Uncertainty and Unemployment Revisited: The Consequences of Financial and Labor Contracting Frictions*

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Abstract

I build a novel search model to study how uncertainty shocks to firm-level productivity affect unemployment through the financial channel of incomplete labor contracts. In my model, the labor contracting friction implies wage insensitivity to firms' idiosyncratic shocks. Hence, wage bills are debt-like commitments by firms to workers, which firms are less likely to take on when high uncertainty raises firm default risks. Therefore, when uncertainty is high, firms hire fewer workers, and unemployment increases. Quantitatively, I find that uncertainty shocks, together with aggregate productivity shocks, explain 90% of the increase in unemployment during the Great Recession. The model's quantitative performance deteriorates greatly if either the financial channel or uncertainty shocks are absent. Given the model's quantitative success, I use it to analyze the impact of the United States and German labor market policies that expanded a lot in recent recessions. The U.S. policy raises unemployment benefits, making it more expensive for firms to pay wages, amplifying the recession. Germany subsidizes firms' wage bills to keep workers employed, which outperforms the U.S. policy but still yields a negative impact since its misallocation losses outweigh its gains from insuring firms.

Keywords: search and matching, financial frictions, incomplete labor contracts, uncertainty, volatility, firm heterogeneity, business cycles, labor market policies.
JEL Codes: E24, E32, E44, D53, D83, J08.

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1 Introduction

In this paper, I construct and analyze a new search model of unemployment. The core of the model is a labor contracting friction which requires wages to be insensitive to firm-level idiosyncratic shocks in the intertemporal firm-worker contracts. The paper introduces this contracting friction, along with a more standard firm financing friction, into Schaal's (2017) directed search framework with time-varying uncertainty of firm-level shocks. I document that the labor contracting and financial frictions together (and only together) allow the uncertainty shocks with aggregate productivity shocks to account for 90% of the increase in unemployment during the Great Recession.

The labor market friction does not require wages to be sticky: they can adjust fully in response to workers' outside opportunities. What is key is that wages cannot respond to firm-level idiosyncratic shocks. The paper shows this kind of contracting friction can be micro-founded in an intuitive fashion by assuming that firms have private information about their shocks. The restriction is also consistent with existing empirical evidence showing that the pass-through from firms' idiosyncratic transitory shocks to workers' earnings is insignificant (Guiso, Pistaferri and Schivardi, 2005; Rute Cardoso and Portela, 2009).

This labor market friction affects unemployment through a financial channel. The firm financial friction makes default costly, so an increase in idiosyncratic risk makes them more averse to borrowing. And the insensitivity of wages to firm-specific shocks implies that wage bills are debt-like commitments to workers, so hiring a worker is isomorphic to borrowing more. Therefore, when uncertainty is high, firms hire fewer workers, and unemployment increases.

Formally, my model is built on Schaal (2017). He in turn extends the directed search framework in Menzio and Shi (2010) to have multi-worker firms and decreasing returns to scale production technology, which enables within-firm endogenous hirings, separations, and on-the-job search. My model allows for both aggregate productivity shocks and uncertainty shocks, as in Schaal (2017). Despite two aggregates shocks and firm heterogeneity, the model keeps its tractability because directed search provides block recursivity (Kaas and Kircher, 2015; Menzio and Shi, 2011; Schaal, 2017).

Then, as described above, I extend the model by incorporating the labor contracting friction as well as the firm financial friction. The latter assumes firms can only borrow through state-uncontingent debt, and default is costly because it leads to liquidation. The price of debt reflects the firm's default probability and the post-default recovery from the firm's value. I also model the agency friction that managers can divert firms' funds for their private interests, which constrains the firms' incentives to save, so the default risk will not be offset by a large stock of savings. The financial friction interacts with the labor contracting friction to generate risk. Neither friction is effective individually. If labor contracts are complete, firms can use them as state-contingent instruments to hedge against shocks and eliminate idiosyncratic risk. If the financial market is complete, how wages are paid within labor contracts is inconsequential because it is the present value of wages that determines the incentives of hiring and firing.[1]

I use the model for three quantitative analyses. First, I show it accounts for 90% of unemployment volatility in the data. The model is calibrated by matching labor market flows, financial market moments, and the business cycle statistics of GDP and the interquartile range (IQR) of firm sales growth rates for the processes of aggregate productivity shocks and uncertainty shocks. I find uncertainty shocks, rather than aggregate productivity shocks, are essential for the financial impact of wage commitments on unemployment volatility. This is because equilibrium wages decrease much for aggregate productivity shocks as in Shimer's (2005) puzzle, but not for uncertainty shocks. Specifically, an increase in uncertainty spreads the distribution of firm-level productivity, maintaining firm values as well as the wage level.[2]

Second, I show the model explains much of the unemployment fluctuations in U.S. past recessions, particularly 90% of the increase in unemployment during the Great Recession. For this exercise, I first apply the particle filter to my model and estimate the historical aggregate productivity shocks and uncertainty shocks using the data of GDP and the IQR of firm sales growth.[3] Then, I feed the estimated structural shocks to the model, and it generates much of the increases in unemployment during recessions. In particular, the financial channel contributed more to the Great Recession than other recessions because it had the largest increase in uncertainty, driving the financial channel to amplify the rise in unemployment. Without financial frictions, the model collapses to a canonical search framework, which underestimates the effect of uncertainty shocks, so it predicts only about half of the increase in unemployment during the Great Recession, as in Schaal (2017).

Finally, I assess the welfare consequences of the U.S. and German labor market policies that expanded during recent recessions. The U.S. policy increased unemployment benefits during Covid. Higher unemployment benefits cause higher wages, making hiring even riskier for firms, so the recession deepens and welfare decreases. In contrast, Germany subsidizes firms to pay wages and keep workers employed. It insures firms against idiosyncratic risk and thus outperforms the U.S. policy. However, wage subsidies encourage labor hoarding and cause misallocation, which outweighs the gains from insuring firms, so welfare decreases. And if financial frictions are ignored when evaluating the two policies, their welfare losses will be greatly underestimated. In particular, the counterfactual model will misleadingly suggest the U.S. policy is better.

[1] See Pissarides (2009) for similar neutrality of incumbent workers' wage rigidity with respect to aggregate shocks.

[2] This positive impact of increasing volatility on firm values is called the Oi-Hartman-Abel effect (Oi (1961), Hartman (1972), and Abel (1983)).

[3] A particle filter is a Monte Carlo Bayesian estimator for the posterior distribution of structural shocks and allows non-linear systems, suitable for my non-linear model.

Related Literature. My paper contributes to three strands of literature. First, I build on the papers that find the crucial role of uncertainty shocks in driving business cycles (Basu and Bundick, 2017; Bloom et al., 2018; Christiano, Motto and Rostagno, 2014; Fajgelbaum, Schaal and Taschereau-Dumouchel, 2017; Fernández-Villaverde et al., 2011).[4] Their findings motivate Schaal (2017) to introduce uncertainty shocks into a canonical search framework to understand the fluctuations of unemployment. My model extends his works by considering the labor contracting and financing frictions, which move unemployment fluctuations closer to the data. This result echoes the finding that financial frictions are a key reason for uncertainty shocks to cause recessions (Alfaro, Bloom and Lin, 2019; Arellano, Bai and Kehoe, 2019; Christiano, Motto and Rostagno, 2014; Gilchrist, Sim and Zakrajšek, 2014).

Second, my work complements the literature that uses wage stickiness to generate unemployment volatility (Bils, Chang and Kim, 2022; Fukui, 2020; Gertler and Trigari, 2009; Hall, 2005; Hall and Milgrom, 2008; Menzio and Moen, 2010; Rudanko, 2019; Schoefer, 2021; Shimer, 2004). Taking a different route from sticky wages, I find wage insensitivity to firms' idiosyncratic shocks explains much unemployment volatility. This exposition is motivated by the empirical evidence that the pass-through from firms' transitory idiosyncratic shocks to workers' earnings is very little (Guiso, Pistaferri and Schivardi, 2005; Rute Cardoso and Portela, 2009). Also, indirect evidence shows that firms with higher labor shares are more sensitive to shocks (Donangelo et al., 2019), and wage growth and labor shares are the primary factors predicting credit spreads (Favilukis, Lin and Zhao, 2020).

Third, there is a growing literature adding firm financial frictions to search models (Blanco and Navarro, 2016; Boz, Durdu and Li, 2015; Christiano, Trabandt and Walentin, 2011; Chugh, 2013; Eckstein, Setty and Weiss, 2019; Garin, 2015; Mumtaz and Zanetti, 2016; Petrosky-Nadeau, 2014; Schoefer, 2021; Sepahsalari, 2016; Wasmer and Weil, 2004; Zanetti, 2019). I model the financial friction as risky debt with endogenous default decisions, following Arellano, Bai and Kehoe (2019); Khan and Thomas (2013); Ottonello and Winberry (2020). My model differs from others in considering the micro-level labor contracting friction. The associated technical challenge is the dimensionality curse stemming from the firm's continuum of historical-dependent labor contracts. But I prove that wage payments are uniquely determined within contracts. Given the expression of wages, labor contracts are no longer a necessary part of the state variables, resolving the dimensionality curse. This approach to tractability is, as far as I know, new to the literature.

Layout. The paper proceeds as follows. I first set up the model in Section 2. Section 3 calibrates the model and presents quantitative results, including the event study for U.S. past recessions and

[4] Uncertainty in my paper specifically refers to micro-level volatility, i.e., the volatility of firm-level idiosyncratic productivity documented by Bloom et al. (2018). For studies on macro-level volatility of aggregate productivity shocks, see, e.g., Leduc and Liu (2016), Freund and Rendahl (2020), Cacciatore and Ravenna (2021), and Den Haan, Freund and Rendahl (2021). My paper does not include macro-level volatility due to the computational burden. But I conjecture it will not change the result much because I do not assume sticky wages and, according to Schaal (2017), the size and impact of macro-level volatility are small.

labor market policy experiments. Lastly, section 4 concludes.

2 Model

To study the impact of aggregate shocks on unemployment, I build a directed search and matching model. The equilibrium is block recursive to provide tractability, following Menzio and Shi (2010, 2011), Kaas and Kircher (2015), and Schaal (2017).[5] The model also features financial frictions with firm default risks, following Arellano, Bai and Kehoe (2019).

2.1 Environment and Timing

There are four types of agents in the economy: workers, firms, managers, and international financial intermediaries. Workers are infinitely lived and risk-neutral. They have the same productivity. The total mass of workers is normalized to one unit. Firms are also risk-neutral. They hire workers to produce homogeneous goods and finance by borrowing from the financial intermediaries.

Firms' idiosyncratic productivity is drawn from the Markov process $\pi_z(z'|z, \sigma)$, where σ is time-varying uncertainty of firm-level productivity. Higher uncertainty implies a more widely spread distribution of tomorrow's idiosyncratic productivity shocks, so firms are more likely to draw a low idiosyncratic productivity. The other aggregate shock in the economy is the aggregate productivity shock A. I use S to summarize the two aggregate shocks (A, σ). Firms also face an i.i.d. random operating cost shock ϵ, which follows a normal distribution $\Phi_\epsilon \equiv \mathcal{N}(\mu_\epsilon, \sigma_\epsilon^2)$. I use s to denote the two firm-specific shocks (z, ϵ).

I assume that job search is directed. Each labor submarket is indexed by a promised utility x, which is the lifetime utility firms promise to workers hired from this submarket. The submarket tightness θ is the ratio of vacancies to the number of workers looking for jobs in each submarket. I use $p(\theta)$ to denote the job-finding rate of workers and $q(\theta)$ to denote the vacancy-filling rate of firms. The relation between x and θ will be determined by the free entry condition in equilibrium.

Following the implicit contract literature, I assume that firms are committed to labor contracts while workers are not. So, workers can leave the firm whenever their outside option is better. One justification for this approach is that firms care about their reputations more than individual workers. I denote the recursive-from labor contract as $C = \{w,\ \tau,\ W'(S', s'),\ d(S', s')\}$, where w is the current wage payment, τ is the layoff probability, $W'(S', s')$ is the next-period employment value promised by the firm, and $d(S', s')$ is the indicator for the firm's exit decision.

[5] As for other search models with multi-worker firms, Acemoglu and Hawkins (2014) and Elsby and Michaels (2013) introduce Nash bargaining into random search. Because my paper focuses on business cycles, I leverage directed search with block recursivity to solve the problem globally out of the steady-state.

Figure 1: Timing

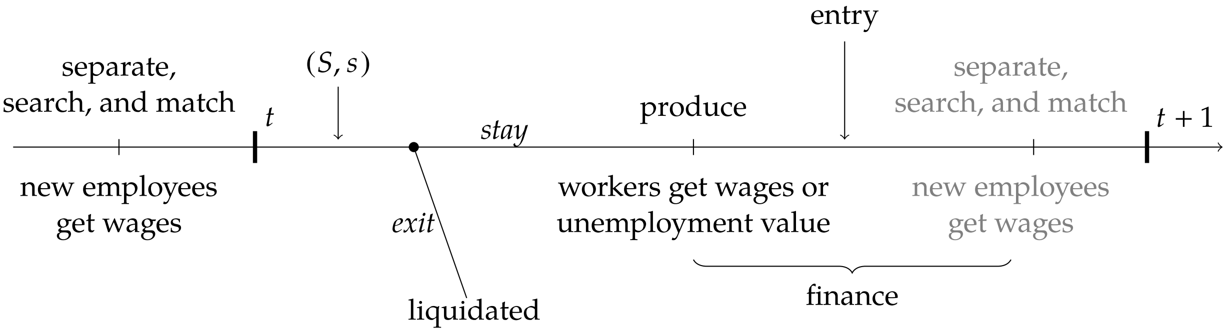


Figure 1 shows the timing. At the end of the preceding period, firms and workers interacted in the labor market to separate, search, and match. They draw up labor contracts in this stage. And newly employed workers receive wages. At the beginning of the current period, all shocks (S, s) realize. Then firms decide to exit or not. If a firm exits, it defaults on all its debts, including labor contracts, and its operations are liquidated. Otherwise, firms produce based on the number of employees as determined at the end of the last period. At the same time, employed workers receive wages according to continuing labor contracts. Unemployed workers also obtain unemployment benefits in this stage. Next, potential new firms can pay an entry cost to enter, after which both new entrants and incumbent firms participate simultaneously in the labor market. Firms borrow from international financial intermediaries to finance the expenditure during the process.

2.2 Worker's Problem

There are two types of workers in the economy: unemployed and employed workers. I abstract from the participation margin.

Unemployed Worker's Problem. An unemployed worker receives unemployment benefits $\bar{u}$ in the current period and chooses a submarket x_u to job search to maximize their unemployment value. The matching probability $p(\theta(S, x_u))$ depends on the aggregate shocks and the promised utility of the submarket. Therefore, the unemployment value is:

$$\boldsymbol{U}(S) = \max_{x_u} \bar{u} + p(\theta(S, x_u))x_u + (1 - p(\theta(S, x_u)))\beta\,\mathbb{E}\,\boldsymbol{U}(S'). \tag{1}$$

Employed Worker's Problem. The value of employment depends on the contingent labor contract $C = \{w,\ \tau,\ W'(S', s'),\ d(S', s')\}$. The worker receives his wage w in the current period and can simultaneously search for other jobs as well. I use x to denote his choice of on-the-job search submarket. If he successfully gets a new job, he receives x as lifetime utility. Notice that the job finding rate $p(\theta(S, x))$ is discounted by the relative on-the-job search efficiency λ, which matches the job-to-job transition rate.

In the next period, if the worker is laid off or the firm exits, he will be unemployed and receive the unemployment value $\boldsymbol{U}(S')$. Otherwise, he can still work for the firm and receive the promised utility $W'(S',s')$. Notice that I assume firms are fully committed to labor contracts, but workers are not. Therefore, for promised utilities lower than the unemployment value, the worker will voluntarily leave the job and become unemployed. The following equation formalizes the value of employment:

$$
\begin{aligned}
\boldsymbol{W}(S,s,\boldsymbol{C}) = \max_{x}\ & w + \lambda p(\theta(S,x))x \\
& + (1-\lambda p(\theta(S,x)))\beta\,\mathbb{E}\Big\{[\tau + (1-\tau)(\pi_d + (1-\pi_d)d(S',s'))]\boldsymbol{U}(S') \\
& + (1-\tau)(1-\pi_d)(1-d(S',s'))\max\{W'(S',s'),\boldsymbol{U}(S')\}\Big\}.
\end{aligned} \tag{2}
$$

where π_d is the exogenous exit rate of firms.

2.3 Firm's Problem

Firms maximize their present values, namely, the discounted cumulative sum of equity payouts.

A firm's states include realized aggregate shocks $S \in \boldsymbol{S}$, realized firm-specific shocks $s \in \boldsymbol{s}$, the number of employees n, and the set of promised utilities to its employees $\{W(S,s;i)\}_{i\in[0,n]}$, where i is the index of incumbent employees within the firm.

Firms optimize over the current equity payout Δ, next-period debt b', next-period employment n', the number of workers to hire n_h, the submarket x_h in which to search, and next-period exit decisions $d(S',s')$. I assume that a firm only posts vacancies in one submarket each period. Firms also choose the current-period wages of incumbent workers $w(i)$, the layoff probability $\tau(i)$, the wages of newly hired workers $w_h(i')$, and the set of next-period lifetime utilities $\{W(S',s';i')\}_{S'\in\boldsymbol{S}',s'\in\boldsymbol{s}';i'\in[0,n']}$, subject to the participation constraint (8) and the promise-keeping constraint (9). I use $w(i)$ for incumbent employee i's wage, $\bar{w}_m$ for the manager's wage[6], and $w_h(i')$ for the wage of a newly hired employee i'.

Equations (3) to (11) summarize the firm's problem starting from the production stage:

$$
\begin{aligned}
\boldsymbol{J}(S,s,b,n,\{W(S,s;i)\}_{i\in[0,n]}) = & \max_{\substack{\Delta,b',n',n_h,x_h,d(S',s'),\\ \{w(i),\tau(i)\}_{i\in[0,n]},\\ \{w_h(i')\}_{i'\in(n'-n_h,n']},\\ \{W'(S',s';i'),\bar{W}(i')\}_{S'\in\boldsymbol{S}',s'\in\boldsymbol{s}';i'\in[0,n']}}} \Delta \\
& + \beta(1-\pi_d)\,\mathbb{E}_{S',s'|S,s}\left\{(1-d(S',s'))\boldsymbol{J}(S',s',b',n',\{W(S',s';i')\}_{S'\in\boldsymbol{S}',s'\in\boldsymbol{s}';i'\in[0,n']})\right\}
\end{aligned} \tag{3}
$$

[6] One manager per firm.

$$\text{s.t. } \Delta = Azn^{\alpha} - \int_0^n w(i)di - \bar{w}_m - \epsilon - b - c\frac{n_h}{q(\theta(S,x_h))} - \int_{n'-n_h}^{n'} w_h(i')di' + Q(S,z,b',n')b' \geq 0, \quad (4)$$

$$n' = \int_0^n (1-\tau(i))(1-\lambda p(\theta(S,x^*(S;i))))di + n_h, \quad (5)$$

$$\begin{aligned} x^*(S;i) = \arg\max_x p(\theta(S,x))\Big\{ & x - \beta\,\mathbb{E}\left\{[\tau + (1-\tau)(\pi_d + (1-\pi_d)d(S',s'))]\boldsymbol{U}(S')\right. \\ & \left. + (1-\tau)(1-\pi_d)(1-d(S',s'))\max\{W'(S',s';i'),\boldsymbol{U}(S')\}\right\}\Big\}, \end{aligned} \quad (6)$$

$$W'(S',s';i') = \boldsymbol{U}(S') + \bar{W}(i'), \quad (7)$$

$$\bar{W}(i') \geq 0, \quad (8)$$

$$\boldsymbol{W}(S,s,\boldsymbol{C}) \geq \begin{cases} W(S,s,i) & \text{for } i \in [0,n], \\ x_h & \text{for newly hired employees}, \end{cases} \quad (9)$$

$$i'(i) = \int_0^i (1-\tau(j))(1-\lambda p(\theta(S,x^*(S))))dj, \forall i \in [0,n], \quad (10)$$

$$Q(S,z,b',n')b' - n_h\frac{c}{q(\theta(S,x_h))} - \int_{n'-n_h}^{n'} w_h(i')di' \geq M(S,z,n) - F_m(S,z), \quad (11)$$

where $F_m(S,z) = \left[\frac{\bar{w}_m + (1-\gamma)\frac{\beta}{1-\beta}\bar{w}_m}{(1-\Phi(A\xi\,\mathbb{E}[A'z'n'^{\alpha} - \int_0^{n'} w(i')di' - \bar{w}_m - \epsilon']))\zeta\,\mathbb{E}\,z'}\right]^{\frac{1}{\alpha}}\bar{u}$.

