# Report Blatt 2

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## 1 Exercise 1

### 1.1 a

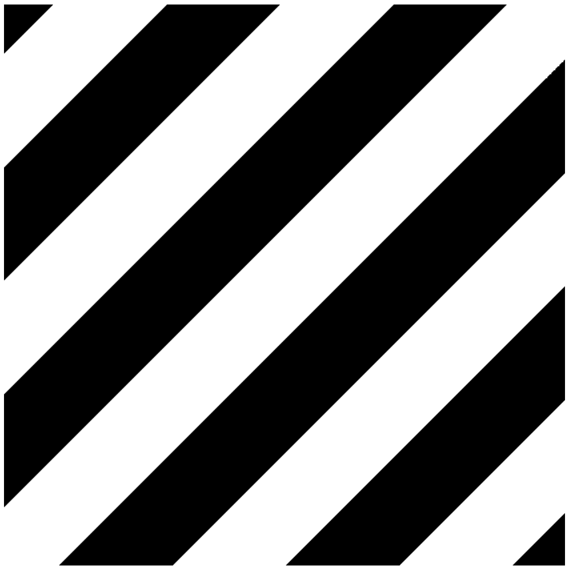
Since fourier transformed image consists of complex numbers, its magnitude and phase show the values of the phase and absolute length of these complex numbers. We've transformed the given image into the fourier domain (see Fig.1) and have displayed its phase and its magnitude specifically its *log-magnitude*, since the magnitude image itself shows a single white dot in the middle of a black picture. The phase image does not contain much interpretable information. The magnitude image, however, shows, which frequencies make up the original image in each direction. In our case, one can clearly see the diagonal line in the magnitude image, representing all the trigonometric functions that make up the striped pattern of the original image.

### 1.2 b

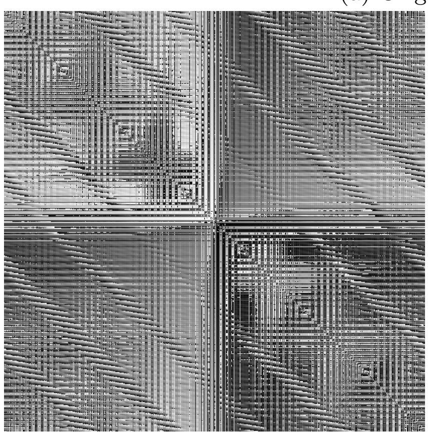
We have convolved the two images in Fig.2 and obtained the result image shown. Note the darker borders, where the convolution calculation includes pixels lying outside the source image, which are then valued 0.

### 1.3 c

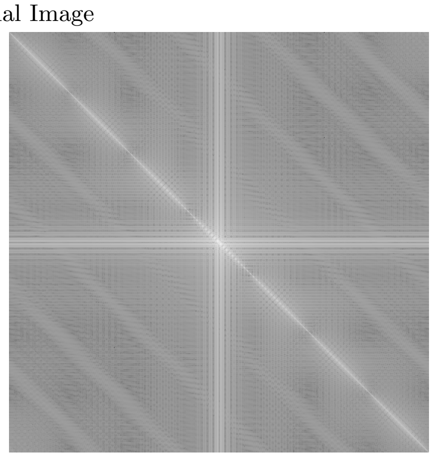
We have convolved the two images in Fig.3, but in the frequency domain. We have transformed both to the frequency domain, and then multiplied them element-wise. The result image can be obtained by transforming the outcome of the multiplication back to the spatial domain. Note that both methods of convolution produce the same result.



(a) Original Image



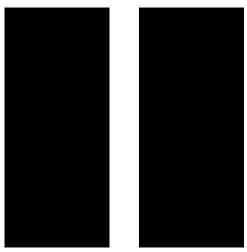
(b) Phase Image



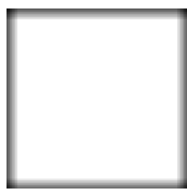
(c) Log Magnitude Image

Figure 1: Fourier transformation of a stripe pattern.

(a) Original Image 1



(b) Original Image 2



(c) Result after convolution

Figure 2: Spatial convolution of two images.

(a) Original Image 1 (b) Original Image 2 (c) Result after convolution

Figure 3: Frequency convolution of two images.

### 1.4 d

We have done the previous two steps for higher resolutions of the same images and compared execution times of spatial and frequency convolution. Times in table 1 are measured in ms.
Theoretically, the spatial convolution should be faster on small images, as the fourier transformation takes time, but slower on large images, as it is less efficient than the frequency convolution complexity-wise.

