. I assume that a firm only posts vacancies in one submarket each period. Firms also choose the current-period wages of incumbent workers $w(i)$, the layoff probability $\tau(i)$, the wages of newly hired workers $w_h(i')$, and the set of next-period lifetime utilities $\{W(S',s';i')\}_{S'\in\boldsymbol{S}',s'\in\boldsymbol{s}';i'\in[0,n']}$, subject to the participation constraint (8) and the promise-keeping constraint (9). I use $w(i)$ for incumbent employee i's wage, $\bar{w}_m$ for the manager's wage[6], and $w_h(i')$ for the wage of a newly hired employee i'.

Equations (3) to (11) summarize the firm's problem starting from the production stage:

$$\begin{aligned}\boldsymbol{J}(S,s,b,n,\{W(S,s;i)\}_{i\in[0,n]}) = \max_{\substack{\Delta,b',n',n_h,x_h,d(S',s'),\\ \{w(i),\tau(i)\}_{i\in[0,n]},\\ \{w_h(i')\}_{i'\in(n'-n_h,n']},\\ \{W'(S',s';i'),\bar{W}(i')\}_{S'\in\boldsymbol{S}',s'\in\boldsymbol{s}';i'\in[0,n']}}} \Delta \\ + \beta(1-\pi_d)\,\mathbb{E}_{S',s'|S,s}\left\{(1-d(S',s'))\boldsymbol{J}(S',s',b',n',\{W(S',s';i')\}_{S'\in\boldsymbol{S}',s'\in\boldsymbol{s}';i'\in[0,n']})\right\}\end{aligned} \tag{3}$$

$$\text{s.t. } \Delta = Azn^{\alpha} - \int_0^n w(i)di - \bar{w}_m - \epsilon - b - c\frac{n_h}{q(\theta(S,x_h))} - \int_{n'-n_h}^{n'} w_h(i')di' + Q(S,z,b',n')b' \geq 0, \tag{4}$$

$$n' = \int_0^n (1-\tau(i))(1-\lambda p(\theta(S,x^*(S;i))))di + n_h, \tag{5}$$

$$\begin{aligned}x^*(S;i) = \arg\max_x p(\theta(S,x))\Big\{x - \beta\,\mathbb{E}\big\{[\tau + (1-\tau)(\pi_d + (1-\pi_d)d(S',s'))]\boldsymbol{U}(S') \\ + (1-\tau)(1-\pi_d)(1-d(S',s'))\max\{W'(S',s';i'),\boldsymbol{U}(S')\}\big\}\Big\},\end{aligned} \tag{6}$$

$$W'(S',s';i') = \boldsymbol{U}(S') + \bar{W}(i'), \tag{7}$$

$$\bar{W}(i') \geq 0, \tag{8}$$

$$\boldsymbol{W}(S,s,\boldsymbol{C}) \geq \begin{cases} W(S,s,i) & \text{for } i \in [0,n], \\ x_h & \text{for newly hired employees,}\end{cases} \tag{9}$$

[6] One manager per firm.

$$i'(i) = \int_0^i (1-\tau(j))(1-\lambda p(\theta(S, x^*(S))))dj, \forall i \in [0, n], \tag{10}$$

$$Q(S,z,b',n')b' - n_h \frac{c}{q(\theta(S,x_h))} - \int_{n'-n_h}^{n'} w_h(i')di' \geq M(S,z,n) - F_m(S,z), \tag{11}$$

where $F_m(S,z) = \left[\frac{\bar{w}_m + (1-\gamma)\frac{\beta}{1-\beta}\bar{w}_m}{(1-\Phi(A\xi\,\mathbb{E}[A'z'n'^{\alpha} - \int_0^{n'} w(i')di' - \bar{w}_m - \epsilon']))\zeta\,\mathbb{E}z'}\right]^{\frac{1}{\alpha}} \bar{u}$.

The firm chooses its equity payouts Δ for the current period. I assume that firms are subject to the non-negative equity payout constraint in equation (4). I adopt this assumption so that firms cannot always raise cash through equity issuance, which ensures the financial friction is effective in my model. Equity payouts Δ equal output Azn^{α} minus the wage payments to incumbent employees $\int_0^n w(i)di$, minus the manager's wage $\bar{w}_m$, minus the stochastic operating cost ϵ, minus debt b, minus vacancy posting costs $c\frac{n_h}{q(\theta(S,x_h))}$, minus wage payments to newly hired workers $\int_{n'-n_h}^{n'} w_h(i')di'$, and plus borrowings $Q(S,z,b',n')b'$. I assume that output is decreasing returns to scale with respect to the number of employees by letting α be smaller than one. This assumption helps generate meaningful firm sizes, essential to capturing firms' downsizing behaviors when uncertainty is high. The parameter c is the posting cost per vacancy. To hire n_h new workers, the firm needs to post $\frac{n_h}{q(\theta(S,x_h))}$ vacancies, where q denotes the vacancy-filling rate. The total vacancy posting cost is correspondingly $c\frac{n_h}{q(\theta(S,x_h))}$. The bond price Q is determined such that the international financial intermediaries break even, which will be defined later.

Equation (5) is the law of motion for employment. The firm's next-period number of employees is the sum of staying employees and new hires. Employees can separate from the firm for two reasons, on-the-job search and layoffs. Employees optimally choose an on-the-job search submarket to maximize their expected lifetime utility as in eq. (6). I use $x^*(S;i)$ to denote worker i's optimal on-the-job search market. Then the probability for a worker to transit to another firm is $\lambda p(\theta(S, x^*(S;i)))$. If the worker does not find a new job, he faces a layoff probability $\tau(i)$. Therefore, eq. (5) means that the staying employees plus new hires sum to the next-period employment.

Eq. (7) assumes a specific contract form for the next-period promised utilities, which are comprised of two parts: the outside option of unemployment $\boldsymbol{U}(S')$ and a utility markup chosen by the firm $\bar{W}(i')$. The promised utility markup can be contingent on workers, but it does not vary across states. The state-uncontingency of $\bar{W}(i')$ is crucial for the financial friction's effectiveness. Suppose firms' future promises to workers could be contingent on states. Labor contracts will then serve as a much better financial instrument than state-uncontingent bonds, which I conceive of as a counterfactual. Appendix D uses asymmetric information to provide a micro-founded model to justify this setup. The idea is based on Hall and Lazear (1984) and Lemieux, MacLeod and Parent (2012), who prove the optimality of predetermined wages when considering information frictions. Specifically, suppose workers do not have information about the firm's conditions. Firms can lie to pay less to workers. Because workers do not know whether the firm is truly facing a bad shock, they will not accept the wage cut. Section 2.10 discusses the assumption of a state-uncontingent promised utility markup in detail.

Recall that I assume firms are committed to labor contracts, but workers are not. Therefore, the participation constraint (8) shows that the firm should promise a non-negative utility markup to retain its workers. Otherwise, the worker would rather be unemployed. Furthermore, the promise-keeping constraint (9) requires the firm to adhere to its commitment that the worker's employment value is as least the promised lifetime utility. For incumbent worker $i \in [0, n]$, his promised utility is $W(S, s, i)$, one of the firm's state variables. For a newly hired worker, his promised utility is x_h, according to the firm's choice of hiring submarket. Finally, eq. (10) formalizes the transition of the employee's index from i to i'.

The last constraint (11) reflects the agency frictions between shareholders and managers, following Jensen (1986) and Arellano, Bai and Kehoe (2019).[7] This constraint dampens the firm's saving incentives so that the financial friction is effective. Otherwise, firms will build up a large cash buffer so that the financial constraint will never bind.

The micro-foundation of the agency frictions is as follows. I assume that there is a pool of potential managers from which each firm can hire one manager to operate the firm. The total mass of managers is much smaller than of workers, so I abstract from managers when calculating unemployment. Each manager can also be self-employed and produce $\bar{w}_m$ units of goods. The market for managers is competitive, so a manager's wage is also $\bar{w}_m$.

Each period consists of a day and night. During the day, managers are monitored by the firm's shareholders, so managers adopt the firm's optimal policies. The manager uses borrowing $Q(S, z, b', n')b'$ and sales to pay dividends, wages of incumbent workers, his own wage, the operating cost, and debt. Search happens overnight, and the manager is supposed to use the remaining resources to pay vacancy posting costs and the wages of new workers. However, what happens during the night cannot be observed by shareholders until the next day. Therefore, the manager can propose an alternative production plan to the financial intermediary to borrow as much as possible at night. To convince the financial intermediary of the new plan $(\bar{b}', \bar{n}')$, the manager needs to provide proof by posting vacancies to have $\bar{n}'$ workers in the next period if hiring is necessary. The manager thus needs to pay vacancy posting costs and wages for newly hired workers for the alternative proposal. In sum, to maximize available funds, the manager will

[7] Workers own firms in the model, so they are shareholders. I do not explicitly model equity payouts in the worker's problem because workers are risk-neutral and the free entry condition implies that the firm's net present value is zero.

come up with a proposal to achieve maximum possible borrowing net of hiring costs:

$$M(S,z,n) = \max_{\substack{b',n',n_h,x_h,d(S',s'),\\ \{\tau(i)\}_{i\in[0,n]},\{w_h(i')\}_{i'\in(n'-n_h,n']},\\ \{W'(S',s';i'),\bar{W}(i')\}_{S'\in\boldsymbol{S}',s'\in\boldsymbol{s}';i'\in[0,n']}}} Q(S,z,b',n')b' - n_h\frac{c}{q(\theta(S,x_h))} - \int_{n'-n_h}^{n'} w_h(i')di' \tag{12}$$

$$\text{s.t. (5), (7), (8), and (9).} \tag{13}$$

Given the maximum net borrowing $M(S,z,n)$, the remaining credit available for the manager is the maximum net borrowing minus the previous borrowing plus the originally planned but unused money for search, i.e., $M(S,z,n) - Q(S,z,b',n')b' + n_h\frac{c}{q(\theta(S,x_h))} + \int_{n'-n_h}^{n'} w_h(i')di'$.

The manager wants to use the remaining resources to hire workers to produce for his own project in the next period. After the next-period production occurs, shareholders learn what has occurred. The extra workers will be laid-off and search for jobs. Because the manager only needs to hire workers for the next-period production, the outside value of unemployment benefits $\bar{u}$ is the lowest wage for the manager to retain workers to produce. The manager will use the rest of the funds to hire as many workers as possible. The number of workers n_s is determined by

$$n_s = \frac{M(S,z,n) - Q(S,z,b',n')b' + n_h\frac{c}{q(\theta(S,x_h))} + \int_{n'-n_h}^{n'} w_h(i')di'}{\bar{u}}. \tag{14}$$

The manager takes advantage of the firm's productivity for his sided project, so the output is

$$\zeta z' n_s^{\alpha},$$

where ζ indicates the profitability of the manager's own project.

I also assume that there is an auditing technology to detect a manager's intention to deviate at night. The effectiveness of the auditing technology, ξA, is based on a measure of auditing quality, ξ, proportional to aggregate productivity. The incentive and available resources to use the auditing technology are approximated by the firm's expected income $\mathbb{E}[A'z'n'^{\alpha} - \int_0^{n'} w(i')di' - \bar{w}_m - \epsilon']$. The more the firm expects to earn, the more it can and should pay for the auditing technology. I assume that the probability of the manager being caught is Gaussian and determined by the amount of auditing:

$$\Phi\Big(\xi A\,\mathbb{E}[A'z'n'^{\alpha} - \int_0^{n'} w(i')di' - \bar{w}_m - \epsilon]\Big). \tag{15}$$

I model the auditing technology to match the correlation between credit spreads and aggregate output, so the financial effect of aggregate productivity shocks is consistent with the empirical covariance. Otherwise, a positive aggregate productivity shock will cause counterfactually higher credit spreads because firms would have higher income and borrow substantially to avoid the managerial deviations. With the auditing technology, firms do not need to borrow that much when aggregate productivity is high, so the credit spreads decrease, as in the data.

Suppose the manager deviates from the firm's optimal policies and works on his side project. In that case, shareholders will find out and fire the manager the next day. Assume the manager faces probability γ of becoming self-employed (else returning to the manager market), which approximates the punishment for deviation. Therefore, to avoid manager deviations, the firm should not operate with significant unused credit so the manager cannot hire many workers and the side project is not attractive. To do so, the firm should satisfy the following constraint such that the manager prefers to be honest:

$$\left(1-\Phi\Big(\xi A\,\mathbb{E}[A'z'n'^{\alpha}-\int_0^{n'}w(i')di'-\bar{w}_m-\epsilon]\Big)\right)\mathbb{E}_t\,\beta\zeta A_{t+1}z_{t+1}n_s^{\alpha}+\gamma\,\mathbb{E}_t\sum_{j=2}^{\infty}\beta^j\bar{w}_m\leq\mathbb{E}_t\sum_{j=1}^{\infty}\beta^j\bar{w}_m,$$

which delivers the agency friction constraint (11) by plugging in equation (14).

The agency friction constraint (11) incentivizes firms to borrow in the spirit of Jensen (1986). Without agency frictions, firms have strong incentives to save and grow out of the financial friction. There are other options to reduce firms' savings, such as using a lower discount factor. But the model requires an unrealistically low discount factor to match observed leverage since firms are very opposed to liquidation. For other ways to make firms borrow in the presence of financial frictions, Quadrini (2011) provides one summary.

2.4 Bond Pricing

I assume that the economy's financial market is small compared with the rest of the world, so the risk-free interest rate in the international financial market is exogenous. This assumption ensures the block recursivity and thus computational tractability.

International financial intermediaries supply one-period bonds to firms. They are risk-neutral and competitive. The opportunity cost of lending is the risk-free interest rate r in the world financial market, equal to $1/\beta-1$. Financial intermediaries break even when lending to firms. If the firm defaults, the recovery of financial intermediaries is proportional to the firm's expected income π', which equals $A'z'n'^{\alpha}-\int_0^{n'}w(i')di'-\bar{w}_m-\mu_\epsilon$, which approximates the firm's value. That is, lenders recover more when the firm has a higher value.

Formally, the break-even bond price $Q(S,z,b',n')$ is determined by the following equation:

$$Q(S,s,b',n')=\beta\,\mathbb{E}_{S',s'|S,s}\left\{(1-\pi_d)(1-d(S',s'))+[1-(1-\pi_d)(1-d(S',s'))]\min\{\eta\frac{\pi'}{b'},1\}\right\},\quad(16)$$

where η denotes the recovery rate and $\pi'=A'z'n'^{\alpha}-\int_0^{n'}w(i')di'-\bar{w}_m-\mu_\epsilon$.

2.5 Wages

The state of the firm's problem (3) is an infinite-dimensional object because of the set of promised utilities. This section shows how to simplify the firm's problem by deriving wages and default decisions.

First, the promise-keeping constraint (9) always binds. Otherwise, firms could lower wages and earn more. Moreover, Proposition 1 shows that the participation constraint (8) also binds.

Proposition 1 *The participation constraint* (8) *binds, i.e.,* $\bar{W}(i') = 0$*, for any worker* i'*.*

Appendix C provides the proof. Here, I explain the intuition. The participation constraint (8) requires that the promised utility markup should be non-negative. Suppose there exists a strictly positive promised utility markup $\bar{W}(i') > 0$. Then the firm can have a relatively low current wage according to the binding promise-keeping constraint (9). Namely, a positive promised utility markup can be understood as borrowing from the employee by backloading wages. However, borrowing from employees is more costly than borrowing from lenders through collateralized bonds. Therefore, promising a positive utility markup is never optimal for firms.

Given the binding promise-keeping constraint (9) and the participation constraint (8), I am able to determine wages. The binding participation constraint (8) implies that the promised utilities always equal the unemployment value $\boldsymbol{U}$. From the worker's problem (1) and (2) and the binding promise-keeping constraint (9), an incumbent worker's wage is

$$\begin{aligned} w(S) &= \boldsymbol{U}(S) - \lambda \max_x p(\theta(S,x))[x - \beta\,\mathbb{E}\,\boldsymbol{U}(S')] - \beta\,\mathbb{E}\,\boldsymbol{U}(S') \\ &= \bar{u} + (1-\lambda)\max_x p(\theta(S,x))[x - \beta\,\mathbb{E}\,\boldsymbol{U}(S')]. \end{aligned} \tag{17}$$

That is, an incumbent worker's wage equals the outside payoff of being unemployed minus gains from on-the-job search.

Similarly, a newly hired worker's wage equals

$$w_h(S) = x_h - \beta\,\mathbb{E}\,\boldsymbol{U}(S'). \tag{18}$$

The uniquely determined wages in (17) and (18) are crucial for solving the problem quantitatively. Given the wages, the infinite-dimensional distribution of promised utilities not informative as a state variable, and the firm's problem can be simplified by removing the implicit contract constraints, (7), (8), and (9).

In terms of employment, the following Lemma 2.1 shows that while the model pins down the firm's total layoffs, the individual worker's layoff probability is undetermined.

Lemma 2.1 *The firm's total layoffs $\int_0^n \tau(i)di$ is uniquely determined, but the individual probability of layoff $\tau(i)$ is not.*

Proof For each optimal policy, eq. (5) determines the firm's total layoffs

$$\int_0^n \tau(i)di = n - \frac{n' - n_h}{1 - \lambda p(\theta(S, x^*(S)))}. \tag{19}$$

As long as total layoffs are constant, any perturbation of individual layoff probabilities $\{\tau(i)\}_{i \in [0,n]}$ does not affect the firm's value. □

The key reason Lemma 2.1 holds is homogeneous workers, so the distribution of layoff probabilities is irrelevant. Therefore, I will focus on the symmetric decision rule that all employees face the same layoff probability throughout the rest of this paper.

2.6 Firm's Default Decision and Cash on Hand

To further reduce the number of dimensions, I next explore the firm's default decision and rewrite the firm's problem using cash on hand as a state variable.

Notice that the outside value is zero when a firm exits, so a firm defaults and exits when it cannot satisfy the non-negative equity payout constraint (4). Define cash on hand X as:

$$X = Azn^{\alpha} - n[\bar{u} + (1-\lambda)\mu(S)] - \bar{w}_m - \epsilon - b, \tag{20}$$

where $\mu(S) \equiv \max_x p(\theta(S, x))[x - \beta \mathbb{E} \boldsymbol{U}(S')]$. Then, a firm defaults if and only if:

$$X + M(S, z, n) < 0, \tag{21}$$

where $M(S, z, n)$ is maximum net borrowing as defined in equation (12). Therefore, a firm's default decision can be summarized by the operating cost cutoff $\bar{\epsilon}(S, z, b, n)$, defined as:

$$\bar{\epsilon}(S, z, b, n) \equiv Azn^{\alpha} - \int_0^n w(i)di - b + M(S, z, n) - \bar{w}_m. \tag{22}$$

So, the firm defaults when the operating cost is higher than the cutoff $\bar{\epsilon}(S, z, b, n)$ such that the firm cannot satisfy the non-negative equity payout constraint, i.e.,

$$d(S, s, b, n) = \begin{cases} 0, & \text{if } \epsilon \le \bar{\epsilon}(S, z, b, n), \\ 1, & \text{if } \epsilon > \bar{\epsilon}(S, z, b, n). \end{cases} \tag{23}$$

Then the bond price can be simplified to the following expression:

$$
\begin{aligned}
Q(S,z,b',n') = \beta\,\mathbb{E}_{S',z'|S,z}\Big\{&(1-\pi_d)\Phi_\epsilon(\bar{\epsilon}(S',z',b',n')) \\
&+[1-(1-\pi_d)\Phi_\epsilon(\bar{\epsilon}(S',z',b',n'))]\min\{\eta\frac{A'z'n'^{\alpha}-\int_0^{n'}w(i')di'-\bar{w}_m-\mu_\epsilon}{b'},1\}\Big\}.
\end{aligned}
\tag{24}
$$

Plugging in the default cutoff (22) and wages (17) and (18), I rewrite the firm's problem (3) using cash on hand X as a state variable:

$$
\boldsymbol{V}(S,z,X,n) = \max_{\substack{\Delta,b',n',\\ \tau,n_h,x_h}} \Delta+\beta(1-\pi_d)\,\mathbb{E}_{S',z'|S,z}\int_{-\infty}^{\bar{\epsilon}(S',z',b',n')}\boldsymbol{V}(S',z',X',n')d\Phi_\epsilon(\epsilon') \tag{25}
$$

$$
\text{s.t. } (6), \tag{26}
$$

$$
n' = (1-\tau)(1-\lambda p(\theta(S,x^*(S))))n+n_h, \tag{27}
$$

$$
\Delta = X+Q(S,z,b',n')b'-n_h\frac{c}{q(\theta(S,x_h))}-n_h[x_h-\beta\,\mathbb{E}\,\boldsymbol{U}(S')]\geq 0, \tag{28}
$$

$$
X' = A'z'n'^{\alpha}-n'[\bar{u}+(1-\lambda)\mu(S')]-\bar{w}_m-\epsilon'-b', \tag{29}
$$

$$
\bar{\epsilon}(S',z',b',n') = A'z'n'^{\alpha}-n'[\bar{u}+(1-\lambda)\mu(S')]-b'+M(S',z',n')-\bar{w}_m, \tag{30}
$$

$$
Q(S,z,b',n')b'-n_h\frac{c}{q(\theta(S,x_h))}-n_h[x_h-\beta\,\mathbb{E}\,\boldsymbol{U}(S')]\geq M(S,z,n)-F_m(S,z). \tag{31}
$$

The non-negative equity payout constraint (28) reveals that firms' decisions depend on cash on hand X. When cash on hand is too low, the firm defaults because it cannot fully pay wages and debts. On the other hand, when cash on hand is sufficiently high, the firm is not constrained by (28). In this case, the firm solves the following relaxed problem:

$$
\begin{aligned}
\hat{\boldsymbol{V}}(S,z,X,n) = \max_{\substack{b',n',\\ \tau,n_h,x_h}} &X+Q(S,z,b',n')b'-n_h\frac{c}{q(\theta(S,x_h))}-n_h[x_h-\beta\,\mathbb{E}\,\boldsymbol{U}(S')] \\
&+\beta(1-\pi_d)\,\mathbb{E}_{S',z'|S,z}\int_{-\infty}^{\bar{\epsilon}(S',z',b',n')}\boldsymbol{V}(S',z',X',n')d\Phi_\epsilon(\epsilon')
\end{aligned}
\tag{32}
$$

$$
\text{s.t. } (6), (27), (29), (30), \text{ and } (31). \tag{33}
$$

For the relaxed problem, cash on hand does not affect the firm's choices. Let $\hat{b}(S,z,n)$, $\hat{n}(S,z,n)$, $\hat{\tau}(S,z,n)$, $\hat{n}_h(S,z,n)$, and $\hat{x}_h(S,z,n)$ denote the optimal policies for the relaxed problem. The following Lemma 2.2 characterizes firms' decisions with respect to cash on hand.