The firm chooses its equity payouts Δ for the current period. I assume that firms are subject to the non-negative equity payout constraint in equation (4). I adopt this assumption so that firms cannot always raise cash through equity issuance, which ensures financial frictions are effective in my model. Equity payouts Δ equal output Azn^{α} minus the wage payments to incumbent employees $\int_0^n w(i)di$, minus the manager's wage $\bar{w}_m$, minus the stochastic operating cost ϵ, minus debt b, minus vacancy posting costs $c\frac{n_h}{q(\theta(S,x_h))}$, minus wage payments to newly hired workers $\int_{n'-n_h}^{n'} w_h(i')di'$, and plus borrowings $Q(S,z,b',n')b'$. I assume that output is decreasing returns to scale with respect to the number of employees by letting α be smaller than one. This assumption helps generate meaningful firm sizes, essential to capturing firms' downsizing behaviors when uncertainty is high. The parameter c is the posting cost per vacancy. To hire n_h new workers, the firm needs to post $\frac{n_h}{q(\theta(S,x_h))}$ vacancies, where q denotes the vacancy-filling rate. The total vacancy posting cost is correspondingly $c\frac{n_h}{q(\theta(S,x_h))}$. The bond price Q is determined such that the international financial intermediaries break even, which will be defined later.

Equation (5) is the law of motion for employment. The firm's next-period number of employees is the sum of staying employees and new hires. Employees can separate from the firm for two reasons, on-the-job search and layoffs. Employees optimally choose an on-the-job search submarket to maximize their expected lifetime utility as in eq. (6). I use $x^*(S;i)$ to denote worker i's optimal on-the-job search market. Then the probability for a worker to transit to another firm is $\lambda p(\theta(S, x^*(S;i)))$. If the worker does not find a new job, he faces a layoff probability $\tau(i)$. Therefore, eq. (5) means that the staying employees plus new hires sum to the next-period employment.

Eq. (7) assumes a specific contract form for the next-period promised utilities, which are comprised of two parts: the outside option of unemployment $\boldsymbol{U}(S')$ and a utility markup chosen by the firm $\bar{W}(i')$. The promised utility markup can be contingent on workers, but it does not vary across states. The state-uncontingency of $\bar{W}(i')$ is crucial for effective financial frictions. Suppose firms' future promises to workers could be contingent on states. Labor contracts will then serve as a much better financial instrument than state-uncontingent bonds, which I conceive of as a counterfactual. Appendix D uses asymmetric information to provide a micro-founded model to justify this setup. The idea is based on Hall and Lazear (1984), who prove the optimality of predetermined wages when considering information frictions. Specifically, suppose workers do not have information about the firm's conditions. Firms can lie to pay less to workers. Because workers do not know whether the firm is truly facing a bad shock, they will not accept the wage cut. Section 2.10 discusses the assumption of a state-uncontingent promised utility markup in detail.

Recall that I assume firms are committed to labor contracts, but workers are not. Therefore, the participation constraint (8) shows that the firm should promise a non-negative utility markup to retain its workers. Otherwise, the worker would rather be unemployed. Furthermore, the promise-keeping constraint (9) requires the firm to adhere to its commitment that the worker's employment value is as least the promised lifetime utility. For incumbent worker $i \in [0, n]$, his promised utility is $W(S, s, i)$, one of the firm's state variables. For a newly hired worker, his promised utility is x_h, according to the firm's choice of hiring submarket. Finally, eq. (10) formalizes the transition of the employee's index from i to i'.

The last constraint (11) reflects the agency frictions between shareholders and managers, following Jensen (1986) and Arellano, Bai and Kehoe (2019).[7] This constraint dampens the firm's saving incentives so that financial frictions are effective in the model. Otherwise, firms will build up a large cash buffer and never default.

The micro-foundation of the agency frictions is as follows. I assume that there is a pool of potential managers from which each firm can hire one manager to operate the firm. The total mass of managers is much smaller than of workers, so I abstract from managers when calculating unemployment. Each manager can also be self-employed and produce $\bar{w}_m$ units of goods. The market for managers is competitive, so a manager's wage is also $\bar{w}_m$.

Each period consists of a day and night. During the day, managers are monitored by the

[7] Workers own firms in the model, so they are shareholders. I do not explicitly model equity payouts in the worker's problem because workers are risk-neutral and the free entry condition implies that the firm's net present value is zero.

firm's shareholders, so managers adopt the firm's optimal policies. The manager uses borrowing $Q(S,z,b',n')b'$ and sales to pay dividends, wages of incumbent workers, his own wage, the operating cost, and debt. Search happens overnight, and the manager is supposed to use the remaining resources to pay vacancy posting costs and the wages of new workers. However, what happens during the night cannot be observed by shareholders until the next day. Therefore, the manager can propose an alternative production plan to the financial intermediary to borrow as much as possible at night. To convince the financial intermediary of the new plan $(\bar{b}', \bar{n}')$, the manager needs to provide proof by posting vacancies to have $\bar{n}'$ workers in the next period if hiring is necessary. The manager thus needs to pay vacancy posting costs and wages for newly hired workers for the alternative proposal. In sum, to maximize available funds, the manager will come up with a proposal to achieve maximum possible borrowing net of hiring costs:

$$M(S,z,n) = \max_{\substack{b',n',n_h,x_h,d(S',s'),\\ \{\tau(i)\}_{i\in[0,n]},\{w_h(i')\}_{i'\in(n'-n_h,n']},\\ \{W'(S',s';i'),\bar{W}(i')\}_{S'\in\boldsymbol{S}',s'\in\boldsymbol{s}';i'\in[0,n']}}} Q(S,z,b',n')b' - n_h\frac{c}{q(\theta(S,x_h))} - \int_{n'-n_h}^{n'} w_h(i')di' \tag{12}$$

$$\text{s.t. (5), (7), (8), and (9).} \tag{13}$$

Given the maximum net borrowing $M(S,z,n)$, the remaining credit available for the manager is the maximum net borrowing minus the previous borrowing plus the originally planned but unused money for search, i.e., $M(S,z,n) - Q(S,z,b',n')b' + n_h\frac{c}{q(\theta(S,x_h))} + \int_{n'-n_h}^{n'} w_h(i')di'$.

The manager wants to use the remaining resources to hire workers to produce for his own project in the next period. After the next-period production occurs, shareholders learn what has occurred. The extra workers will be laid-off and search for jobs. Because the manager only needs to hire workers for the next-period production, the outside value of unemployment benefits $\bar{u}$ is the lowest wage for the manager to retain workers to produce. The manager will use the rest of the funds to hire as many workers as possible. The number of workers n_s is determined by

$$n_s = \frac{M(S,z,n) - Q(S,z,b',n')b' + n_h\frac{c}{q(\theta(S,x_h))} + \int_{n'-n_h}^{n'} w_h(i')di'}{\bar{u}}. \tag{14}$$

The manager takes advantage of the firm's productivity for his sided project, so the output is

$$\zeta z' n_s^{\alpha},$$

where ζ indicates the profitability of the manager's own project.

I also assume that there is an auditing technology to detect a manager's intention to deviate at night. The effectiveness of the auditing technology, ξA, is based on a measure of auditing quality, ξ, proportional to aggregate productivity. The incentive and available resources to use the auditing technology are approximated by the firm's expected income $\mathbb{E}[A'z'n'^{\alpha} - \int_0^{n'} w(i')di' - \bar{w}_m - \epsilon']$. The more the firm expects to earn, the more it can and should pay for the auditing technology. I assume that the probability of the manager being caught is Gaussian and determined by the amount of auditing:

$$\Phi\Big(\xi A\,\mathbb{E}[A'z'n'^{\alpha} - \int_0^{n'} w(i')di' - \bar{w}_m - \epsilon]\Big). \tag{15}$$

I model the auditing technology to match the correlation between credit spreads and aggregate output, so the financial effect of aggregate productivity shocks is consistent with the empirical covariance. Otherwise, a positive aggregate productivity shock will cause counterfactually higher credit spreads because firms would have higher income and borrow substantially to avoid the managerial deviations. With the auditing technology, firms do not need to borrow that much when aggregate productivity is high, so the credit spreads decrease, as in the data.

Suppose the manager deviates from the firm's optimal policies and works on his side project. In that case, shareholders will find out and fire the manager the next day. Assume the manager faces probability γ of becoming self-employed (else returning to the manager market), which approximates the punishment for deviation. Therefore, to avoid manager deviations, the firm should not operate with significant unused credit so the manager cannot hire many workers and the side project is not attractive. To do so, the firm should satisfy the following constraint such that the manager prefers to be honest:

$$\Big(1 - \Phi\Big(\xi A\,\mathbb{E}[A'z'n'^{\alpha} - \int_0^{n'} w(i')di' - \bar{w}_m - \epsilon]\Big)\Big)\,\mathbb{E}_t\,\beta\zeta A_{t+1}z_{t+1}n_s^{\alpha} + \gamma\,\mathbb{E}_t\sum_{j=2}^{\infty}\beta^j\bar{w}_m \le \mathbb{E}_t\sum_{j=1}^{\infty}\beta^j\bar{w}_m,$$

which delivers the agency friction constraint (11) by plugging in equation (14).

The agency friction constraint (11) incentivizes firms to borrow in the spirit of Jensen (1986). Without agency frictions, firms have strong incentives to save and grow out of financial frictions. There are other options to reduce firms' savings, such as using a lower discount factor. But the model requires an unrealistically low discount factor to match observed leverage since firms are very opposed to liquidation. For other ways to make firms borrow in the presence of financial frictions, Quadrini (2011) provides one summary.

2.4 Bond Pricing

I assume that the economy's financial market is small compared with the rest of the world, so the risk-free interest rate in the international financial market is exogenous. This assumption ensures the block recursivity and thus computational tractability.

International financial intermediaries supply one-period bonds to firms. They are risk-neutral and competitive. The opportunity cost of lending is the risk-free interest rate r in the world financial market, equal to $1/\beta - 1$. Financial intermediaries break even when lending to firms. If the firm defaults, the recovery of financial intermediaries is proportional to the firm's expected income π', which equals $A'z'n'^{\alpha} - \int_0^{n'} w(i')di' - \bar{w}_m - \mu_\epsilon$, which approximates the firm's value. That is, lenders recover more when the firm has a higher value.

Formally, the break-even bond price $Q(S, z, b', n')$ is determined by the following equation:

$$Q(S, s, b', n') = \beta\, \mathbb{E}_{S', s'|S,s} \left\{(1 - \pi_d)(1 - d(S', s')) + [1 - (1 - \pi_d)(1 - d(S', s'))] \min\{\eta \frac{\pi'}{b'}, 1\}\right\}, \quad (16)$$

where η denotes the recovery rate and $\pi' = A'z'n'^{\alpha} - \int_0^{n'} w(i')di' - \bar{w}_m - \mu_\epsilon$.

2.5 Wages

The state of the firm's problem (3) is an infinite-dimensional object because of the set of promised utilities. This section shows how to simplify the firm's problem by deriving wages and default decisions.

First, the promise-keeping constraint (9) always binds. Otherwise, firms could lower wages and earn more. Moreover, Proposition 1 shows that the participation constraint (8) also binds.

Proposition 1 *The participation constraint (8) binds, i.e., $\bar{W}(i') = 0$, for any worker i'.*

Appendix C provides the proof. Here, I explain the intuition. The participation constraint (8) requires that the promised utility markup should be non-negative. Suppose there exists a strictly positive promised utility markup $\bar{W}(i') > 0$. Then the firm can have a relatively low current wage according to the binding promise-keeping constraint (9). Namely, a positive promised utility markup can be understood as borrowing from the employee by backloading wages. However, borrowing from employees is more costly than borrowing from lenders through collateralized bonds. Therefore, promising a positive utility markup is never optimal for firms.

Given the binding promise-keeping constraint (9) and the participation constraint (8), I am able to determine wages. The binding participation constraint (8) implies that the promised utilities always equal the unemployment value $\boldsymbol{U}$. From the worker's problem (1) and (2) and the binding promise-keeping constraint (9), an incumbent worker's wage is

$$\begin{aligned} w(S) &= \boldsymbol{U}(S) - \lambda \max_x p(\theta(S, x))[x - \beta\, \mathbb{E}\, \boldsymbol{U}(S')] - \beta\, \mathbb{E}\, \boldsymbol{U}(S') \\ &= \bar{u} + (1 - \lambda) \max_x p(\theta(S, x))[x - \beta\, \mathbb{E}\, \boldsymbol{U}(S')]. \end{aligned} \quad (17)$$

That is, an incumbent worker's wage equals the outside payoff of being unemployed minus gains from on-the-job search.

Similarly, a newly hired worker's wage equals

$$w_h(S) = x_h - \beta\,\mathbb{E}\,\boldsymbol{U}(S'). \tag{18}$$

The uniquely determined wages in (17) and (18) are crucial for solving the problem quantitatively. Given the wages, the infinite-dimensional distribution of promised utilities not informative as a state variable, and the firm's problem can be simplified by removing the implicit contract constraints, (7), (8), and (9).

In terms of employment, the following Lemma 2.1 shows that while the model pins down the firm's total layoffs, the individual worker's layoff probability is undetermined.

Lemma 2.1 *The firm's total layoffs $\int_0^n \tau(i)di$ is uniquely determined, but the individual probability of layoff $\tau(i)$ is not.*

Proof For each optimal policy, eq. (5) determines the firm's total layoffs

$$\int_0^n \tau(i)di = n - \frac{n' - n_h}{1 - \lambda p(\theta(S, x^*(S)))}. \tag{19}$$

As long as total layoffs are constant, any perturbation of individual layoff probabilities $\{\tau(i)\}_{i\in[0,n]}$ does not affect the firm's value. □

The key reason Lemma 2.1 holds is homogeneous workers, so the distribution of layoff probabilities is irrelevant. Therefore, I will focus on the symmetric decision rule that all employees face the same layoff probability throughout the rest of this paper.

2.6 Firm's Default Decision and Cash on Hand

To further reduce the number of dimensions, I next explore the firm's default decision and rewrite the firm's problem using cash on hand as a state variable.

Notice that the outside value is zero when a firm exits, so a firm defaults and exits when it cannot satisfy the non-negative equity payout constraint (4). Define cash on hand X as:

$$X = Azn^{\alpha} - n[\bar{u} + (1-\lambda)\mu(S)] - \bar{w}_m - \epsilon - b, \tag{20}$$

where $\mu(S) \equiv \max_x p(\theta(S, x))[x - \beta\,\mathbb{E}\,\boldsymbol{U}(S')]$. Then, a firm defaults if and only if:

$$X + M(S, z, n) < 0, \tag{21}$$

where $M(S, z, n)$ is maximum net borrowing as defined in equation (12). Therefore, a firm's default decision can be summarized by the operating cost cutoff $\bar{\epsilon}(S,z,b,n)$, defined as:

$$\bar{w}_m + \bar{\epsilon}(S,z,b,n) \equiv Azn^{\alpha} - \int_0^n w(i)di - b + M(S,z,n). \tag{22}$$

So, the firm defaults when the operating cost is higher than the cutoff $\bar{\epsilon}(S,z,b,n)$ such that the firm cannot satisfy the non-negative equity payout constraint, i.e.,

$$d(S,s,b,n) = \begin{cases} 0, & \text{if } \epsilon \leq \bar{\epsilon}(S,z,b,n), \\ 1, & \text{if } \epsilon > \bar{\epsilon}(S,z,b,n). \end{cases} \tag{23}$$

Then the bond price can be simplified to the following expression:

$$\begin{aligned} Q(S,z,b',n') = \beta\, \mathbb{E}_{S',z'|S,z} \Big\{ &(1-\pi_d)\Phi(\bar{\epsilon}(S',z',b',n')) \\ &+ [1-(1-\pi_d)\Phi(\bar{\epsilon}(S',z',b',n'))] \min\{\eta \frac{A'z'n'^{\alpha} - \int_0^{n'} w(i')di' - \bar{w}_m - \mu_\epsilon}{b'}, 1\} \Big\}. \end{aligned} \tag{24}$$

Plugging in the default cutoff (22) and wages (17) and (18), I rewrite the firm's problem (3) using cash on hand X as a state variable:

$$\boldsymbol{V}(S,z,X,n) = \max_{\substack{\Delta,b',n',\\ \tau,n_h,x_h}} \Delta + \beta(1-\pi_d)\, \mathbb{E}_{S',z'|S,z} \int_{-\infty}^{\bar{\epsilon}(S',z',b',n')} \boldsymbol{V}(S',z',X',n')d\Phi(\epsilon') \tag{25}$$

$$\text{s.t. } (6), \tag{26}$$

$$n' = (1-\tau)(1-\lambda p(\theta(S,x^*(S))))n + n_h, \tag{27}$$

$$\Delta = X + Q(S,z,b',n')b' - n_h \frac{c}{q(\theta(S,x_h))} - n_h[x_h - \beta\, \mathbb{E}\, \boldsymbol{U}(S')] \geq 0, \tag{28}$$

$$X' = A'z'n'^{\alpha} - n'[\bar{u} + (1-\lambda)\mu(S')] - \bar{w}_m - \epsilon' - b', \tag{29}$$

$$\bar{w}_m + \bar{\epsilon}(S',z',b',n') = A'z'n'^{\alpha} - n'[\bar{u} + (1-\lambda)\mu(S')] - b' + M(S',z',n'), \tag{30}$$

$$Q(S,z,b',n')b' - n_h \frac{c}{q(\theta(S,x_h))} - n_h[x_h - \beta\, \mathbb{E}\, \boldsymbol{U}(S')] \geq M(S,z,n) - F_m(S,z). \tag{31}$$

The non-negative equity payout constraint (28) reveals that firms' decisions depend on cash on hand X. When cash on hand is too low, the firm defaults because it cannot fully pay wages and debts. On the other hand, when cash on hand is sufficiently high, the firm is not constrained by

(28). In this case, the firm solves the following relaxed problem:

$$
\begin{aligned}
\hat{\boldsymbol{V}}(S,z,X,n) = \max_{\substack{b',n',\\ \tau,n_h,x_h}} & X + Q(S,z,b',n')b' - n_h \frac{c}{q(\theta(S,x_h))} - n_h[x_h - \beta\,\mathbb{E}\,\boldsymbol{U}(S')] \\
& + \beta(1-\pi_d)\,\mathbb{E}_{S',z'|S,z} \int_{-\infty}^{\bar{\epsilon}(S',z',b',n')} \boldsymbol{V}(S',z',X',n')d\Phi(\epsilon')
\end{aligned} \tag{32}
$$

$$
\text{s.t. (6), (27), (29), (30), and (31).} \tag{33}
$$

For the relaxed problem, cash on hand does not affect the firm's choices. Let $\hat{b}(S,z,n)$, $\hat{n}(S,z,n)$, $\hat{\tau}(S,z,n)$, $\hat{n}_h(S,z,n)$, and $\hat{x}_h(S,z,n)$ denote the optimal policies for the relaxed problem. The following Lemma 2.2 characterizes firms' decisions with respect to cash on hand.

