Gradient et laplacien

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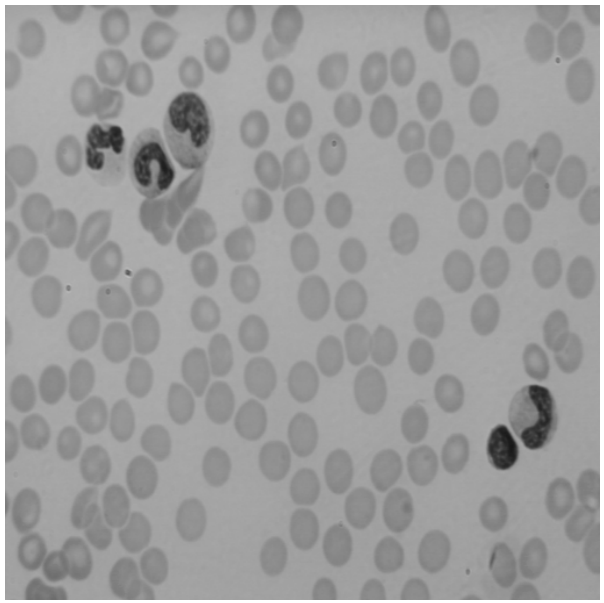
Segmentation des images : quels critères utiliser ?



Qu'est ce qu'un contour ?