Lemma 2.2 *(Decision Cutoffs): If $X < -M(S,z,n)$, the firm cannot satisfy the nonnegative external equity payout condition and has to default. If $X \geq \hat{X}(S,z,n) \equiv -\{Q(S,z,\hat{b},\hat{n})\hat{b}-\hat{n}_h\frac{c}{q(\theta(S,\hat{x}_h))}-\hat{n}_h[\hat{x}_h-\beta\,\mathbb{E}\,\boldsymbol{U}(S')]\}$, the firm solves the relaxed problem (32), and the level of cash on hand does not affect the optimal* *decisions.*

Proof If the firm's cash on hand X is less than $-M(S,z,n)$, even though the firm borrows as much as possible, it cannot make nonnegative external equity payouts. So, the firm defaults and exits. If the firm's cash on hand X is more than $\hat{X}(S,z,n)$, then $(\hat{b},\hat{n},\hat{\tau},\hat{n}_h,\hat{x}_h)$ is also the solution to the firm's problem (25), because constraint (28) holds automatically. In this case, cash on hand does not affect any constraints, and the optimal decisions do not depend on cash on hand. □

Lemma 2.2 provides the method to solve the firm's problem by level of cash on hand. This partitioning method has been used by Khan and Thomas (2013), Arellano, Bai and Kehoe (2019), and Ottonello and Winberry (2020).

2.7 Firm Entry

Potential new firms pay a fixed cost k_e to enter. New entrants' productivity will be drawn from the stationary distribution of idiosyncratic productivity $g_z(\cdot)$. New entrants do not produce in the entry period but hire workers as do incumbent firms. New firms start with zero debt and no labor. Then the new entrant's problem is:

$$\boldsymbol{J}_e(S,z) = \max_{n_h,x_h} -n_h \frac{c}{q(\theta(S,x_h))} - n_h[x_h - \beta\,\mathbb{E}\,\boldsymbol{U}(S')] \tag{34}$$

$$+\beta(1-\pi_d)\,\mathbb{E}_{S',z'|S,z}\int_{-\infty}^{\bar{\epsilon}(S',z',b_0,n_h)} \boldsymbol{V}(S',z',X',n_h)d\Phi_\epsilon(\epsilon'), \tag{35}$$

$$\text{s.t. } b_0 = 0,\ \ (29), \text{ and } (30). \tag{36}$$

I use n_e, x_e, and d_e to denote the new entrant's optimal policies.

Notice that both incumbent firms and new entrants only post vacancies in the markets with the lowest hiring cost. Define the minimum hiring cost per worker as

$$\kappa(S) \equiv \min_{x_h}[x_h + \frac{c}{q(\theta(S,x_h))}]. \tag{37}$$

In equilibrium, only submarkets with the lowest hiring cost are active. Given the equilibrium hiring cost $\kappa(S)$,the mapping from the market's promised utility x to the market intensity θ is

$$\theta(S,x) = \begin{cases} q^{-1}\left(\frac{c}{\kappa(S)-x}\right), & \text{if } x \le \kappa(S)-c, \\ 0, & \text{if } x \ge \kappa(S)-c. \end{cases} \tag{38}$$

Notice that the upper bound of the vacancy filling probability q is one. When the submarket's promised utility x is higher than $\kappa - c$, no firm posts vacancies there because the vacancy filling probability cannot be greater than one to compensate for the hiring cost. In this case, the market is inactive, and the market tightness is zero.

The value of $\kappa(S)$ is determined by the free entry condition, which requires that the entry cost equals the expected entry value:

$$k_e = \sum_z \boldsymbol{J}_e(S,z) g_z(z), \forall S. \tag{39}$$

Therefore, the free entry condition closes the model by pinning down the hiring cost $\kappa(S)$ for all aggregate states S.

2.8 Equilibrium

This section defines the block recursive equilibrium of the model.

Definition 2.1 *Let s^f summarize the firm's state variables (S, z, X, n). The block recursive equilibrium consists of the policy and value functions of unemployed workers $\{x_u(S), \boldsymbol{U}(S)\}$; of employed workers $\{x(S, s, \boldsymbol{C}), \boldsymbol{W}(S, s, \boldsymbol{C})\}$; of incumbent firms $\{\Delta(s^f),\ b'(s^f),\ n'(s^f),\ \tau(s^f),\ n_h(s^f),\ x_h(s^f),\ w(S), w_h(S)\}$; of new firms $\{n_e(S),\ x_e(S),\ \boldsymbol{J}_e(S)\}$; the hiring cost per worker $\kappa(S)$; the labor market tightness function $\theta(S, x; \kappa(S))$; and bond price schedules $Q(S, z, b', n')$ such that*

1. *Given the bond price schedules, the hiring cost, and the labor market tightness, the policy and value functions of unemployed workers, employed workers, incumbent firms, and entering firms solve their respective problems (1), (2), (17), (18), (25), and (34).*
2. *The bond price schedule satisfies (24).*
3. *The hiring cost per worker and the labor market tightness function satisfy (37) and (38).*
4. *The free entry condition (39) holds.*

2.9 Aggregate Transitions

Let $\Upsilon(z, X, n)$ denote the mass of firms with states (z, X, n), which is the sum of incumbent firms and new entrants which do not default. The law of motion of the firm distribution is:

$$\begin{aligned}
&\Upsilon'(z', X', n') \\
&= \sum_{z,X,n,\epsilon'} (1-\pi_d)(1-d(S', s'; S, z, X, n))\mathbb{1}\{X'(S', s'; S, z, X, n) = X'\}\phi_\epsilon(\epsilon')\pi_z(z'|z,\sigma)\mathbb{1}\{n'(S, z, X, n) = n'\}\Upsilon(z, X, n) \\
&\quad + m_e(S, \Upsilon)\sum_{z,\epsilon'}(1-\pi_d)(1-d_e(S', s'; S, z))\mathbb{1}\{X'_e(S', s'; S, z) = X'\}\phi_\epsilon(\epsilon')\pi_z(z'|z,\sigma)\mathbb{1}\{n_e(S) = n'\}g_z(z).
\end{aligned} \tag{40}$$

Although a firm's value reduces to zero once it defaults, I assume it still produces in the period of default, and the output adds to GDP. And since its employees participate in production, they are not counted in unemployment in the current period. This setup relieves the concern that varying default rates mechanically drive the fluctuations of output and unemployment. The firm's employees are laid off after the production stage and receive unemployment benefits, and they can search for new jobs in the labor market in the current period. I use Use $\Upsilon^p(z,n)$ to denote the distribution of producing firms, which thus evolves per:

$$\begin{aligned}\Upsilon^p(z',n') = &\sum_{z,X,n,\epsilon'}(1-\pi_d)\pi_z(z'|z,\sigma)\mathbb{1}\{n'(S,z,X,n)=n'\}\Upsilon(z,X,n)\\ &+ m_e(S,\Upsilon)\sum_{z,\epsilon'}(1-\pi_d)\pi_z(z'|z,\sigma)\mathbb{1}\{n_e(S)=n'\}g_z(z).\end{aligned} \tag{41}$$

The mass of entrants $m_e(S,\Upsilon)$ is determined such that total jobs found by workers equals the total jobs created by incumbent firms and new entrants:[8]

$$JF_{\text{workers}}(S,\Upsilon) = JC_{\text{incumbents}}(S,\Upsilon) + m_e(S,\Upsilon)JC_{\text{entrants}}(S,\Upsilon), \tag{42}$$

where

$$JF_{\text{workers}}(S,\Upsilon) = p(\theta(S,x_u^*(S)))\Big(1-\sum_{z,X,n}n\Upsilon(z,X,n)\Big) + \sum_{z,X,n}\lambda p(\theta(S,x^*(S)))n\Upsilon(z,X,n), \tag{43}$$

$$JC_{\text{incumebents}}(S,\Upsilon) = \sum_{z,X,n}n_h(S,z,X,n)\Upsilon(z,X,n), \tag{44}$$

$$JC_{\text{entrants}}(S,\Upsilon) = \sum_{z}g_z(z)n_e(S,z). \tag{45}$$

Aggregate output is the sum of all firms' output:

$$Y = \sum_{z,n}Azn^{\alpha}\Upsilon^p(z,n), \tag{46}$$

[8] Over the business cycle, jobs created by incumbent firms, $JC_{\text{incumbents}}$, can occasionally be larger than jobs found by workers, JF_{workers}. If the total mass of workers is restricted to one, then entry will be negative and not well-defined. To deal with this issue, I assume that when incumbent firms hire more workers than find jobs, the entry m_e is zero, and the mass of workers increases such that equation (42) holds. Then I normalize the economy so that the mass of workers is one unit again. This setup can be understood as an increase in labor force participation. Simulation shows that the average annual population growth rate is less than 0.5%, implying that the potential problem of negative entry is small. Another way to solve this problem is to assign different entry costs for different aggregate states. See Kaas and Kircher (2015) for this treatment.

and the unemployment rate u is the share of workers who do not produce:

$$u = 1 - \sum_{z,n} n \Upsilon^{p}(z,n). \tag{47}$$

2.10 Discussions of the Assumptions

My model's key assumption and driving force is the incompleteness of financial instruments and labor contracts. This section discusses these two assumptions in greater detail.

First, I assume that firms can only borrow through state-uncontingent debt. Beyond the inherent plausibility given the widespread real-world existence of state-uncontingent debt, much related literature uses this assumption to model firms' default risks (Arellano, Bai and Kehoe, 2019; Guntin, 2022; Khan and Thomas, 2013; Ottonello and Winberry, 2020). I also assume that firms can only have non-negative equity payouts. This assumption ensures that firms do not have deep pockets by issuing large amounts of equity, which in turn guarantees that the financial friction is meaningful in my model. Empirically, according to the calculations in Schoefer (2021) using U.S. Flow of Funds data, the average equity raised by U.S. firms from 1951Q4 to 2019Q4 is negative because of share repurchases. Therefore, on average equity issuance is not a primary financing tool for U.S. firms. Admittedly, a more realistic model could have equity issuance, but the recalibration will indicate sizable equity issuance costs. I conjecture that my model's implications would thus not change much. Therefore, I follow the literature to exclude equity financing for simplicity.

Second, novel to the search literature, I assume that labor contracts do not insure firms from idiosyncratic shocks. Specifically, the promised utility is assumed to have a state-uncontingent utility markup over the unemployment value. The state-uncontingency implies that wages are do not change in response to firm-specific idiosyncratic shocks.

The assumption of state-uncontingency distinguishes my framework from the textbook Diamond-Mortensen–Pissarides search models and a subsequent group of models assuming wage rigidity to generate unemployment volatility. Instead of requiring wage rigidity, I allow wages to vary with the outside value of unemployment. That is, wages in my model change flexibly in response to aggregate shocks according to the free entry condition. Moreover, my model does not impose any restrictions on how offers are posted for hiring new workers. Search is competitive in my model. Firms can flexibly post vacancies to hire workers.

Instead, the core of my model's labor contracting friction is that wages are insensitive to firm-specific shocks, preventing firms from hedging against idiosyncratic risks and generating higher unemployment. This restriction is supported by the existing empirical evidence that the pass-through from firms' idiosyncratic transitory shocks to workers' earnings is insignificant (Guiso, Pistaferri and Schivardi, 2005; Rute Cardoso and Portela, 2009).

I also provide a theory of information frictions to micro-found the labor contracting friction in Appendix D. The idea follows Hall and Lazear (1984) and Lemieux, MacLeod and Parent (2012), who use information frictions to rationalize wage stickiness. The intuition is the difficulty of contracting on firm-specific shocks when workers do not have the information. If it is an aggregate shock, everyone knows about it. Workers accept wage cuts based on the observed outside options. But if it is an idiosyncratic shock, workers do not have the information whether the firm is really in a worse situation. So, it is rational for them to suspect the firm is lying to cut their wages. Because workers do not trust firms, the incentive-compatible labor contracts are not contingent on firms' idiosyncratic shocks.

3 Quantitative Analysis

In this section, I first parametrize the model by matching moments. Then I explain the mechanism and show the connections among uncertainty shocks, contracting frictions, and unemployment. Next, I apply the model to U.S. business cycles to see to what degree the model can explain unemployment dynamics during recessions. Finally, I conduct policy experiments to investigate the impacts of labor market policies in the context of elevated uncertainty.

I use the global method o grid search to solve the problem numerically. Despite aggregate shocks and rich heterogeneity, the model is computationally tractable because of the block recursivity. Appendix E describes the computational algorithm in greater detail.

3.1 Parameterization

There are four shocks (A, σ, z, ϵ) in the economy. The logs of aggregate productivity and uncertainty both follow AR(1) processes:

$$\log A_{t+1} = \rho_A \log A_t + \sigma_A \sqrt{1-\rho_A^2}\epsilon_t^A, \tag{48}$$

$$\log \sigma_{t+1} = (1-\rho_\sigma)\log\bar{\sigma} + \rho_\sigma \log \sigma_t + \sigma_\sigma \sqrt{1-\rho_\sigma^2}\epsilon_t^\sigma, \tag{49}$$

where the innovations ϵ_t^A and ϵ_t^σ follow the standard normal distribution. I follow Schaal (2017) and allow ϵ_t^A and ϵ_t^σ to be correlated with the correlation coefficient $\rho_{A\sigma}$.

Firm j's idiosyncratic productivity also follows an AR(1) process:

$$\log z_{jt+1} = \rho_z \log z_{jt} + \sigma_t \sqrt{1-\rho_z^2}\epsilon_{jt}^z, \tag{50}$$

where ϵ_{jt}^z follows the standard normal distribution, and the time-varying uncertainty σ_t controls the standard deviations of the innovation.

The i.i.d. operating cost shock ϵ's distribution, $\Phi(\cdot)$, is normally distributed with mean μ_ϵ and standard deviation σ_ϵ.

I follow Menzio and Shi (2010) and Schaal (2017) in using the following job finding probability function:

$$p(\theta) = \theta(1 + \theta^\gamma)^{-1/\gamma}. \tag{51}$$

Accordingly, the vacancy-filling rate $q(\theta)$ is $p(\theta)/\theta$.

I calibrate the parameters as closely as possible to Schaal (2017) for comparison. Table 1 shows the parameter values. The parameters in Panel A are exogenously assigned, following the literature. The quarterly discount factor β equals 0.988, corresponding to a 5% annual risk-free interest rate. The labor coefficient α is set as 0.66 to be consistent with the wage share. I follow Khan and Thomas (2008) to set the persistence of idiosyncratic productivity ρ_z as 0.95.

The remaining parameters in Panel B are calibrated by matching moments using U.S. data. Table 2 shows the moments in the data and the model. Because of the model's non-linearity, all parameters influence all moments jointly. However, each moment is primarily affected by certain parameters, and I organize them into four groups accordingly. The first two groups of parameters are related to aggregate shocks and the labor market, which are calibrated according to Schaal (2017). The last group of parameters, associated with the financial market, are added upon Schaal's (2017) calibration for my financial channel.

The first set of parameters controls the AR(1) processes of aggregate shocks. For the aggregate productivity parameters (ρ_A, σ_A), I use the autocorrelation and standard deviation of output as target moments. The data moments are calculated by Schaal (2017) using real GDP from the Bureau of Economic Analysis. He detrends the time series of output by an HP-filter with a parameter of 1,600 to obtain the log deviations.

To calibrate the process of uncertainty shocks to firm-level productivity, I follow Bloom et al. (2018) to use the interquartile range of sales growth rates across firms (IQR) to reflect the degree of volatility in the economy. I obtain the firm-level sales data from Compustat. I use the Consumer Price Index for All Urban Consumers (CPI) to deflate sales. To avoid the composition of firms influencing the IQR, I follow Bloom et al. (2018) and use only firms with at least 100 quarters of observations. I also drop firms in the finance and public administration sector. I also follow Davis and Haltiwanger (1992) by measuring the sales growth rate at quarter t as $(y_t - y_{t-4})/((y_t + y_{t-4})/2)$, so growth rates are less affected by extreme values of sales. Next, because firms may respond heterogeneously to shocks in different industries, the IQR of the original sales growth rates may reflect not only the underlying uncertainty shocks but also heterogeneous responses. Therefore, I follow Bloom et al. (2018) and Schaal (2017) and measure volatility controlling for firms' permanent

Table 1: Parameter Values

Parameters	Notations	Values	Sources/Matched Moments
Panel A: Assigned Parameters			
Discount factor	β	0.988	5% annual interest rate
Decreasing returns to scale coefficient	α	0.66	Labor share
Persistence of productivity	ρ_z	0.95	Khan and Thomas (2008)
Panel B: Parameters from Moment Matching			
Aggregate shocks			
Persistence of aggregate productivity	ρ_A	0.920	Autocorrelation of output
SD of aggregate productivity	σ_A	0.024	SD of output
Mean of uncertainty	$\bar{\sigma}$	0.248	Mean of IQR
Persistence of uncertainty	ρ_σ	0.880	Autocorrelation of IQR
SD of uncertainty	σ_σ	0.092	SD of IQR
Correlation between ϵ_t^A and ϵ_t^σ	$\rho_{A\sigma}$	−0.020	Correlation (output, IQR)
Labor market			
Unemployment benefits	$\bar{u}$	0.142	EU rate
Vacancy posting cost	c	0.001	UE rate
Relative on-the-job search efficiency	λ	0.100	EE rate
Matching function elasticity	γ	1.600	$\epsilon_{UE/\theta}$
Entry cost	k_e	15.21	Entry/Total job creation
Mean operating cost	$\bar{w}_m + \mu_\epsilon$	0.001	Average establishment size
Financial market			
SD of production costs	σ_ϵ	0.080	Mean credit spread
Agency friction	$\tilde{\zeta}$	2.400	Median leverage
Auditing quality	ξ	1.780	Correlation (output, spreads)
Recovery rate	η	2.410	Correlation (IQR, spreads)
Exogenous exit rate	π_d	0.021	Annual exit rate

Notes: Panel A shows parameters exogenously assigned. Panel B shows parameters calibrated to match the targeted data moments in Table 2.

heterogeneity and industry heterogeneity over business cycles. Specifically, I project firms' sales growth on firm-level fixed effects and industry-quarter fixed effects to obtain residuals of sales growth[9] and use these residuals as my measure of volatility to construct the IQR. Given the time series of IQR, I compute its mean, detrend the time series with an HP-filter, and compute the autocorrelation and standard deviations to serve as targets for the uncertainty shock parameters $(\mu_\sigma, \rho_\sigma, \sigma_\sigma)$. I also use the correlation between output and IQR to pin down the correlation between aggregate productivity shocks and uncertainty shocks $\rho_{A\sigma}$. The output data is quarterly real GDP per capita from the Bureau of Economic Analysis, retrieved from FRED. It is detrended by the HP-filter with 1,600 as the parameters to obtain the log deviations.

The second group of parameters is related to the labor market. The unemployment utility $\bar{u}$ is the opportunity cost of working, affecting wages and thus firms' firing decisions; the vacancy

[9] Firm industry is based on the Standard Industrial Classification (SIC) at the 3-digit level.

Table 2: Matched Moments

Moments	Data	Benchmark Model $A+\sigma$	Benchmark Model A	No Contracting Frictions $A+\sigma$	No Contracting Frictions A
Aggregate shocks					
Autocorrelation of output	0.839	0.868	0.877	0.838	0.867
SD of output	0.016	0.015	0.015	0.019	0.017
Mean of IQR	0.171	0.169	0.160	0.161	0.169
Autocorrelation of IQR	0.647	0.611	-	0.623	-
SD of IQR	0.013	0.011	-	0.010	-
Correlation (output, IQR)	-0.351	-0.305	-	-0.314	-
Labor market					
UE rate	0.834	0.814	0.817	0.840	0.832
EU rate	0.076	0.083	0.080	0.063	0.070
EE rate	0.085	0.081	0.082	0.044	0.044
$\epsilon_{UE/\theta}$	0.720	0.717	0.707	0.711	0.705
Average establishment size	15.6	15.4	15.3	15.5	15.6
Entry/Total job creation	0.21	0.18	0.18	0.27	0.25
Financial market					
Mean credit spread (%)	1.09	0.96	0.97	-	-
Median leverage (%)	26	21	21	-	-
Correlation (output, spreads)	-0.549	-0.503	-	-	-
Correlation (IQR, spreads)	0.462	0.448	-	-	-
Annual exit rate (%)	8.9	9.0	9.2	9.0	9.0

Notes: This table shows the targeted data moments and moments matched by the benchmark model and the model without contracting frictions. $A+\sigma$ means the model has both aggregate productivity shocks and uncertainty shocks, and A means the model only has aggregate productivity shocks. Table 6 reports the recalibrated parameters of the four models.

posting cost c primarily affects firms' hiring decisions; and the relative on-the-job search efficiency λ influences the probability of job-to-job transitions. I calibrate these three parameters using the transition probability from employment to unemployment (EU), the transition probability from unemployment to employment (UE), and the transition probability from employment to employment (EE). The data moments for EU, UE, and EE are the quarterly versions of the monthly ones in Schaal (2017), who obtains the monthly EU and UE rates from Shimer (2005) and the EE rate from Nagypál (2007). The matching function elasticity γ is calibrated to match the elasticity of UE rates to the labor market tightness θ, which Schaal (2017) obtains from Shimer (2005). The entry cost k_e is calibrated to match the share of jobs created by entrants, which is calculated by Schaal (2017) using Business Employment Dynamics (BED). The mean operating cost affects firms' exit decisions and thus can be pinned down by the average establishment size, measured by Schaal (2017) using the 2002 Economic Census. Notice that the mean operating cost μ_ϵ and the manager's wage $\bar{w}_m$ symmetrically influence firms' cash on hand, so I calibrate $\mu_\epsilon + \bar{w}_m$ using the average establishment size.