Lemma 2.2 *(Decision Cutoffs): If $X < -M(S,z,n)$, the firm cannot satisfy the nonnegative external equity payout condition and has to default. If $X \geq \hat{x}(S,z,n) \equiv -\{Q(S,z,\hat{b},\hat{n})\hat{b} - \hat{n}_h \frac{c}{q(\theta(S,\hat{x}_h))} - \hat{n}_h[\hat{x}_h - \beta\,\mathbb{E}\,\boldsymbol{U}(S')]\}$, the firm solves the relaxed problem (32), and the level of cash on hand does not affect the optimal decisions.*

Proof If the firm's cash on hand X is less than $-M(S,z,n)$, even though the firm borrows as much as possible, it cannot make nonnegative external equity payouts. So, the firm defaults and exits. If the firm's cash on hand X is more than $\hat{x}(S,z,n)$, then $(\hat{b},\hat{n},\hat{\tau},\hat{n}_h,\hat{x}_h)$ is also the solution to the firm's problem (25), because constraint (28) holds automatically. In this case, cash on hand does not affect any constraints, and the optimal decisions do not depend on cash on hand. □

Lemma 2.2 provides the method to solve the firm's problem by level of cash on hand. This partitioning method has been used by Khan and Thomas (2013), Arellano, Bai and Kehoe (2019), and Ottonello and Winberry (2020).

2.7 Firm Entry

Potential new firms pay a fixed cost k_e to enter. New entrants' productivity will be drawn from the stationary distribution of idiosyncratic productivity $g_z(\cdot)$. New entrants do not produce in the entry period but hire workers as do incumbent firms. New firms start with zero debt and no labor. Then the new entrant's problem is:

$$
\boldsymbol{J}_e(S,z) = \max_{n_h,x_h} -n_h \frac{c}{q(\theta(S,x_h))} - n_h[x_h - \beta\,\mathbb{E}\,\boldsymbol{U}(S')] \tag{34}
$$

$$
+ \beta(1-\pi_d)\,\mathbb{E}_{S',z'|S,z} \int_{-\infty}^{\bar{\epsilon}(S',z',b_0,n_h)} \boldsymbol{V}(S',z',X',n_h)d\Phi(\epsilon'), \tag{35}
$$

$$
\text{s.t. } b_0 = 0, \ \ \text{(29), and (30).} \tag{36}
$$

I use n_e, x_e, and d_e to denote the new entrant's optimal policies.

Notice that both incumbent firms and new entrants only post vacancies in the markets with the lowest hiring cost. Define the minimum hiring cost per worker as

$$\kappa(S) \equiv \min_{x_h}[x_h + \frac{c}{q(\theta(S, x_h))}]. \tag{37}$$

In equilibrium, only submarkets with the lowest hiring cost are active. Given the equilibrium hiring cost $\kappa(S)$,the mapping from the market's promised utility x to the market intensity θ is

$$\theta(S, x) = \begin{cases} q^{-1}\left(\frac{c}{\kappa(S)-x}\right), & \text{if } x \leq \kappa(S) - c, \\ 0, & \text{if } x \geq \kappa(S) - c. \end{cases} \tag{38}$$

Notice that the upper bound of the vacancy filling probability q is one. When the submarket's promised utility x is higher than $\kappa - c$, no firm posts vacancies there because the vacancy filling probability cannot be greater than one to compensate for the hiring cost. In this case, the market is inactive, and the market intensity is zero.

The value of $\kappa(S)$ is determined by the free entry condition, which requires that the entry cost equals the expected entry value:

$$k_e = \sum_z \boldsymbol{J}_e(S, z) g_z(z), \forall S. \tag{39}$$

Therefore, the free entry condition closes the model by pinning down the hiring cost $\kappa(S)$ for all aggregate states S.

2.8 Equilibrium

This section defines the block recursive equilibrium of the model.

Definition 2.1 *Let s^f summarize the firm's state variables (S, z, X, n). The block recursive equilibrium consists of the policy and value functions of unemployed workers $\{x_u(S), \boldsymbol{U}(S)\}$; of employed workers $\{x(S, s, C), \boldsymbol{W}(S, s, C)\}$; of incumbent firms $\{\Delta(s^f)$, $b'(s^f)$, $n'(s^f)$, $\tau(s^f)$, $n_h(s^f)$, $x_h(s^f)$, $w(S)$, $w_h(S)\}$; of new firms $\{n_e(S)$, $x_e(S)$, $\boldsymbol{J}_e(S)\}$; the hiring cost per worker $\kappa(S)$; the labor market tightness function $\theta(S, x; \kappa(S))$; and bond price schedules $Q(S, z, b', n')$ such that*

1. *Given the bond price schedules, the hiring cost, and the labor market tightness, the policy and value functions of unemployed workers, employed workers, incumbent firms, and entering firms solve their respective problems (1), (2), (17), (18), (25), and (34).*

2. *The bond price schedule satisfies (24).*

3. *The hiring cost per worker and the labor market tightness function satisfy* (37) *and* (38).

4. *The free entry condition* (39) *holds.*

2.9 Aggregate Transitions

Let $\Upsilon(z, X, n)$ denote the mass of firms with states (z, X, n), which is the sum of incumbent firms and new entrants which do not default. The law of motion of the firm distribution is:

$$\begin{aligned}
&\Upsilon'(z', X', n') \\
&= \sum_{z,X,n,\epsilon'} (1-\pi_d)(1-d(S', s'; S, z, X, n))\mathbb{1}\{X'(S', s'; S, z, X, n) = X'\}\phi(\epsilon')\pi_z(z'|z,\sigma)\mathbb{1}\{n'(S, z, X, n) = n'\}\Upsilon(z, X, n) \\
&\quad + m_e(S, \Upsilon)\sum_{z,\epsilon'}(1-\pi_d)(1-d_e(S', s'; S, z))\mathbb{1}\{X'_e(S', s'; S, z) = X'\}\phi(\epsilon')\pi_z(z'|z,\sigma)\mathbb{1}\{n_e(S) = n'\}g_z(z).
\end{aligned} \tag{40}$$

Following Arellano, Bai and Kehoe (2019), I assume that defaulting firms can still produce as long as their sales can cover the minimum wage required to retain workers for production. As discussed in Section 2.3, the necessary wage equals the unemployment benefits. These firms' employees will be laid off after production occurs and search for new jobs in the labor market, so they are not counted in unemployment in the current period. This setup relieves the concern that varying default rates mechanically drive the fluctuations of output and unemployment. Let $d^p(S', s'; S, z, X, n)$ be the production indicator, which equals one if and only if the firm does not default or satisfies $A'z'n'^{\alpha} > \bar{u}n'$ and zero otherwise. I use Use $\Upsilon^p(z, n)$ to denote the distribution of producing firms, which thus evolves per:

$$\begin{aligned}
\Upsilon^p(z', n') = &\sum_{z,X,n,\epsilon'} (1-\pi_d)(1-d^p(S', s'; S, z, X, n))\pi_z(z'|z,\sigma)\mathbb{1}\{n'(S, z, X, n) = n'\}\Upsilon(z, X, n) \\
&+ m_e(S, \Upsilon)\sum_{z,\epsilon'}(1-\pi_d)(1-d^p_e(S', s'; S, z))\pi_z(z'|z,\sigma)\mathbb{1}\{n_e(S) = n'\}g_z(z).
\end{aligned} \tag{41}$$

The mass of entrants $m_e(S, \Upsilon)$ is determined such that total jobs found by workers equals the total jobs created by incumbent firms and new entrants:[8]

$$JF_{\text{workers}}(S,\Upsilon) = JC_{\text{incumbents}}(S,\Upsilon) + m_e(S,\Upsilon)JC_{\text{entrants}}(S,\Upsilon), \tag{42}$$

where

$$JF_{\text{workers}}(S,\Upsilon) = p(\theta(S,x_u^*(S)))\Big(1-\sum_{z,X,n} n\Upsilon(z,X,n)\Big) + \sum_{z,X,n} \lambda p(\theta(S,x^*(S)))n\Upsilon(z,X,n), \tag{43}$$

$$JC_{\text{incumebents}}(S,\Upsilon) = \sum_{z,X,n} n_h(S,z,X,n)\Upsilon(z,X,n), \tag{44}$$

$$JC_{\text{entrants}}(S,\Upsilon) = \sum_{z} g_z(z)n_e(S,z). \tag{45}$$

Aggregate output is the sum of all firms' output:

$$Y = \sum_{z,n} Azn^{\alpha}\Upsilon^p(z,n), \tag{46}$$

and the unemployment rate u is the share of workers who do not produce:

$$u = 1 - \sum_{z,n} n\Upsilon^p(z,n). \tag{47}$$

2.10 Discussions of the Assumptions

My model's key assumption and driving force is the incompleteness of financial instruments and labor contracts. This section discusses and justifies this assumption.

First, I assume that firms can only borrow through state-uncontingent debt. Beyond the inherent plausibility given the widespread real-world existence of state-uncontingent debt, much related literature uses this assumption to model firms' default risks (Arellano, Bai and Kehoe, 2019; Khan and Thomas, 2013; Ottonello and Winberry, 2020). I also assume that firms can only have non-negative equity payouts. This assumption ensures that firms do not have deep pockets by issuing large amounts of equity, which in turn guarantees that financial frictions are meaningful in my model. Empirically, according to the calculations in Schoefer (2021) using U.S. Flow of Funds

[8] Over the business cycle, jobs created by incumbent firms, $JC_{\text{incumbents}}$, can occasionally be larger than jobs found by workers, JF_{workers}. If the total mass of workers is restricted to one, then entry will be negative and not well-defined. To deal with this issue, I assume that when incumbent firms hire more workers than find jobs, the entry m_e is zero, and the mass of workers increases such that equation (42) holds. Then I normalize the economy so that the mass of workers is one unit again. This setup can be understood as an increase in labor force participation. Simulation shows that the average annual population growth rate is less than 0.5%, implying that the potential problem of negative entry is small. Another way to solve this problem is to assign different entry costs for different aggregate states. See Kaas and Kircher (2015) for this treatment.

data, the average equity raised by U.S. firms from 1951Q4 to 2019Q4 is negative because of share repurchases. Therefore, on average equity issuance is not a primary financing tool for U.S. firms. Admittedly, a more realistic model could have equity issuance, but the recalibration will indicate sizable equity issuance costs. I conjecture that my model's implications would thus not change much. Therefore, I follow the literature to exclude equity financing for simplicity.

Second, novel to the search literature, I assume that labor contracts do not insure firms from idiosyncratic shocks. Specifically, the promised utility is assumed to have a state-uncontingent utility markup over the unemployment value. The state-uncontingency makes wages debt-like commitments that firms are less likely to take on when uncertainty is high. This mechanism helps my model explain unemployment better than Schaal (2017).

The assumption of state-uncontingency distinguishes my framework from the textbook Diamond-Mortensen–Pissarides search models and a subsequent group of models assuming wage rigidity. As Shimer (2005) points out, typical search models cannot generate the observed unemployment volatility because wage movements largely cancel out the employment impacts of aggregate productivity shocks in equilibrium. One of the solutions to Shimer's puzzle is wage rigidity (Hall, 2005). However, whether wage rigidity is a proper assumption to explain unemployment is still an open question (Kudlyak, 2014; Pissarides, 2009; Rudanko, 2009). To clarify, instead of making the debatable assumption of wage rigidity, I allow wages to vary with the outside value of unemployment. That is, wages can still decrease to offset the effects of aggregate shocks as in textbook search model. The difference is that I assume labor contracts do not allow firms to hedge against idiosyncratic risks. My quantitative results show that the micro-level risk caused by wage commitments is the key to explaining unemployment dynamics over business cycles. On the other hand, suppose the labor contract is state-contingent. Then it will be a much better financial instrument than state-uncontingent debt. I conceive this implication as counterfactual.

To justify the state-uncontingency of labor contracts, Appendix D provides a micro-founded theory of information frictions. The idea is based on Hall and Lazear (1984). When workers do not clearly observe firm shocks, firms have the incentive to lie to workers to pay lower wages. Workers know this, so they do not accept wage cuts when the firm declares a bad shock. Therefore, the incentive-compatible lifetime promised utility features a state-uncontingent utility markup over the worker's outside value.

The state-uncontingency of labor contracts can be tested empirically. One implication or potential criticism is that it will cause inefficient separations. However, Jäger, Schoefer and Zweimüller (2019) show that separations are indeed inefficient in the data. Using a quasi-experimental repeal of an unemployment insurance extension in Austria, they do not find that surviving jobs display higher resilience. This implies that separated jobs are not simply the ones with the lowest surplus. In line with their conclusion of inefficient separations, my assumption of state-uncontingent labor contracts can also have similar implications.

With both types of incompleteness, my financial channel of wage commitments can improve a search model's performance in explaining unemployment. Moreover, there has been empirical evidence in the existing literature to support the interaction between finance and labor. Favilukis, Lin and Zhao (2020) find that wage growth and labor shares are the primary factors predicting credit spreads. Donangelo et al. (2019) use Census data and find that firms with higher labor shares are more sensitive to shocks. Schoefer (2021) shows that industries with higher labor shares are associated with more procyclical cash flows.

3 Quantitative Analysis

In this section, I first parametrize the model by matching moments. Then I explain the mechanism and show the critical connection between financial frictions and uncertainty shocks and unemployment volatility. Next, I apply the model to U.S. business cycles to see to what degree the model can explain unemployment dynamics during recessions. Finally, I conduct policy experiments to investigate the labor market policies of the U.S. and Germany during the recent Covid recession.

I use grid search to solve the problem numerically. The global solution is computationally tractable because of block recursivity.

3.1 Parameterization

There are four shocks (A, σ, z, ϵ) in the economy. The logs of aggregate productivity and uncertainty both follow AR(1) processes:

$$\log A_{t+1} = \rho_A \log A_t + \sigma_A \sqrt{1 - \rho_A^2} \epsilon_t^A, \tag{48}$$

$$\log \sigma_{t+1} = (1 - \rho_\sigma) \log \bar{\sigma} + \rho_\sigma \log \sigma_t + \sigma_\sigma \sqrt{1 - \rho_\sigma^2} \epsilon_t^\sigma, \tag{49}$$

where the innovations ϵ_t^A and ϵ_t^σ follow the standard normal distribution. I follow Schaal (2017) and allow ϵ_t^A and ϵ_t^σ to be correlated with the correlation coefficient $\rho_{A\sigma}$.

Firm j's idiosyncratic productivity also follows an AR(1) process:

$$\log z_{jt+1} = \rho_z \log z_{jt} + \sigma_t \sqrt{1 - \rho_z^2} \epsilon_{jt}^z, \tag{50}$$

where ϵ_{jt}^z follows the standard normal distribution, and the time-varying uncertainty σ_t controls the standard deviations of the innovation.

The i.i.d. operating cost shock ϵ's distribution, $\Phi(\cdot)$, is normally distributed with mean μ_ϵ and standard deviation σ_ϵ.

I follow Menzio and Shi (2010) and Schaal (2017) in using the following job finding probability function:

$$p(\theta) = \theta(1 + \theta^{\gamma})^{-1/\gamma}. \tag{51}$$

Accordingly, the vacancy-filling rate $q(\theta)$ is $p(\theta)/\theta$.

I calibrate the parameters as closely as possible to Schaal (2017) for comparison. Table 1 shows the parameter values. The parameters in Panel A are exogenously assigned, following the literature. The quarterly discount factor β equals 0.988, corresponding to a 5% annual risk-free interest rate. The labor coefficient α is set as 0.66 to be consistent with the wage share. I follow Khan and Thomas (2008) to set the persistence of idiosyncratic productivity ρ_z as 0.95.

The remaining parameters in Panel B are calibrated by matching moments using U.S. data. Table 2 shows the moments in the data and the model. Because of the model's non-linearity, all parameters influence all moments jointly. However, each moment is primarily affected by certain parameters, and I organize them into four groups accordingly. The first two groups of parameters are related to aggregate shocks and the labor market, which are calibrated according to Schaal (2017). The other two groups of parameters, associated with the financial market and firm exit, are added upon Schaal's (2017) calibration for my financial channel.

The first set of parameters controls the AR(1) processes of aggregate shocks. For the aggregate productivity parameters (ρ_A, σ_A), I use the autocorrelation and standard deviation of output as target moments. The data moments are calculated by Schaal (2017) using real GDP from the Bureau of Economic Analysis. He detrends the time series of output by an HP-filter with a parameter of 1,600 to obtain the log deviations.

To calibrate the process of uncertainty shocks to firm-level productivity, I follow Bloom et al. (2018) to use the interquartile range of sales growth rates across firms (IQR) to reflect the degree of volatility in the economy. I obtain the sales of firms from CRSP-Compustat, which provides balance sheet data of publicly listed firms. I use the Consumer Price Index for All Urban Consumers (CPI) to deflate sales. To avoid the composition of firms influencing the IQR, I follow Bloom et al. (2018) and use only firms with at least 100 quarters of observations. I also drop firms in the finance and public administration sector. I also follow Davis and Haltiwanger (1992) by measuring the sales growth rate at quarter t as $(y_t - y_{t-4})/((y_t + y_{t-4})/2)$, so growth rates are less affected by extreme values of sales. Next, because firms may respond heterogeneously to shocks in different industries, the IQR of the original sales growth rates may reflect not only the underlying uncertainty shocks but also heterogeneous responses. Therefore, I follow Bloom et al. (2018) and Schaal (2017) and measure volatility controlling for firms' permanent heterogeneity and industry heterogeneity over business cycles. Specifically, I project firms' sales growth on firm-level fixed effects and industry-quarter fixed effects to obtain residuals of sales growth[9] and use these residuals as my measure of volatility

[9] Firm industry is based on the Standard Industrial Classification (SIC) at the 3-digit level.

Table 1: Parameter Values

Parameters	Notation	Value	Source/Matched Moment
Panel A: Assigned Parameters			
Discount factor	β	0.988	5% annual interest rate
Decreasing returns to scale coefficient	α	0.66	Labor share
Persistence of productivity	ρ_z	0.95	Khan and Thomas (2008)
Panel B: Parameters from Moment Matching			
Aggregate shocks			
Persistence of aggregate productivity	ρ_A	0.920	Autocorrelation of output
SD of aggregate productivity	σ_A	0.024	SD of output
Mean of uncertainty	$\bar{\sigma}$	0.248	Mean of IQR
Persistence of uncertainty	ρ_σ	0.880	Autocorrelation of IQR
SD of uncertainty	σ_σ	0.092	SD of IQR
Correlation between ϵ_t^A and ϵ_t^σ	$\rho_{A\sigma}$	−0.020	Correlation (output, IQR)
Labor market			
Unemployment benefits	$\bar{u}$	0.142	EU rate
Vacancy posting cost	c	0.001	UE rate
Relative on-the-job search efficiency	λ	0.100	EE rate
Matching function elasticity	γ	1.600	$\epsilon_{UE/\theta}$
Entry cost	k_e	15.21	Entry/Total job creation
Mean operating cost	$\bar{w}_m + \mu_\epsilon$	0.001	Average establishment size
Financial market			
SD of production costs	σ_ϵ	0.080	Mean credit spread
Agency friction	$\tilde{\zeta}$	2.400	Median leverage
Auditing quality	ξ	1.780	Correlation (output, spreads)
Recovery rate	η	2.410	Correlation (IQR, spreads)
Exit			
Exogenous exit rate	π_d	0.021	Annual exit rate

Notes: Panel A shows parameters exogenously assigned. Panel B shows parameters calibrated to match the targeted data moments in Table 2.

to construct the IQR. Given the time series of IQR, I compute its mean, detrend the time series with an HP-filter, and compute the autocorrelation and standard deviations to serve as targets for the uncertainty shock parameters ($\mu_\sigma, \rho_\sigma, \sigma_\sigma$). I also use the correlation between output and IQR to pin down the correlation between aggregate productivity shocks and uncertainty shocks $\rho_{A\sigma}$. The output data is quarterly real GDP per capita from the Bureau of Economic Analysis, retrieved from FRED. It is detrended by the HP-filter with 1,600 as the parameters to obtain the log deviations.