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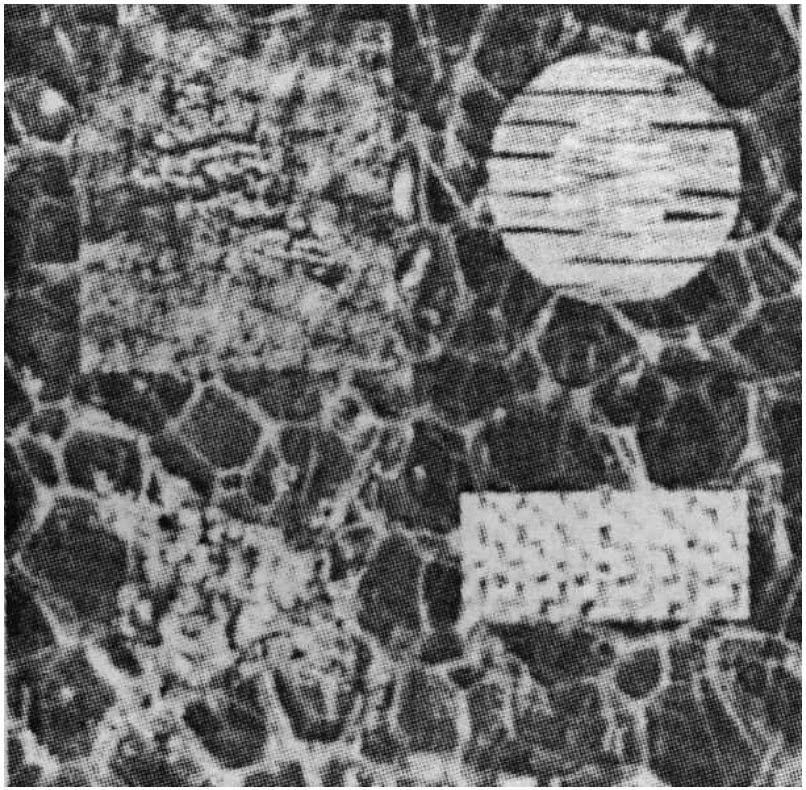
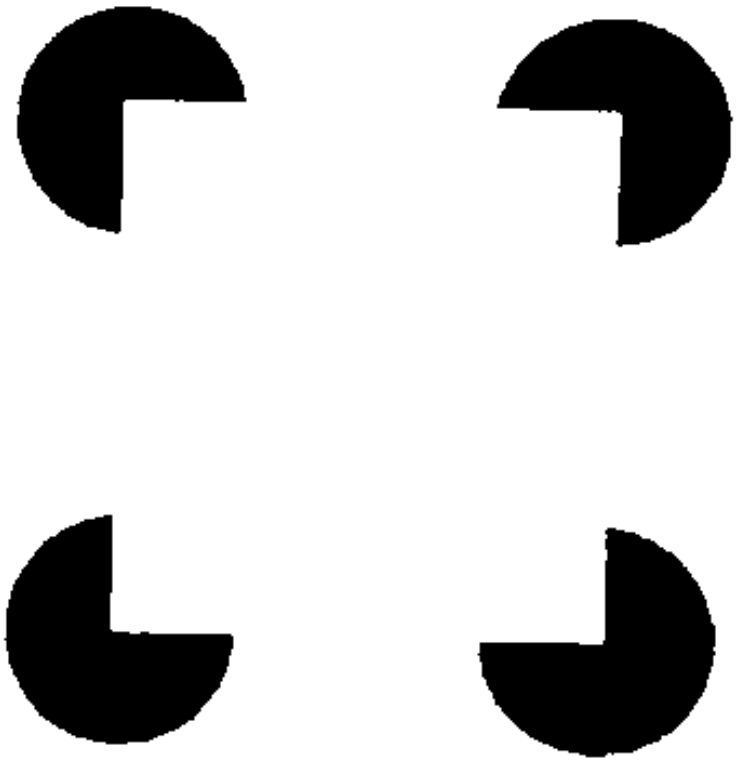
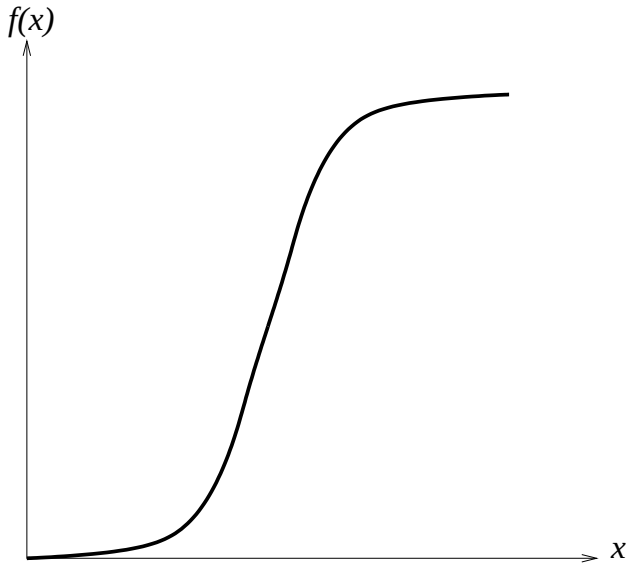
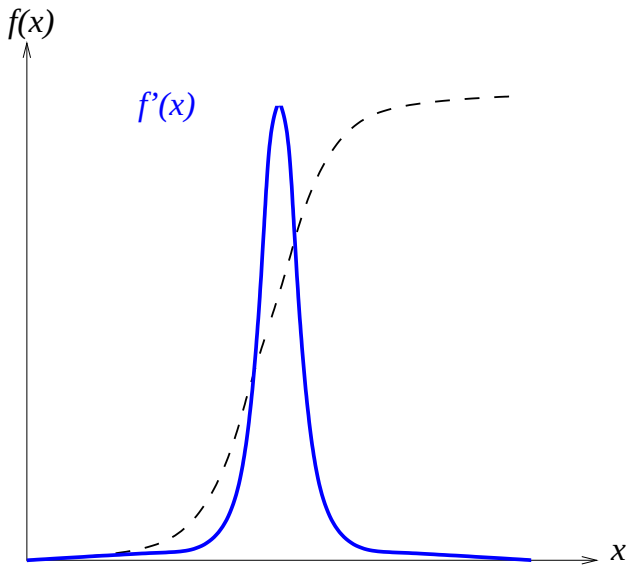