Parameters in the last group deal with the financial market. First, I use the average credit spread to calibrate the standard deviation of the operating cost, σ_ϵ. The credit spread is the difference between the yield on Baa and Aaa corporate bonds. The data source is Moody's, retrieved from FRED, Federal Reserve Bank of St. Louis. Correspondingly, the credit spread in the model is the annualized difference between the actual borrowing cost and the risk-free interest rate:

$$\frac{1}{Q(S, z, b', n')} - \frac{1}{\beta}. \tag{52}$$

Because the agency friction constraint incentivizes firms to borrow, I use the median leverage of firms to calibrate the agency friction parameter $\tilde{\zeta} \equiv \zeta/(\bar{w}_m + (1-\lambda)\frac{\beta}{1-\beta}\bar{w}_m)$. Leverage is the ratio of the firm's total debt to its annualized sales. The data moment of median leverage is from Arellano, Bai and Kehoe (2019). Next, I use the correlation between output and credit spreads to parameterize the auditing technology ξ, and I use the correlation between IQR and credit spreads for the recovery rate η. Targeting the two correlations anchors the financial impacts of aggregate productivity shocks and uncertainty shocks. The last parameter is the exogenous exit rate π_d, which helps generate exits beyond defaults. I use the annual exit rate calculated from Business Dynamics Statistics (BDS) to calibrate π_d.

3.2 Differences from the Calibration of Schaal (2017)

My parametrization is based on Schaal (2017) when estimating parameters related to aggregate shocks and the labor market. I follow his calibration closely except for the following three differences.

First, Schaal (2017) uses a monthly frequency, while my model's frequency is quarterly. I choose the quarterly frequency to accommodate the data moments related to the financial market. As is common in the finance literature, leverage should be one of the target moments, defined as a firm's debt over annualized sales. In a quarterly model, annualized sales in the denominator equal four times the quarterly sales. However, suppose the model is monthly. Annualized sales in the denominator will be 12 times the monthly sales. Therefore, when targeting the same median leverage in the data, the monthly model implies the firm's debt is much higher than its per-period sales, and the default risks will be counterfactually high. Thus, I follow the finance literature and use a quarterly model.

Second, Schaal (2017) uses 0.85 as the decreasing returns to scale coefficient α, and I use 0.66. Neither of us explicitly models capital, while Schaal (2017) chooses 0.85 to approximate the total decreasing returns. But he also points out that the results are unaffected when targeting a labor share of 0.66. Because my mechanism is about wage commitments, I choose to target the wage share so that the size of firm commitments is consistent with the data. If I used 0.85 as the decreasing returns to scale coefficient, wage commitments would be larger, increasing the risk to firms and generating counterfactually high credit spreads.

Third, to calibrate the uncertainty shock process, Schaal (2017) uses the interquartile range (IQR) of innovations to idiosyncratic productivity calculated by Bloom et al. (2018). Instead, I follow both Bloom et al. (2018) and Arellano, Bai and Kehoe (2019) and use the IQR of firms' sales growth rates. I make this deviation because targeting the IQR of innovations to idiosyncratic productivity leads to a counterfactually high sales volatility. Specifically, the IQR of sales growth in the model will be more than five times the data. Because sales volatility determines firm default probability, the counterfactually high volatility of sales leads to counterfactually large default rates and extremely high credit spreads. To keep the magnitude of financial effects reasonable, I use the IQR of firms' sales growth rates as in Arellano, Bai and Kehoe (2019), who also model uncertainty shocks and the firm financial friction simultaneously. The main difference between using the IQR of firms' sales growth rates and the IQR of idiosyncratic productivity innovations is the level of uncertainty $\bar{\sigma}$.[10] But, they have very similar business cycle behaviors in terms of innovations to uncertainty, i.e., ϵ_t^{σ}. In particular, Figure 9 in Appendix B compares the log deviations of estimated aggregate productivity shocks and uncertainty shocks of the model without contracting frictions with Schaal (2017), showing that the two uncertainty shocks have similar variations over business cycles.

As a validation of this calibration choice, Table 3 shows that my counterfactual model without contracting frictions has very similar business cycle statistics to Schaal (2017). Further, Figure 7 displays the changes in unemployment during recessions, and the model without contracting frictions also yields very similar patterns to Schaal (2017).

3.3 Business Cycle Statistics

To assess how well my model can explain business cycles, I report simulated business cycle statistics in Table 3. To compute the moments, I simulate the model for 3,000 quarters and use the log deviations from an HP-filter trend with a smoothing parameter of 1,600. Beyond the benchmark model, I consider three alternative models for comparison. All models are recalibrated by matching the same moments. Table 2 contains the calibration results and Table 6 reports the recalibrated parameters.

Since all models are calibrated, they have similar predictions for output and labor productivity in the first two columns, and I will focus on their differences in terms of unemployment volatility in brief. The next section will investigate the mechanism in greater detail.

Benchmark Model With Both Shocks. The standard deviation of unemployment is 0.121 in the data (Panel A), and my benchmark model with both aggregate productivity shocks and uncertainty

[10] One concern about the idiosyncratic productivity measured by Bloom et al. (2018) is that they use revenue TFP, which can reflect firm pricing power instead of productivity (Bils, Klenow and Ruane, 2021; Hsieh and Klenow, 2009).

Table 3: Business Cycle Statistics

	Y	Y/L	U	V	Hirings	Quits	Layoffs	Wages
Panel A: Data								
Std Dev.	0.016	0.012	0.121	0.138	0.058	0.102	0.059	0.008
cor(Y,x)	1	0.590	-0.859	0.702	0.677	0.720	-0.462	0.555
Panel B: Benchamark Model								
Both A and σ Shocks								
Std Dev.	0.015	0.013	0.106	0.097	0.048	0.029	0.111	0.011
cor(Y,x)	1	0.910	-0.500	0.774	0.140	0.884	-0.202	0.876
Only A Shocks								
Std Dev.	0.015	0.011	0.079	0.081	0.019	0.028	0.053	0.010
cor(Y,x)	1	0.988	-0.901	0.904	0.010	0.964	-0.853	0.980
Panel C: Model Without Contracting Frictions								
Both A and σ Shocks								
Std Dev.	0.019	0.016	0.090	0.085	0.060	0.079	0.068	-
cor(Y,x)	1	0.990	-0.797	0.485	-0.101	0.401	-0.602	-
Only A Shocks								
Std Dev.	0.017	0.014	0.076	0.061	0.041	0.057	0.053	-
cor(Y,x)	1	0.994	-0.882	0.658	-0.158	0.610	-0.813	-

Notes: Panel A shows the business cycle moments in the data. Panels B and C report moments of 3,000-quarter simulations of the benchmark model and the model without contracting frictions, with and without uncertainty shocks. "Both A and σ Shocks" means the model has both aggregate productivity shocks and uncertainty shocks, and "Only A Shocks" means the model has only aggregate shocks. Both the data and the model simulations are log-detrended by the HP filter with smoothing parameter 1600. To be consistent with the notations in Schaal (2017), Y denotes output, Y/L is output per worker, U represents unemployment, and V is vacancies.

shocks can generate a standard deviation of 0.106 (Panel B), which indicates that my model can explain much of the unemployment volatility in the data. Next, I use three reference models to explain the roles of both financial frictions and uncertainty shocks in this quantitative performance.

Benchmark Model With Only Aggregate Productivity Shocks. The second part of Panel B calibrates the same model but keeps only the aggregate productivity shocks. This is the uncertainty of firms' idiosyncratic productivity is no longer time-varying, and there are only aggregate productivity shocks in driving the business cycles. Now the model only generates a standard deviation of unemployment of 0.079. Compared to the the number of 0.106 in the benchmark case, adding uncertainty shocks generates 22% of the unemployment volatility n the data. So, I conclude that uncertainty shocks are crucial for understanding the fluctuations of unemployment.

The interaction between financial and labor contracting frictions is the key. To quantify their roles, notice that neither of them is effective individually. That is, suppose either friction is absent. The model will collapse to the one without contracting frictions at all. Specifically, if labor contracts are complete, firms can use them as state-contingent instruments to hedge against shocks. Actually, firms can just borrow from workers through complete labor contracts without needing to borrow through incomplete financial instruments. And the financial friction induced by state-uncontingent debt is irrelevant. On the other hand, if the financial market is complete, the within-contract labor market friction has no impact because how wages are paid within labor contracts is irrelevant, given that it is the present value of wages that determines the incentives of hiring and firing.

Therefore, I solve and recalibrate the model without contracting frictions. The model is solved by joint surplus maximization as in Schaal (2017). Table 2 and Table 6 show the recalibration results of the moment matching and the values of parameters. Because there are no financial variables to pin down the standard deviation of operating costs, I use the same σ_ϵ as in the benchmark. Financial parameters, including the agency friction $\tilde{\zeta}$, auditing quality ξ, and recovery rate η, are not applicable in this case. The recalibrated parameters in Table 6 suggests that the standard deviations of both shocks need to increase to match the variations of aggregate output and IQR in the data. The need for larger shocks suggests the model without contracting frictions underestimate the impact of aggregate shocks.

Notice that Table 2 shows this counterfactual model's job-to-job transition rate (EE rate) is lower than the data. The reason is that firms can now control workers' on-the-job behaviors. So, when firms do not want to have separations, they can prevent any employee from leaving through on-the-job search. This unrealistic feature lets the model have a lower job-to-job transition rate. In Appendix F, I solve another counterfactual model without the financial friction but kept the benchmark labor contracting outcomes. That is, I allow workers to do on-the-job search as in the benchmark case, where the optimal on-the-job decision is determined by eq. (17). Table 8 shows that this model's EE rate is consistent with the data. Despite this difference in on-the-job search, the results in Appendix F indicate that this model has a very similar quantitative performance to the model without contracting frictions. Therefore, I keep focusing on the counterfactual model without contracting frictions throughout the paper.

Model Without Contracting Frictions and With Both Shocks. Panel C in Table 3 reports the business cycle statistics in this case.[11] The first part of Panel C shows the results when the model has both aggregate productivity shocks and uncertainty shocks. It generates only a 0.090 standard deviation of unemployment, consistent with the number in Schaal (2017). Comparing it with the 0.106 of the benchmark model reveals the important role of contracting frictions in driving unemployment fluctuations.

Model Without Contracting Frictions and With Only Aggregate Productivity Shocks. The second part of Panel C shows the model statistics without contracting frictions and with only

[11] Panel C in Table 3 does not report the statistics of wages because wages are undetermined when there is no contracting friction. Table 9 in Appendix F reports the wage statistics generated by the counterfactual model that does not have the financial friction but keeps the benchmark labor contracting outcomes.

the aggregate productivity shock. It generates a standard deviation of unemployment of 0.076. This number is similar to the 0.079 in the benchmark case with contracting frictions. It suggests that contracting frictions need to interact with uncertainty shocks to have a significant impact on unemployment volatility. The key is the equilibrium response of wages. Distinguished from aggregate productivity shocks, the offsetting effect of equilibrium wages is much smaller for uncertainty shocks. High uncertainty spreads the distribution of firms' idiosyncratic productivity, and high-productivity firms may still benefit from elevated volatility, which maintains wages at a high level. This is called the Oi-Hartman-Abel effect (Oi (1961), Hartman (1972), Abel (1983)) in the volatility literature. Section 3.4.3 discusses this in greater detail.

3.4 Inspecting the Mechanism

In this section, I first explain the mechanism via firm-level decision rules. Next, I show the impulse responses at the macro level to illustrate the impact of aggregate productivity shocks and uncertainty shocks.

3.4.1 Firm-level Decisions

I use the median firm's decision rules to explain how high uncertainty leads firms to downsize employment.

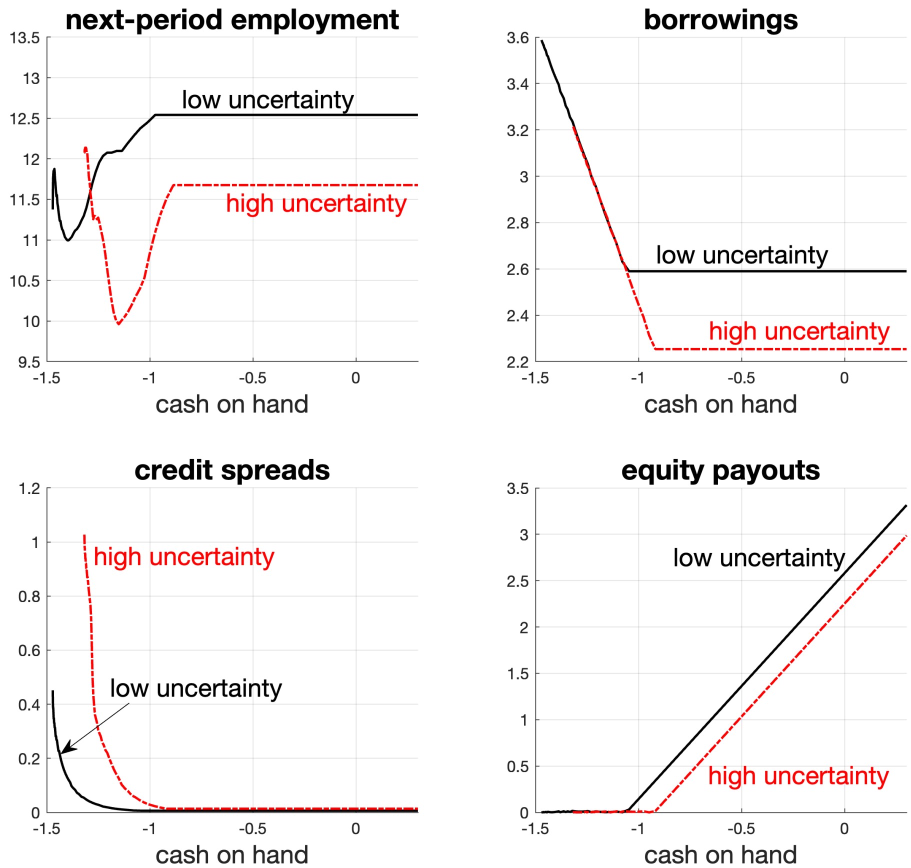
Panel A in Figure 3 shows how firms' decisions depend on cash on hand X and the level of uncertainty. I vary the cash on hand on the horizontal axis and fix the firm's idiosyncratic productivity and employment at their median levels. The decision rules are next-period employment n', borrowing Qb', credit spread $1/Q - 1/\beta$, and equity payouts Δ. The solid black lines are for the low uncertainty state, one unconditional standard deviation below the mean uncertainty. The dash-dot red lines are for the high uncertainty state, one unconditional standard deviation higher than the mean.

First, the relations between the decision rules and cash on hand are consistent with Lemma 2.2. When cash on hand is higher than a cutoff, firms' employment, borrowing, and credit spreads no longer depend on cash on hand. The equity payouts increase with cash on hand one for one in this case. When below the cutoff, the equity payout is zero because the non-negative equity payout constraint binds.[12] As cash on hand decreases, firms need to borrow more to satisfy the non-negative equity payout constraint. Credit spreads subsequently rise. Employment for the next period decreases as cash on hand decreases because firms face higher default risks and cut employment to avoid defaulting on wage payments next period. But when cash on hand is very low, firms hire slightly more workers. The reason is the increased default probability. Conditional

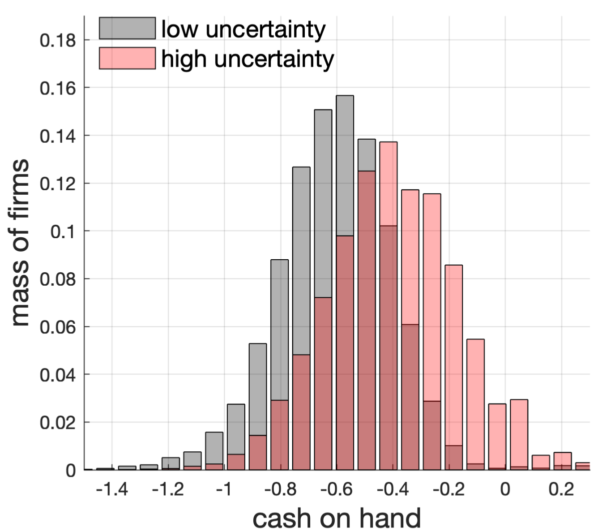
[12] The slight difference between zero is due to computational errors of grid search.

Figure 3: The Effect of Different Levels of Uncertainty

Panel A: Firm's Decisions Rules



Panel B: Distribution of Firms' Cash on Hand



Notes: Panel A shows the median firm's decision rules for next-period employment, borrowing, credit spread, and equity payouts with respect to cash on hand. The solid black lines are for the low uncertainty state, and the dash-dot red lines are for the high uncertainty state. The firm's idiosyncratic productivity and current employment are fixed at their median levels. Panel B shows the stochastic stationary distribution of firms' cash on hand when uncertainty is fixed at the low level (black) and the high level (red). Aggregate productivity is set at a high level for both panels. The "high" or "low" state means one unconditional standard deviation above or below the mean.

on survival, firms have higher expected productivity. Thus, firms decide to take on more risk and hire more workers.

Second, firms' decisions depend on the level of uncertainty. Higher uncertainty implies greater default risks because of the larger probability of drawing low productivity, resulting in higher credit spreads. Therefore, firms are averse to borrowing and equity payouts. And the insensitivity of wages to firm-specific shocks implies that wage bills are debt-like commitments to workers, so hiring a worker is isomorphic to borrowing more. Therefore, firms decrease the number of employees in high-uncertainty states.

Risk aversion also reflects on the distribution of firms' cash on hand. Panel B depicts the stochastic stationary distribution when uncertainty is kept at the high and low levels, respectively. It shows that the distribution of cash on hand shifts to the right when uncertainty is high. Firms want to hold a higher level of cash on hand because they now face higher idiosyncratic risk. A safer portfolio is desirable in this case.

3.4.2 Aggregate Dynamics

I use the impulse responses in Figures 4 and 5 to illustrate the macro-level implications of aggregate productivity shocks and uncertainty shocks.

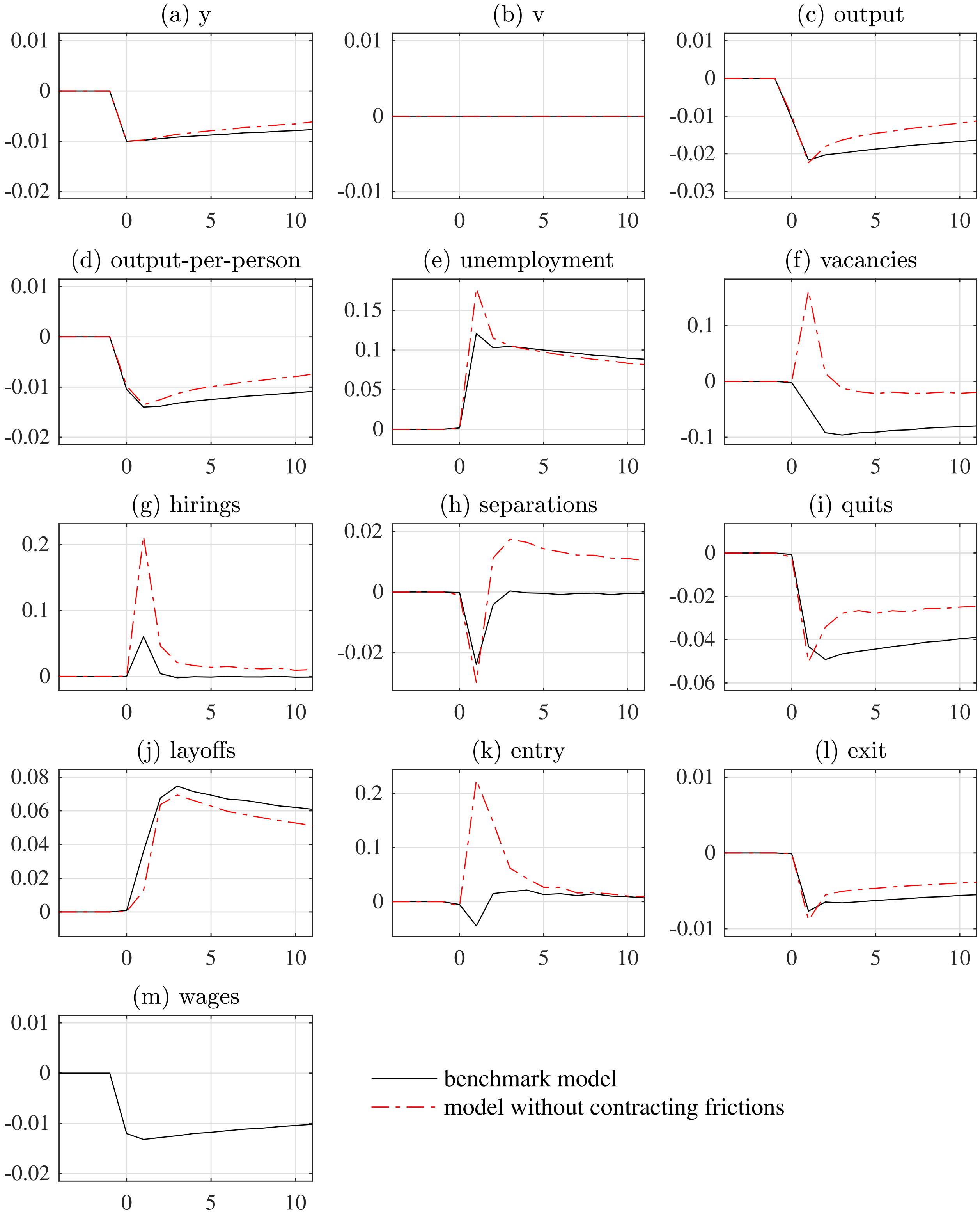
A Negative Aggregate Productivity Shock.