The second group of parameters is related to the labor market. The unemployment utility $\bar{u}$ is the opportunity cost of working, affecting wages and thus firms' firing decisions; the vacancy posting cost c primarily affects firms' hiring decisions; and the relative on-the-job search efficiency λ influences the probability of job-to-job transitions. I calibrate these three parameters using the transition probability from employment to unemployment (EU), the transition probability from unemployment to employment (UE), and the transition probability from employment to

Table 2: Matched Moments

Moments	Data	Benchmark Model $A+\sigma$	Benchmark Model A	No Financial Frictions $A+\sigma$	No Financial Frictions A
Aggregate shocks					
Autocorrelation of output	0.839	0.868	0.877	0.840	0.870
SD of output	0.016	0.015	0.015	0.019	0.017
Mean of IQR	0.171	0.169	0.160	0.167	0.165
Autocorrelation of IQR	0.647	0.611	-	0.614	-
SD of IQR	0.013	0.011	-	0.012	-
Correlation (output, IQR)	-0.351	-0.305	-	-0.318	-
Labor market					
UE rate	0.834	0.814	0.817	0.845	0.824
EU rate	0.076	0.083	0.080	0.073	0.074
EE rate	0.085	0.081	0.082	0.085	0.082
$\epsilon_{UE/\theta}$	0.720	0.717	0.707	0.705	0.714
Average establishment size	15.6	15.4	15.3	15.0	15.6
Entry/Total job creation	0.21	0.18	0.18	0.17	0.17
Financial market					
Mean credit spread (%)	1.09	0.96	0.97	-	-
Median leverage (%)	26	21	21	-	-
Correlation (output, spreads)	-0.549	-0.503	-	-	-
Correlation (IQR, spreads)	0.462	0.448	-	-	-
Exit					
Annual exit rate (%)	8.9	9.0	9.2	9.0	9.1

Notes: This table shows the targeted data moments and moments matched by the benchmark model and the model without financial frictions. $A+\sigma$ means the model has both aggregate productivity shocks and uncertainty shocks, and A means the model only has aggregate productivity shocks. Table 6 reports the recalibrated parameters of the four models.

employment (EE). The data moments for EU, UE, and EE are the quarterly versions of the monthly ones in Schaal (2017), who obtains the monthly EU and UE rates from Shimer (2005) and the EE rate from Nagypál (2007). The matching function elasticity γ is calibrated to match the elasticity of UE rates to the labor market intensity θ, which Schaal (2017) obtains from Shimer (2005). The entry cost k_e is calibrated to match the share of jobs created by entrants, which is calculated by Schaal (2017) using Business Employment Dynamics (BED). The mean operating cost affects firms' exit decisions and thus can be pinned down by the average establishment size, measured by Schaal (2017) using the 2002 Economic Census. Notice that the mean operating cost μ_ϵ and the manager's wage $\bar{w}_m$ symmetrically influence firms' cash on hand, so I calibrate $\mu_\epsilon + \bar{w}_m$ using the average establishment size.

Parameters in the third group deal with the financial market. First, I use the average credit spread to calibrate the standard deviation of the operating cost, σ_ϵ. The credit spread is the difference between the yield on Baa and Aaa corporate bonds. The data source is Moody's, retrieved from

FRED, Federal Reserve Bank of St. Louis. Correspondingly, the credit spread in the model is the annualized difference between the actual borrowing cost and the risk-free interest rate:

$$\frac{1}{Q(S,z,b',n')} - \frac{1}{\beta}. \tag{52}$$

Because the agency friction constraint incentivizes firms to borrow, I use the median leverage of firms to calibrate the agency friction parameter $\tilde{\zeta} \equiv \zeta/(\bar{w}_m + (1-\lambda)\frac{\beta}{1-\beta}\bar{w}_m)$. Leverage is the ratio of the firm's total debt to its annualized sales. The data moment of median leverage is from Arellano, Bai and Kehoe (2019). Next, I use the correlation between output and credit spreads to parameterize the auditing technology ξ, and I use the correlation between IQR and credit spreads for the recovery rate η. Targeting the two correlations anchors the financial impacts of aggregate productivity shocks and uncertainty shocks.

The last parameter is the exogenous exit rate π_d, which helps generate exits beyond defaults. I use the annual exit rate calculated from Business Dynamics Statistics (BDS) to calibrate π_d.

3.2 Differences from the Calibration of Schaal (2017)

My parametrization is based on Schaal (2017) when estimating parameters related to aggregate shocks and the labor market. I follow his calibration closely except for the following three differences.

First, Schaal (2017) uses a monthly frequency, while my model's frequency is quarterly. I choose the quarterly frequency to accommodate the data moments related to the financial market. As is common in the finance literature, leverage should be one of the target moments, defined as a firm's debt over annualized sales. In a quarterly model, annualized sales in the denominator equal four times the quarterly sales. However, suppose the model is monthly. Annualized sales in the denominator will be 12 times the monthly sales. Therefore, when targeting the same median leverage in the data, the monthly model implies the firm's debt is much higher than its per-period sales, and the default risks will be counterfactually high. Thus, I follow the finance literature and use a quarterly model.

Second, Schaal (2017) uses 0.85 as the decreasing returns to scale coefficient α, and I use 0.66. Neither of us explicitly models capital, while Schaal (2017) chooses 0.85 to approximate the total decreasing returns. But he also points out that the results are unaffected when targeting a labor share of 0.66. Because my mechanism is about wage commitments, I choose to target the wage share so that the size of firm commitments is consistent with the data. If I used 0.85 as the decreasing returns to scale coefficient, wage commitments would be larger, increasing the risk to firms and generating counterfactually high credit spreads.

Third, to calibrate the uncertainty shock process, Schaal (2017) uses the interquartile range (IQR) of innovations to idiosyncratic productivity calculated by Bloom et al. (2018). Instead, I follow both Bloom et al. (2018) and Arellano, Bai and Kehoe (2019) and use the IQR of firms' sales growth rates. I make this deviation because targeting the IQR of innovations to idiosyncratic productivity leads to a counterfactually high sales volatility. Specifically, the IQR of sales growth in the model will be more than five times the data. Because sales volatility determines firm default probability, the counterfactually high volatility of sales leads to counterfactually large default rates and extremely high credit spreads. To keep the magnitude of financial effects reasonable, I use the IQR of firms' sales growth rates as in Arellano, Bai and Kehoe (2019), who also model uncertainty shocks and financial frictions simultaneously. The main difference between using the IQR of firms' sales growth rates and the IQR of idiosyncratic productivity innovations is the level of uncertainty $\bar{\sigma}$.[10] But, they have very similar business cycle behaviors in terms of innovations to uncertainty, i.e., ϵ_t^{σ}. In particular, Figure 7 in the appendix compares the log deviations of estimated aggregate productivity shocks and uncertainty shocks of the model without financial frictions with Schaal (2017), showing that the two uncertainty shocks have similar variations over business cycles.

As a validation of this calibration choice, Table 3 shows that my counterfactual model without financial frictions has very similar business cycle statistics to Schaal (2017). Further, Figure 5 displays the changes in unemployment during recessions, and the model without financial frictions also yields very similar patterns to Schaal (2017). Therefore, our divergence in calibration is not the main reason for our different quantitative performances.

3.3 Business Cycle Statistics

To assess how well my model can explain business cycles, I report simulated business cycle statistics in Table 3. To compute the moments, I simulate the model for 3,000 quarters and use the log deviations from an HP-filter trend with a smoothing parameter of 1,600. Beyond the benchmark model, I consider three alternative models for comparison. All models are recalibrated by matching the same moments. Table 2 contains the calibration results and Table 6 reports the recalibrated parameters.

Since all models are calibrated, they have similar predictions for output and labor productivity in the first two columns, and I will focus on their differences in terms of unemployment volatility in brief. The next section will investigate the mechanism in greater detail.

Benchmark Model With Both Shocks. The standard deviation of unemployment is 0.121 in the data (Panel A), and my benchmark model with both aggregate productivity shocks and uncertainty shocks can generate a standard deviation of 0.106 (Panel B), which is 10 percent higher than the

[10] One concern about the idiosyncratic productivity measured by Bloom et al. (2018) is that they use revenue TFP, which can reflect firm pricing power instead of productivity (Bils, Klenow and Ruane, 2021; Hsieh and Klenow, 2009).

Table 3: Business Cycle Statistics

	Y	Y/L	U	V	Hirings	Quits	Layoffs	Wages
Panel A: Data								
Std Dev.	0.016	0.012	0.121	0.138	0.058	0.102	0.059	0.008
cor(Y,x)	1	0.590	-0.859	0.720	0.677	0.720	-0.462	0.555
Panel B: Benchamark Model								
Both A and σ Shocks								
Std Dev.	0.015	0.013	0.106	0.097	0.048	0.029	0.111	0.011
cor(Y,x)	1	0.910	-0.500	0.774	0.140	0.884	-0.202	0.876
Only A Shock								
Std Dev.	0.015	0.011	0.079	0.081	0.019	0.028	0.053	0.010
cor(Y,x)	1	0.988	-0.901	0.904	0.010	0.964	-0.853	0.980
Panel C: Model Without Financial Frictions								
Both A and σ Shocks								
Std Dev.	0.019	0.016	0.088	0.084	0.038	0.026	0.069	0.014
cor(Y,x)	1	0.986	-0.781	0.826	0.051	0.919	-0.569	0.964
Only A Shock								
Std Dev.	0.017	0.014	0.069	0.064	0.023	0.023	0.047	0.012
cor(Y,x)	1	0.994	-0.876	0.906	-0.024	0.975	-0.787	0.985

Notes: Panel A shows the business cycle moments in the data. Panels B and C report moments of 3,000-quarter simulations of the benchmark model and the model without financial frictions, with and without uncertainty shocks. Both the data and the model simulations are log-detrended by the HP filter with smoothing parameter 1600. To be consistent with Schaal (2017), Y denotes output, Y/L is output per worker, U represents unemployment, and V is vacancies.

0.090 in Schaal (2017). Next, I use three alternative models to explain the roles of both financial frictions and uncertainty shocks in my model's quantitative performance.

Benchmark Model With Only Aggregate Productivity Shocks. The second part of Panel B calibrates the same model but keeps only the aggregate productivity shocks, i.e., the uncertainty of firms' idiosyncratic productivity is no longer time-varying. Now the model only generates a standard deviation of unemployment of 0.079. This finding implies that uncertainty shocks are necessary for the financial channel's effectiveness. Distinguished from aggregate productivity shocks, the offsetting effect of equilibrium wages is much smaller for uncertainty shocks. High uncertainty spreads the distribution of firms' idiosyncratic productivity, and high productivity firms may still benefit from elevated volatility, which props up wages. This is called as the Oi-Hartman-Abel effect (Oi (1961), Hartman (1972), Abel (1983)) in the volatility literature. Therefore, though wages still decline when uncertainty is high, the decline is much smaller than when it is a negative aggregate productivity shock.

Model Without Financial Frictions and With Both Shocks. The first part of Panel C shows the results when the model does not have financial frictions but has both shocks. It generates a 0.088

standard deviation of unemployment, in line with the 0.090 in Schaal (2017). Comparing it with the 0.106 of the benchmark model reveals the important role of financial frictions in the model. Without financial frictions, my model is similar to Schaal (2017). Adding the financial channel amplifies the effect of aggregate shocks on unemployment and generates more unemployment volatility.

Model Without Financial Frictions and With Only Aggregate Productivity Shocks. I show the model statistics without financial frictions and with only the aggregate productivity shock in the second part of Panel C. It generates a standard deviation of unemployment of 0.069, consistent with the 0.067 in the corresponding model of Schaal (2017). This number is similar to the benchmark model with only the aggregate productivity shock, and confirms the complementarity between uncertainty shocks and the financial channel of wage commitments.

3.4 Inspecting the Mechanism

In this section, I explain the mechanism in my model. First, I show how the interaction between the financial channel of wage commitments and uncertainty shocks works. Next, I illustrate the importance of the financial channel by comparing it with the recalibrated model without financial frictions. Finally, I explain why the mechanism needs to interact with uncertainty shocks.

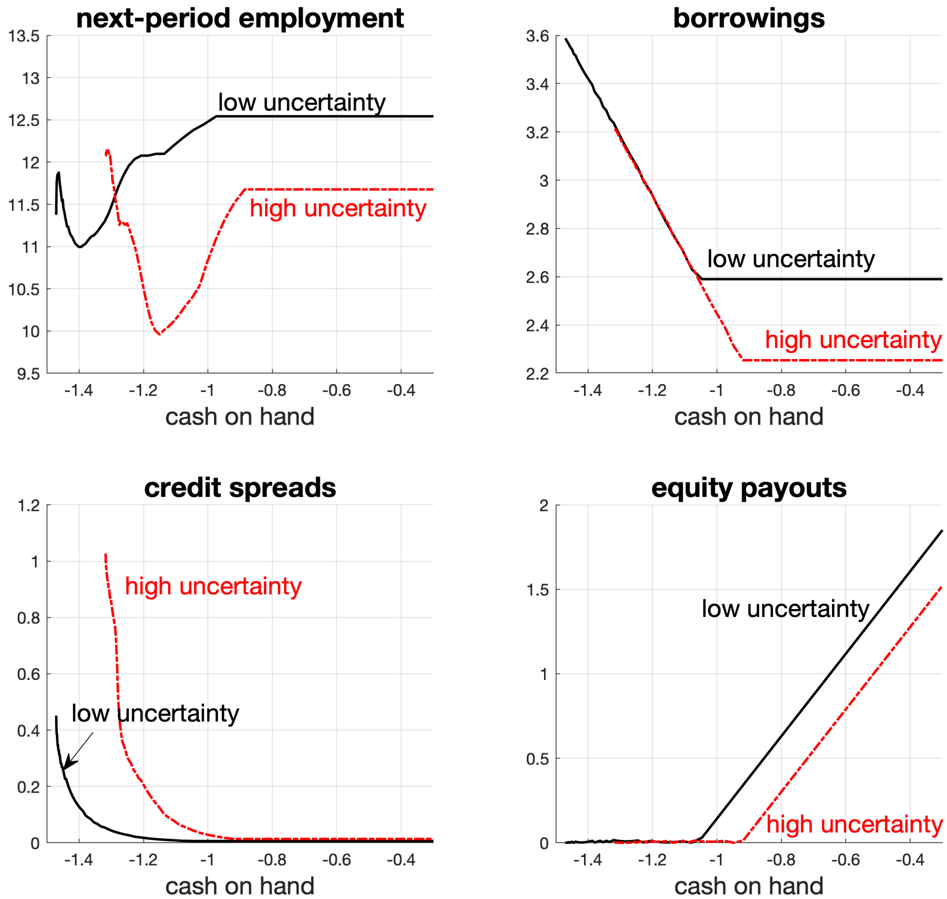
3.4.1 The Financial Channel of Wage Commitments

I use the firm's decision rules to explain how high uncertainty leads firms to cut employment due to financial concerns over wage payments.

Figure 2 shows how firms' decisions depend on cash on hand X and the level of uncertainty. I vary the cash on hand on the horizontal axis and fix the firm's idiosyncratic productivity and employment at their median levels. The decision rules are next-period employment n', borrowing Qb', credit spread $1/Q - 1/\beta$, and equity payouts Δ. The solid black lines are for the low uncertainty state, one unconditional standard deviation below the mean uncertainty. The dash-dot red lines are for the high uncertainty state, one unconditional standard deviation higher than the mean. The decision rules are in line with the patterns in Arellano, Bai and Kehoe (2019) because both models have similar financial channels.

First, the relations between the decision rules and cash on hand are consistent with Lemma 2.2. When cash on hand is higher than a cutoff, firms' employment, borrowing, and credit spreads no longer depend on cash on hand. The equity payouts increase with cash on hand one for one in this case. When below the cutoff, the equity payout is zero because the non-negative equity

Figure 2: Decision Rules Under Different Levels of Uncertainty



Notes: The four panels plot a median firm's decision rules for next-period employment, borrowing, credit spread, and equity payouts with respect to cash on hand. The solid black lines are for the low uncertainty state, and the dash-dot red lines are for the high uncertainty state. Aggregate productivity is set at the high level. "High" or "low" means one unconditional standard deviation above or below the mean. The firm's idiosyncratic productivity and current employment are fixed at their median levels.

payout constraint binds.[11] As cash on hand decreases, firms need to borrow more to satisfy the non-negative equity payout constraint. Credit spreads subsequently rise. Employment for the next period decreases as cash on hand decreases because firms face higher default risks and cut employment to avoid defaulting on wage payments next period. But when cash on hand is very low, firms hire slightly more workers. The reason is the increased default probability. Conditional on survival, firms have higher expected productivity. Thus, firms decide to take on more risk and hire more workers.

Second, firms' decisions depend on the level of uncertainty. Higher uncertainty implies greater default risks because of the larger probability of drawing low productivity, resulting in higher

[11] The slight difference between zero is due to computational errors of grid search.

credit spreads. Therefore, firms are tentative about borrowing and equity payouts. For a similar reason, firms also decrease the number of employees in high uncertainty states. Intuitively, firms make employment decisions, i.e., search, matching, and separation, in the labor market before knowing tomorrow's productivity. The committed wage payments are like debts in this sense, which firms have to pay or default in the next period. If the firm draws low productivity next period, wages can be too high compared to sales, and firms may default and be liquidated. Because of these concerns over meeting payroll, firms are precautionary in making commitments during high uncertainty periods. Low demand leads to higher unemployment. This is the key mechanism in my model.

3.4.2 Role of Financial Frictions

The model contains four sources of financial frictions. First, firms only have access to state-uncontingent debt to finance operations. Otherwise, firms would be able to allocate assets across states, repay less in bad states, and never default. Second, firms can default on their debts, and the debt prices reflect firms' default probabilities. Third, negative external equity payouts are not allowed, i.e., firms cannot use equity issuance to get rid of financial constraint. Fourth, firms are subject to agency frictions, so they cannot build up a savings buffer to grow out of default risks.