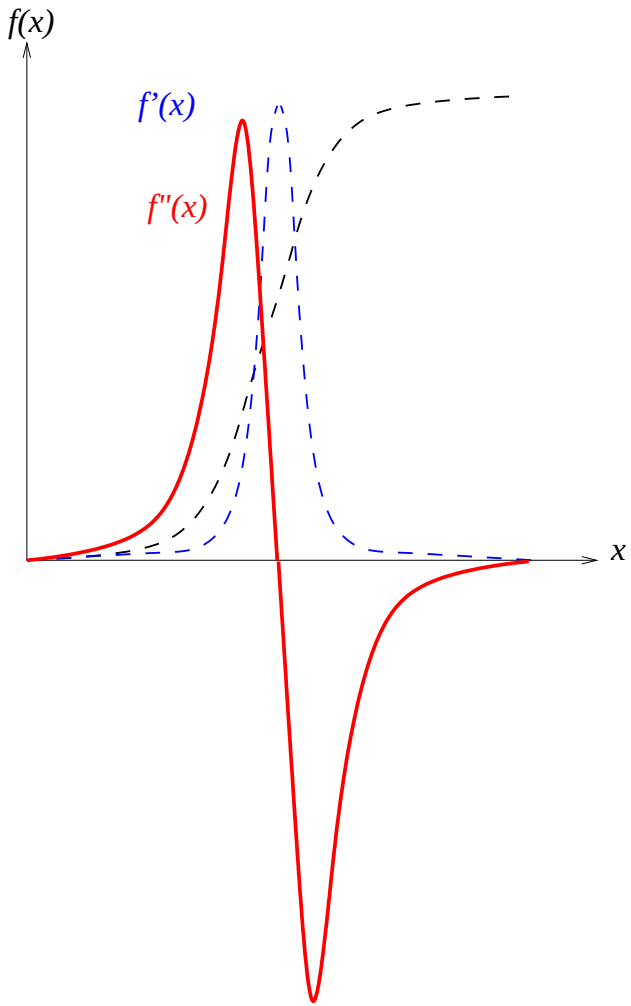
Contour idéal dans le cas continu



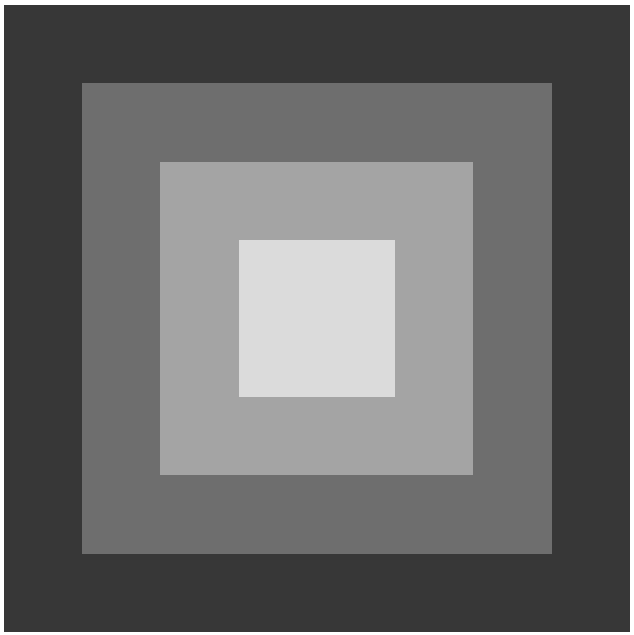
Contour idéal dans le cas continu : dérivée première