Figure 4 plots the impulse responses to a negative aggregate productivity shock. To draw the impulse responses, I simulate the economy's distribution 4,000 times with stochastic aggregate shocks. At quarter 0, I impose a 1% negative aggregate productivity shock. Then I let the economy evolve stochastically again. The impulse responses in Figure 4 are the average of the 4,000 simulated paths. The solid black lines are from the benchmark model, and the dash-dot red lines are without contracting frictions.

For the benchmark model in solid black lines, a 1% negative aggregate productivity shock leads to a 2% decline in output and 10% higher unemployment. And the dash-dot red lines show the results when there are no contracting frictions, showing that the changes in output and unemployment are similar to those in the benchmark. This finding implies that the financial channel of incomplete contracts primarily operates through uncertainty shocks. The reason is that equilibrium wages decline more in response to the aggregate productivity shock when there are contracting frictions (Panel (m)), which offsets the negative impact of lower aggregate productivity.

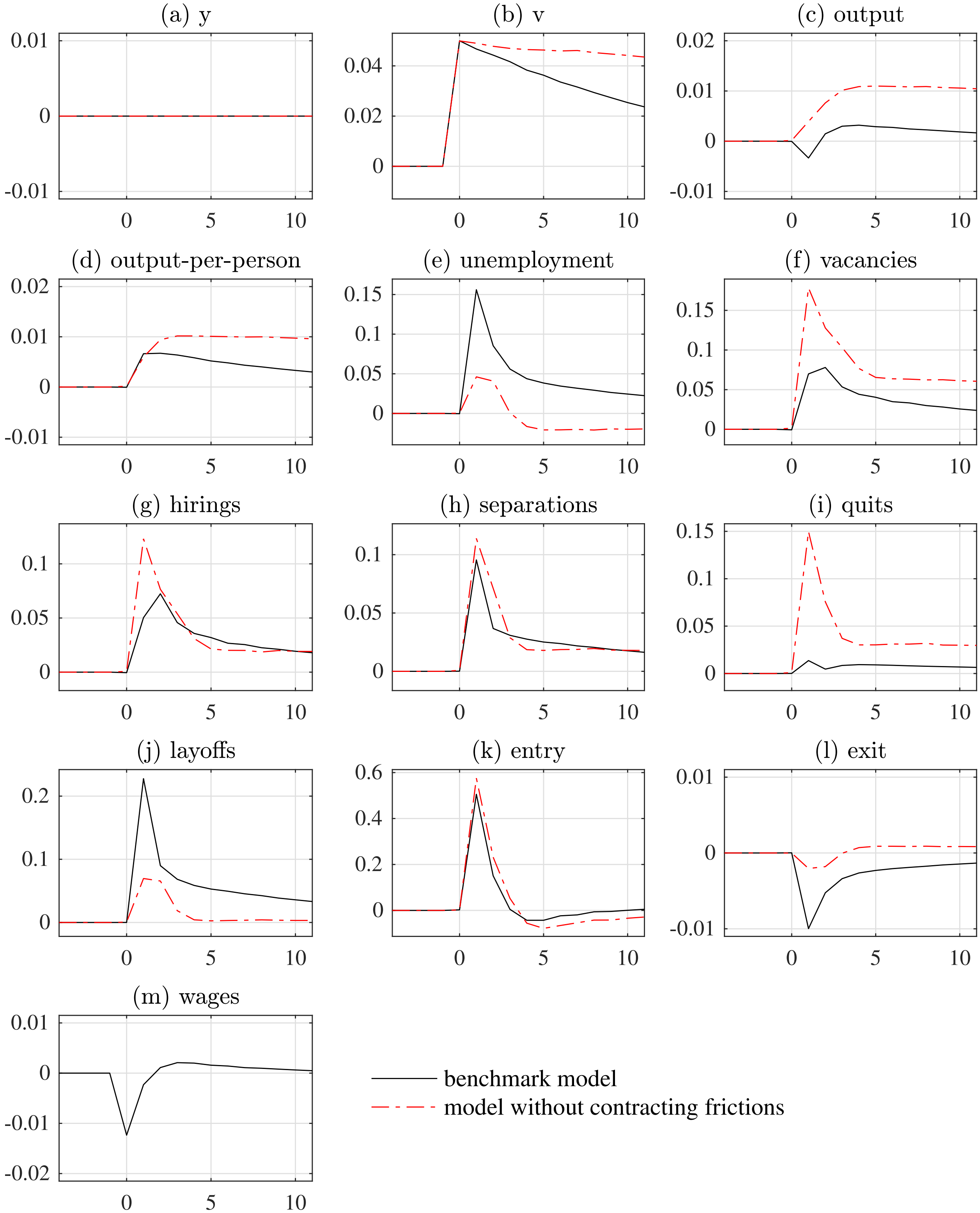
A Positive Uncertainty Shock. Figure 5 displays the impulse responses following a 5% positive uncertainty shock. The methodology to draw the impulse responses is the same, except I shock the simulations with a 5% positive uncertainty shock at quarter 0.

Figure 4: Aggregate Impulse Responses to a 1% Negative Aggregate Productivity Shock



Notes: The panels are impulse responses to a 1% transitory negative aggregate productivity shock at quarter 0. The impulse responses are the average of 4,000 simulated paths, presented as log deviations from the mean. Solid black lines are the benchmark results. Dash-dot red lines are for the model without contracting frictions. I use Schaal's (2017) code when plotting this figure.

Figure 5: Aggregate Impulse Responses to a 5% Positive Uncertainty Shock



Notes: The panels are impulse responses to a 5% positive uncertainty shock at quarter 0. The impulse responses are the average of 4,000 simulated paths, presented as log deviations from the mean. Solid black lines are the benchmark results. Dash-dot red lines are for the model without contracting frictions. I use Schaal's (2017) code when plotting this figure.

In the benchmark model, a 5% positive uncertainty shock lowers output slightly and raises unemployment by 15%, while the model without contracting frictions generates an output boom and much less of an increase in unemployment. This result explains why Schaal (2017) finds it difficult for a search model to generate a sufficient increase in unemployment during the Great Recession. His model behaves in the same way as my counterfactual model without contracting frictions, where elevated uncertainty generates unemployment mainly through the reallocation of workers across firms. My work builds on his model by considering the financial channel of incomplete labor contracts, which interacts with uncertainty shocks and improves the model's ability to explain unemployment fluctuations.

3.4.3 Specialness of Uncertainty Shocks

In this section, I explain what is special about uncertainty shocks compared to aggregate productivity shocks, such that contracting frictions operate mainly through elevated uncertainty instead of lower aggregate productivity.

In the last panels of Figures 4 and 5, I show the impulse responses of wages. The decline of wages is larger and more persistent in response to a negative aggregate productivity shock than a uncertainty shock.[13] Because the lower wages offset the effect of the negative aggregate productivity shock, adding contracting frictions does not amplify unemployment volatility much. The offsetting equilibrium effect of wages is also the reason for the unemployment volatility puzzle in Shimer (2005). He finds that the calibrated standard Diamond-Mortensen–Pissarides model generates less than 10% of the observed standard deviation of unemployment. Because of the free entry condition, the decline in wages largely absorbs the effect of aggregate productivity shocks. Similarly, the free entry condition in my model also leads to a large decrease in wages to offset the impact of aggregate productivity shocks.

Nonetheless, the offsetting effect of wage dynamics is smaller for uncertainty shocks. The reason wages do not decrease much is simply that an uncertainty shock is also a dispersion shock. A positive uncertainty shock spreads the distribution of firm-level productivity. Since a firm's profit is convex in terms of its idiosyncratic productivity, a wider distribution delivers a higher expected profit. The uncertainty literature calls this property the Oi-Hartman-Abel effect (Oi (1961), Hartman (1972), Abel (1983)). The Oi-Hartman-Abel effect is stronger for firms with high productivity because firm productivity is persistent. These firms' high expected values indicate that wages do not need to decrease much to satisfy the free entry condition. Since the equilibrium wage is not low enough for firms to offset the higher risk of drawing low idiosyncratic productivity, they hire fewer workers. Therefore, at the aggregate level, higher uncertainty translates into higher

[13] Actually, if we look into individual workers' wages instead of average wages, both the wages of incumbent workers and newly hired workers increase when uncertainty increases. The average wage in Figure 5 decreases because the share of newly hired workers increases, who have lower wages.

unemployment.

In sum, unlike a typical search model with only aggregate productivity shocks and homogeneous firms, I argue that uncertainty shocks are crucial to understanding unemployment because firms face idiosyncratic risk and do not have an instrument to borrow against it.

3.5 Event Study for U.S. Recessions

The preceding section shows my model can explain much of the unconditional variance of unemployment. In this section, I use my model to understand five U.S. past recessions from the 70s to the Great Recession. For this exercise, I first apply the particle filter to my calibrated model, equipped with the time series data, to estimate the historical aggregate productivity shocks and uncertainty shocks, following the approach in Bocola and Dovis (2019). Then I let the model predict unemployment with the estimated shocks and examine its performance in accounting for the increases in unemployment during recessions.

A particle filter is a Monte Carlo Bayesian estimator for the posterior distribution of structural shocks, which suits non-linear systems like mine. However, directly applying the particle filter to my model is infeasible because one of the model's state variables, the distribution of heterogeneous firms, is infinite-dimensional. Therefore, the first step is to follow Krusell and Smith (1998) and approximate my infinite-dimensional model by an auxiliary non-linear state-space system with a finite number of states:

$$\begin{aligned} \boldsymbol{Y}_t &= g(\boldsymbol{X}_t) + \epsilon_t^Y, \\ \boldsymbol{X}_t &= f(\boldsymbol{X}_{t-1}, \epsilon_t^X), \end{aligned} \tag{53}$$

where $\boldsymbol{Y}_t$ is a vector of observables, and $\boldsymbol{X}_t$ is an auxiliary finite-dimensional state vector. Function f is the transition of states, and function g is the mapping from states to observations. ϵ_t^X is a vector of shocks to state variables, and ϵ_t^Y is a vector of independent and serially uncorrelated Gaussian measurement errors.

The goal is that, given the observables $\boldsymbol{Y}$, estimate the underlying states $\boldsymbol{X}$, including aggregate productivity A and uncertainty σ. So, the state vector should be sufficiently informative such that its mapping to observables is accurate. For this purpose, I include five groups of state variables in $\boldsymbol{X}_t$: (i) a constant; (ii) logged aggregate productivity A and uncertainty σ up to five-quarter lags, $\{\log A_{t-p}, \log \sigma_{t-p}\}_{p=0}^5$; ($iii$) the interactions between aggregate productivity and uncertainty, $\left\{\log A_{t-p} \cdot \log \sigma_{t-p}, \left\{\log A_{t-p} \cdot \log \sigma_{t-q}, \log A_{t-q} \cdot \log \sigma_{t-p}\right\}_{q=p+1}^3\right\}_{p=0}^2$; ($iv$) the squared logged changes of aggregate productivity and uncertainty and their interactions with the levels, $\left\{(\Delta \log A_{t-p})^2, (\Delta \log \sigma_{t-p})^2, (\Delta \log A_{t-p})^2 \cdot \log \sigma_{t-1}, (\Delta \log \sigma_{t-p})^2 \cdot \log A_{t-1}\right\}_{p=0}^3$; ($v$) lagged logged aggregate credit spreads and their interactions with aggregate productivity and uncertainty, $\Big\{ \log \mathrm{spr}_{t-1} \cdot \log A_t, \log \mathrm{spr}_{t-1} \cdot \log \sigma_t, \Big\{ \log \mathrm{spr}_{t-p}, \log \mathrm{spr}_{t-p} \cdot \log A_{t-1}, \log \mathrm{spr}_{t-p} \cdot$ $\log \sigma_{t-1}, \Big\{ \log \mathrm{spr}_{t-p} \cdot (\Delta \log A_{t-q})^2, \log \mathrm{spr}_{t-p} \cdot (\Delta \log \sigma_{t-q})^2 \Big\}_{q=0}^{2} \Big\}_{p=1}^{5} \Big\}$.

The next step is to obtain the mapping from this set of state variables to observables. Specifically, I choose aggregate output and the interquartile range (IQR) of firm sales growth as the observables since they have clear and distinct relations with aggregate productivity and uncertainty. To obtain the mapping $g(\cdot)$, I project the model-simulated aggregate output and IQR on the set of state variables, respectively. Their R^2s are 0.999998 and 0.9997, indicating the mapping's accuracy and validating the choice of state variables. Then the regression error variance is used to model the measurement errors ϵ_t^Y.

On the other hand, the transition function $f(\cdot)$ is set up according to the evolution of states. First, the transitions of aggregate productivity and uncertainty are defined by eq. (48) and eq. (49). Second, the transition from the states to the next-period credit spread is obtained by projecting the model-simulated credit spreads on the state variables, which also displays a high R^2 of 0.9998. Then the remaining transitions can be derived exactly from the definition of state variables. For example, the state variable $\log A_t \cdot \log \sigma_t$ is simply the state $\log A_t$ multiplied by another state $\log \sigma_t$. Lastly, state shocks in ϵ_t^X are the innovations to aggregate productivity ϵ_t^A, the innovations to uncertainty ϵ_t^σ, and the error term from the projection for credit spreads.

Given the finite-dimensional state-space system (53), I can apply the particle filter to it and estimate the underlying states from the data.[14] Specifically, I use the times series of GDP per capita from the Bureau of Economic Analysis (BEA) and the IQR of firm sales growth from Compustat as observable variables. The data is from 1972 to 2018. And the series are detrended by the band-pass filter for business cycle fluctuations between 6 and 32 quarters, consistent with Schaal (2017). Given the data, I use the particle filter to estimate the underlying states from the state-space system. Figure 6 plots the estimated aggregate productivity and uncertainty. It shows that the estimated aggregate productivity is closely related to aggregate output and the estimated uncertainty is tightly associated with the IQR of firm sales growth.

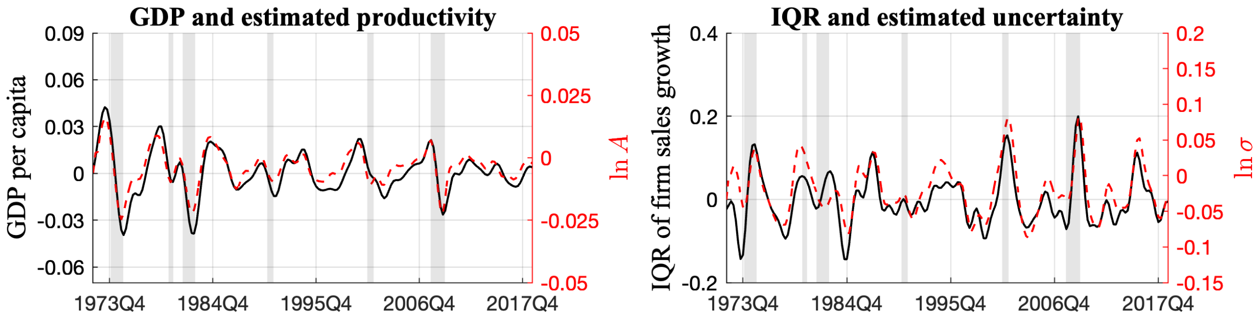
Next, I let the state-space model predict unemployment by feeding the estimated states, where the mapping from states to unemployment is also obtained by projection ($R^2 = 0.99998$). Figure 7 compares the model-predicted unemployment and the data, displayed as the peak-to-trough log deviations of unemployment during recessions.[15] Panel A shows the baseline results. The

[14] The particle filter's algorithm uses a set of particles to approximate the underlying states. Particles evolve and predict observables according to the state-space system (53). The data of observables correct the state estimates of particles by calculating their likelihoods. This process repeats recursively till the end of the data. I set the number of particles as $10,000$.

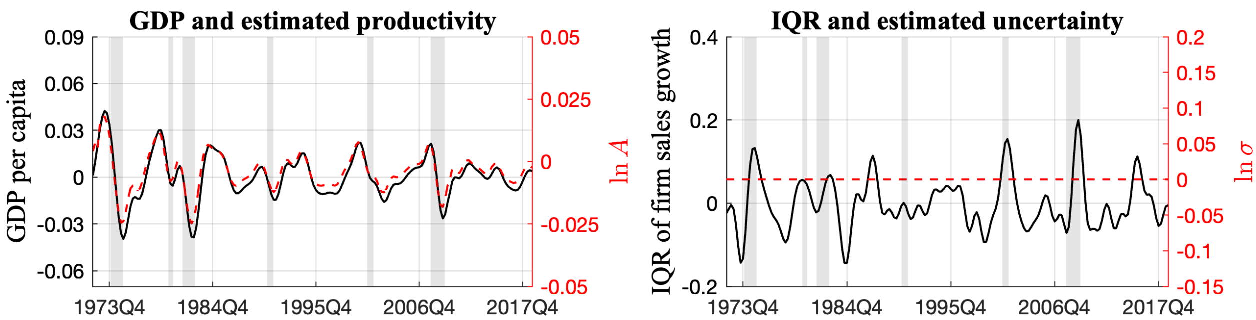
[15] Figure 10 plots the variation of aggregate output during each recession, where all models display similar output declines as in the data, implying that the particle filter performs well in estimating the underlying shocks by matching aggregate output.

Figure 6: Estimated Aggregate Productivity and Uncertainty

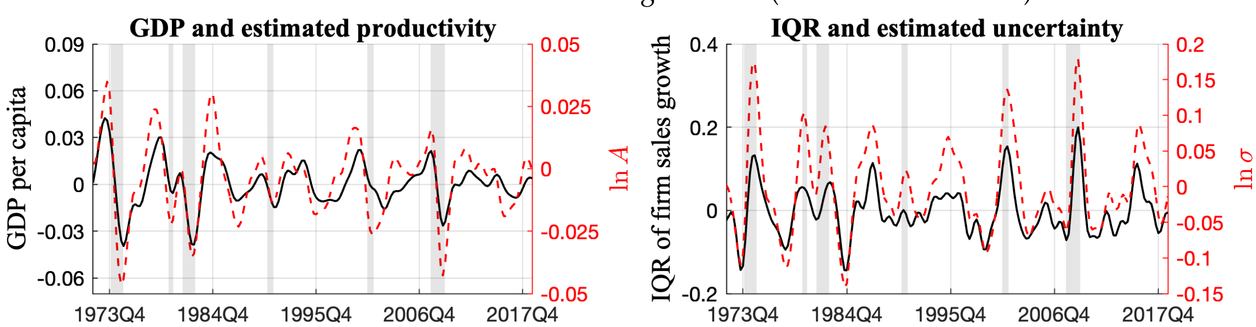
Panel A: Benchmark Model (Both A and σ Shocks)



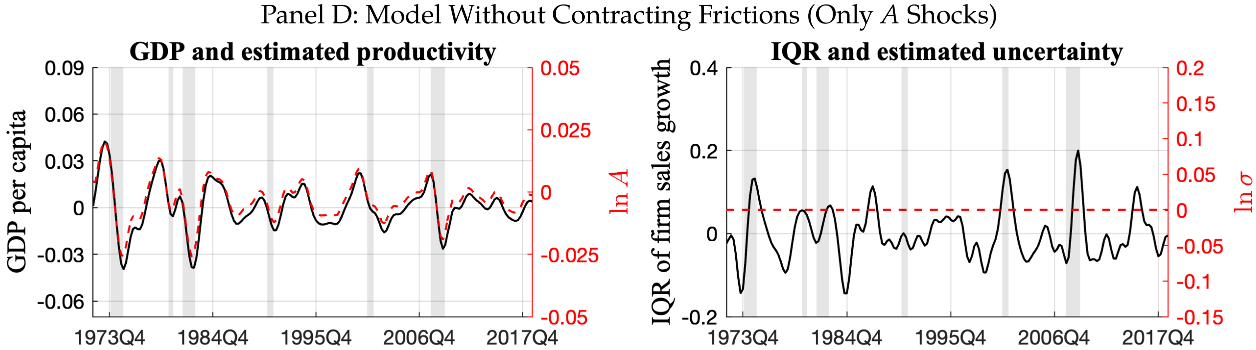
Panel B: Benchmark Model (Only A Shocks)



Panel C: Model Without Contracting Frictions (Both A and σ Shocks)



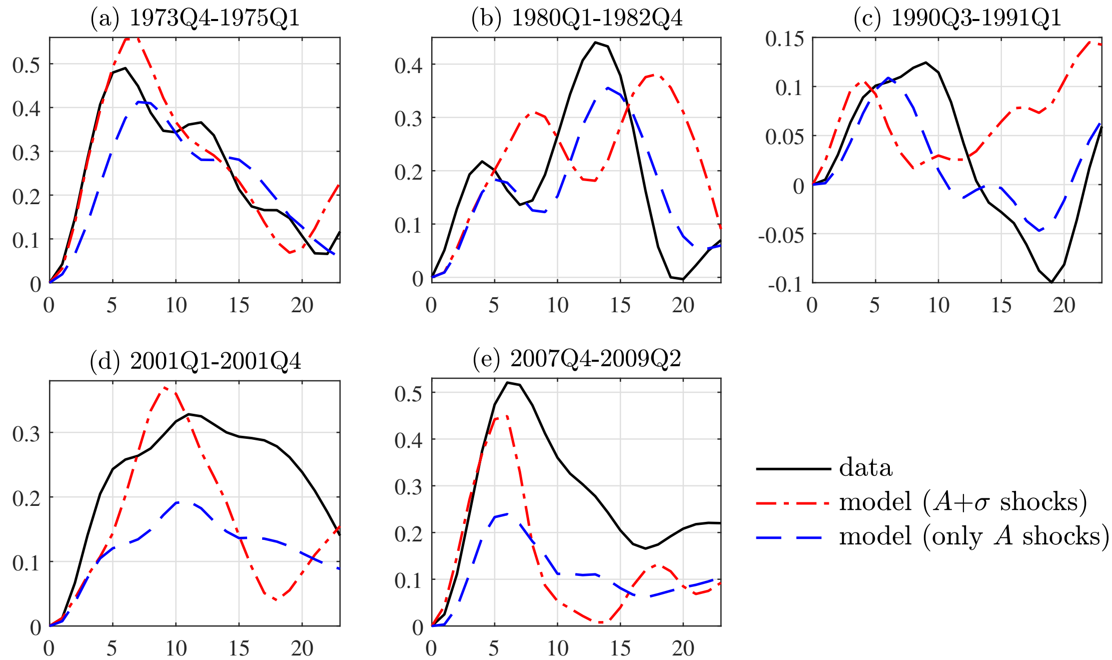
Panel D: Model Without Contracting Frictions (Only A Shocks)