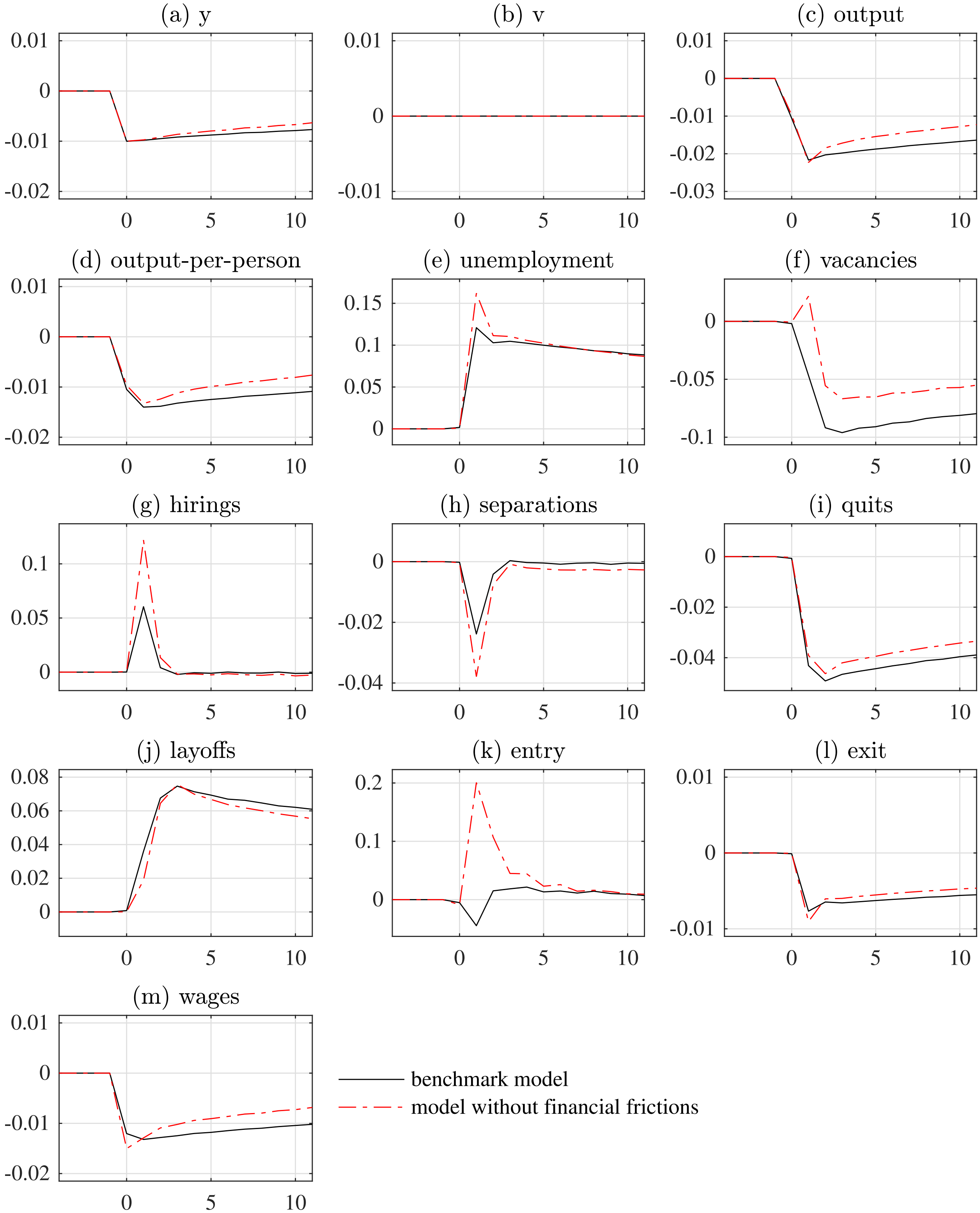
To understand the role of financial frictions in the model, I shut down all the financial incompleteness in a reference model. In this case, the financial market is complete with state-contingent assets and perfect enforcement of debts. Firms are not subject to the non-negative equity payout constraint. The monitor is perfect, so there is no agency problem. This model without financial frictions resembles Schaal (2017), who studies the effect of uncertainty shocks on unemployment under search and matching frictions.

I recalibrate the model to match the targeted moments except for the financial variables. Columns "No Financial Frictions, $A + \sigma$" of Table 2 and Table 6 show the results of the moment matching and the values of parameters, respectively. Because there are no financial variables to pin down the standard deviation of operating costs, I use the same σ_ϵ as in the benchmark. Financial parameters, including the agency friction $\tilde{\zeta}$, auditing quality ξ, and recovery rate η, are not applicable in this case.

The recalibrated parameters in Table 6 suggest that the standard deviations of both shocks need to increase to match the variations of aggregate output and IQR in the data, in particular the standard deviation of aggregate productivity shocks. The need for larger standard deviations of shocks for the model without financial frictions suggests that financial frictions amplify the effect of shocks.

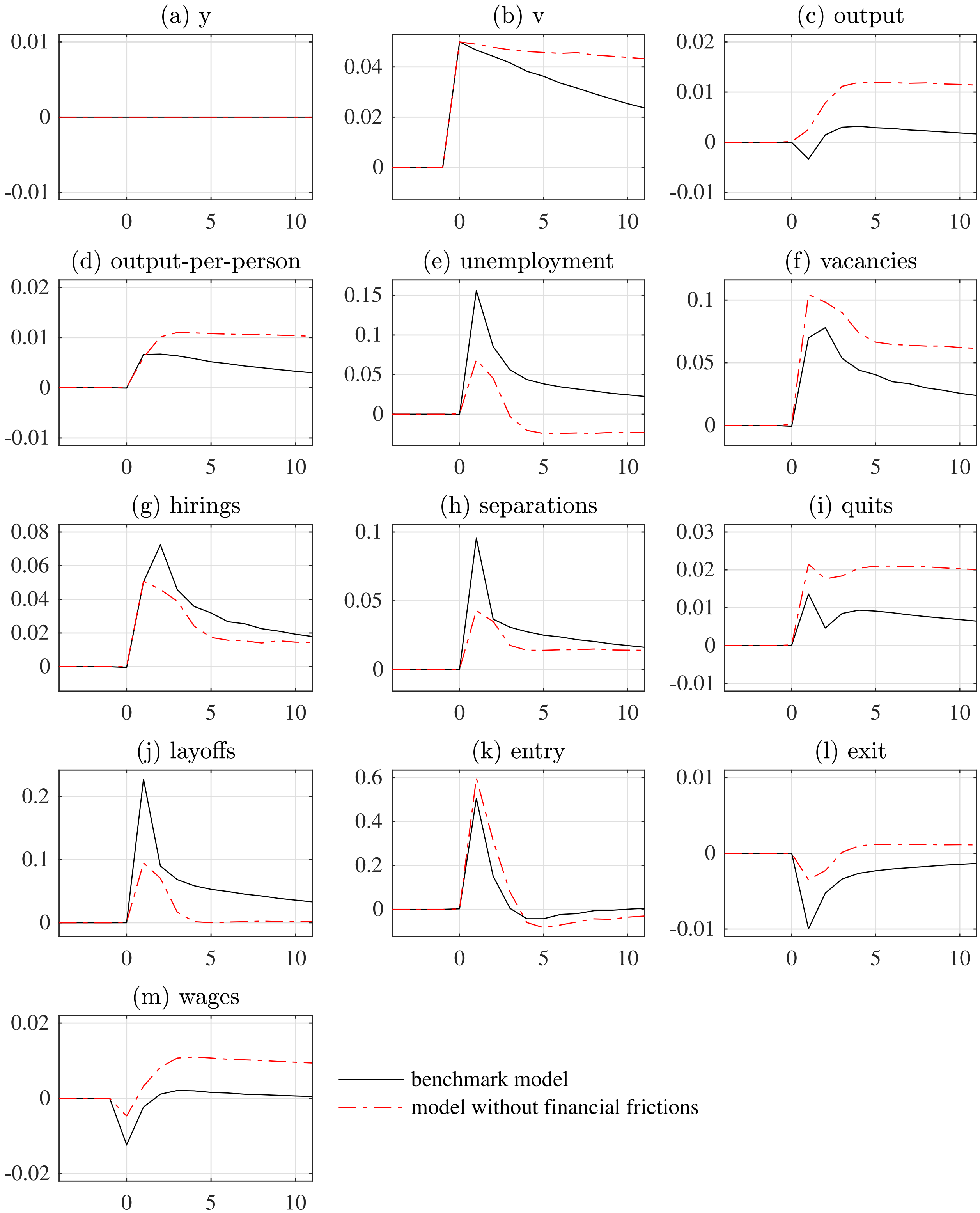
I use the impulse responses in Figures 3 and 4 to illustrate how financial frictions affect aggregate

Figure 3: Aggregate Impulse Responses to a 1% Negative Aggregate Productivity Shock



Notes: The panels are impulse responses to a 1% transitory negative aggregate productivity shock at quarter 0. The impulse responses are the average of 4,000 simulated paths, presented as log deviations from the mean. Solid black lines are the benchmark results. Dash-dot red lines are for the model without financial frictions. I use Schaal's (2017) code when plotting.

Figure 4: Aggregate Impulse Responses to a 5% Positive Uncertainty Shock



Notes: The panels are impulse responses to a 5% positive uncertainty shock at quarter 0. The impulse responses are the average of 4,000 simulated paths, presented as log deviations from the mean. Solid black lines are the benchmark results. Dash-dot red lines are for the model without financial frictions. I use Schaal's (2017) code when plotting.

outcomes.

A Negative Aggregate Productivity Shock. Figure 3 plots the impulse responses to a negative aggregate productivity shock. To draw the impulse responses, I simulate the economy's distribution 4,000 times with stochastic aggregate shocks. At quarter 0, I impose a 1% negative aggregate productivity shock. Then I let the economy evolve stochastically again. The impulse responses in Figure 3 are the average of the 4,000 simulated paths. The solid black lines are from the benchmark model, and the dash-dot red lines are without financial frictions.

For the benchmark model in solid black lines, a 1% negative aggregate productivity shock leads to a 2% decline in output and 10% higher unemployment.

The dash-dot red lines show the results when there are no financial frictions, showing that the changes in output and unemployment are similar to in the benchmark. This finding implies that the financial channel primarily operates through uncertainty shocks. The reason is that wages decline more with financial frictions (Panel (m)), which offsets the negative impact of lower aggregate productivity.

A Positive Uncertainty Shock. Figure 4 displays the impulse responses following a 5% positive uncertainty shock. The methodology to draw the impulse responses is the same, except I shock the simulations with a 5% positive uncertainty shock at quarter 0.

In the benchmark model, a 5% positive uncertainty shock lowers output slightly and raises unemployment by 15%, while the model without financial frictions generates an output boom and much less of an increase in unemployment. This result explains why Schaal (2017) finds it difficult for a search model to generate a sufficient increase in unemployment during the Great Recession. In his model, the effect of uncertainty on unemployment is mainly through the reallocation of workers across firms. My work builds on his by incorporating a financial channel of wage commitments into the model, which interacts with uncertainty shocks and improves the model's ability to explain unemployment fluctuations.

3.4.3 Role of Uncertainty Shocks

Recall that Table 3 shows that with only the aggregate productivity shock, the standard deviation of unemployment is significantly smaller. In this section, I explain why the financial channel of wage commitments is particularly influential when interacting with uncertainty shocks instead of aggregate productivity shocks.

In the last panels of Figures 3 and 4, I show the impulse responses of wages. The decline of wages is much larger in response to a negative aggregate productivity shock than a uncertainty shock.[12]

[12] Actually, both the wages of incumbent workers and newly hired workers increase when uncertainty increases. The average wage in Figure 4 decreases because the share of newly hired workers increases, who have lower wages.

Because the lower wages offset the effect of the negative aggregate productivity shock, the financial channel of wage commitments does not amplify unemployment volatility much. The offsetting equilibrium effect of wages is also the reason for the unemployment volatility puzzle in Shimer (2005). He finds that the calibrated standard Diamond-Mortensen–Pissarides model generates less than 10% of the observed standard deviation of unemployment. In that model, because of the free entry condition, the decline in wages largely absorbs the effect of aggregate productivity shocks. Similarly, the free entry condition in my model also leads to a large decrease in wages to offset the impact of aggregate productivity shocks.

Nonetheless, the offsetting effect of wage dynamics is smaller for uncertainty shocks. The reason why wages do not decrease much is due to a property of uncertainty shocks. A positive uncertainty shock spreads the distribution of firm-level productivity. Since a firm's profit is convex in terms of its idiosyncratic productivity, a wider distribution delivers a higher expected profit. The uncertainty literature calls this property the Oi-Hartman-Abel effect (Oi (1961), Hartman (1972), Abel (1983)). The Oi-Hartman-Abel effect is stronger for firms with high productivity because firm productivity is persistent. These firms' high expected values means that wages do not need to decrease much to satisfy the free entry condition. Since the equilibrium wage is not low enough for a median firm to offset the higher risk of drawing low idiosyncratic productivity, it hires less. At the aggregate level, unemployment increases.

Therefore, the financial channel of wage commitment needs to interact with uncertainty shocks to improve the model's prediction of unemployment volatility significantly. This finding provides a new way to understand the fluctuations of unemployment. Unlike a typical search model with only aggregate productivity shocks and homogeneous firms, I argue that uncertainty shocks are crucial to understanding unemployment because firms have limited ability to hedge against idiosyncratic risks.

3.5 Event Study for U.S. Recessions

The preceding section shows how my model can generate most of the unemployment volatility. In this section, I use my model to understand five U.S. past recessions from the 70s to the Great Recession. For this exercise, I first apply the particle filter to my calibrated model, equipped with the time series data, to estimate the historical aggregate productivity shocks and uncertainty shocks, following the approach in Bocola and Dovis (2019). Then I let the model predict unemployment with the estimated shocks and examine its performance in accounting for the increases in unemployment during recessions.

A particle filter is a Monte Carlo Bayesian estimator for the posterior distribution of structural shocks, which suits non-linear systems like mine. However, directly applying the particle filter to my model is infeasible because one of the model's state variables, the distribution of heterogeneous firms, is infinite-dimensional. Therefore, the first step is to follow Krusell and Smith (1998) and approximate my infinite-dimensional model by an auxiliary non-linear state-space system with a finite number of states:

$$\begin{aligned} \boldsymbol{Y}_t &= g(\boldsymbol{X}_t) + \epsilon_t^Y, \\ \boldsymbol{X}_t &= f(\boldsymbol{X}_{t-1}, \epsilon_t^X), \end{aligned} \tag{53}$$

where $\boldsymbol{Y}_t$ is a vector of observables, and $\boldsymbol{X}_t$ is an auxiliary finite-dimensional state vector. Function f is the transition of states, and function g is the mapping from states to observations. ϵ_t^X is a vector of shocks to state variables, and ϵ_t^Y is a vector of independent and serially uncorrelated Gaussian measurement errors.

The goal is that, given the observables $\boldsymbol{Y}$, estimate the underlying states $\boldsymbol{X}$, including aggregate productivity A and uncertainty σ. So, the state vector should be sufficiently informative such that its mapping to observables is accurate. For this purpose, I include five groups of state variables in $\boldsymbol{X}_t$: (i) a constant; (ii) logged aggregate productivity A and uncertainty σ up to five-quarter lags, $\{\log A_{t-p}, \log \sigma_{t-p}\}_{p=0}^5$; ($iii$) the interactions between aggregate productivity and uncertainty, $\left\{\log A_{t-p} \cdot \log \sigma_{t-p}, \left\{\log A_{t-p} \cdot \log \sigma_{t-q}, \log A_{t-q} \cdot \log \sigma_{t-p}\right\}_{q=p+1}^3\right\}_{p=0}^2$; ($iv$) the squared logged changes of aggregate productivity and uncertainty and their interactions with the levels, $\left\{(\Delta \log A_{t-p})^2, (\Delta \log \sigma_{t-p})^2, (\Delta \log A_{t-p})^2 \cdot \log \sigma_{t-1}, (\Delta \log \sigma_{t-p})^2 \cdot \log A_{t-1}\right\}_{p=0}^3$; ($v$) lagged logged aggregate credit spreads and their interactions with aggregate productivity and uncertainty, $\Big\{\log \mathrm{spr}_{t-1} \cdot \log A_t, \log \mathrm{spr}_{t-1} \cdot \log \sigma_t, \Big\{\log \mathrm{spr}_{t-p}, \log \mathrm{spr}_{t-p} \cdot \log A_{t-1}, \log \mathrm{spr}_{t-p} \cdot \log \sigma_{t-1}, \left\{\log \mathrm{spr}_{t-p} \cdot (\Delta \log A_{t-q})^2, \log \mathrm{spr}_{t-p} \cdot (\Delta \log \sigma_{t-q})^2\right\}_{q=0}^2\Big\}_{p=1}^5\Big\}$.

The next step is to obtain the mapping from this set of state variables to observables. Specifically, I choose aggregate output and the interquartile range (IQR) of firm sales growth as the observables since they have clear and distinct relations with aggregate productivity and uncertainty. To obtain the mapping $g(\cdot)$, I project the model-simulated aggregate output and IQR on the set of state variables, respectively. Their R^2s are 0.999998 and 0.9997, indicating the mapping's accuracy and validating the choice of state variables. Then the regression error variance is used to model the measurement errors ϵ_t^Y.

On the other hand, the transition function $f(\cdot)$ is set up according to the evolution of states. First, the transitions of aggregate productivity and uncertainty are defined by eq. (48) and eq. (49). Second, the transition from the states to the next-period credit spread is obtained by projecting the model-simulated credit spreads on the state variables, which also displays a high R^2 of 0.9998. Then the remaining transitions can be derived exactly from the definition of state variables. For example, the state variable $\log A_t \cdot \log \sigma_t$ is simply the state $\log A_t$ multiplied by another state $\log \sigma_t$. Lastly, state shocks in ϵ_t^X are the innovations to aggregate productivity ϵ_t^A, the innovations to uncertainty ϵ_t^σ, and the error term from the projection for credit spreads.

Given the finite-dimensional state-space system (53), I can apply the particle filter to it and estimate the underlying states from the data.[13] Specifically, I use the times series of GDP per capita from the Bureau of Economic Analysis and the IQR of firm sales growth from Compustat as observable variables. The data is from 1972 to 2018. And the series are detrended by the band-pass filter for business cycle fluctuations between 6 and 32 quarters, consistent with Schaal (2017). Given the data, I use the particle filter to estimate the realized states from the state-space system. Figure 8 in the appendix plots the data series and the estimated aggregate productivity and uncertainty, showing that aggregate output is closely related to aggregate productivity and the IQR of firm sales growth is tightly associated with uncertainty.

Next, I let the state-space model predict unemployment by feeding the estimated states, where the mapping from states to unemployment is also obtained by projection ($R^2 = 0.99998$). Figure 5 compares the model-predicted unemployment and the data, displayed as the peak-to-trough log deviations of unemployment during recessions. Panel A shows the results with both aggregate productivity shocks and uncertainty shocks. The black lines are the data, and the dash-dotted red lines are the predictions of the benchmark model. They display similar patterns and magnitudes, indicating the benchmark model accounts for a great share of the increase in unemployment during recessions. Notably, it generates about 90% of the Great Recession.