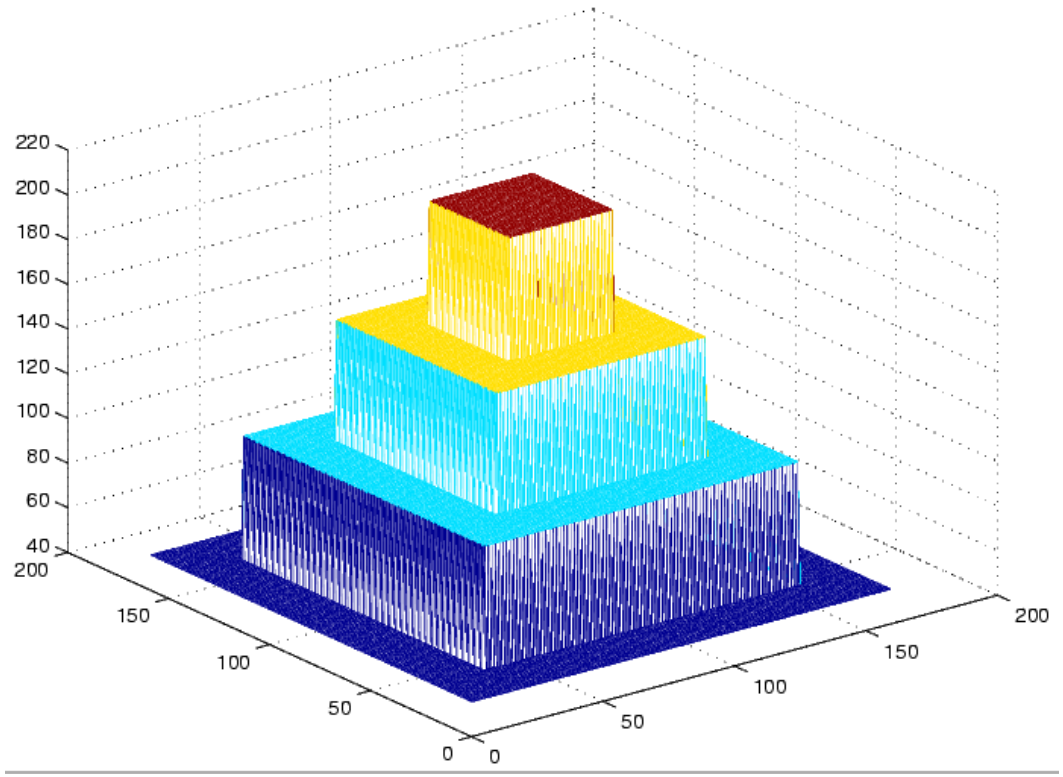


Contour idéal dans le cas continu : dérivée seconde



Cas d'une image





Approche continue des contours : gradient

image supposée continue $i(x, y)$

gradient

$$\overrightarrow{G} = \overrightarrow{\nabla i} = \begin{bmatrix} \frac{\partial i}{\partial x} \\ \\ \frac{\partial i}{\partial y} \end{bmatrix}$$

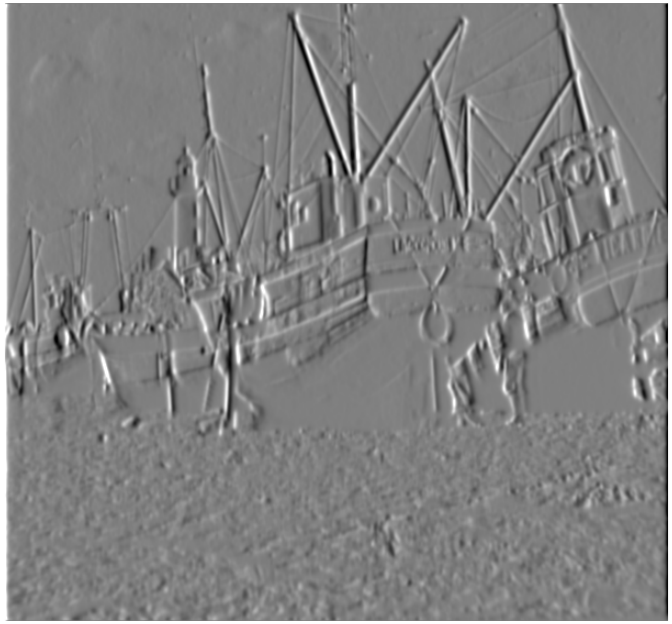
module du gradient

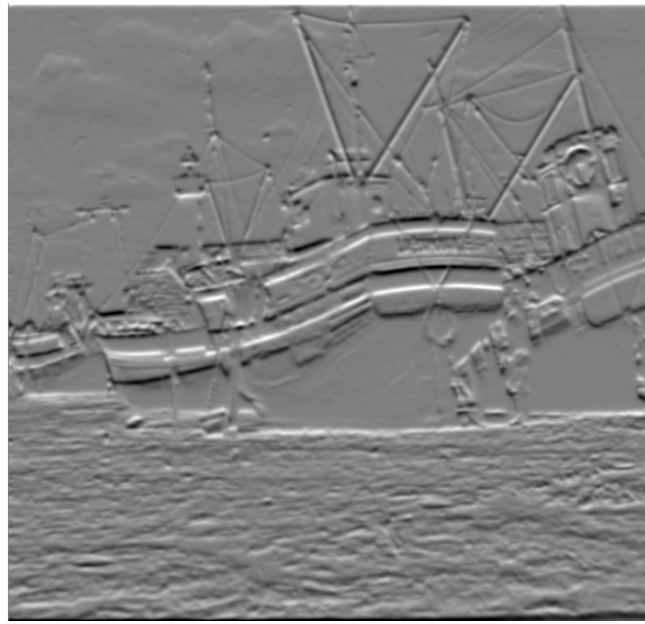
$$G = |\overrightarrow{\nabla i}| = \left[\left(\frac{\partial i}{\partial x} \right)^2 + \left(\frac{\partial i}{\partial y} \right)^2 \right]^{1/2}$$