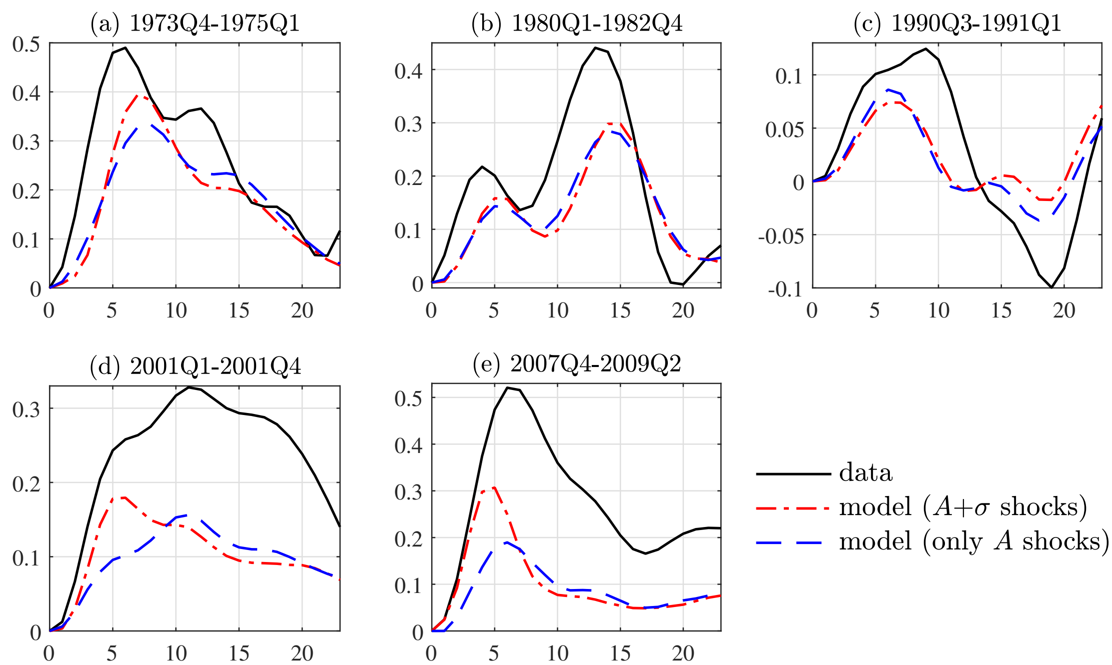
Notes: This figure shows the estimated aggregate productivity and uncertainty of two benchmark models and two reference models without contracting frictions. I apply the particle filter to my model and estimate the states of aggregate productivity, A, and uncertainty, σ, from the data series of GDP per capita and the IQR of firm sales growth, which are detrended by a band-pass filter to focus on fluctuations between 6 and 32 quarters, following Schaal (2017). Panels on the left-hand side display log deviations of GDP (solid black lines) and the estimated demeaned logged aggregate productivity (dashed red lines). Panels on the right-hand side present the log deviations of the interquartile range (IQR) of firm sales growth (solid black lines) and the estimated demeaned logged uncertainty (dashed red lines). The logged uncertainty is demeaned for the comparison of its fluctuations across models.

Figure 7: Unemployment Series With and Without Modeling Contracting Frictions

Panel A: Benchmark Models



Panel B: Models Without Contracting Frictions



Notes: The panels show the model's predictions for unemployment during recessions. Panel A is for the benchmark models. Panel B is for the models contracting frictions. All models are (re-)calibrated to match the data moments. I use the particle filter to jointly estimate the time series of aggregate productivity shocks and uncertainty shocks by matching output and the IQR of firm sales growth in the data. The data are detrended by a band-pass filter to focus on fluctuations between 6 and 32 quarters, following Schaal (2017). Given the estimated shocks, I show the model-predicted unemployment. The data on unemployment is the solid black lines. The unemployment fluctuations predicted by the models with both aggregate productivity shocks and uncertainty shocks are the dash-dotted red lines (labeled as $A + \sigma$ shocks), and predictions without contracting frictions are the dashed blue line. Series are depicted in terms of log deviations from the peak preceding the recession. I use Schaal's (2017) code when plotting this figure.

black lines are the data, and the dash-dotted red lines are the predictions of the benchmark model with both aggregate productivity shocks and uncertainty shocks. They display similar patterns and magnitudes of the increase in unemployment, indicating the benchmark model accounts for a great share of the increase in unemployment during recessions.

To understand the role of uncertainty shocks, I also show the predictions for unemployment of the models with only aggregate productivity shocks using dashed blue lines. It is clear that the model's performance in explaining recessions deteriorate in general. And the deterioration is particularly significant for the early 2000s recession and the Great Recession. The reason is that the two recessions had the largest increase in uncertainty while only mild decreases in aggregate productivity (Figure 6), which interacted with the financial channel of incomplete contracts and generated the sizable increases in unemployment.

Also, Panel B plots the predictions of counterfactual models without contracting frictions. It shows that both the model with uncertainty shocks (dash-dot red lines) and the model without uncertainty shocks (dashed blue lines) explain much less the increase in unemployment. In particular, the prediction for the Great Recession deteriorates greatly. This result is consistent with Schaal (2017), who finds that the canonical search framework alone cannot generate enough increase in unemployment during the Great Recession. The reason is that the model without contracting frictions largely underestimates the impact of elevated uncertainty. Given that the Great Recession has the largest increase in uncertainty, its unemployment is underestimated by the most. Therefore, I conclude that contracting frictions are the key driving force for uncertainty shocks to explain the rise of unemployment during recessions.

Table 4 reports the numerical peak-to-trough changes in unemployment for each recession. The upper panel shows the results for the benchmark. With both aggregate productivity shocks and uncertainty shocks, the benchmark model explains much of the increase in unemployment. Without uncertainty shocks, the model's performance deteriorates greatly. On average, adding uncertainty shocks generates 26% of the increase in unemployment during the past five recessions. The lower panel of Table 4 reports the results for counterfactual models without contracting frictions. In this case, adding uncertainty shocks only generates 7% of the increase in unemployment.

From Table 4, we can also learn that contracting frictions operate through uncertainty shocks more than through aggregate productivity shocks. The Great Recession is the most striking example. When the model only considers aggregate productivity shocks, the benchmark explains 46% of the increase in unemployment during the Great Recession, and the model without contracting frictions can also explain 36%. So, adding contracting frictions only generates an additional 10% of the rise in unemployment. By contrast, when there are uncertainty shocks on top of aggregate productivity shocks, adding contracting frictions causes an additional 27% increase in unemployment, i.e., from 59% to 86%.

Table 4: Peak-To-Trough Changes of Unemployment During Recessions

	1973-1975	1980-1982	1990-1991	2001	2007-2009
Data	0.490	0.441	0.124	0.328	0.521
Benchmark models					
Both A and σ shocks	0.557	0.382	0.107	0.370	0.449
Only A shocks	0.413	0.355	0.109	0.193	0.239
$\Rightarrow$ Data explained by adding σ shocks	29.5%	5.9%	-1.7%	53.9%	40.2%
	25.6% on average				
Models without contracting frictions					
Both A and σ shocks	0.395	0.298	0.074	0.179	0.307
Only A shocks	0.333	0.285	0.086	0.156	0.190
$\Rightarrow$ Data explained by adding σ shocks	12.6%	3.0%	-9.6%	7.1%	22.6%
	7.1% on average				

Notes: The table shows the peak-to-trough changes in unemployment during recessions for the data, two benchmark models, and two models without contracting frictions. "Both A and σ Shocks" means the model has both aggregate productivity shocks and uncertainty shocks, and "Only A Shocks" means the model has only aggregate shocks. To obtain the model's predictions of unemployment, I first use the particle filter to jointly estimate the time series of aggregate productivity shocks and uncertainty shocks by matching output and the IQR of firm sales growth in the data. The data are detrended by a band-pass filter to focus on fluctuations between 6 and 32 quarters, following Schaal (2017). Given the estimated shocks, I show the peak-to-trough changes predicted by the model during recessions. Consistent with Schaal (2017), series are depicted in terms of log deviations from the peak preceding the recession.

3.6 Policy Implications

Given the important role of elevated uncertainty in driving unemployment fluctuations, I use my model to analyze the impacts of labor market policies that target high-uncertainty periods. Specifically, I consider two policies that expanded a lot during recent recessions: increasing unemployment benefits and subsiding wage payments. Also, I discuss how the effects of these policies are biased when contracting frictions are not omitted from the analysis.

Increasing Unemployment Benefits. In the recent 2020 Covid-19 pandemic, the U.S. market uncertainty increased dramatically. Specifically, Altig et al. (2020) show that business executives are much more uncertain about their firms' future sales growth rates during the COVID-19 pandemic, according to the U.S. monthly panel Survey of Business Uncertainty (SBU) and the U.K. monthly Decision Maker Panel (DMP). At the same time, the U.S. government deployed economic support policies. One notable response was the U.S. Federal Pandemic Unemployment Compensation (FPUC) program, which increased unemployment benefits by an extra \$600 per week.

To figure out the aggregate impacts of raising unemployment, I modify my model such that the government increases unemployment benefits by 1% when uncertainty is high. Given the policy, I re-solve the model quantitatively. So, the policy is anticipated by the agents in the economy. For simplicity, I assume that the government collects tax revenue through a lump-sum tax, and this policy costs 4.81 basis points of output in the simulation.

Figure 8 shows the impulse responses to a 5% positive uncertainty shock. Panel A is the benchmark model's results. The solid black lines are the ones without policies, the same as in Figure 5. And the dashed red lines are for the policy of raising unemployment benefits, where unemployment benefits increase by 1% when the uncertainty shock hits period 0. It is clear that this policy amplifies the recession by generating lower output and raising unemployment by an additional 5%. The reason is that increased unemployment value requires higher wages, which not only increases the cost of production but also strengthens the financial concern of wage commitments. Therefore, the recession deepens.

Next, Table 5 summarizes the impact of the unemployment insurance (UI) policy on business cycles. Panel A describes the experiments of labor market policies. And Panel B compares the model-simulated moments of the benchmark model and the model with the policy. It shows output decreases by 0.41%, unemployment increases by 0.39 percentage points, the standard deviation of unemployment increases by 16%, and total surplus decreases by 4.3 basis points.[16] The reason is that the increased unemployment benefits distort the labor market by pushing wages higher by around 6.1 basis points in the simulation. Therefore, firms hire fewer workers, making it harder for the unemployed to find jobs, as the unemployment-to-employment transition rate decreases by 1.5 percentage points. Overall, since higher unemployment benefits distort the economy by increasing the marginal cost of the labor force, aggregate efficiency decrease.

Subsiding Wage Payments. Another representative labor market policy is Germany's social insurance program, Kurzarbeit, which is a very different social security system from the U.S. In Germany, firms cut workers' hours. Then the government compensates part of the workers' earnings losses, so firms can keep workers employed. In other words, the government subsides firms to pay wages when there is a bad shock.[17] During the Great Recession and the Covid recession, Germany expanded this program and provided more generous wage subsidies.

I model this policy by allowing the firm to have an option to let part of its workforce idle when uncertainty is high. The government pays 84.4% of the idle workers' wages, and the firm pays the rest.[18] This replacement rate is chosen to have the same share of government expenditure to output ratio as the UI policy, so it costs 4.86 basis points of output.

The dash-dot blue lines in Figure 8 show the impulse responses to a transitory positive uncertainty shock given the wage subsidies. The results show that output decreases slightly more and unemployment increases slightly less. The difference from the ones without policy is tiny because the policy's pros and cons offset each other. Specifically, Table 5 reports the model-simulated

[16] Total surplus is the sum of worker and firm surplus.

[17] Cooper, Meyer and Schott (2017) provide information on this system.

[18] Although my model does not have an explicit component of working hours, it does not affect the results since each firm has a continuum of the workforce to be idle, isomorphic to modeling hours cut.

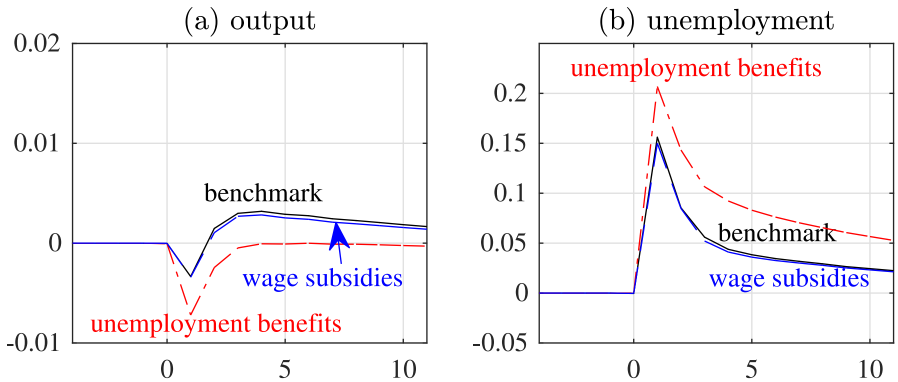
Table 5: The Aggregate Outcomes of Labor Market Policies

	No Policy	UI Policy	Wage Policy
Panel A: Policies			
Increase in unemployment benefits	-	1%	-
The replacement rate of wage subsidies	-	-	84.4%
Panel B: Aggregate Outcomes			
Benchmark Model			
Mean of output	100	99.593	99.938
SD of output	0.015	0.015	0.015
Mean of unemployment (%)	5.823	6.210	5.804
SD of unemployment	0.106	0.123	0.104
Mean of average wages	100	100.061	100.014
SD of average wages	0.011	0.011	0.011
UE rate	0.814	0.799	0.814
EU rate	0.083	0.085	0.083
EE rate	0.081	0.080	0.081
Mean credit spread (%)	0.96	0.96	0.97
Median leverage (%)	21	21	21
Annual exit rate (%)	9.0	9.0	9.0
Fiscal cost share of output (basis points)	-	4.809	4.862
Total surplus	100	99.957	99.974
Model Without Contracting Frictions			
Mean of output	100	99.963	99.992
SD of output	0.019	0.019	0.019
Mean of unemployment (%)	4.306	4.334	4.275
SD of unemployment	0.090	0.091	0.089
Mean of average wages	-	-	-
SD of average wages	-	-	-
UE rate	0.840	0.839	0.840
EU rate	0.063	0.064	0.063
EE rate	0.044	0.044	0.044
Mean credit spread (%)	-	-	-
Median leverage (%)	-	-	-
Annual exit rate (%)	9.0	9.0	9.0
Fiscal cost share of output (basis points)	-	3.274	0.000
Total surplus	100	99.99993	99.996

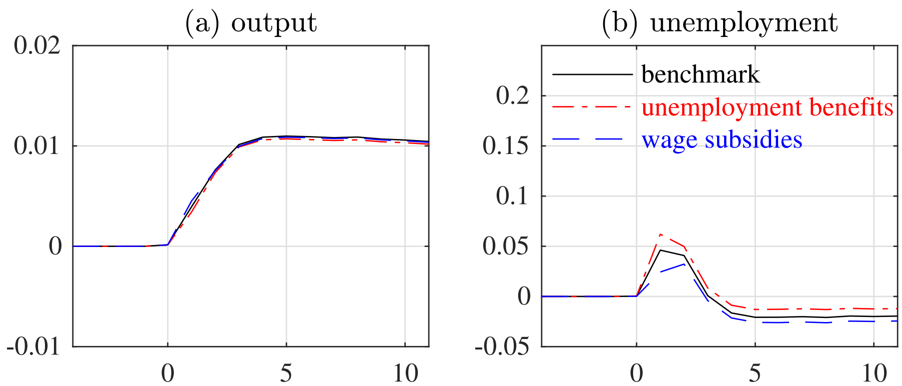
Notes: The table shows the model-simulated moments without and with the labor market policies. Panel A specifies the policies, and Panel B displays the moments of 3,000-quarter simulations of the benchmark model and the model without contracting frictions. Policies are implemented conditional on uncertainty higher than its average. Given each policy, I re-solve the model. That is, policies are anticipated by the agents in the economy. The output, average wages, and total surplus are normalized to 100 for the two models without policy. The standard deviations of output, unemployment, and average wages use the log deviations from the HP-filter with parameter 1,600.

Figure 8: Output and Unemployment Responses to a 5% Uncertainty Shock Under Policies

Panel A: Benchmark Model



Panel B: Model Without Contracting Frictions



Notes: The panels are impulse responses of aggregate output and unemployment to a 5% positive uncertainty shock at quarter 0. Panel A shows the results of the benchmark model, and Panel B displays the results of the reference model without contracting frictions. The models have both aggregate productivity shocks and uncertainty shocks. Solid black lines are the results without policy intervention (labeled as the benchmark). Dash-dot red lines are for the model with the policy of enhanced unemployment benefits. Dashed blue lines are for the model with the policy of wage subsidies. Both policies are implemented conditional on uncertainty higher than its average. The impulse responses are the average of 4,000 simulated paths, presented as log deviations from the mean. I use Schaal's (2017) code when plotting this figure.

moments with wage subsidies. On the positive side, the unemployment rate decreases by 1.9 basis points, and its standard deviation over business cycles is 1.9 percent lower. That is, wage subsidies lower and stabilize unemployment by providing state-contingent insurance to firms to help them pay wages and retain employees, weakening the financial concern of wage commitments. It also avoids separations, so it can save the resources spent on search.

However, the policy's overall impact is negative—aggregate output decreases by 6.2 basis points, and total surplus decreases by 2.6 basis points. The negative effect is due to policy-induced distortions. The wage subsidies encourage labor hoarding, which misallocates the labor force to low-productivity firms that are supposed to separate from their employees. As a result, the labor market is less efficient, so aggregate output and total surplus decrease.

Biased Policy Evaluation Without Consideration of Contracting Frictions. I have already shown that contracting frictions are crucial for uncertainty shocks to affect unemployment. In this section, I document that contracting frictions are also important for policy evaluation in the context of elevated unemployment. Specifically, I apply the above two policy experiments to the recalibrated model without contracting frictions.

Panel B of Figure 8 displays the impulse response to a 5% positive uncertainty shock without considering contracting frictions. The dashed red lines show that the more generous UI policy causes a much smaller decrease in output and a much smaller increase in unemployment. The reason is that when contracting frictions are absent, the policy's negative influence is weakened substantially by the equilibrium response of wages. On the other hand, the policy of wage subsidies displays a stronger stabilization effect by generating slightly lower unemployment. In this case, the policy-induced distortion is smaller because wage subsidies do not twist firms' liquidity incentives. Plus, the lower part of Table 5 reports the model's moments when contracting frictions are absent. The last row of total surplus suggests that the negative impacts of both labor market policies diminish dramatically. The efficiency loss induced by the UI policy dramatically decreases from 4.3 to 7×10^{-5} basis points. And the efficiency loss caused by wage subsidies decreases from 2.6 to $4{\times}10^{-3}$ basis points. So, I conclude that the model with contracting frictions greatly underestimates the efficiency losses, and it misleadingly suggests that the UI policy is better.

To sum up, the UI policy pays unemployed workers more during high uncertainty states, making it more expensive for firms to pay wages; wage subsidies help firms keep workers when facing transitory negative shocks. My quantitative results show that the UI policy substantially amplifies recessions and wage subsidies have mild negative impacts. Both policies' negative effects are largely underestimated when contracting frictions are absent when evaluating policies. It is worth noting that my model focuses on the labor demand mechanism. It does not include worker-side risk aversion or demand effects, which may provide additional benefits for the two policies through other channels.

4 Conclusion

In this paper, I build a new search model to assess to what extent uncertainty shocks affect the fluctuations of unemployment. My quantitative results show that, on top of aggregate productivity shocks, uncertainty shocks generate 26% of the increases in unemployment during past recessions. The key is the interaction of financial and labor market incompleteness, such that firms have limited ability to hedge against the risks of idiosyncratic productivity variations.

I also use my model to quantify the impact of labor market policies in the context of elevated uncertainty. The results show that raising unemployment benefits, as in the U.S. during Covid, amplified the recession. On the other hand, a German approach of subsidizing firm wage bills provides insurance but causes misallocation losses.