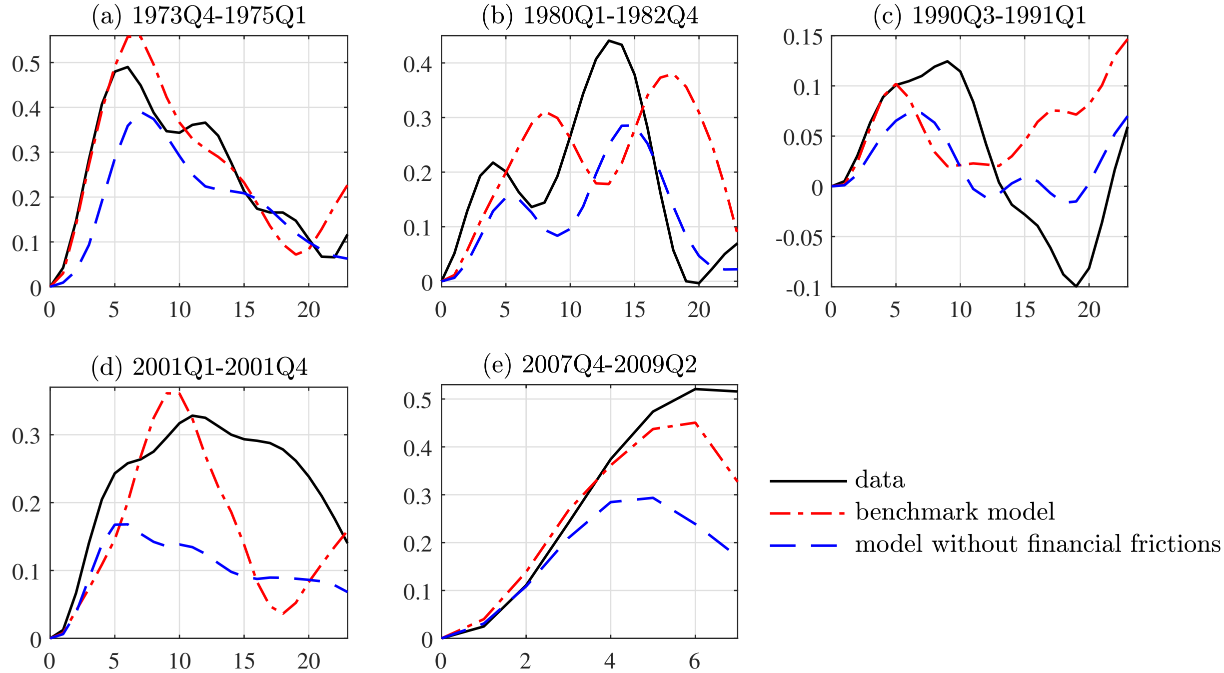
To understand the role of financial frictions, I also plot the predictions of the counterfactual model without financial frictions using dashed blue lines, which display less of the increase in unemployment during recessions. In particular, the prediction for the Great Recession deteriorates greatly. This result is consistent with Schaal (2017), who finds that the canonical search framework alone cannot generate enough increase in unemployment during the Great Recession. The reason is that the Great Recession had the largest increase in uncertainty (Figure 8 in the appendix), which interacted with the financial channel of incomplete contracts and amplified the increase in unemployment. So, financial frictions contribute more to the Great Recession.

A related question is whether the amplification effect of contracting frictions needs to interact with uncertainty shocks in particular. I find the answer is yes. Specifically, I show the predictions for unemployment of the models with only aggregate productivity shocks in Panel B. The first observation is that the model's performance in explaining recessions deteriorate in general. Second, the difference between with and without financial frictions shrinks a lot, implying that the amplification effect of financial frictions on unemployment is much smaller for aggregate productivity shocks. The reason is similar to Shimer's (2005) unemployment volatility puzzle. In the equilib-

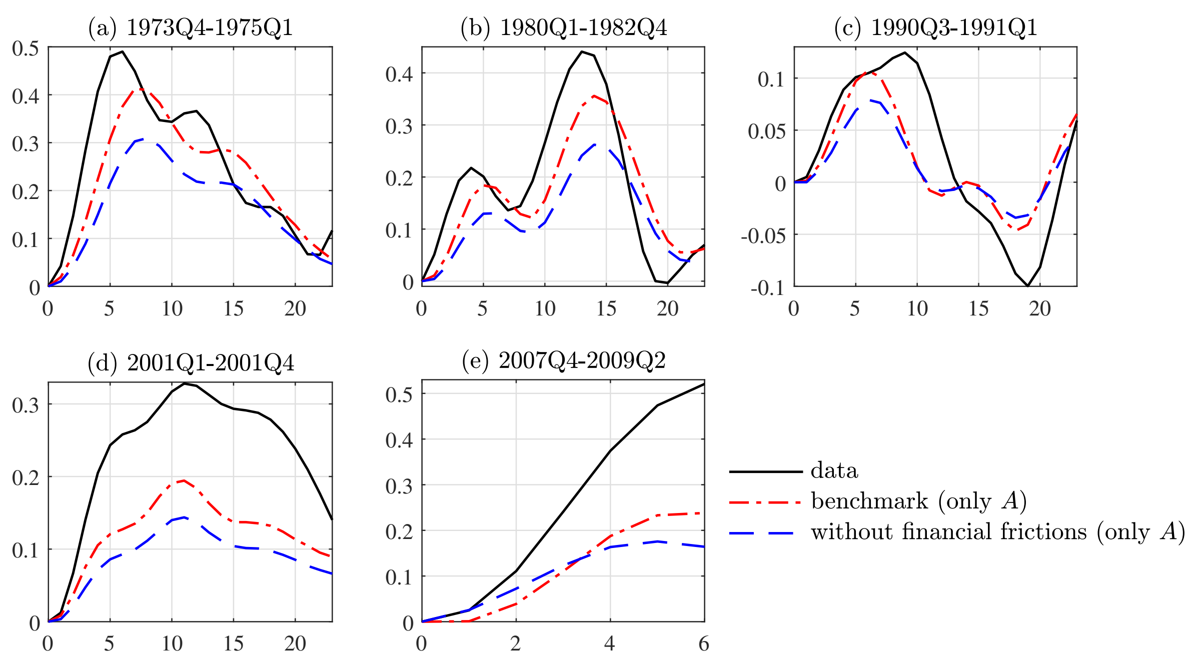
[13] The particle filter's algorithm uses a set of particles to approximate the underlying states. Particles evolve and predict observables according to the state-space system (53). The data of observables correct the state estimates of particles by calculating their likelihoods. This process repeats recursively till the end of the data. I set the number of particles as $10,000$.

Figure 5: Unemployment Series With and Without Modeling Financial Frictions

Panel A: Models With Both Aggregate Productivity Shocks and Uncertainty Shocks



Panel B: Models With Only Aggregate Productivity Shocks



Notes: The panels show the model's predictions for unemployment during recessions. Models in Panel A have both aggregate productivity shocks, A, and uncertainty shocks, σ. I use the particle filter to jointly estimate the time series of aggregate productivity shocks and uncertainty shocks by matching output and the IQR of firm sales growth in the data. The data are detrended by a band-pass filter to focus on fluctuations between 6 and 32 quarters, following Schaal (2017). Models in Panel B have only aggregate productivity shocks. Given the estimated shocks, I show the model-predicted unemployment. The data on unemployment is the solid black lines. The unemployment fluctuations predicted by the benchmark model are the dash-dotted red lines, and model unemployment without financial frictions is the dashed blue line. Series are depicted in terms of log deviations from the peak preceding the recession. I use Schaal's (2017) code when plotting.

Table 4: Peak-To-Trough Changes During Recessions

	1973-1975	1980-1982	1990-1991	2001	2007-2009
Output					
Data	-0.082	-0.069	-0.021	-0.038	-0.048
Both A and σ shocks					
Benchmark model	-0.081	-0.068	-0.021	-0.036	-0.047
Model without financial frictions	-0.078	-0.067	-0.020	-0.037	-0.045
Only A shock					
Benchmark model	-0.082	-0.069	-0.021	-0.038	-0.048
Model without financial frictions	-0.081	-0.068	-0.021	-0.038	-0.047
Unemployment					
Data	0.490	0.441	0.124	0.328	0.521
Both A and σ shocks					
Benchmark model	0.577	0.380	0.147	0.361	0.451
Model without financial frictions	0.390	0.285	0.073	0.168	0.294
Only A shock					
Benchmark model	0.413	0.356	0.107	0.194	0.238
Model without financial frictions	0.309	0.262	0.079	0.144	0.176

Notes: The table shows the peak-to-trough changes in output and unemployment during recessions for both the data and models. I use the particle filter to jointly estimate the time series of aggregate productivity shocks and uncertainty shocks by matching output and the IQR of firm sales growth in the data. The data are detrended by a band-pass filter to focus on fluctuations between 6 and 32 quarters, following Schaal (2017). Given the estimated shocks, I show the peak-to-trough changes predicted by the model during recessions. Consistent with Schaal (2017), series are depicted in terms of log deviations from the peak preceding the recession.

rium, according to the free entry condition, wages decrease a lot to absorb the effect of negative aggregate productivity shocks, which are common to all firms. However, this offsetting effect of wages is much smaller for uncertainty shocks because high volatility spreads the distribution of firms' idiosyncratic productivity. And the probability of drawing high productivity also increases, which makes wages stay at a high level. Therefore, the financial channel of wage commitments is effective when interacting with uncertainty shocks.

Table 4 reports the peak-to-trough changes in aggregate output and unemployment during recessions. Because the particle filter estimates the underlying shocks by matching aggregate output, all models display similar output declines as in the data, validating that the particle filter performs well.[14] On the other hand, different models display distinct predictions for unemployment. Specifically, the data shows that unemployment increased by 50% during the Great Recession. The model without financial frictions can only explain about half of it, while the benchmark model can generate 90% of the increase in unemployment, suggesting that financial frictions are essential. Meanwhile, uncertainty shocks are also important. Without it, the model generates less than half of the increase in unemployment during the Great Recession.

[14] Figure 9 plots the variation of aggregate output during each recession.

3.6 Policy Implications

Given the quantitative performance of my model, I apply it to the labor market policies of the U.S. and Germany during the recent Covid recession. I also discuss how the measured effects of these policies are biased when financial frictions are not considered.

U.S. Policy. In the recent 2020 Covid-19 pandemic, the U.S. market uncertainty increased dramatically. Specifically, Altig et al. (2020) show that business executives are much more uncertain about their firms' future sales growth rates during the COVID-19 pandemic, according to the U.S. monthly panel Survey of Business Uncertainty (SBU) and the U.K. monthly Decision Maker Panel (DMP). At the same time, the U.S. government deployed economic support policies. One notable response was the U.S. Federal Pandemic Unemployment Compensation (FPUC) program, which increased unemployment benefits by an extra \$600 per week.

To figure out the aggregate impacts of this policy, I modify my model to incorporate it. Specifically, I model that the government increases unemployment benefits by 1% when uncertainty is high. Given the policy, I re-solve the model quantitatively. So, the policy is anticipated by the agents in the economy. For simplicity, I assume that the government collects tax revenue through a lump-sum tax, and this policy costs 4.81 basis points of output in the simulation.

Table 5 summarizes the impact of the U.S. policy on business cycles. Panel A displays the policy and its costs as a share of output. And Panel B compares the model-simulated moments of the benchmark model and the model with the U.S. policy. It is clear that raising unemployment benefits has negative impacts through the model's labor demand channel. In particular, output decreases by 0.41%, unemployment increases by 0.39 percentage points, the standard deviation of unemployment increases by 16%, and welfare decreases by 3.9 basis points. The reason is that the increased unemployment benefits distort the labor market by pushing wages higher by around 6.1 basis points in the simulation. Therefore, firms hire fewer workers, making it harder for the unemployed to find jobs. Specifically, the policy decreases the unemployment-to-employment transition rate by 1.5 percentage points. Overall, since the U.S. policy distorts the economy by increasing the marginal cost of the labor force, aggregate output and welfare decrease.

Next, Panel A of Figure 6 shows the impulse responses to a 5% positive uncertainty shock. The solid black lines are the results of the benchmark model as shown in Figure 4, and the dashed red lines are for the model with the U.S. policy, which raises unemployment benefits by 1% when the uncertainty shock hits. It turns out that this policy amplifies the recession by generating lower output and raising unemployment by an additional 5%. The reason is that increased unemployment value requires higher wages, which not only increases the cost of production but also strengthens the financial concern of wage payments. Therefore, the recession is deepened.

German Policy. Germany has a very different social security system from the U.S. As a part of

Table 5: The Impact of Labor Market Policies on Economic Performance

	Without Policy	U.S. Policy	German Policy
Panel A: Policies			
Increase in unemployment benefits	-	1%	-
The replacement rate of wage subsidies	-	-	84.4%
Basis point shares of policy costs to output	-	4.81	4.86
Panel B: Economic Performance			
Benchmark Model			
Mean of output	100	99.593	99.938
SD of output	0.015	0.015	0.015
Mean of unemployment (%)	5.823	6.210	5.804
SD of unemployment	0.106	0.123	0.104
Mean of average wages	100	100.061	100.014
SD of average wages	0.011	0.011	0.011
UE rate	0.814	0.799	0.814
EU rate	0.083	0.085	0.083
EE rate	0.081	0.080	0.081
Mean credit spread (%)	0.96	0.96	0.97
Median leverage (%)	21	21	21
Annual exit rate (%)	9.0	9.0	9.0
Welfare	100	99.961	99.976
Model Without Financial Frictions			
Mean of output	100	99.951	99.985
SD of output	0.019	0.019	0.019
Mean of unemployment	4.834	4.863	4.806
SD of unemployment	0.088	0.089	0.086
Mean of average wages	100	100.017	100.008
SD of average wages	0.014	0.014	0.014
UE rate	0.845	0.844	0.845
EU rate	0.073	0.073	0.072
EE rate	0.085	0.084	0.085
Annual exit rate (%)	9.0	9.0	9.0
Welfare	100	99.998	99.996

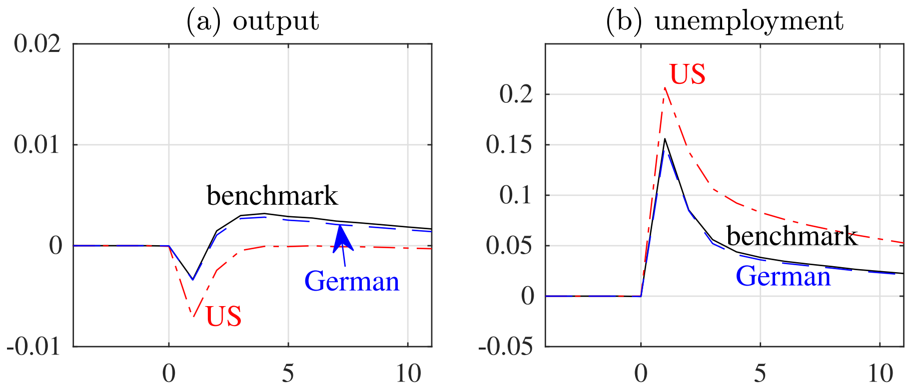
Notes: The table shows the model-simulated moments without and with the labor market policies. Panel A specifies the policies, and Panel B displays the moments of 3,000-quarter simulations of the benchmark model and the model without financial frictions. Given each policy, I re-solve the model. Therefore, policies are anticipated by the agents in the economy. The output, average wages, and welfare levels are normalized to 100 for the two models without policy. The standard deviations of output, unemployment, and average wages use the log deviations from the HP-filter with parameter 1,600 as before.

Germany's social insurance program, Kurzarbeit, the short-time work (STW) policy compensates part of workers' earnings losses when firms cut workers' hours. In other words, the government helps firms pay wages when there is a bad shock.[15] I model Germany's policy by allowing the firm to have an option to let part of its workforce idle when uncertainty is high. The government pays

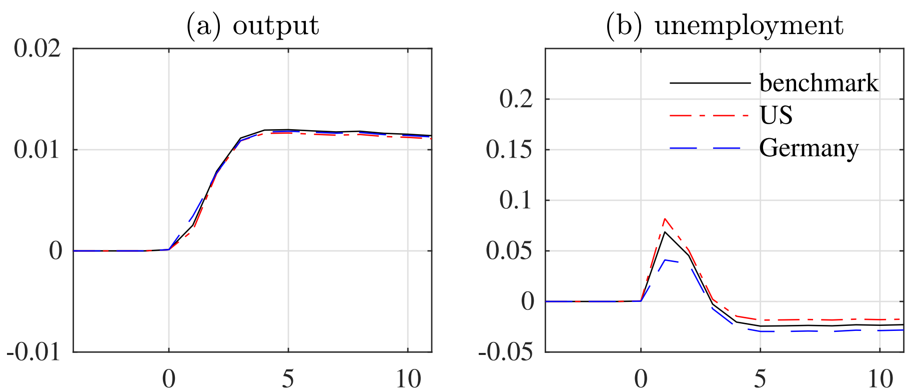
[15] Cooper, Meyer and Schott (2017) provide information on this system.

Figure 6: Policy Implications of a Transitory Positive Uncertainty Shock

Panel A: Benchmark Model



Panel B: Model Without Financial Frictions



Notes: The panels are impulse responses to a 5% positive uncertainty shock at quarter 0. The impulse responses are the average of 4,000 simulated paths, presented as log deviations from the mean. Solid black lines are the benchmark results. Dash-dot red lines are for the model with enhanced U.S. unemployment benefits. Dashed blue lines are for the model with German wage subsidies. I use Schaal's (2017) code when plotting.

84.4% of the idle workers' wages, and the firm pays the rest.[16] This replacement rate is chosen to have the same share of government expenditure to output ratio as the U.S. Table 5 shows that this policy costs 4.86 basis points of output.

Table 5 also reports the model-simulated moments without and with the German policy. It turns out that this policy has its pros and cons. On the positive side, the unemployment rate decreases by 1.9 basis points, and its standard deviation over business cycles is 1.9 percent lower. In other words, the German labor market policy can lower and stabilize unemployment by providing state-contingent insurance to firms to help them pay wages and retain employees, weakening the financial channel of wage commitments. Plus, since the policy reduces separations, it can save the

[16] Although my model does not have an explicit component of working hours, it does not affect the results since each firm has a continuum of the workforce to be idle, equivalent to modeling hours cut.

resources spent on search.

However, the policy's overall impact is negative—aggregate output decreases by 6.2 basis points, and welfare decreases by 2.4 basis points. The negative effect is due to the policy-induced distortions. The wage subsidies encourage labor hoarding, which misallocates the labor force to low-productivity firms that are supposed to separate from their employees. As a result, the labor market is less efficient, so aggregate output and welfare decrease.

The dash-dot blue lines in Figure 6 show the impulse responses to a transitory positive uncertainty shock under Germany's policy. The impulse responses almost do not change because the policy's distortive costs offset the stabilization benefits from insuring firms. But it can still be observed that output decreases slightly more and unemployment increases slightly less, consistent with the findings in Table 5.

Biased Policy Evaluation Without Consideration of Financial Frictions. I have already shown that financial frictions are crucial for my model's implications for business cycles. Another question is how important financial frictions are for policy evaluation. To answer this question, I apply the above two policy experiments to the recalibrated model without financial frictions.

The lower part of Table 5 reports the model's moments when financial frictions are absent, without and with the policies. As we can see, the negative impacts of both labor market policies diminish dramatically. The welfare loss induced by the U.S. policy decreases from 3.9 to 0.2 basis points. And the German policy-induced welfare loss decreases from 2.4 to 0.4 basis points. Panel B of Figure 6 also displays similar results with the impulse response to a 5% positive uncertainty shock. Without considering financial frictions, the U.S. policy generates a much smaller decrease in output and a much smaller increase in unemployment. The reason is that when the financial channel of wage commitments is absent, the policy's negative influence is weakened substantially. On the other hand, the German policy displays a stronger stabilization effect by generating slightly higher output and lower unemployment. In this case, the policy-induced distortion is much smaller because wage subsidies do not twist firms' liquidity incentives. Therefore, the policy's benefits outweighed its drawbacks, and the recession is dampened.

To sum up, the U.S. policy pays unemployed workers more during high uncertainty states, making it more expensive for firms to pay wages; Germany's policy provides wage subsidies when firms cannot afford to retain workers, which helps them keep workers when facing shocks. Both policies expanded aggressively during the Covid recession. My quantitative results show that the U.S. policy significantly amplifies recessions and the German policy has mild negative impacts. Both policies' negative effects are largely underestimated when the financial channel is omitted. It is worth noting that my model focuses on the labor demand mechanism. It does not include worker-side risk aversion or demand effects, which may provide additional benefits for the two policies through other channels.

4 Conclusion

This paper uses a novel model to study the financial effect of wage commitments on unemployment in the presence of uncertainty shocks. When uncertainty rises, firms are less likely to make wage commitments, so labor demand decreases and unemployment increases.