direction du gradient

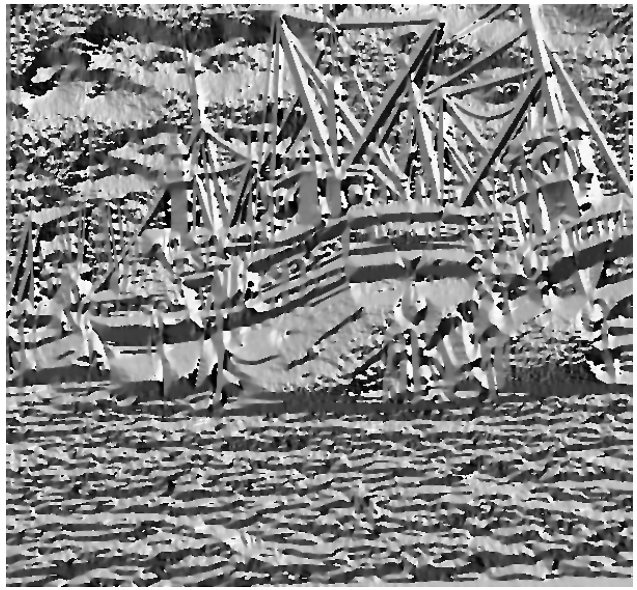
$$\overrightarrow{g} = \frac{\overrightarrow{\nabla i}}{|\overrightarrow{\nabla i}|}$$

Gradient : dérivées partielles suivant x et y





Gradient : norme et direction



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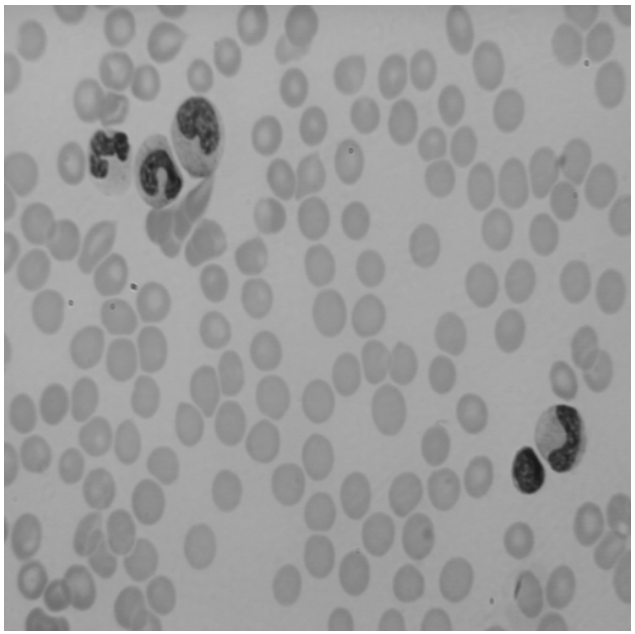
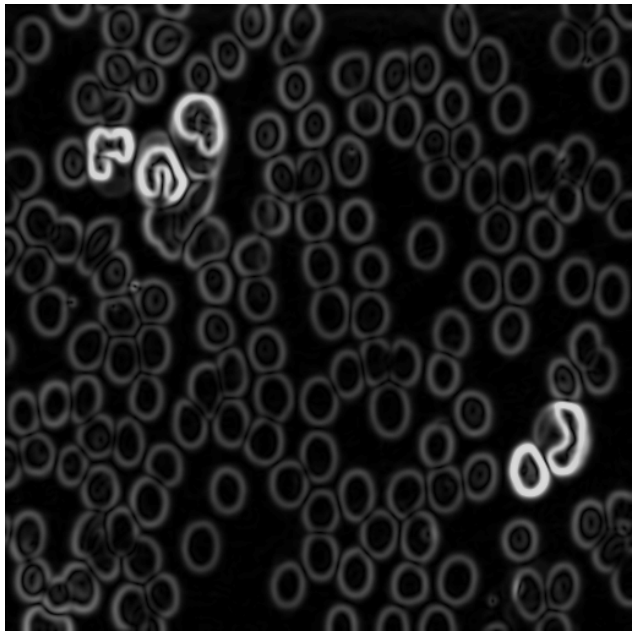
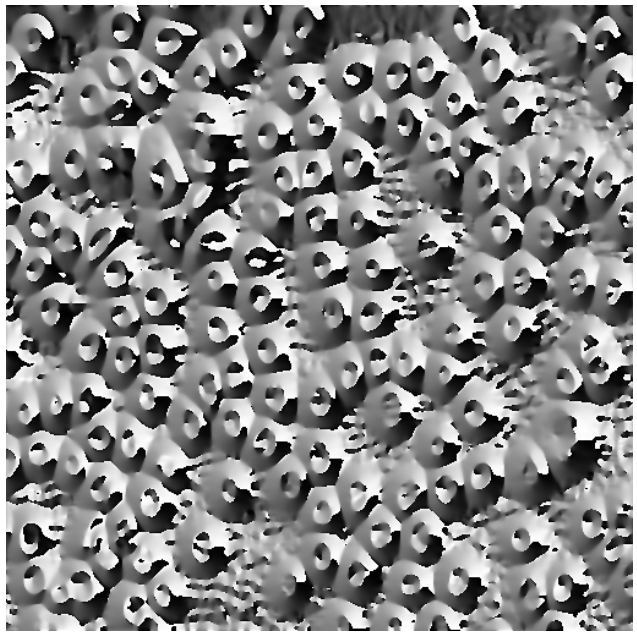