### 1.5 e

In this excercise we were asked to apply an ideal low pass filter in the frequency domain to a striped image (see Fig. 1. Conceptually the striped can be thought of as square wave. The equation for a 1D square wave is as follows:

$$x(t) = \frac{4}{\pi}(sin(2 * \pi * f * t) + \frac{1}{3}sin(6 * \pi * f * t) * ...) \tag{1}$$

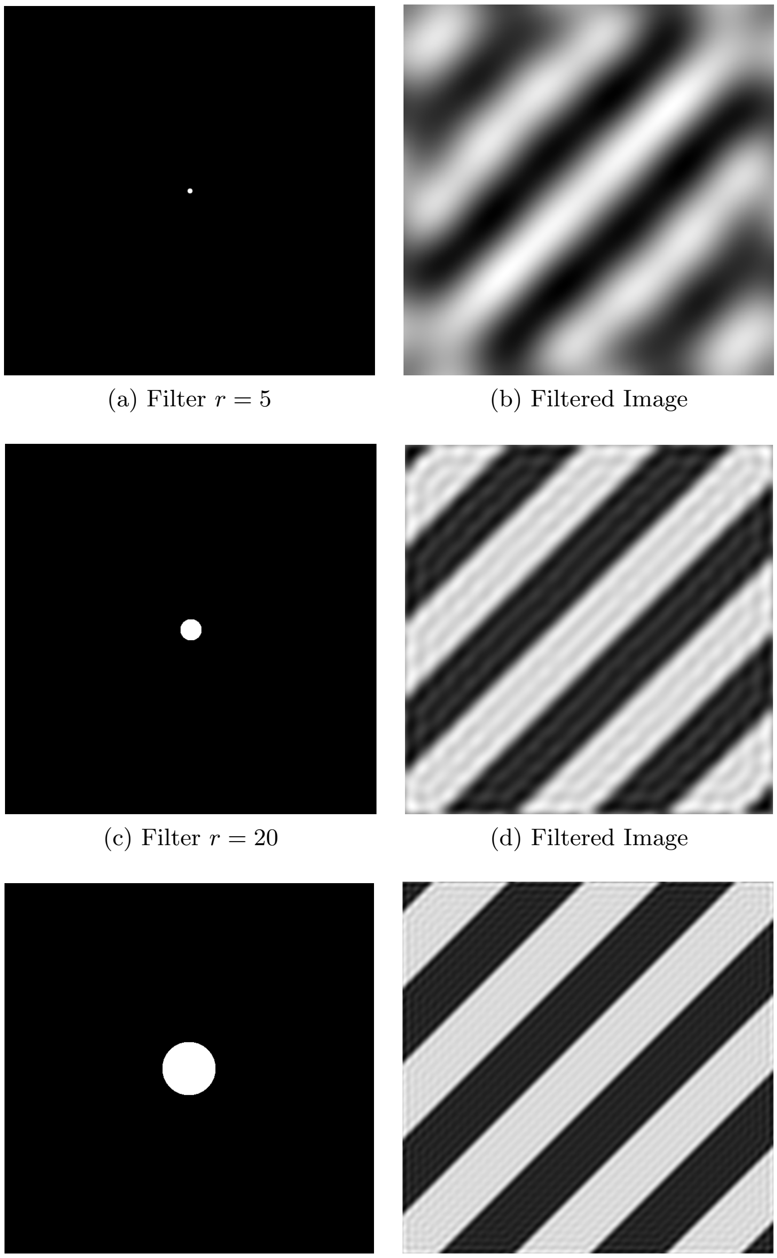
From this it becomes obvious, that removing the higher frequency terms will result in a smoother wave like pattern. This is what we observe in Fig. 4. If only very low frequencies are allowed to pass, such as in Fig.4a), the image becomes very blury (see Fig.4b)). Whereas for larger filters, effects are less pronounced (Fig.4e) and f)).

Also observed are ringing artefacts. These are due to the sharp cut off in the frequency domain filter, therefore inherent to the filter design. Ringing

| imagesize | spatial domain | frequency domain |
|---|---:|---:|
| 128x128 | 0 | 0 |
| 1000x1000 | 280 | 80 |
| 10000x10000 | 68090 | 9540 |

Table 1: Comparing spatial and frequency domain filtering speeds on differently resoluted images.

can be reduced by using filters which have a smoother cutoff edge.



(a) Filter $r = 5$ (b) Filtered Image

(c) Filter $r = 20$ (d) Filtered Image

(e) Filter $r = 50$ (f) Filtered Image

Figure 4: Application of a low pass filter to a striped image.The original image is a rotated square wave pattern. Removing higher frequencies results in a pattern resembling a smoother wave. Also notice the ringing artefacts.