CSCI 677 HW 1

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1. Rotation Matrix.

X-Axis

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & \sin\psi \\ 0 & -\sin\psi & \cos\psi \end{pmatrix}$$

Y-Axis

$$\begin{pmatrix} \cos\varphi & 0 & -\sin\varphi \\ 0 & 1 & 0 \\ \sin\varphi & 0 & \cos\varphi \end{pmatrix}$$

Z-Axis

$$\begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

2. Reverse Projection of Points and Lines on Image.

a) Let a point in camera coordinate system that will project to $(\hat{x}, \hat{y})$ be (x, y, z).

Then,

$$\begin{cases} \hat{x} = x/z \\ \hat{y} = y/z \end{cases}.$$

Therefore, all points that will project to $(\hat{x}, \hat{y})$ can be expressed in form $(\hat{x}z, \hat{y}z, z)$, where $z \in \mathbb{R}$.

Therefore, the equation of the line is, in vector form,

$$\vec{r} = \langle 0,0,0 \rangle + t \langle \hat{x}, \hat{y}, 1 \rangle.$$

b) For a point (x, y, z) to be projected to line given by parameters (a, b, c), the following equation must hold:

$$a \cdot \frac{x}{z} + b \cdot \frac{y}{z} + c = 0.$$

Therefore, the equation of the plane on which points can be projected to the specified line is

$$ax + by + cz = 0.$$

Written in homogeneous coordinates, the equation of the plane is

$$(a, b, c, 0)^T.$$

3. Intrinsic Parameters, Extrinsic Parameters, and Vanishing Points

a) Intrinsic Matrix.

From the equation for K,

$$K = \begin{pmatrix} \alpha & -\alpha\cot\theta & x_0 \\ 0 & \frac{\beta}{\sin\theta} & y_0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Plugging in $\alpha = \beta = 200(px/mm) \times 50 = 1000$, $\theta = -\frac{\pi}{2}$, $x_0 = y_0 = 500$, we get

$$K = \begin{pmatrix} 1000 & 0 & 500 \\ 0 & -1000 & 500 \\ 0 & 0 & 1 \end{pmatrix}.$$

b) Extrinsic Parameters

i) Compute matrix M.

The rotation matrix R is

$$R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 \\ 0 & 1/2 & \sqrt{3}/2 \end{pmatrix}.$$

Using the fact that $(5000, 4000, 2000)$ in world coordinate system is equivalent to $(0,0,0)$ in camera coordinate system, we build an equation

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & t_1 \\ 0 & \sqrt{3}/2 & -1/2 & t_2 \\ 0 & 1/2 & \sqrt{3}/2 & t_3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 5000 \\ 4000 \\ 2000 \\ 1 \end{pmatrix},$$

where $(t_1\ t_2\ t_3)^T$ is the vector $\vec{t}$.

By solving the equation above, we get

$$\vec{t} = (-5000,\ 1000 - 2000\sqrt{3},\ -2000 - 1000\sqrt{3}).$$

Applying $M = K(R\ t)$, we get

$$M = \begin{pmatrix} 1000 & 250 & 250\sqrt{3} & -15{,}000{,}000 - 5{,}000{,}000\sqrt{3} \\ 0 & -500\sqrt{3} + 250 & 500 + 250\sqrt{3} & -2{,}000{,}000 + 1{,}500{,}000\sqrt{3} \\ 0 & 1/2 & \sqrt{3}/2 & -2000\sqrt{3} - 1000\sqrt{3} \end{pmatrix}.$$

ii) The infinity point on every line in vertical direction is

$$(0, 1, 0, 0)^T.$$

By projecting the infinity point on the image plane using $p = M \cdot P_W$, we get

$$(250 \quad -500\sqrt{3}+250 \quad 1/2)^T$$

in homogeneous coordinates.

Converting back to normal coordinate system, we get the coordinates for the vanishing point:

$$(500, \ -1000\sqrt{3}+500).$$

iii) Similarly, using infinity point $(x_0 \ 0 \ z_0 \ 0)^T$, where $\vec{r} = (x_0 \ 0 \ z_0)^T$ is the direction of the lines satisfying $\|\vec{r}\| = 1$, we get the homogeneous coordinates of the vanishing point:

$$(1000x_0 + 250\sqrt{3}z_0 \quad (500+250\sqrt{3})z_0 \quad \tfrac{\sqrt{3}}{2}z_0)^T.$$

Converting back to normal coordinate system, we get

$$\left(\frac{2000\sqrt{3}}{3} \cdot \frac{x_0}{z_0} + 500, \ 500 + \frac{1000}{3}\sqrt{3}\right).$$

iv) Since all points in part (iii) are on the line $y = 500 + \frac{1000}{3}\sqrt{3}$, also, for all $x \in \mathbb{R}$, solving

$$\begin{cases} \frac{x_0}{z_0} = \frac{\sqrt{3}}{2000}(x-500) \\ x_0^2 + z_0^2 = 1 \end{cases}$$

will yield a solution for x_0 and z_0, whose vanishing point is at $(x, 500 + \frac{1000}{3}\sqrt{3})$.

Therefore, the equation of the horizontal line is

$$y = 500 + \frac{1000}{3}\sqrt{3}.$$