Approches classiques : gradient

- Approche dérivative : approximation discrète $\Rightarrow$ masques de gradient

différence

1	
−1	

Roberts

1	
	−1

Prewitt

1		−1
1		−1
1		−1

Sobel

1		−1
2		−2
1		−1

- Approximation de la norme :

$$\hat{G} = |\frac{\partial i}{\partial x}| + |\frac{\partial i}{\partial y}|.$$

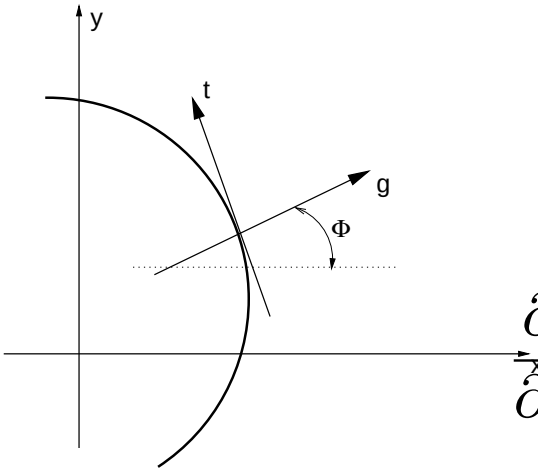
  - Pré-traitements : filtrage (médian, filtre bilatéral, ...)
  - Post-traitements : seuillage, max local du gradient, hystérésis, poursuite, fermeture

Norme du gradient : différence, Roberts, Prewitt, Sobel

Direction du gradient : différence, Roberts, Prewitt, Sobel

Contours : différence, Roberts, Prewitt, Sobel

Approche continue des contours : dérivée seconde



$$\Phi = Arctg[\frac{\frac{\partial i}{\partial x}}{\frac{\partial i}{\partial y}}]$$

$$\frac{\partial i}{\partial g} = \frac{\partial i}{\partial x}\cos\Phi + \frac{\partial i}{\partial y}\sin\Phi$$

- contour :

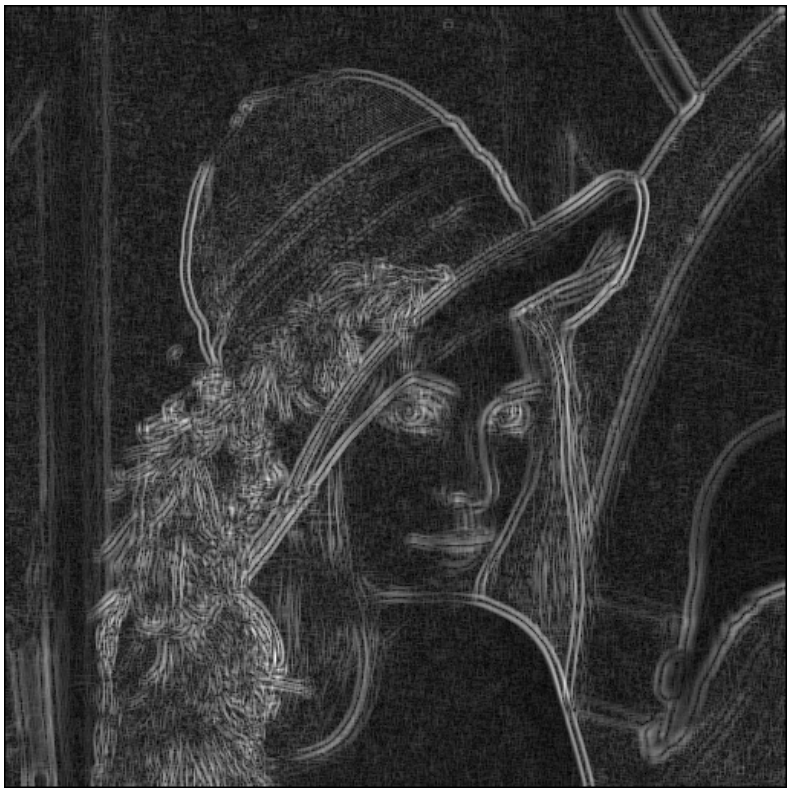
$$\frac{\partial^2 i}{\partial g^2} = 0 \Leftrightarrow \frac{\partial^2 i}{\partial x^2}\cos^2\Phi + 2.\frac{\partial^2 i}{\partial x \partial y}\cos\Phi\sin\Phi + \frac{\partial^2 i}{\partial y^2}\sin^2\Phi = 0$$