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Appendix A Tables

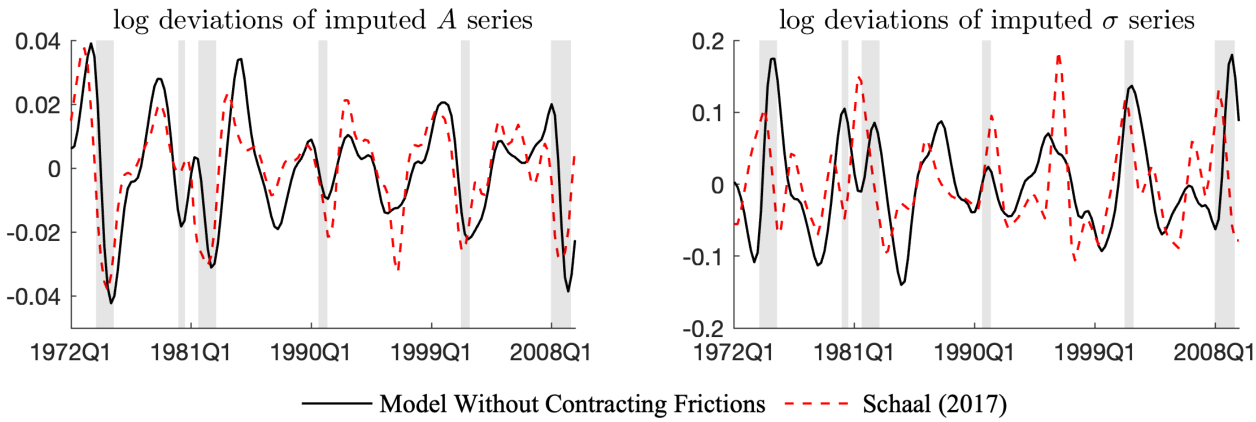
Table 6: Parameters of Reference Models

Parameters	Notations	Benchmark Model		No Contracting Frictions	
		$A+\sigma$	A only	$A+\sigma$	A only
Aggregate shocks					
Persistence of aggregate productivity	ρ_A	0.920	0.920	0.912	0.912
SD of aggregate productivity	σ_A	0.024	0.028	0.042	0.035
Mean of uncertainty	$\bar{\sigma}$	0.248	0.250	0.300	0.280
Persistence of uncertainty	ρ_σ	0.880	-	0.926	-
SD of uncertainty	σ_σ	0.092	-	0.186	-
Correlation between ϵ_t^A and ϵ_t^σ	$\rho_{A\sigma}$	-0.020	-	-0.920	-
Labor market					
Unemployment benefits	$\bar{u}$	0.142	0.142	0.150	0.155
Vacancy posting cost	c	0.001	0.001	0.002	0.002
Relative on-the-job search efficiency	λ	0.100	0.100	0.120	0.120
Matching function elasticity	γ	1.600	1.600	1.600	1.600
Entry cost	k_e	15.21	14.87	14.70	15.21
Mean operating cost	$\bar{w}_m + \mu_\epsilon$	0.001	0.001	0.100	0.100
Financial market					
SD of production costs	σ_ϵ	0.080	0.071	0.080	0.080
Agency friction	$\tilde{\zeta}$	2.400	2.400	-	-
Auditing quality	ξ	1.780	1.780	-	-
Recovery rate	η	2.410	2.410	-	-
Exogenous exit rate	π_d	0.021	0.022	0.022	0.022

Notes: This table reports the calibrated parameters of the benchmark model and the model without contracting frictions. $A+\sigma$ means the model has both aggregate productivity shocks and uncertainty shocks, and A means the model only has aggregate productivity shocks. Table 2 shows the targeted data moments and the model-simulated moments.

Appendix B Figures

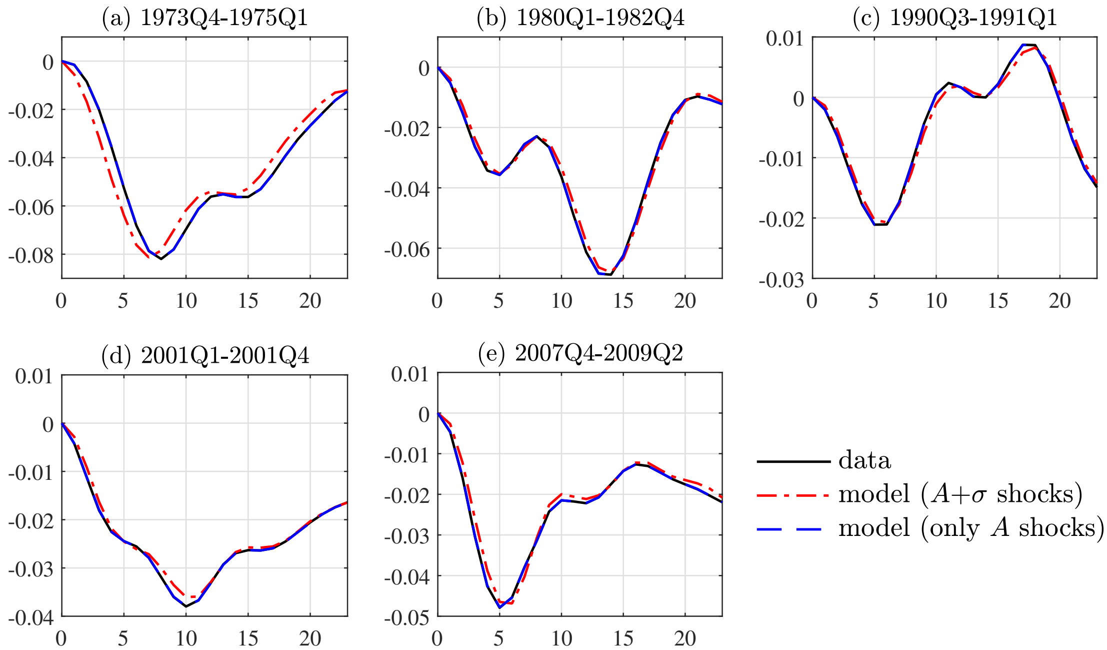
Figure 9: Comparison of Estimated Shocks with Schaal (2017)



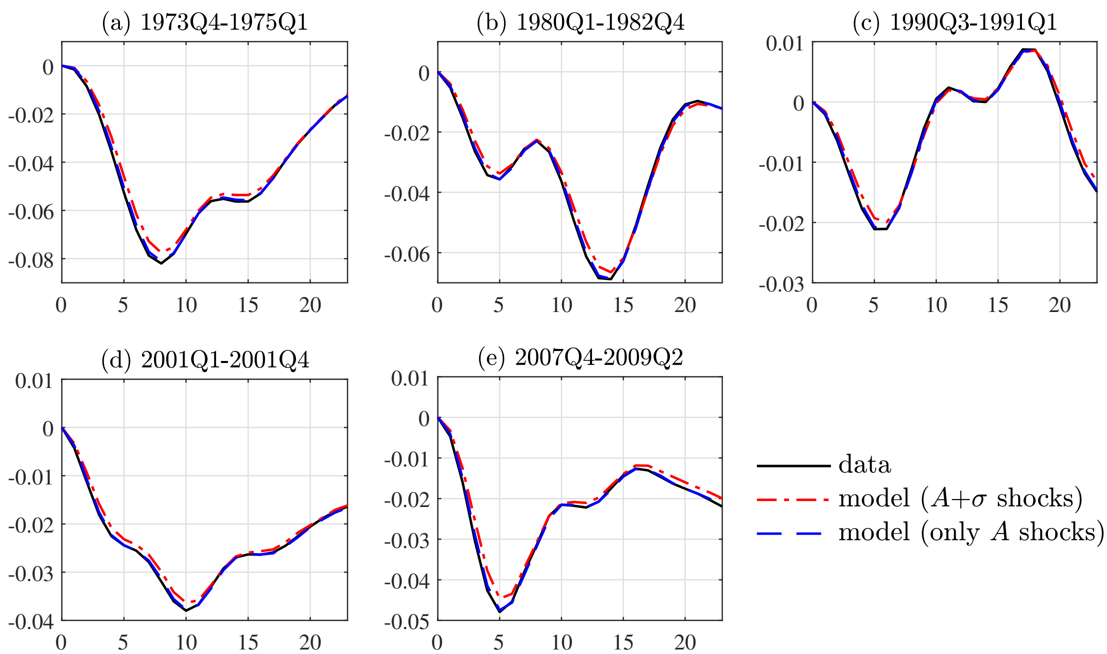
Notes: This figure compares the estimated shocks by my reference model without contracting frictions and the estimation by Schaal (2017). I apply the particle filter to my model and estimate the states of aggregate productivity and uncertainty from the data series of GDP per capita and the IQR of firm sales growth. The black lines show my estimated log deviations of aggregate productivity, A, and uncertainty, σ. The red dashed lines are the imputed shocks directly obtained from Schaal (2017). The series end at 2009Q4, which is the last period studied in Schaal (2017).

Figure 10: Output Series With and Without Modeling Contracting Frictions

Panel A: Benchmark Models



Panel B: Models Without Contracting Frictions



Notes: The panels show the model's predictions for output during recessions. Panel A is for the benchmark models. Panel B is for the models contracting frictions. All models are (re-)calibrated to match the data moments. I use the particle filter to jointly estimate the time series of aggregate productivity shocks and uncertainty shocks by matching output and the IQR of firm sales growth in the data. The data are detrended by a band-pass filter to focus on fluctuations between 6 and 32 quarters, following Schaal (2017). Given the estimated shocks, I show the model-predicted output. The data on output is the solid black lines. The output fluctuations predicted by the models with both aggregate productivity shocks and uncertainty shocks are the dash-dotted red lines (labeled as $A + \sigma$ shocks), and predictions without contracting frictions are the dashed blue line. Series are depicted in terms of log deviations from the peak preceding the recession. I use Schaal's (2017) code when plotting this figure.

Appendix C Proofs

Proposition 1 *The participation constraint binds, i.e., $\bar{W}(i') = 0$, for any worker i'.*

Proof I prove this proposition by contradiction. Suppose, in the firm's optimal policy, there exists a worker whose index in the next period is i' and $\bar{W}(i') > 0$ in his labor contract. Then I can construct an alternative policy by letting $\bar{W}(i') = 0$ and deliver a higher firm's value at the same time. I first discuss the case where the worker is an incumbent employee and then show the case where the worker is newly hired.

Case 1. Suppose i' refers to an incumbent worker. Use i to denote the worker's index in the current period and ϵ^m to denote the worker's mass.

I construct an alternative policy by making the following four changes to the original policy. The idea is to frontload wages and borrow more simultaneously:

1. Decrease the promised utility markup $\bar{W}(i')$ to zero, which just satisfies the participation constraint (8). To simply the notation, I use δ to denote $\bar{W}(i')$ from now on.

2. Decrease the worker's next-period wage $w(i')$ by exactly δ. Since the wage decreases as much as the promised utility, the next-period promise-keeping constraint (9) holds as before.

3. Promise to pay the worker $\tilde{w}$ today conditional on not leaving the firm by on-the-job search, where $\tilde{w}$ equals $\beta\,\mathbb{E}[(1-\tau(i))(1-\pi_d)(1-d(S',s'))]\delta$. This additional payment guarantees that the worker has the same lifetime promised utility today, so today's promise-keeping constraint (9) is unaffected. Importantly, the worker's on-the-job search decision is not affected because the payment is given to the worker conditional on not transiting to another firm. From the firm's perspective, its labor expense today increases by $\epsilon^m(1-\lambda p(\theta(S,x^*(S;i))))\tilde{w}$.[19]

4. Increase the debt b' by $\epsilon^m(1-\tau(i))(1-\lambda p(\theta(S,x^*(S;i))))\delta$, which equals the decrease in the firm's wage bills in the next-period.[20] So, the next-period cash on hand of the firm does not change.

Given these four changes, I next show the firm's value increases. First, because the next-period cash on hand is the same, the next-period default decisions are unchanged. Also, the next-period employment n' does not change, so neither is the expected value of the firm in the next period.

Second, because the borrowing increases more than the increase in today's wage payments, today's equity payouts increase. Formally, the change in current equity payouts equals:

$$\Delta^{\text{new}} - \Delta = Q(S,s,b'^{\text{new}},n)b'^{\text{new}} - Q(S,s,b',n)b' - \epsilon^m(1-\lambda p(\theta(S,x^*(S;i))))\tilde{w}$$

[19] Notice that this additional payment is conditional on the worker does not leave the firm by on-the-job search.

[20] Notice that the firm pays the wage in the next-period conditional on the worker was not separated by firing or on-the-job search in the previous period.

$$
\begin{aligned}
&= \beta\,\mathbb{E}\left\{(1-\pi_d)(1-d(S',s'))\right\}b'^{\text{new}} + \beta\,\mathbb{E}\left\{[1-(1-\pi_d)(1-d(S',s'))]\min\{\eta\frac{\pi'^{\text{new}}}{b'^{\text{new}}},1\}\right\}b'^{\text{new}} \\
&\quad - \beta\,\mathbb{E}\left\{(1-\pi_d)(1-d(S',s'))\right\}b' - \beta\,\mathbb{E}\left\{[1-(1-\pi_d)(1-d(S',s'))]\min\{\eta\frac{\pi'}{b'},1\}\right\}b' \\
&\quad - \epsilon^m(1-\lambda p(\theta(S,x^*(S;i))))\tilde{w} \\
&= \beta\,\mathbb{E}\left\{(1-\pi_d)(1-d(S',s'))\right\}b'^{\text{new}} - \beta\,\mathbb{E}\left\{(1-\pi_d)(1-d(S',s'))\right\}b' \\
&\quad + \beta\,\mathbb{E}\left\{[1-(1-\pi_d)(1-d(S',s'))]\min\{\eta\pi'^{\text{new}},b'^{\text{new}}\}\right\} \\
&\quad - \beta\,\mathbb{E}\left\{[1-(1-\pi_d)(1-d(S',s'))]\min\{\eta\pi',b'\}\right\} - \epsilon^m(1-\lambda p(\theta(S,x^*(S;i))))\tilde{w} \\
&= \beta\,\mathbb{E}\left\{(1-\pi_d)(1-d(S',s'))\right\}(b'^{\text{new}}-b') - \epsilon^m(1-\lambda p(\theta(S,x^*(S;i))))\tilde{w} \\
&\quad + \beta\,\mathbb{E}\left\{[1-(1-\pi_d)(1-d(S',s'))]\min\{\eta\pi'^{\text{new}},b'^{\text{new}}\}\right\} \\
&\quad - \beta\,\mathbb{E}\left\{[1-(1-\pi_d)(1-d(S',s'))]\min\{\eta\pi',b'\}\right\} \\
&= \beta\,\mathbb{E}\left\{(1-\pi_d)(1-d(S',s'))\right\}\epsilon^m(1-\tau(i))(1-\lambda p(\theta(S,x^*(S;i))))\delta \\
&\quad - \epsilon^m(1-\lambda p(\theta(S,x^*(S;i))))\beta\,\mathbb{E}[(1-\tau(i))(1-\pi_d)(1-d(S',s'))]\delta \\
&\quad + \beta\,\mathbb{E}_{S',s'|S,s}\left\{[1-(1-\pi_d)(1-d(S',s'))]\min\{\eta\pi'^{\text{new}},b'^{\text{new}}\}\right\} \\
&\quad - \beta\,\mathbb{E}_{S',s'|S,s}\left\{[1-(1-\pi_d)(1-d(S',s'))]\min\{\eta\pi',b'\}\right\} \\
&= \beta\,\mathbb{E}_{S',s'|S,s}\left\{[1-(1-\pi_d)(1-d(S',s'))]\min\{\eta\pi'^{\text{new}},b'^{\text{new}}\}\right\} \\
&\quad - \beta\,\mathbb{E}_{S',s'|S,s}\left\{[1-(1-\pi_d)(1-d(S',s'))]\min\{\eta\pi',b'\}\right\}.
\end{aligned}
$$

Notice that $b'^{\text{new}} \geq b'$ by construction and $\pi'^{\text{new}} \geq \pi'$ because the next-period wage bills decrease. Therefore, $\min\{\eta\pi'^{\text{new}}, b'^{\text{new}}\} \geq \min\{\eta\pi', b'\}$. So,

$$
\begin{aligned}
\Delta^{\text{new}} - \Delta \geq &\beta\,\mathbb{E}_{S',s'|S,s}\left\{[1-(1-\pi_d)(1-d(S',s'))]\min\{\eta\pi',b'\}\right\} \\
&- \beta\,\mathbb{E}_{S',s'|S,s}\left\{[1-(1-\pi_d)(1-d(S',s'))]\min\{\eta\pi',b'\}\right\} \\
=&0.
\end{aligned}
$$

Lastly, the agency friction constraint (11) holds under this constructed policy. The constraint's left-hand side increases as the borrowing increases, and its right-hand side decreases because of lower next-period wage bills.

Case 2. Suppose i' refers to a newly hired worker in the current period. As before, construct an alternative policy by making the following four changes to the original policy:

1. Decrease the promised utility markup $\bar{W}(i')$ to zero, just satisfying the participation constraint. I use δ to denote $\bar{W}(i')$.

2. Decrease the worker's next-period wage $w(i')$ by δ, so the next-period promise-keeping constraint still holds.

3. Increase the newly hired workers' wage $w_h(i')$ by $\beta\,\mathbb{E}[(1-\pi_d)(1-d(S',s'))]\delta$, guaranteeing that

the worker still has the same lifetime promised utility x_h, so today's promise-keeping constraint still holds. On the firm-side, today's labor expense increases by $\epsilon^m \tilde{w}$, where ϵ^m denotes the worker's mass.

4. Increase the debt b' by $\epsilon^m \delta$, which equals the decrease in the firm's wage bills in the next-period. Thus, the next-period cash on hand does not change.

Given these four changes, the firm's value increases for the following reasons. First, the firm's value in the next period is unaffected because the cash on hand and labor force are unchanged.

Second, because borrowing increases more than the increase in wage payments, the equity payouts increase. Formally,

$$\begin{aligned}
\Delta^{\text{new}} - \Delta &= Q(S, s, b'^{\text{new}}, n)b'^{\text{new}} - Q(S, s, b', n)b' - \epsilon^m \tilde{w} \\
&= \beta \, \mathbb{E} \left\{(1-\pi_d)(1-d(S', s'))\right\}(b'^{\text{new}} - b') - \epsilon^m \tilde{w} \\
&\quad + \beta \, \mathbb{E} \left\{[1-(1-\pi_d)(1-d(S', s'))] \min\{\eta \pi'^{\text{new}}, b'^{\text{new}}\}\right\} \\
&\quad - \beta \, \mathbb{E} \left\{[1-(1-\pi_d)(1-d(S', s'))] \min\{\eta \pi', b'\}\right\} \\
&= \beta \, \mathbb{E} \left\{(1-\pi_d)(1-d(S', s'))\right\}\epsilon^m \delta - \epsilon^m \beta \, \mathbb{E} \left\{(1-\pi_d)(1-d(S', s'))\right\}\delta \\
&\quad + \beta \, \mathbb{E}_{S', s' | S, s} \left\{[1-(1-\pi_d)(1-d(S', s'))] \min\{\eta \pi'^{\text{new}}, b'^{\text{new}}\}\right\} \\
&\quad - \beta \, \mathbb{E}_{S', s' | S, s} \left\{[1-(1-\pi_d)(1-d(S', s'))] \min\{\eta \pi', b'\}\right\} \\
&= \beta \, \mathbb{E}_{S', s' | S, s} \left\{[1-(1-\pi_d)(1-d(S', s'))] \min\{\eta \pi'^{\text{new}}, b'^{\text{new}}\}\right\} \\
&\quad - \beta \, \mathbb{E}_{S', s' | S, s} \left\{[1-(1-\pi_d)(1-d(S', s'))] \min\{\eta \pi', b'\}\right\} \\
&\geq 0,
\end{aligned}$$

where the last inequality is due to $b'^{\text{new}} \geq b'$ and $\pi'^{\text{new}} \geq \pi'$.

Lastly, the agency friction constraint (11) holds. The constraint's left-hand side increases as the borrowing increases more than the increase in newly hired workers' wages, and its right-hand side decreases as next-period wage bills decrease.

In sum, I construct a feasible and better alternative policy, which contradicts the optimality of the original policy with a loose participation constraint. Therefore, the participation constraint always binds in the equilibrium. □

Appendix D Micro-Foundations of Incomplete Labor Contracts

In this section, I use asymmetric information between the firm and its employees to show that the promised utility markup $\bar{W}$ is state-uncontingent. The key assumption is that firms know the realized shocks immediately, but employees know the shocks later in the production stage. This information friction explains why labor contracts are not completely state-contingent, i.e., constraint (7). The logic follows Hall and Lazear (1984) and Lemieux, MacLeod and Parent (2012). They use asymmetric information to justify the optimality of pre-determined wages. We share the mechanism that firms can lie about the states, so the only incentive-compatible result is state-uncontingent promises. I will first set up the model with asymmetric information and then prove the optimality of state un-contingency.

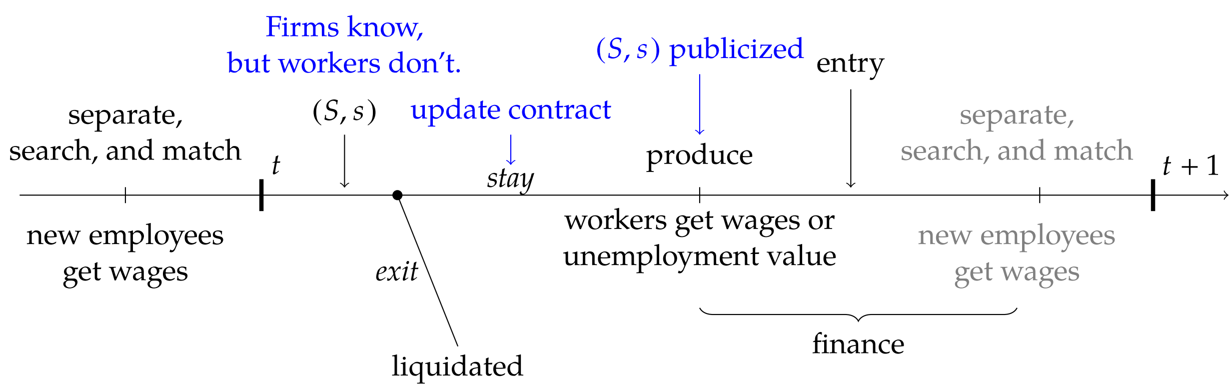
On top of the timeline in Figure 2, Figure 11 adds the timing for asymmetric information. When shocks (S, s) realize at the beginning of each period, firms know the shocks, but workers do not. If a worker leaves the firm now, he is unemployed and obtains the unemployment value in the current period. Given the shocks, firms choose to exit or stay. Staying firms declare their current shocks are $\tilde{S}$ and $\tilde{s}$ and update contracts. Notice that the declaration can differ from the true state since workers do not observe the information now. I allow the declarations to differ across the firm's employees. Given that the labor contract has been updated, the worker gets nothing in the current period if he leaves the firm now.[21] At the production stage, the shocks (S, s) become public information. Workers receive wage payments according to the labor contract, which depends on the firm's declaration of the state $(\tilde{S}, \tilde{s})$. At the end of the period, firms separate, search, and match.