The model builds on Schaal (2017) by generating more unemployment volatility and a larger increase in unemployment during the Great Recession. This result is driven by the interaction between uncertainty shocks and the financial channel of wage commitment. The key is that firms have limited ability to hedge against the risks of idiosyncratic productivity variations.

With rich micro-foundations and realistic quantitative performance, my model has potential for wide application. As an example, I use this framework to investigate labor market policies. I find that raising unemployment benefits as the United States did during the Covid recession amplified the recession. On the other hand, a German approach of subsidizing firm wage bills has a mildly negative impact because of misallocation.

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Appendix A Tables

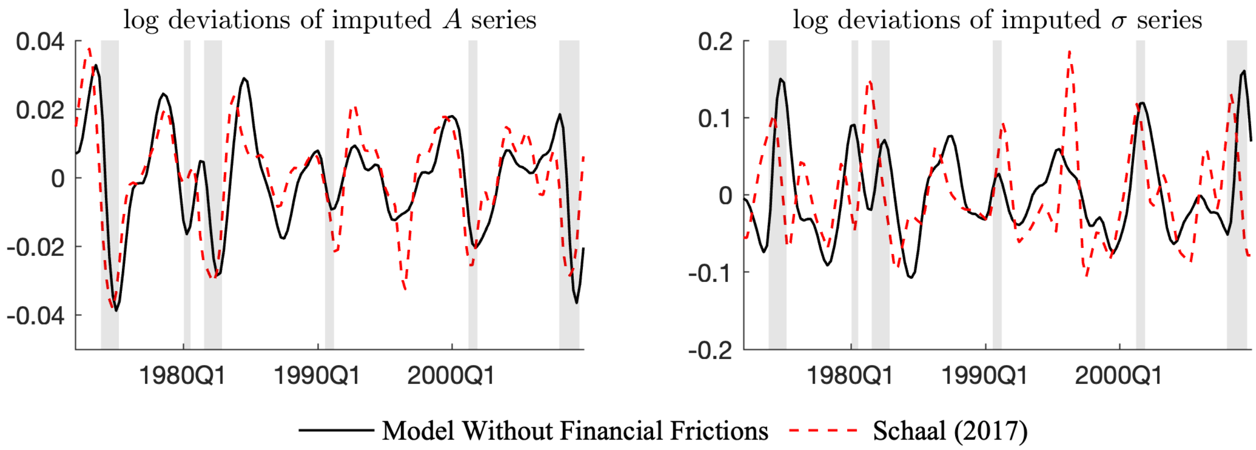
Table 6: Parameters of Reference Models

Parameters	Benchmark Model $A+\sigma$	Benchmark Model A only	No Financial Frictions $A+\sigma$	No Financial Frictions A only
Aggregate shocks				
ρ_A	0.920	0.920	0.913	0.913
σ_A	0.024	0.028	0.041	0.035
$\bar{\sigma}$	0.248	0.250	0.313	0.280
ρ_σ	0.880	-	0.924	-
σ_σ	0.092	-	0.186	-
$\rho_{A\sigma}$	-0.020	-	-0.900	-
Labor market				
$\bar{u}$	0.142	0.142	0.141	0.141
c	0.001	0.001	0.001	0.002
λ	0.100	0.100	0.100	0.100
γ	1.600	1.600	1.600	1.600
k_e	15.21	14.87	11.19	11.47
$\bar{w}_m+\mu_\epsilon$	0.001	0.001	0.100	0.100
π_d	0.021	0.022	0.022	0.022
Financial market				
σ_ϵ	0.080	0.071	0.080	0.080
$\tilde{\zeta}$	2.400	2.400	-	-
ξ	1.780	1.780	-	-
η	2.410	2.410	-	-

Notes: The first column shows the calibrated parameters of the benchmark model. The other columns report the recalibrated parameters for reference models. $A+\sigma$ means the model has both aggregate productivity shocks and uncertainty shocks, and A means the model only has aggregate productivity shocks.

Appendix B Figures

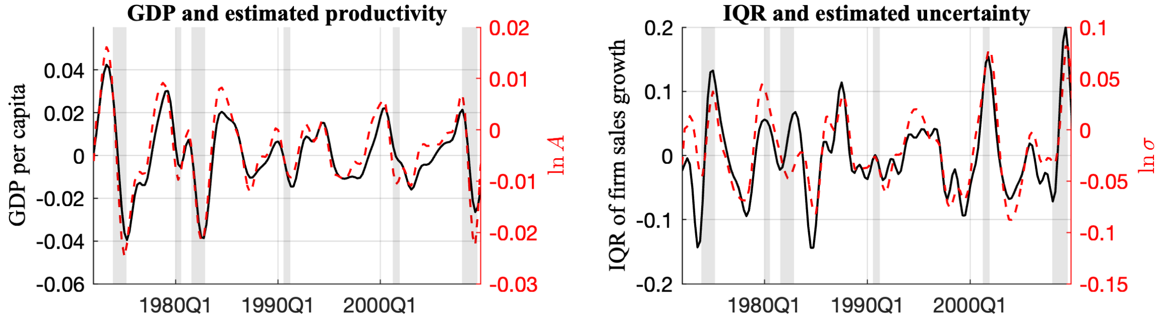
Figure 7: Comparison of Estimated Shocks with Schaal (2017)



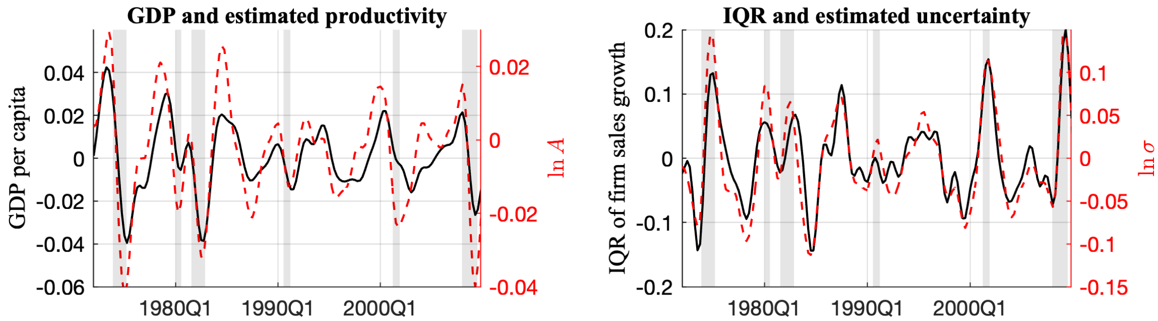
Notes: This figure compares the estimated aggregate productivity shocks and uncertainty shocks with Schaal (2017). I apply the particle filter to my model and estimate the states from the data series of GDP per capita and the IQR of firm sales growth. The black lines show the log deviations of aggregate productivity, A, and uncertainty, σ. The red dashed lines are the imputed shocks from Schaal (2017).

Figure 8: Estimated Aggregate Productivity and Uncertainty

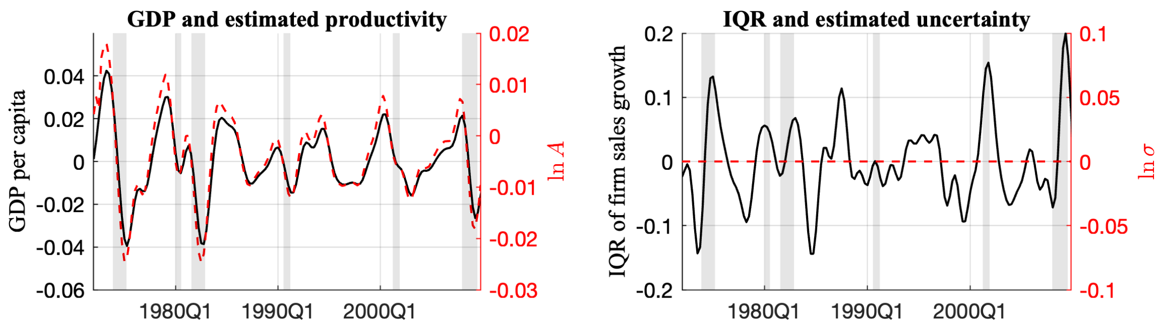
Panel A: Benchmark Model (Both A and σ Shocks)



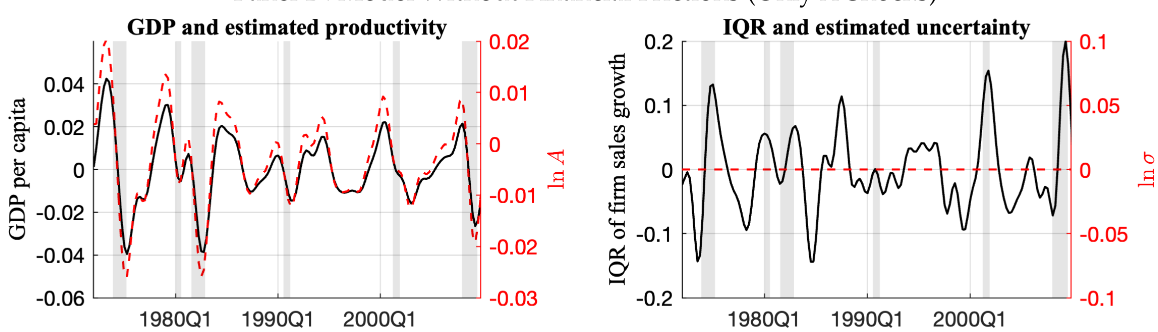
Panel B: Model Without Financial Frictions (Both A and σ Shocks)



Panel C: Benchmark Model (Only A Shocks)



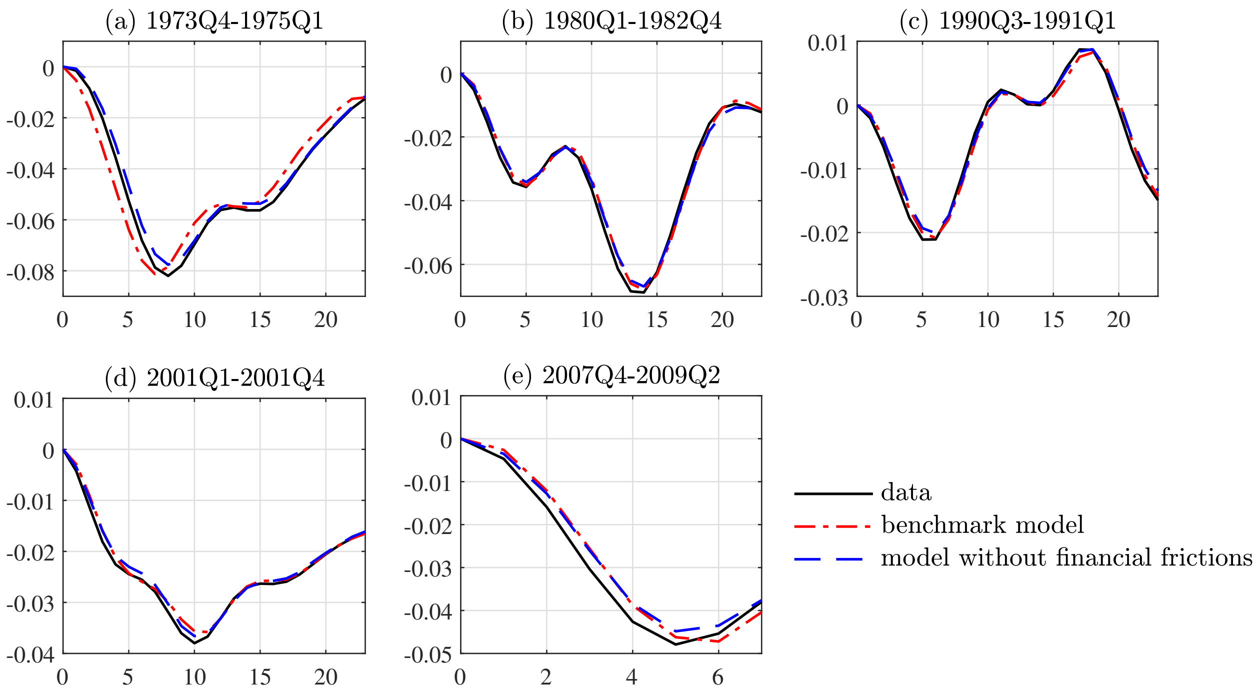
Panel D: Model Without Financial Frictions (Only A Shocks)



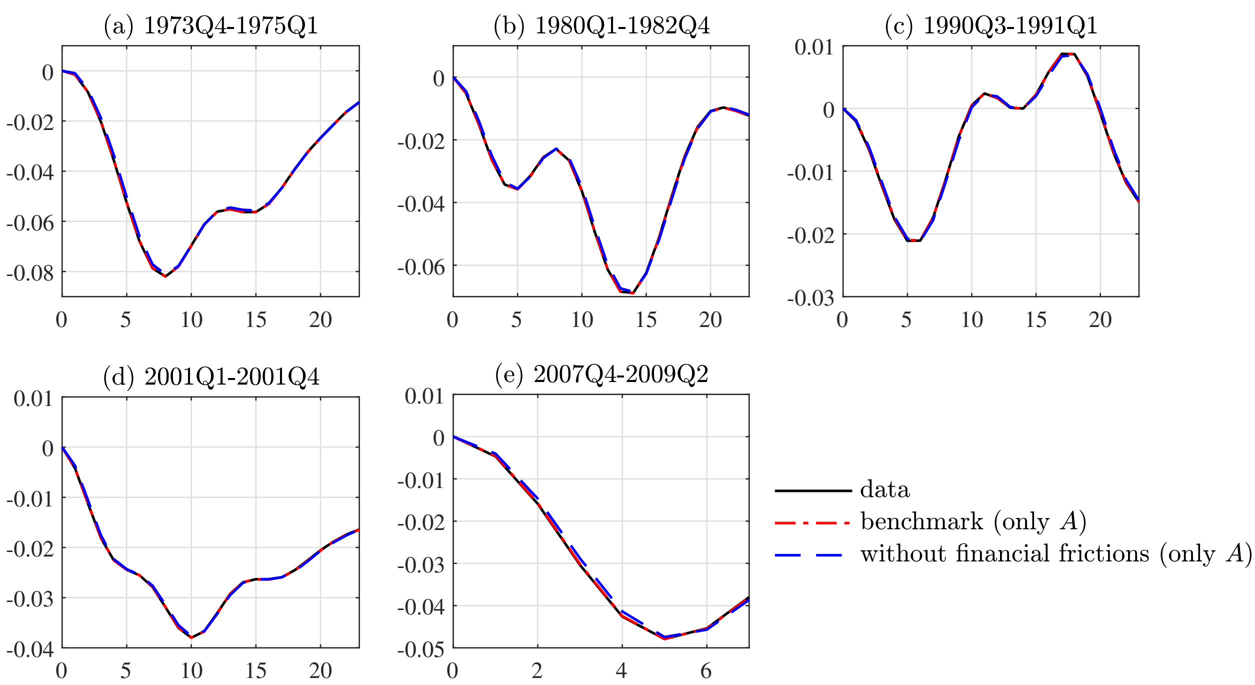
Notes: This figure shows the estimated demeaned aggregate productivity and uncertainty using the benchmark model and three counterfactual models. I apply the particle filter to my model and estimate the states from the data series of GDP per capita and the IQR of firm sales growth, which are detrended by a band-pass filter to focus on fluctuations between 6 and 32 quarters, following Schaal (2017). Panels on the left-hand side display log deviations of GDP (solid black lines) and the estimated demeaned logged aggregate productivity (dashed red lines). Panels on the right-hand side present the log deviations of the interquartile range (IQR) of firm sales growth (solid black lines) and the estimated demeaned logged uncertainty (dashed red lines).

Figure 9: Output Series With and Without Modeling Financial Frictions

Panel A: Models With Both A And σ Shocks



Panel B: Models With Only A Shock



Notes: The panels show the model's predictions for output during recessions. Models in Panel A have both aggregate productivity shocks, A, and uncertainty shocks, σ. I use the particle filter to jointly estimate the time series of aggregate productivity shocks and uncertainty shocks by matching output and the IQR of firm sales growth in the data. The data are detrended by a band-pass filter to focus on fluctuations between 6 and 32 quarters, following Schaal (2017). Models in Panel B have only aggregate productivity shocks. Given the estimated shocks, I show the model-predicted output. The data on output is the solid black lines. The output fluctuations predicted by the benchmark model are the dash-dotted red lines. The output of the model without financial frictions is the dashed blue line. Series are depicted in terms of log deviations from the peak preceding the recession. I use Schaal's (2017) code when plotting.

Appendix C Proofs

Proposition 1 *The participation constraint binds, i.e.,* $\bar{W}(i') = 0$*, for any worker* i'*.*

Proof I prove this proposition by contradiction. Suppose, in the firm's optimal policy, there exists a worker whose index in the next period is i' and $\bar{W}(i') > 0$ in his labor contract. Then I can construct an alternative policy by letting $\bar{W}(i') = 0$ and deliver a higher firm's value at the same time. I first discuss the case where the worker is an incumbent employee and then show the case where the worker is newly hired.

Case 1. Suppose i' refers to an incumbent worker. Use i to denote the worker's index in the current period and ϵ^m to denote the worker's mass.

I construct an alternative policy by making the following four changes to the original policy. The idea is to frontload wages and borrow more simultaneously:

1. Decrease the promised utility markup $\bar{W}(i')$ to zero, which just satisfies the participation constraint (8). To simply the notation, I use δ to denote $\bar{W}(i')$ from now on.

2. Decrease the worker's next-period wage $w(i')$ by exactly δ. Since the wage decreases as much as the promised utility, the next-period promise-keeping constraint (9) holds as before.

3. Promise to pay the worker $\tilde{w}$ today conditional on not leaving the firm by on-the-job search, where $\tilde{w}$ equals $\beta\,\mathbb{E}[(1-\tau(i))(1-\pi_d)(1-d(S',s'))]\delta$. This additional payment guarantees that the worker has the same lifetime promised utility today, so today's promise-keeping constraint (9) is unaffected. Importantly, the worker's on-the-job search decision is not affected because the payment is given to the worker conditional on not transiting to another firm. From the firm's perspective, its labor expense today increases by $\epsilon^m(1-\lambda p(\theta(S,x^*(S;i))))\tilde{w}$.[17]

4. Increase the debt b' by $\epsilon^m(1-\tau(i))(1-\lambda p(\theta(S,x^*(S;i))))\delta$, which equals the decrease in the firm's wage bills in the next-period.[18] So, the next-period cash on hand of the firm does not change.

Given these four changes, I next show the firm's value increases. First, because the next-period cash on hand is the same, the next-period default decisions are unchanged. Also, the next-period employment n' does not change, so neither is the expected value of the firm in the next period.

Second, because the borrowing increases more than the increase in today's wage payments, today's equity payouts increase. Formally, the change in current equity payouts equals:

$$\Delta^{\text{new}} - \Delta = Q(S,s,b'^{\text{new}},n)b'^{\text{new}} - Q(S,s,b',n)b' - \epsilon^m(1-\lambda p(\theta(S,x^*(S;i))))\tilde{w}$$

[17] Notice that this additional payment is conditional on the worker does not leave the firm by on-the-job search.