- Laplacien :

$$\Delta i = \frac{\partial^2 i}{\partial g^2} + \frac{\partial^2 i}{\partial t^2} = \frac{\partial^2 i}{\partial x^2} + \frac{\partial^2 i}{\partial^2 y}$$

$\frac{\partial^2 i}{\partial t^2} \approx 0$ (courbure très faible) $\Rightarrow \Delta i = 0 \Leftrightarrow \frac{\partial^2 i}{\partial g^2} = 0$

Dérivée seconde : le laplacien



Utilisation du laplacien

image originale - filtrage - laplacien - binarisation (+/-) - contours fermés





Laplacien

- Intérêt : réseau de lignes fermées (frontières positif / négatif)
- Masque :

	-1	
-1	4	-1
	-1	

- Inconvénient : grande sensibilité au bruit : fort filtrage

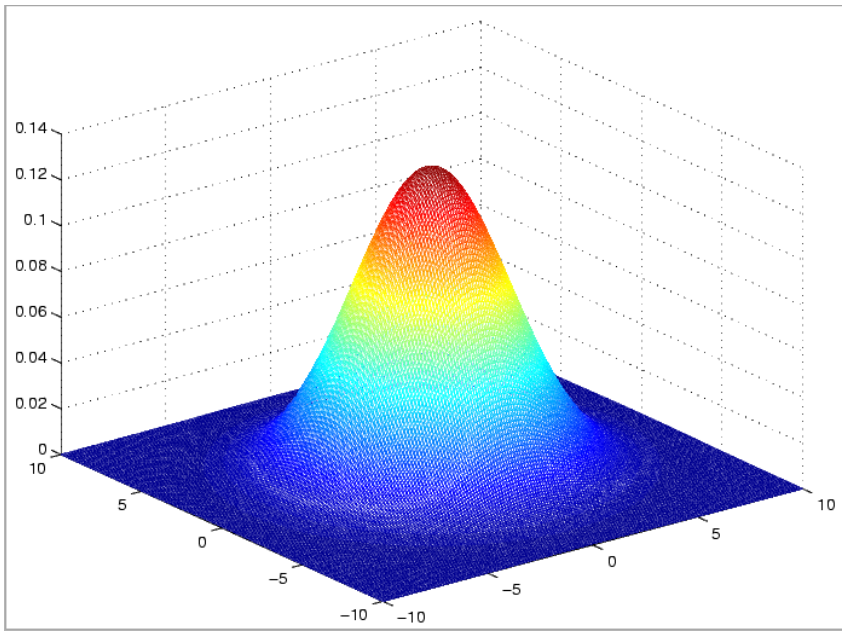
$$contour = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) [filtre\ passe\ bas * image]$$