The labor contract C contains $\left\{w, \tau, \bar{W}(S', s'), d(S', s')\right\}$. Notice I assume that the contact directly specifics the markup $\bar{W}(S', s')$ between the lifetime promised utility $W'(S', s')$ and the outside value of unemployment $\boldsymbol{U}(S')$, which facilitates proving the markup's state-uncontingency.[22]

[21] This assumption facilitates the proof because workers have no incentive to threaten to leave the firm (Proposition 2(i).)

[22] In the traditional implicit contract literature, the labor contract specifies the lifetime utility. Instead, I assume that the contract specifies the promised utility markup in this paper, which is a weaker assumption in my context. Specifically, by asymmetric information, I will prove that the promised value in the contract is state-uncontingent. When the contract specifies the promised utility markup, it implies only the markup part is state-uncontingent, and the promised lifetime utility can still vary with aggregate states, similar to a standard Diamond-Mortensen-Pissarides search model. If the contract specifies the lifetime utility, then the whole lifetime utility is state-uncontingent, implying greater rigidity. This is a stronger assumption than is unnecessary for my model.

Figure 11: Timing With Asymmetric Information



Given the labor contract, the worker's employment value is:

$$\begin{aligned}\boldsymbol{W}(S,s,C) = &\max_{x}\ w + \lambda p(\theta(S,x))x + (1-\lambda p(\theta(S,x)))\tau\beta\,\mathbb{E}_{S'|S}\,\boldsymbol{U}(S') \\ &+ (1-\lambda p(\theta(S,x)))(1-\tau)\beta\max\Big\{\underbrace{\mathbb{E}_{S'|S}\,\boldsymbol{U}(S'),}_{\text{leave before the contract is updated}}\ \mathbb{E}_{S',s'|S,s}\{(\pi_d+(1-\pi_d)d(S',s'))\boldsymbol{U}(S') \\ &+ (1-\pi_d)(1-d(S',s'))\max\{\boldsymbol{U}(S')+\bar{W}(\tilde{S}'^{*},\tilde{s}'^{*}),\underbrace{0+\beta\,\mathbb{E}_{S''|S'}\,\boldsymbol{U}(S'')}_{\text{leave after the contract is updated}}\}\}\Big\}.\end{aligned} \tag{54}$$

As before, the worker receives the wage w at the production stage. The worker can conduct on-the-job search and leave the firm. If the worker stays but gets laid off, he will be unemployed in the next period and receive the unemployment value $\boldsymbol{U}(S')$.

If the worker is not laid off, he can still leave the firm when the outside value is high enough. But the outside value depends on the timing of leaving the firm. If the worker leaves the firm before the contract is renewed, he is counted as unemployed and receives the unemployment value just like a laid-off worker. However, if he leaves the firm after the contract is renewed, he receives zero and gets the unemployment value one period later. This setup can be understood as the worker being ineligible to receive unemployment benefits after the labor relation renews, and drawing up contracts is time-consuming, so he does not have time to produce at home in the same period. Hence, the utility is zero in that period. This assumption will imply that workers have no incentive to threaten to quit when they find the firm lies (Proposition 2(i)), facilitating the proof.

If the labor relation persists, the worker will receive the lifetime utility $\boldsymbol{U}(S')+\bar{W}(\tilde{S}'^{*},\tilde{s}'^{*})$. Notice that because of asymmetric information, the promised utility markup $\bar{W}$ to the worker depends on the firm's declaration of states $(\tilde{S}'^{*},\tilde{s}'^{*})$. To clarify, $\{\bar{W}(S',s')\}$ in the labor contract is the set of utility markups for the next period. However, how much the worker can get in the next period depends on the firm's declaration of states $(\tilde{S}'^{*},\tilde{s}'^{*};i)$.

A firm's states include realized aggregate shocks $S \in \boldsymbol{S}$, realized firm-specific shocks $s \in \boldsymbol{s}$, the number of employees n, and the set of promised utility markups to its employees $\{\bar{W}(S,s;i)\}_{S\in\boldsymbol{S},s\in\boldsymbol{s};i\in[0,n]}$, where i is the index of incumbent employees within the firm. In a slight abuse of notation, S and s inside $\bar{W}(\cdot,\cdot;i)$ refer to the possible shocks instead of the realized shocks.

Besides the choice variables in the original firm's problem (3), the firm now also chooses to declare the current shocks, $\tilde{S}(i)$ and $\tilde{s}(i)$, to each employee i. The following equations (55) to (61) summarize the firm's problem:

$$\boldsymbol{J}(S,s,b,n,\{\bar{W}(S,s;i)\}_{S\in\boldsymbol{S},s\in\boldsymbol{s};i\in[0,n]}) = \max_{\substack{\Delta,b',n',n_h,x_h,d(S',s')\\ \{\tilde{S}(i),\tilde{s}(i),w(i),\tau(i)\}_{i\in[0,n]},\\ \{w_h(i')\}_{i'\in(n'-n_h,n']},\\ \{\bar{W}(S',s';i')\}_{S'\in\boldsymbol{S}',s'\in\boldsymbol{s}';i'\in[0,n']}}} \Delta$$
$$+\beta(1-\pi_d)\,\mathbb{E}_{S',s'|S,s}\left\{(1-d(S',s'))\boldsymbol{J}(S',s',b',n',\{\bar{W}(S',s';i')\}_{S'\in\boldsymbol{S}',s'\in\boldsymbol{s}';i'\in[0,n']})\right\} \tag{55}$$

$$\text{s.t. } (4),(5),(6),(10),(11), \tag{56}$$

$$\begin{aligned}\boldsymbol{W}^E(i') \equiv \mathbb{E}_{S',s'|S,s}\{&(\pi_d+(1-\pi_d)d(S',s'))\boldsymbol{U}(S')\\ &+(1-\pi_d)(1-d(S',s'))\max\{\boldsymbol{U}(S')+\bar{W}(\tilde{S}'^*,\tilde{s}'^*;i'),0+\beta\,\mathbb{E}_{S''|S'}\boldsymbol{U}(S'')\}\},\end{aligned} \tag{57}$$

$$\boldsymbol{W}^E(i') \geq \mathbb{E}_{S'|S}\boldsymbol{U}(S'), \forall i' \in [0,n'], \tag{58}$$

$$\max_x \; w(i)+\lambda p(\theta(S,x))x+(1-\lambda p(\theta(S,x)))\tau(i)\beta\,\mathbb{E}_{S'|S}\boldsymbol{U}(S') \tag{59}$$

$$+(1-\lambda p(\theta(S,x)))(1-\tau(i))\beta\boldsymbol{W}^E(i') \geq \boldsymbol{U}(S)+\bar{W}(\tilde{S},\tilde{s};i), \;\; \text{for } i' \in [0,n'-n_h], \tag{60}$$

$$w_h(i')+\beta\boldsymbol{W}^E(i') \geq x_h, \;\; \text{for } i' \in (n'-n_h,n']. \tag{61}$$

Equations (57) to (61) describe the new implicit contract constraints in the presence of asymmetric information. First, equation (57) uses W^E to denote the worker's expected lifetime utility if he stays with the firm. Notice that W^E is also the last part of the employment value (54). Constraint (58) is the new participation constraint, meaning that the worker's expected utility is at least the expected unemployment value so that he will stay. Equation (59) is the new promise-keeping constraint for incumbent workers. This constraint requires the firm to commit to paying the employee at least the promised lifetime utility. The left-hand side is the incumbent worker's employment value, i.e., equation (54). The right-hand side is the promised lifetime utility, comprised of two parts—the unemployment value $\boldsymbol{U}(S)$ and the promised utility markup $\bar{W}(\tilde{S},\tilde{s};i)$. Notice that $\tilde{S}(i)$ and $\tilde{s}(i)$ are the firm's declarations of shocks, two of the firm's choice variables. They can be different from the true shocks because firms know the realized shocks when renewing labor contracts, but workers do not. The declarations can be different across the firm's employees. Equation (61) is the new promise-keeping constraint for newly hired workers. Its left-hand side is the newly hired worker's employment value. On the right-hand side, x_h is the submarket where the firm employs new workers, and x_h is also the promised lifetime utility of the vacancies posted in that submarket. Thus, equation (61) means the firm should guarantee that newly hired workers receive at least the lifetime utility promised by the offer.

The following Proposition 2 proves that the promised utility markup $\bar{W}$ is state-uncontingent.

Proposition 2 *The labor relation between the firm and its employees has the following properties:*

(i) Workers do not leave the firm even if they find the firm lied.

(ii) The promised utility markup $\bar{W}$ is state-uncontingent.

Proof As for point (i), recall that employees discover whether the firm lied about shocks in the production stage, i.e. after the contract is updated. If they leave the firm now, they get nothing today and start receiving the unemployment value in the next period. So, even if the firm gives the worker zero wages and fires them right after the production stage, the worker is willing to stay with the firm.

As for point (ii), because employees will not leave the firm regardless, according to point (i), lying about the shocks has no consequences for the firm. Thus, firms always declare the lowest employment surplus in $\{\bar{W}(S, s; i)\}_{S \in \boldsymbol{S}, s \in \boldsymbol{s}}$ to each employee i. Therefore, the incentive-compatible labor contract requires the promised utility markup $\bar{W}$ to be state-uncontingent. $\square$

Appendix E Computational Algorithm

This section explains the computational algorithm for solving the model. I use Fortran as the programming language and parallelize to run the code with 20 cores.

First, I define $h(A,\sigma)$ as the vacancy posting cost plus a newly hired worker's wage:

$$h(A,\sigma) \equiv \min_{x_h} \left[\frac{c}{q(\theta(A,\sigma,x_h))} + w_h(A,\sigma,x_h) \right] \tag{62}$$

$$= \min_{x_h} \left[\frac{c}{q(\theta(A,\sigma,x_h))} + x_h - \beta\,\mathbb{E}\,\boldsymbol{U}(A',\sigma') \right] \tag{63}$$

$$= \kappa(A,\sigma) - \beta\,\mathbb{E}\,\boldsymbol{U}(A',\sigma'). \tag{64}$$

$h(A,\sigma)$ represents the costs paid in the current period to hire a new worker, which is the price I use to solve the labor market equilibrium.

Second, I discretize the state space. Aggregate productivity, A, is discretized into two points, i.e., high and low, the same for uncertainty, σ. The number of grids for firm-level idiosyncratic productivity, z, equals 13. The grids of z depend on the last-period uncertainty, σ_{-1}. Therefore, both σ and σ_{-1} are firms' state variables in the numerical implementation. I use Tauchen's method to discretize A, σ, and z. Cash on hand, X, has 64 grids. Debt, b, has 301 grids. Employment, n, has 240 grids.

Then I use the following steps to solve the problem:

1. Initialize the iteration counter $k = 0$. Make the initial guess for the current-period hiring cost $h^{(0)}(A,\sigma)$.

2. Given $h^{(k)}(A,\sigma)$, solve the unemployment value $\boldsymbol{U}^{(k)}(A,\sigma)$ by the value function iteration, along with the first-order condition with respect to x_u:

$$\boldsymbol{U}^{(k)}(A,\sigma) = \max_{x_u} \bar{u} + p(\theta^{(k)}(A,\sigma,x_u))x_u + (1 - p(\theta^{(k)}(A,\sigma,x_u)))\beta\,\mathbb{E}\,\boldsymbol{U}^{(k)}(A',\sigma') \tag{65}$$

$$= \bar{u} + \max_{x_u} p(\theta^{(k)}(A,\sigma,x_u))[x_u - \beta\,\mathbb{E}\,\boldsymbol{U}^{(k)}(A',\sigma')] + \beta\,\mathbb{E}\,\boldsymbol{U}^{(k)}(A',\sigma') \tag{66}$$

Given the following mapping from eq. (37):

$$x(A,\sigma,\theta) = \kappa(A,\sigma) - \frac{c}{q(\theta)}, \tag{67}$$

derive the first-order condition with respect to x_u that indicates the optimal choice of the labor market to search:

$$\theta_u^*(A,\sigma) = \left\{ \left[\frac{c}{\max\{\kappa(A,\sigma) - \beta\,\mathbb{E}\,\boldsymbol{U}(A',\sigma'), c\}} \right]^{-\frac{\gamma}{1+\gamma}} - 1 \right\}^{\frac{1}{\gamma}} \tag{68}$$

$$= \left\{ \left[\frac{c}{\max\{h(A,\sigma), c\}} \right]^{-\frac{\gamma}{1+\gamma}} - 1 \right\}^{\frac{1}{\gamma}} \tag{69}$$

When $h(A,\sigma) < c$, workers choose $\theta_u^* = 0$ to stay unemployed because the value of working offered in every submarket is less than the value of unemployment. On the other hand, as long as $h(A,\sigma) \geq c$, there always exists a market with θ close to 0 such that the value of employment is higher than unemployment, so workers want to search for jobs.

Plug the search decision $\theta_u^*(A,\sigma)$ into equation (66) and get the updated $\boldsymbol{U}(A,\sigma)$. Repeat this process until $\boldsymbol{U}(A,\sigma)$ converges.

3. Given $h^{(k)}(A,\sigma)$, solve the bond pricing schedule $Q^{(k)}(A,\sigma,\sigma_{-1},z,b',n')$ using the following iteration.

First, guess the bond pricing schedule $Q^{\text{old}}(A,\sigma,\sigma_{-1},z,b',n') = \beta$ and the maximum net borrowing $M^{\text{old}}(A,\sigma,\sigma_{-1},z,n) = \beta * b_{\max}$, where $b_{\max}$ denotes the upper-bound of the grids of debt.

Next, update Q and M. Then repeat until the relative difference between M^{old} and M^{new} is less than 10^{-7} and that between Q^{old} and Q^{new} is less than 10^{-10}.

(a) Update $Q(A,\sigma,\sigma_{-1},z,b',n')$ according to the following equation:

$$\begin{aligned} Q^{\text{new}}(A,\sigma,\sigma_{-1},z,b',n') = \beta\,\mathbb{E}\Big\{&(1-\pi_d)\Phi_\epsilon(\bar{\epsilon}(A',\sigma',\sigma,z',b',n')) \\ &+ [1-(1-\pi_d)\Phi_\epsilon(\bar{\epsilon}(A',\sigma',\sigma,z',b',n'))]\min\{\eta\frac{A'z'n'^{\alpha} - n'w(A',\sigma') - \bar{w}_m - \mu_\epsilon}{b'}, 1\}\Big\}, \end{aligned} \tag{70}$$

where the default cutoff, $\bar{\epsilon}(A',\sigma',\sigma,z',b',n')$, is calculated as follows

$$\bar{\epsilon}(A',\sigma',\sigma,z',b',n') \equiv A'z'n'^{\alpha} - n'w(A',\sigma') - b' + M^{\text{old}}(A',\sigma',\sigma,z',n') - \bar{w}_m, \tag{71}$$

and the incumbent worker's wage, $w(A',\sigma')$, is computed according to eq. (17).

(b) Update $M(A,\sigma,\sigma_{-1},z,n)$:

$$M^{\text{new}}(A,\sigma,\sigma_{-1},z,n) \equiv \max_{b',n',n_h,x_h} Q^{\text{new}}(A,\sigma,\sigma_{-1},z,b',n')b' - n_h\frac{c}{q(\theta(A,\sigma,x_h))} - n_h w_h(A,\sigma,x_h) \tag{72}$$

$$= \max_{b',n',n_h} Q^{\text{new}}(A,\sigma,\sigma_{-1},z,b',n')b' - n_h h^{(k)}(A,\sigma) \tag{73}$$

$$= \max_{b',n'} Q^{\text{new}}(A,\sigma,\sigma_{-1},z,b',n')b' - H^{(k)}(A,\sigma,n,n') \tag{74}$$

where $H(A,\sigma,n,n')$ denotes the matrix of hiring costs

$$H^{(k)}(A,\sigma,n,n') \equiv \begin{cases} [n' - (1-\lambda p(\theta^*(A,\sigma)))n]h^{(k)}(A,\sigma), & \text{if } n' > (1-\lambda p(\theta^*(A,\sigma)))n, \\ 0, & \text{if } n' \leq (1-\lambda p(\theta^*(A,\sigma)))n, \end{cases} \tag{75}$$

where the optimal on-the-job search market, $\theta^*(A,\sigma)$, is the same as the choice of unemployed workers, $\theta^*_u(A,\sigma)$.

4. Given $h^{(k)}(A,\sigma)$ and $Q^{(k)}(A,\sigma,\sigma_{-1},z,b',n')$, solve the firm's problem by value function iteration as follows.

(a) Guess the firm's value function $\boldsymbol{V}^{\text{old}}(A,\sigma,\sigma_{-1},z,X,n)$.

(b) Compute the expected future value:

$$G(A,\sigma,\sigma_{-1},z,b',n') \equiv \mathbb{E}\int_{-\infty}^{\bar{\epsilon}(A',\sigma',\sigma,z',b',n')} \boldsymbol{V}^{\text{old}}(A',\sigma',\sigma,z',X',n')d\Phi_\epsilon(\epsilon'), \tag{76}$$

where the default cutoff, $\bar{\epsilon}(A',\sigma',\sigma,z',b',n')$, is from eq. (71) and tomorrow's cash on hand is determined by

$$X' = A'z'n'^{\alpha} - n'w(A',\sigma') - \bar{w}_m - \epsilon' - b', \tag{77}$$

Then the firm's problem can be simplified into

$$\boldsymbol{V}^{\text{new}}(A,\sigma,\sigma_{-1},z,X,n) = \max_{\Delta,b',n'} \Delta + \beta(1-\pi_d)G(A,\sigma,\sigma_{-1},z,b',n')$$

$$\text{s.t. } \Delta = X + Q(A,\sigma,\sigma_{-1},z,b',n')b' - H(A,\sigma,n,n') \geq 0,$$

$$Q(A,\sigma,\sigma_{-1},z,b',n')b' - H(A,\sigma,n,n') \geq M(A,\sigma,\sigma_{-1},z,n) - F_m(A,\sigma,\sigma_{-1},z).$$

(c) Before solving $\boldsymbol{V}^{\text{new}}$, solve the relaxed problem first:

The relaxed problem is

$$\hat{\boldsymbol{V}}(A,\sigma,\sigma_{-1},z,n) = \max_{b',n'} Q(A,\sigma,\sigma_{-1},z,b',n')b' - H(A,\sigma,n,n') + \beta(1-\pi_d)G(A,\sigma,\sigma_{-1},z,b',n')$$

$$\text{s.t. } Q(A,\sigma,\sigma_{-1},z,b',n')b' - H(A,\sigma,n,n') \geq M(A,\sigma,\sigma_{-1},z,n) - F_m(A,\sigma,\sigma_{-1},z).$$

Let $\hat{b}(A,\sigma,\sigma_{-1},z,n)$ and $\hat{n}(A,\sigma,\sigma_{-1},z,n)$ denote the optimal policies of the relaxed problem.

(d) Given $\hat{b}(A,\sigma,\sigma_{-1},z,n)$ and $\hat{n}(A,\sigma,\sigma_{-1},z,n)$, update the grids of cash on hand. The grids of cash on hand X are equidistantly distributed on $[X_{\min}, X_{\max}]$. The lower bound, $X_{\min}$, equals $-M(A,\sigma,\sigma_{-1},z,n)$. The upper bound, $X_{\max}$, equals the maximum of $\hat{X}(A,\sigma,\sigma_{-1},z,n) = -[Q(A,\sigma,\sigma_{-1},z,\hat{b},\hat{n})\hat{b} - H(A,\sigma,\sigma_{-1},n,\hat{n})]$.

(e) Update the firm's value function, $\boldsymbol{V}(A, \sigma, \sigma_{-1}, z, X, n)$, by grid search. For each state $(A, \sigma, \sigma_{-1}, z, X, n)$ of $\boldsymbol{V}(\cdot)$, I go though the combinations of choices (b', n') to find the maximum objective value to update $\boldsymbol{V}^{\text{new}}(A, \sigma, \sigma_{-1}, z, X, n)$, where (b', n') should satisfy the non-negative equity payout constraint and the agency friction constraint. The grid search for optimal b' and n' in value function iterations is around the frictionless optimal levels of b' and n'.

(e) Given $\boldsymbol{V}^{\text{new}}(A, \sigma, \sigma_{-1}, z, X, n)$, update the expected future value, $G(A, \sigma, \sigma_{-1}, z, b', n')$. For each state $(A, \sigma, \sigma_{-1}, b', n')$ of $G(\cdot)$, I use Gauss-Legendre method to compute the integration with respect to ϵ', with the linear interpolation of $\boldsymbol{V}^{\text{new}}(A', \sigma', \sigma, z', X', n')$ with respect to X'. Denote the updated expected future value as $G^{\text{new}}(A, \sigma, \sigma_{-1}, z, b', n')$.

5. Renew the current-period hiring cost, $h^{(k+1)}(A, \sigma)$, such that the free entry condition holds for each aggregate state (A, σ):

$$k_e = \sum_z \boldsymbol{J}_e(A, \sigma, z) g_z(z), \forall (A, \sigma), \tag{78}$$

where the new entrant's value is solved by

$$\begin{aligned} \boldsymbol{J}_e(A, \sigma, z) &= \max_{n_h, x_h} -n_h \frac{c}{q(\theta(A, \sigma, x_h))} - n_h w_h(A, \sigma, x_h) + \beta(1-\pi_d) G^{\text{new}}(A, \sigma, \sigma_{-1}, z, b_0, n_h) \quad (79) \\ &= \max_{n_h} -n_h h^{(k+1)}(A, \sigma) + \beta(1-\pi_d) G^{\text{new}}(A, \sigma, \sigma_{-1}, z, b_0, n_h), \quad (80) \end{aligned}$$

where the initial debt, b_0, equals zero.