[18] Notice that the firm pays the wage in the next-period conditional on the worker was not separated by firing or on-the-job search in the previous period.

$$
\begin{aligned}
&= \beta\,\mathbb{E}\left\{(1-\pi_d)(1-d(S',s'))\right\}b'^{\text{new}} + \beta\,\mathbb{E}\left\{[1-(1-\pi_d)(1-d(S',s'))]\min\{\eta\frac{\pi'^{\text{new}}}{b'^{\text{new}}},1\}\right\}b'^{\text{new}} \\
&\quad - \beta\,\mathbb{E}\left\{(1-\pi_d)(1-d(S',s'))\right\}b' - \beta\,\mathbb{E}\left\{[1-(1-\pi_d)(1-d(S',s'))]\min\{\eta\frac{\pi'}{b'},1\}\right\}b' \\
&\quad - \epsilon^m(1-\lambda p(\theta(S,x^*(S;i))))\tilde{w} \\
&= \beta\,\mathbb{E}\left\{(1-\pi_d)(1-d(S',s'))\right\}b'^{\text{new}} - \beta\,\mathbb{E}\left\{(1-\pi_d)(1-d(S',s'))\right\}b' \\
&\quad + \beta\,\mathbb{E}\left\{[1-(1-\pi_d)(1-d(S',s'))]\min\{\eta\pi'^{\text{new}},b'^{\text{new}}\}\right\} \\
&\quad - \beta\,\mathbb{E}\left\{[1-(1-\pi_d)(1-d(S',s'))]\min\{\eta\pi',b'\}\right\} - \epsilon^m(1-\lambda p(\theta(S,x^*(S;i))))\tilde{w} \\
&= \beta\,\mathbb{E}\left\{(1-\pi_d)(1-d(S',s'))\right\}(b'^{\text{new}}-b') - \epsilon^m(1-\lambda p(\theta(S,x^*(S;i))))\tilde{w} \\
&\quad + \beta\,\mathbb{E}\left\{[1-(1-\pi_d)(1-d(S',s'))]\min\{\eta\pi'^{\text{new}},b'^{\text{new}}\}\right\} \\
&\quad - \beta\,\mathbb{E}\left\{[1-(1-\pi_d)(1-d(S',s'))]\min\{\eta\pi',b'\}\right\} \\
&= \beta\,\mathbb{E}\left\{(1-\pi_d)(1-d(S',s'))\right\}\epsilon^m(1-\tau(i))(1-\lambda p(\theta(S,x^*(S;i))))\delta \\
&\quad - \epsilon^m(1-\lambda p(\theta(S,x^*(S;i))))\beta\,\mathbb{E}[(1-\tau(i))(1-\pi_d)(1-d(S',s'))]\delta \\
&\quad + \beta\,\mathbb{E}_{S',s'|S,s}\left\{[1-(1-\pi_d)(1-d(S',s'))]\min\{\eta\pi'^{\text{new}},b'^{\text{new}}\}\right\} \\
&\quad - \beta\,\mathbb{E}_{S',s'|S,s}\left\{[1-(1-\pi_d)(1-d(S',s'))]\min\{\eta\pi',b'\}\right\} \\
&= \beta\,\mathbb{E}_{S',s'|S,s}\left\{[1-(1-\pi_d)(1-d(S',s'))]\min\{\eta\pi'^{\text{new}},b'^{\text{new}}\}\right\} \\
&\quad - \beta\,\mathbb{E}_{S',s'|S,s}\left\{[1-(1-\pi_d)(1-d(S',s'))]\min\{\eta\pi',b'\}\right\}.
\end{aligned}
$$

Notice that $b'^{\text{new}} \geq b'$ by construction and $\pi'^{\text{new}} \geq \pi'$ because the next-period wage bills decrease. Therefore, $\min\{\eta\pi'^{\text{new}}, b'^{\text{new}}\} \geq \min\{\eta\pi', b'\}$. So,

$$
\begin{aligned}
\Delta^{\text{new}} - \Delta \geq& \beta\,\mathbb{E}_{S',s'|S,s}\left\{[1-(1-\pi_d)(1-d(S',s'))]\min\{\eta\pi',b'\}\right\} \\
&- \beta\,\mathbb{E}_{S',s'|S,s}\left\{[1-(1-\pi_d)(1-d(S',s'))]\min\{\eta\pi',b'\}\right\} \\
=&0.
\end{aligned}
$$

Lastly, the agency friction constraint (11) holds under this constructed policy. The constraint's left-hand side increases as the borrowing increases, and its right-hand side decreases because of lower next-period wage bills.

Case 2. Suppose i' refers to a newly hired worker in the current period. As before, construct an alternative policy by making the following four changes to the original policy:

1. Decrease the promised utility markup $\bar{W}(i')$ to zero, just satisfying the participation constraint. I use δ to denote $\bar{W}(i')$.

2. Decrease the worker's next-period wage $w(i')$ by δ, so the next-period promise-keeping constraint still holds.

3. Increase the newly hired workers' wage $w_h(i')$ by $\beta\,\mathbb{E}[(1-\pi_d)(1-d(S',s'))]\delta$, guaranteeing that

the worker still has the same lifetime promised utility x_h, so today's promise-keeping constraint still holds. On the firm-side, today's labor expense increases by $\epsilon^m \tilde{w}$, where ϵ^m denotes the worker's mass.

4. Increase the debt b' by $\epsilon^m \delta$, which equals the decrease in the firm's wage bills in the next-period. Thus, the next-period cash on hand does not change.

Given these four changes, the firm's value increases for the following reasons. First, the firm's value in the next period is unaffected because the cash on hand and labor force are unchanged.

Second, because borrowing increases more than the increase in wage payments, the equity payouts increase. Formally,

$$
\begin{aligned}
\Delta^{\text{new}} - \Delta &= Q(S, s, b'^{\text{new}}, n) b'^{\text{new}} - Q(S, s, b', n) b' - \epsilon^m \tilde{w} \\
&= \beta \, \mathbb{E} \left\{ (1 - \pi_d)(1 - d(S', s')) \right\} (b'^{\text{new}} - b') - \epsilon^m \tilde{w} \\
&\quad + \beta \, \mathbb{E} \left\{ [1 - (1 - \pi_d)(1 - d(S', s'))] \min\{\eta \pi'^{\text{new}}, b'^{\text{new}}\} \right\} \\
&\quad - \beta \, \mathbb{E} \left\{ [1 - (1 - \pi_d)(1 - d(S', s'))] \min\{\eta \pi', b'\} \right\} \\
&= \beta \, \mathbb{E} \left\{ (1 - \pi_d)(1 - d(S', s')) \right\} \epsilon^m \delta - \epsilon^m \beta \, \mathbb{E} \left\{ (1 - \pi_d)(1 - d(S', s')) \right\} \delta \\
&\quad + \beta \, \mathbb{E}_{S', s' | S, s} \left\{ [1 - (1 - \pi_d)(1 - d(S', s'))] \min\{\eta \pi'^{\text{new}}, b'^{\text{new}}\} \right\} \\
&\quad - \beta \, \mathbb{E}_{S', s' | S, s} \left\{ [1 - (1 - \pi_d)(1 - d(S', s'))] \min\{\eta \pi', b'\} \right\} \\
&= \beta \, \mathbb{E}_{S', s' | S, s} \left\{ [1 - (1 - \pi_d)(1 - d(S', s'))] \min\{\eta \pi'^{\text{new}}, b'^{\text{new}}\} \right\} \\
&\quad - \beta \, \mathbb{E}_{S', s' | S, s} \left\{ [1 - (1 - \pi_d)(1 - d(S', s'))] \min\{\eta \pi', b'\} \right\} \\
&\geq 0,
\end{aligned}
$$

where the last inequality is due to $b'^{\text{new}} \geq b'$ and $\pi'^{\text{new}} \geq \pi'$.

Lastly, the agency friction constraint (11) holds. The constraint's left-hand side increases as the borrowing increases more than the increase in newly hired workers' wages, and its right-hand side decreases as next-period wage bills decrease.

In sum, I construct a feasible and better alternative policy, which contradicts the optimality of the original policy with a loose participation constraint. Therefore, the participation constraint always binds in the equilibrium. □

Appendix D Micro-Foundations of Incomplete Labor Contracts

In this section, I use asymmetric information between the firm and its employees to show that the promised utility markup $\bar{W}$ is state-uncontingent. The key assumption is that firms know the realized shocks immediately, but employees know the shocks later in the production stage. This information friction explains why labor contracts are not completely state-contingent, i.e., constraint (7). The logic follows Hall and Lazear (1984) and Lemieux, MacLeod and Parent (2012). They use asymmetric information to justify the optimality of pre-determined wages. We share the mechanism that firms can lie about the states, so the only incentive-compatible result is state-uncontingent promises. I will first set up the model with asymmetric information and then prove the optimality of state un-contingency.

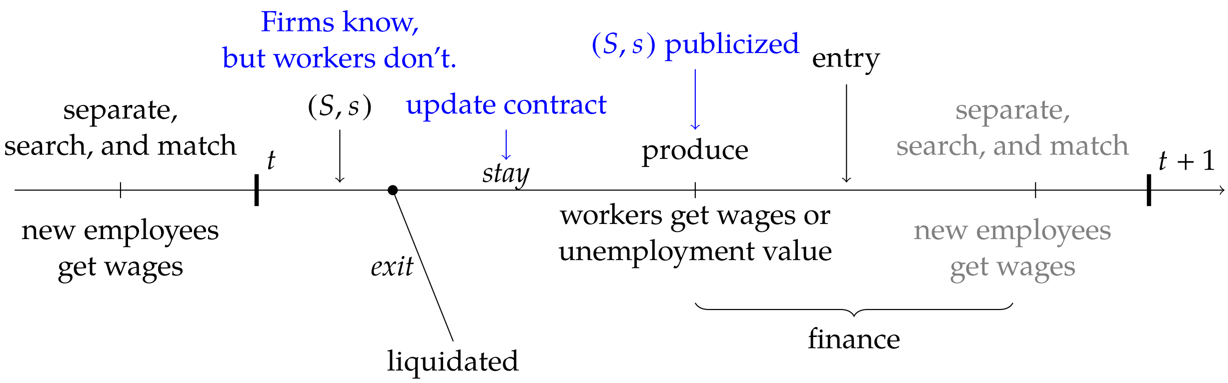
On top of the timeline in Figure 1, Figure 10 adds the timing for asymmetric information. When shocks (S, s) realize at the beginning of each period, firms know the shocks, but workers do not. If a worker leaves the firm now, he is unemployed and obtains the unemployment value in the current period. Given the shocks, firms choose to exit or stay. Staying firms declare their current shocks are $\tilde{S}$ and $\tilde{s}$ and update contracts. Notice that the declaration can differ from the true state since workers do not observe the information now. I allow the declarations to differ across the firm's employees. Given that the labor contract has been updated, the worker gets nothing in the current period if he leaves the firm now.[19] At the production stage, the shocks (S, s) become public information. Workers receive wage payments according to the labor contract, which depends on the firm's declaration of the state $(\tilde{S}, \tilde{s})$. At the end of the period, firms separate, search, and match.

The labor contract C contains $\left\{w, \tau, \bar{W}(S', s'), d(S', s')\right\}$. Notice I assume that the contact directly specifics the markup $\bar{W}(S', s')$ between the lifetime promised utility $W'(S', s')$ and the outside value of unemployment $\boldsymbol{U}(S')$, which facilitates proving the markup's state-uncontingency.[20]

[19] This assumption facilitates the proof because workers have no incentive to threaten to leave the firm (Proposition 2(i).)

[20] In the traditional implicit contract literature, the labor contract specifies the lifetime utility. Instead, I assume that the contract specifies the promised utility markup in this paper, which is a weaker assumption in my context. Specifically, by asymmetric information, I will prove that the promised value in the contract is state-uncontingent. When the contract specifies the promised utility markup, it implies only the markup part is state-uncontingent, and the promised lifetime utility can still vary with aggregate states, similar to a standard Diamond-Mortensen-Pissarides search model. If the contract specifies the lifetime utility, then the whole lifetime utility is state-uncontingent, implying greater rigidity. This is a stronger assumption than is unnecessary for my model.

Figure 10: Timing With Asymmetric Information



Given the labor contract, the worker's employment value is:

$$
\begin{aligned}
\boldsymbol{W}(S,s,C) = \max_x\ & w + \lambda p(\theta(S,x))x + (1-\lambda p(\theta(S,x)))\tau\beta\,\mathbb{E}_{S'|S}\,\boldsymbol{U}(S') \\
&+ (1-\lambda p(\theta(S,x)))(1-\tau)\beta \max\Big\{ \underbrace{\mathbb{E}_{S'|S}\,\boldsymbol{U}(S'),}_{\text{leave before the contract is updated}}\ \mathbb{E}_{S',s'|S,s}\,\{(\pi_d + (1-\pi_d)d(S',s'))\boldsymbol{U}(S') \\
&+ (1-\pi_d)(1-d(S',s'))\max\{\boldsymbol{U}(S') + \bar{W}(\tilde{S}'^*, \tilde{s}'^*), \underbrace{0 + \beta\,\mathbb{E}_{S''|S'}\,\boldsymbol{U}(S'')}_{\text{leave after the contract is updated}}\}\}\Big\}.
\end{aligned}
\tag{54}
$$

As before, the worker receives the wage w at the production stage. The worker can conduct on-the-job search and leave the firm. If the worker stays but gets laid off, he will be unemployed in the next period and receive the unemployment value $\boldsymbol{U}(S')$.

If the worker is not laid off, he can still leave the firm when the outside value is high enough. But the outside value depends on the timing of leaving the firm. If the worker leaves the firm before the contract is renewed, he is counted as unemployed and receives the unemployment value just like a laid-off worker. However, if he leaves the firm after the contract is renewed, he receives zero and gets the unemployment value one period later. This setup can be understood as the worker being ineligible to receive unemployment benefits after the labor relation renews, and drawing up contracts is time-consuming, so he does not have time to produce at home in the same period. Hence, the utility is zero in that period. This assumption will imply that workers have no incentive to threaten to quit when they find the firm lies (Proposition 2(i)), facilitating the proof.

If the labor relation persists, the worker will receive the lifetime utility $\boldsymbol{U}(S') + \bar{W}(\tilde{S}'^*, \tilde{s}'^*)$. Notice that because of asymmetric information, the promised utility markup $\bar{W}$ to the worker depends on the firm's declaration of states $(\tilde{S}'^*, \tilde{s}'^*)$. To clarify, $\{\bar{W}(S', s')\}$ in the labor contract is the set of utility markups for the next period. However, how much the worker can get in the next period depends on the firm's declaration of states $(\tilde{S}'^*, \tilde{s}'^*; i)$.

A firm's states include realized aggregate shocks $S \in \boldsymbol{S}$, realized firm-specific shocks $s \in \boldsymbol{s}$, the number of employees n, and the set of promised utility markups to its employees $\{\bar{W}(S,s;i)\}_{S\in\boldsymbol{S},s\in\boldsymbol{s};i\in[0,n]}$, where i is the index of incumbent employees within the firm. In a slight abuse of notation, S and s inside $\bar{W}(\cdot,\cdot;i)$ refer to the possible shocks instead of the realized shocks.

Besides the choice variables in the original firm's problem (3), the firm now also chooses to declare the current shocks, $\tilde{S}(i)$ and $\tilde{s}(i)$, to each employee i. The following equations (55) to (61) summarize the firm's problem:

$$\begin{aligned}\boldsymbol{J}(S,s,b,n,\{\bar{W}(S,s;i)\}_{S\in\boldsymbol{S},s\in\boldsymbol{s};i\in[0,n]}) = &\max_{\substack{\Delta,b',n',n_h,x_h,d(S',s')\\ \{\tilde{S}(i),\tilde{s}(i),w(i),\tau(i)\}_{i\in[0,n]},\\ \{w_h(i')\}_{i'\in(n'-n_h,n']},\\ \{\bar{W}(S',s';i')\}_{S'\in\boldsymbol{S}',s'\in\boldsymbol{s}';i'\in[0,n']}}} \Delta \\ &+\beta(1-\pi_d)\,\mathbb{E}_{S',s'|S,s}\left\{(1-d(S',s'))\boldsymbol{J}(S',s',b',n',\{\bar{W}(S',s';i')\}_{S'\in\boldsymbol{S}',s'\in\boldsymbol{s}';i'\in[0,n']})\right\}\end{aligned} \tag{55}$$

$$\text{s.t. } (4), (5), (6), (10), (11), \tag{56}$$

$$\begin{aligned}\boldsymbol{W}^E(i') \equiv \mathbb{E}_{S',s'|S,s}\{&(\pi_d+(1-\pi_d)d(S',s'))\boldsymbol{U}(S')\\ &+(1-\pi_d)(1-d(S',s'))\max\{\boldsymbol{U}(S')+\bar{W}(\tilde{S}'^*,\tilde{s}'^*;i'),0+\beta\,\mathbb{E}_{S''|S'}\boldsymbol{U}(S'')\}\},\end{aligned} \tag{57}$$

$$\boldsymbol{W}^E(i') \geq \mathbb{E}_{S'|S}\boldsymbol{U}(S'), \forall i' \in [0,n'], \tag{58}$$

$$\begin{aligned}\max_x\ & w(i)+\lambda p(\theta(S,x))x+(1-\lambda p(\theta(S,x)))\tau(i)\beta\,\mathbb{E}_{S'|S}\boldsymbol{U}(S') && (59)\\ &+(1-\lambda p(\theta(S,x)))(1-\tau(i))\beta\boldsymbol{W}^E(i') \geq \boldsymbol{U}(S)+\bar{W}(\tilde{S},\tilde{s};i), \quad \text{for } i' \in [0,n'-n_h], && (60)\end{aligned}$$

$$w_h(i')+\beta\boldsymbol{W}^E(i') \geq x_h, \quad \text{for } i' \in (n'-n_h,n']. \tag{61}$$

Equations (57) to (61) describe the new implicit contract constraints in the presence of asymmetric information. First, equation (57) uses W^E to denote the worker's expected lifetime utility if he stays with the firm. Notice that W^E is also the last part of the employment value (54). Constraint (58) is the new participation constraint, meaning that the worker's expected utility is at least the expected unemployment value so that he will stay. Equation (59) is the new promise-keeping constraint for incumbent workers. This constraint requires the firm to commit to paying the employee at least the promised lifetime utility. The left-hand side is the incumbent worker's employment value, i.e., equation (54). The right-hand side is the promised lifetime utility, comprised of two parts—the unemployment value $\boldsymbol{U}(S)$ and the promised utility markup $\bar{W}(\tilde{S},\tilde{s};i)$. Notice that $\tilde{S}(i)$ and $\tilde{s}(i)$ are the firm's declarations of shocks, two of the firm's choice variables. They can be different from the true shocks because firms know the realized shocks when renewing labor contracts, but workers do not. The declarations can be different across the firm's employees. Equation (61) is the new promise-keeping constraint for newly hired workers. Its left-hand side is the newly hired worker's employment value. On the right-hand side, x_h is the submarket where the firm employs new workers, and x_h is also the promised lifetime utility of the vacancies posted in that submarket. Thus, equation (61) means the firm should guarantee that newly hired workers receive at least the lifetime utility promised by the offer.

The following Proposition 2 proves that the promised utility markup $\bar{W}$ is state-uncontingent.

Proposition 2 *The labor relation between the firm and its employees has the following properties:*

(i) Workers do not leave the firm even if they find the firm lied.

(ii) The promised utility markup $\bar{W}$ is state-uncontingent.

Proof As for point (i), recall that employees discover whether the firm lied about shocks in the production stage, i.e. after the contract is updated. If they leave the firm now, they get nothing today and start receiving the unemployment value in the next period. So, even if the firm gives the worker zero wages and fires them right after the production stage, the worker is willing to stay with the firm.

As for point (ii), because employees will not leave the firm regardless, according to point (i), lying about the shocks has no consequences for the firm. Thus, firms always declare the lowest employment surplus in $\{\bar{W}(S, s; i)\}_{S \in \boldsymbol{S}, s \in \boldsymbol{s}}$ to each employee i. Therefore, the incentive-compatible labor contract requires the promised utility markup $\bar{W}$ to be state-uncontingent. $\square$