6. The iteration stops when the expected future value converges, i.e., $\text{dist}(G^{\text{new}}, G^{\text{old}}) < 10^{-6}$, where I follow Judd (1998) and define the distance function as $\text{dist}(f^{(k+1)}, f^{(k)}) = \frac{(\sum_x (f^{(k+1)}(x) - f^{(k)}(x))^2)^{\frac{1}{2}}}{1 + (\sum_x f^{(k)}(x)^2)^{\frac{1}{2}}}$. If the problem does not converge, assign k with $k+1$ and start from Step 2 again.

Appendix F The Counterfactual Model Without the Financial Friction but Kept the Benchmark Labor Contracting Outcomes

Table 7: Parameters of the Counterfactual Model Without the Financial Friction but Kept the Benchmark Labor Contracting Outcomes

		No Financial Friction	
Parameters	**Notations**	$A + \sigma$	A only
Aggregate shocks			
Persistence of aggregate productivity	ρ_A	0.913	0.913
SD of aggregate productivity	σ_A	0.041	0.035
Mean of uncertainty	$\bar{\sigma}$	0.313	0.280
Persistence of uncertainty	ρ_σ	0.924	-
SD of uncertainty	σ_σ	0.186	-
Correlation between ϵ_t^A and ϵ_t^σ	$\rho_{A\sigma}$	-0.900	-
Labor market			
Unemployment benefits	$\bar{u}$	0.141	0.141
Vacancy posting cost	c	0.001	0.002
Relative on-the-job search efficiency	λ	0.100	0.100
Matching function elasticity	γ	1.600	1.600
Entry cost	k_e	11.19	11.47
Mean operating cost	$\bar{w}_m + \mu_\epsilon$	0.100	0.100
Financial market			
SD of production costs	σ_ϵ	0.080	0.080
Agency friction	$\tilde{\zeta}$	-	-
Auditing quality	ξ	-	-
Recovery rate	η	-	-
Exogenous exit rate	π_d	0.022	0.022

Notes: This table reports the recalibrated parameters of the model without the financial friction but kept the benchmark labor contracting outcomes. $A + \sigma$ means the model has both aggregate productivity shocks and uncertainty shocks, and A means the model only has aggregate productivity shocks. Table 8 shows the targeted data moments and the model-simulated moments.

Table 8: Matched Moments of the Counterfactual Model Without the Financial Friction but Kept the Benchmark Labor Contracting Outcomes

		No Financial Friction	
Moments	**Data**	$A + \sigma$	A
Aggregate shocks			
Autocorrelation of output	0.839	0.840	0.870
SD of output	0.016	0.019	0.017
Mean of IQR	0.171	0.167	0.165
Autocorrelation of IQR	0.647	0.614	-
SD of IQR	0.013	0.012	-
Correlation (output, IQR)	-0.351	-0.318	-
Labor market			
UE rate	0.834	0.845	0.824
EU rate	0.076	0.073	0.074
EE rate	0.085	0.085	0.082
$\epsilon_{UE/\theta}$	0.720	0.705	0.714
Average establishment size	15.6	15.0	15.6
Entry/Total job creation	0.21	0.17	0.17
Financial market			
Mean credit spread (%)	1.09	-	-
Median leverage (%)	26	-	-
Correlation (output, spreads)	-0.549	-	-
Correlation (IQR, spreads)	0.462	-	-
Annual exit rate (%)	8.9	9.0	9.1

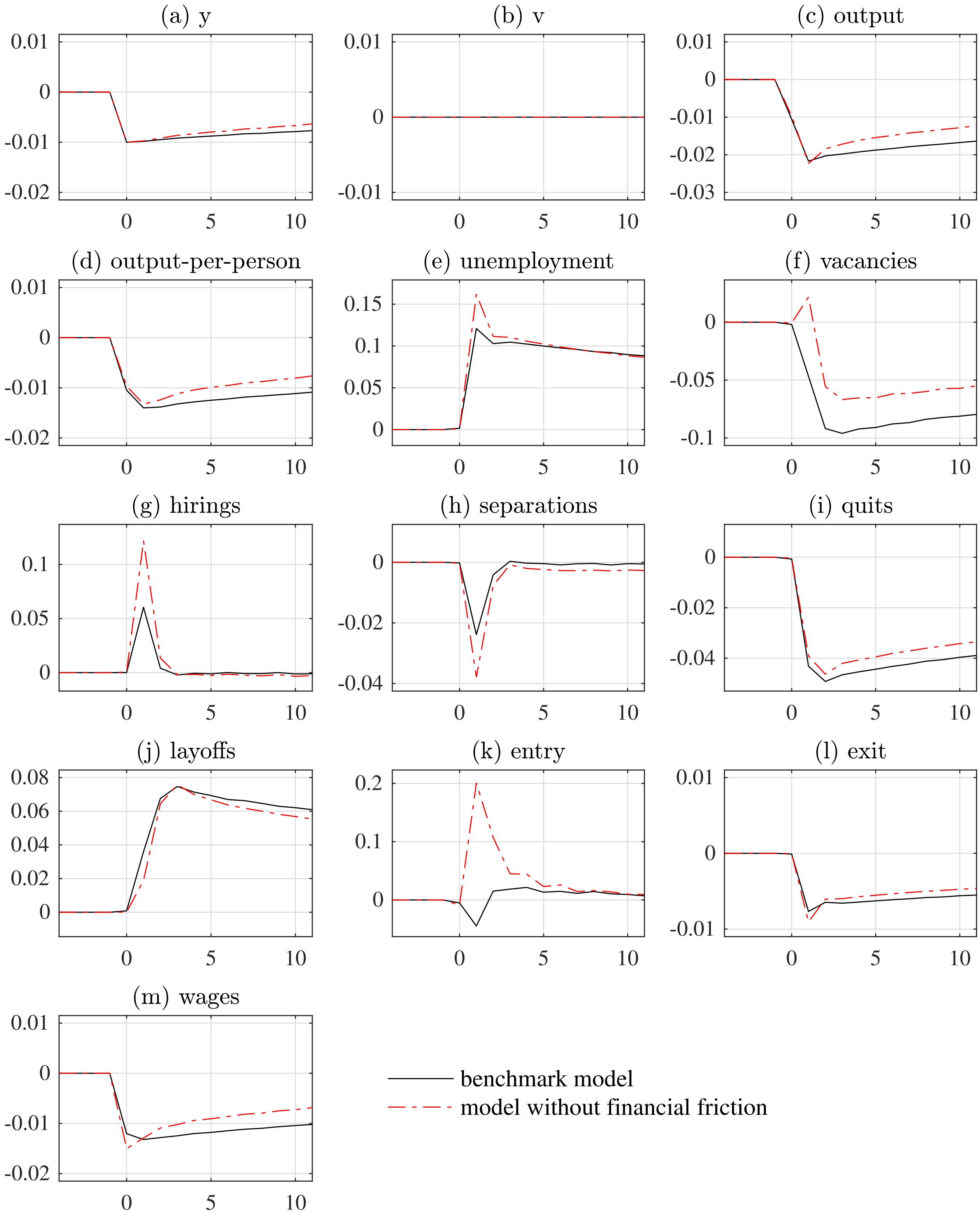
Notes: This table shows the targeted data moments and moments matched by the model without the financial friction but kept the benchmark labor contracting outcomes. $A + \sigma$ means the model has both aggregate productivity shocks and uncertainty shocks, and A means the model only has aggregate productivity shocks. Table 7 reports the recalibrated parameters.

Table 9: Business Cycle Statistics of the Counterfactual Model Without the Financial Friction but Kept the Benchmark Labor Contracting Outcomes

	Y	Y/L	U	V	Hirings	Quits	Layoffs	Wages
				Panel A: Data				
Std Dev.	0.016	0.012	0.121	0.138	0.058	0.102	0.059	0.008
cor(Y,x)	1	0.590	-0.859	0.702	0.677	0.720	-0.462	0.555
				Panel B: Model Without Financial Friction				
				Both A and σ Shocks				
Std Dev.	0.019	0.016	0.088	0.084	0.038	0.026	0.069	0.014
cor(Y,x)	1	0.986	-0.781	0.826	0.051	0.919	-0.569	0.964
				Only A Shocks				
Std Dev.	0.017	0.014	0.069	0.064	0.023	0.023	0.047	0.012
cor(Y,x)	1	0.994	-0.876	0.906	-0.024	0.975	-0.787	0.985

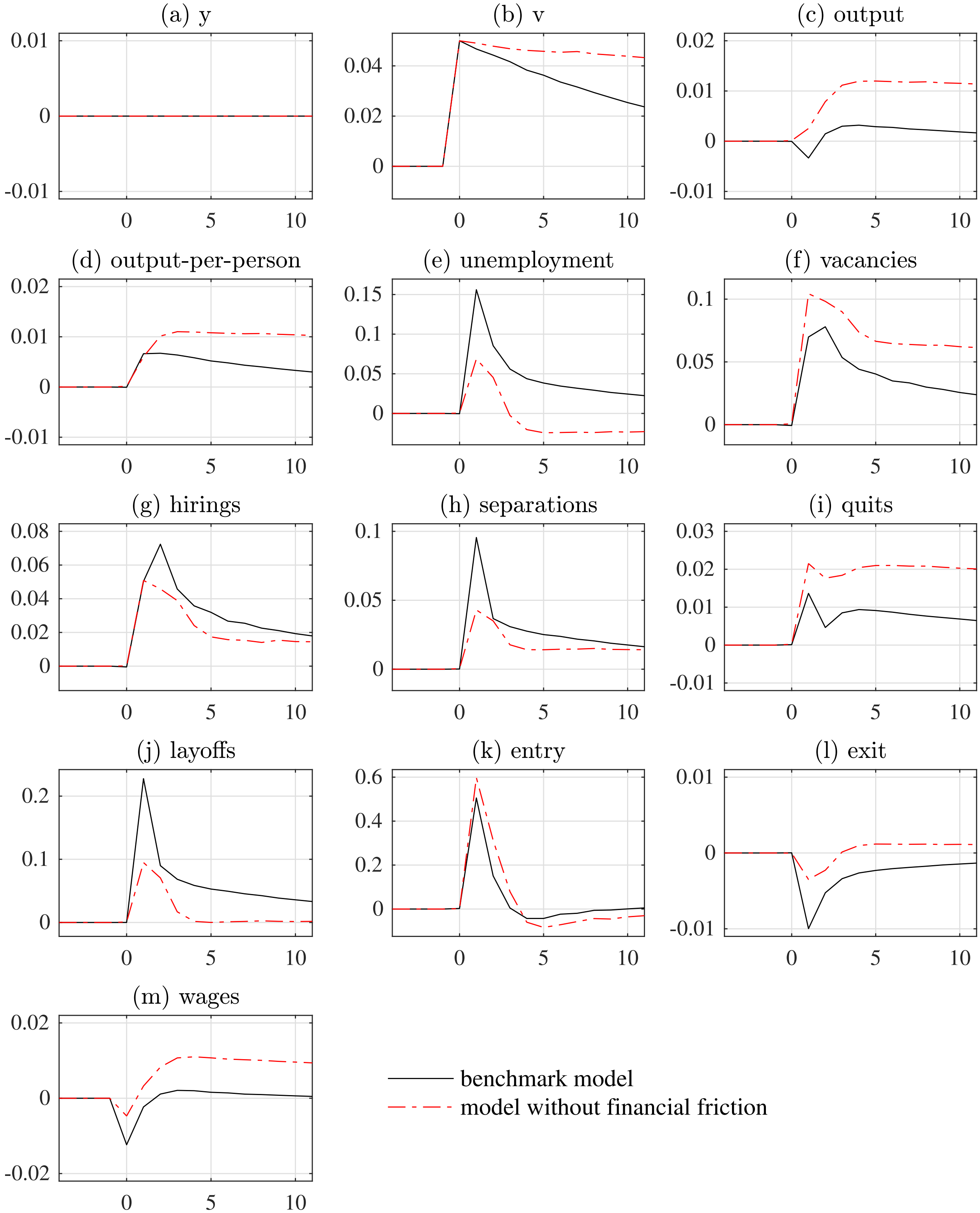
Notes: Panel A shows the business cycle moments in the data. Panels B report moments of 3,000-quarter simulations of the model without the financial friction but kept the benchmark labor contracting outcomes, with and without uncertainty shocks. Both the data and the model simulations are log-detrended by the HP filter with smoothing parameter 1600. To be consistent with Schaal (2017), Y denotes output, Y/L is output per worker, U represents unemployment, and V is vacancies.

Figure 12: Aggregate Impulse Responses to a 1% Negative Aggregate Productivity Shock



Notes: The panels are impulse responses to a 1% transitory negative aggregate productivity shock at quarter 0. The impulse responses are the average of 4,000 simulated paths, presented as log deviations from the mean. Solid black lines are the benchmark results. Dash-dot red lines are for the model without the financial friction but kept the benchmark labor contracting outcomes. I use Schaal's (2017) code when plotting this figure.

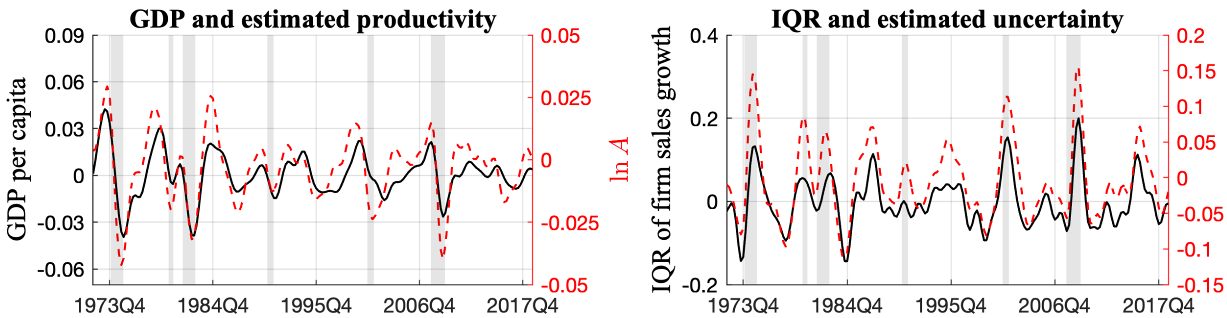
Figure 13: Aggregate Impulse Responses to a 5% Positive Uncertainty Shock



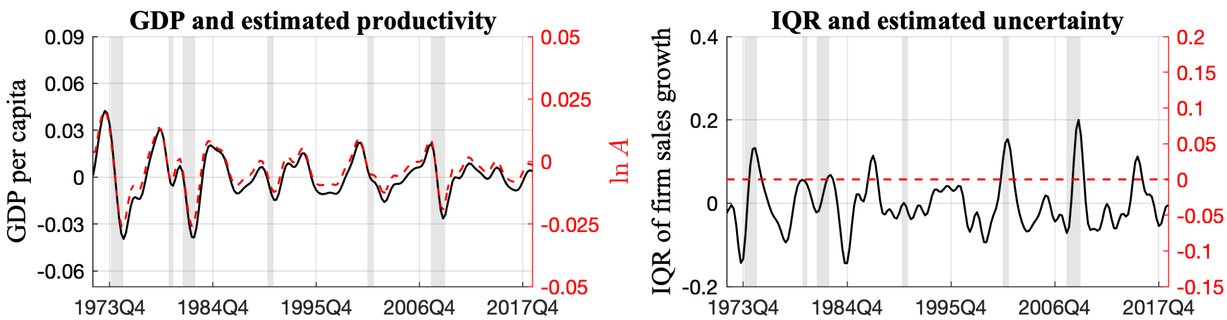
Notes: The panels are impulse responses to a 5% positive uncertainty shock at quarter 0. The impulse responses are the average of 4,000 simulated paths, presented as log deviations from the mean. Solid black lines are the benchmark results. Dash-dot red lines are for the model without the financial friction but kept the benchmark labor contracting outcomes. I use Schaal's (2017) code when plotting this figure.

Figure 14: Estimated Aggregate Productivity and Uncertainty of the Counterfactual Model Without the Financial Friction but Kept the Benchmark Labor Contracting Outcomes

Panel A: Model Without Financial Friction (Both A and σ Shocks)

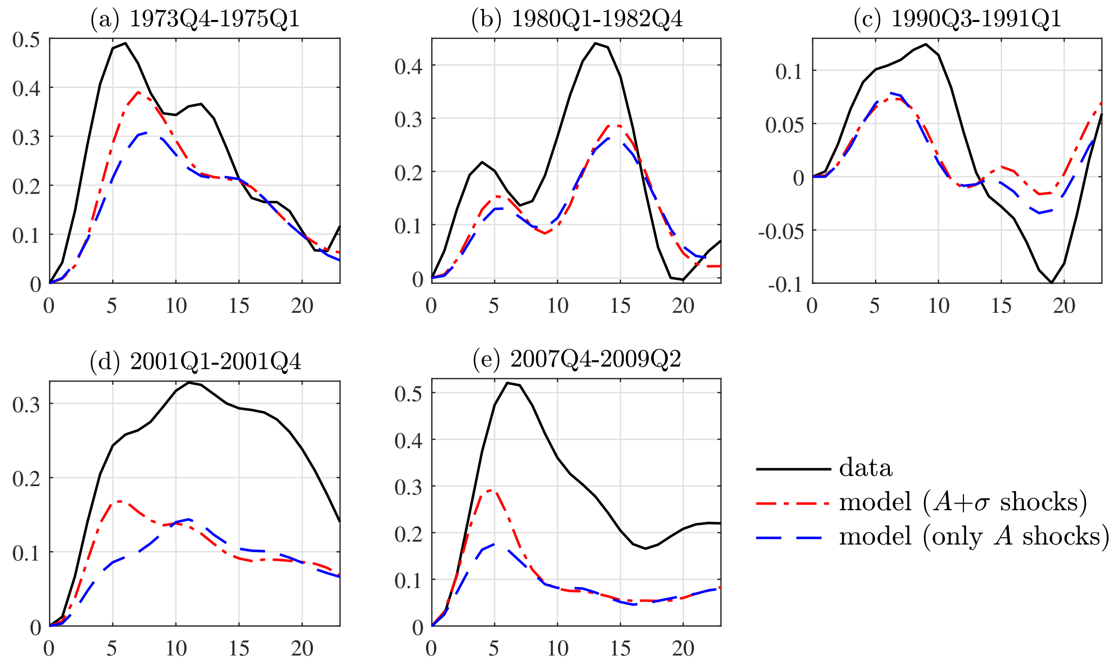


Panel B: Model Without Financial Friction (Only A Shocks)



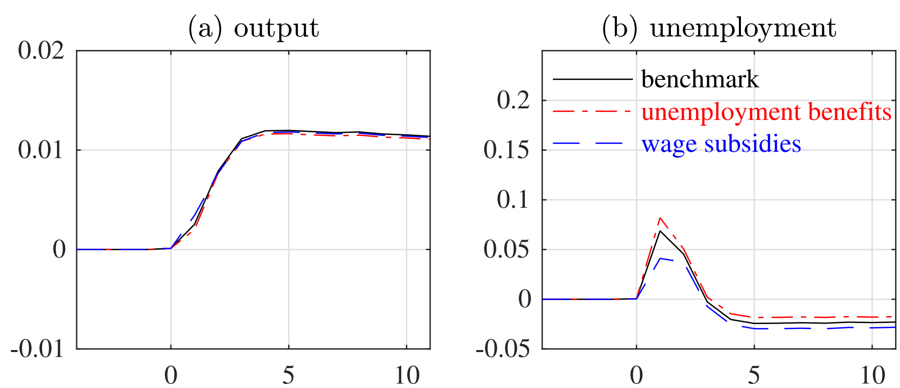
Notes: This figure shows the estimated aggregate productivity and uncertainty of the model without the financial friction but kept the benchmark labor contracting outcomes. I apply the particle filter to my model and estimate the states of aggregate productivity, A, and uncertainty, σ, from the data series of GDP per capita and the IQR of firm sales growth, which are detrended by a band-pass filter to focus on fluctuations between 6 and 32 quarters, following Schaal (2017). Panels on the left-hand side display log deviations of GDP (solid black lines) and the estimated demeaned logged aggregate productivity (dashed red lines). Panels on the right-hand side present the log deviations of the interquartile range (IQR) of firm sales growth (solid black lines) and the estimated demeaned logged uncertainty (dashed red lines). The logged uncertainty is demeaned for the comparison of its fluctuations across models.

Figure 15: Unemployment Series of the Counterfactual Model Without the Financial Friction but Kept the Benchmark Labor Contracting Outcomes



Notes: The panels show the model's predictions for unemployment during recessions of the model without the financial friction but kept the benchmark labor contracting outcomes. The model is recalibrated to match the data moments. I use the particle filter to jointly estimate the time series of aggregate productivity shocks and uncertainty shocks by matching output and the IQR of firm sales growth in the data. The data are detrended by a band-pass filter to focus on fluctuations between 6 and 32 quarters, following Schaal (2017). Given the estimated shocks, I show the model-predicted unemployment. The data on unemployment is the solid black lines. The unemployment fluctuations predicted by the models with both aggregate productivity shocks and uncertainty shocks are the dash-dotted red lines (labeled as $A + \sigma$ shocks), and predictions without contracting frictions are the dashed blue line. Series are depicted in terms of log deviations from the peak preceding the recession. I use Schaal's (2017) code when plotting this figure.

Figure 16: Output and Unemployment Responses to a 5% Uncertainty Shock Under Policies of the Counterfactual Model Without the Financial Friction but Kept the Benchmark Labor Contracting Outcomes



Notes: The picture shows the impulse responses of aggregate output and unemployment to a 5% positive uncertainty shock at quarter 0. This counterfactual model does not have the financial friction but keeps the benchmark labor contracting outcomes. The model has both aggregate productivity shocks and uncertainty shocks. Solid black lines are the results without policy intervention (labeled as the benchmark). Dash-dot red lines are for the model with the policy of enhanced unemployment benefits. Dashed blue lines are for the model with the policy of wage subsidies. Both policies are implemented conditional on uncertainty higher than its average. The impulse responses are the average of 4,000 simulated paths, presented as log deviations from the mean. I use Schaal's (2017) code when plotting this figure.