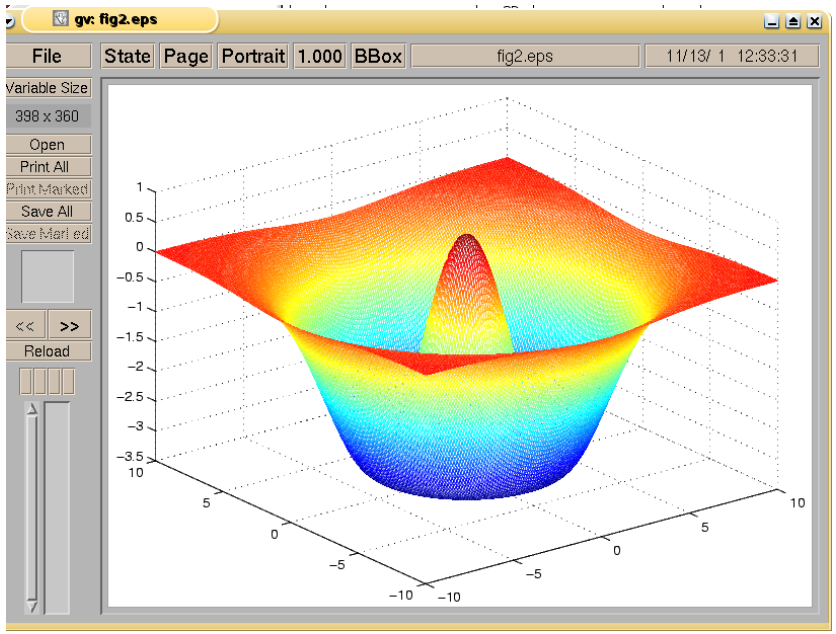
$$contour = image * \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) [filtre\ passe\ bas]$$

$$g(x,y) = \frac{1}{2\pi\sigma^2} \exp(-\frac{(x^2+y^2)}{2\sigma^2})$$

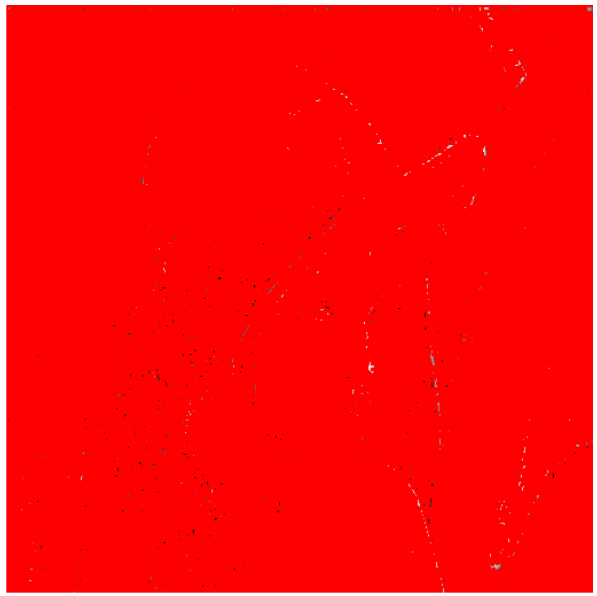
$$\Delta g = \frac{4}{2\pi\sigma^2}(\frac{x^2+y^2}{2\sigma^2} - 1) \exp(-\frac{(x^2+y^2)}{2\sigma^2})$$

Filtre gaussien et son laplacien





Exemple de filtre LoG ($\sigma = 0$)



Exemple de filtre LoG ($\sigma = 1$)



Exemple de filtre LoG ($\sigma = 2$)



Exemple de filtre LoG ($\sigma = 3$)



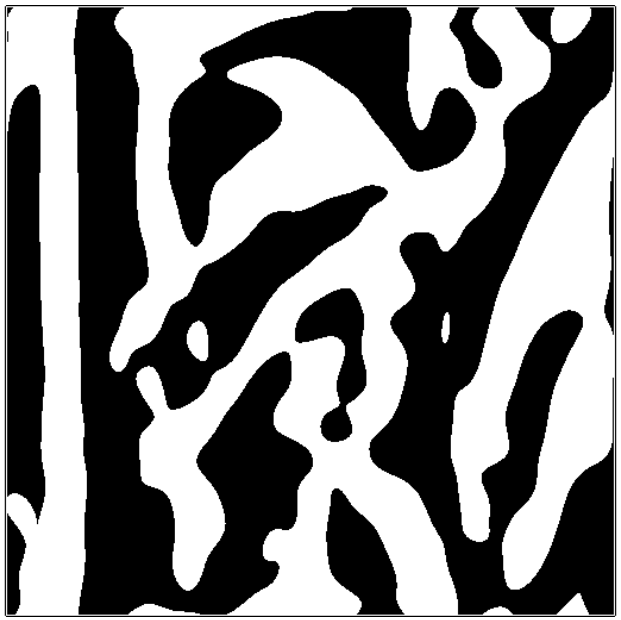
Exemple de filtre LoG ($\sigma = 5$)



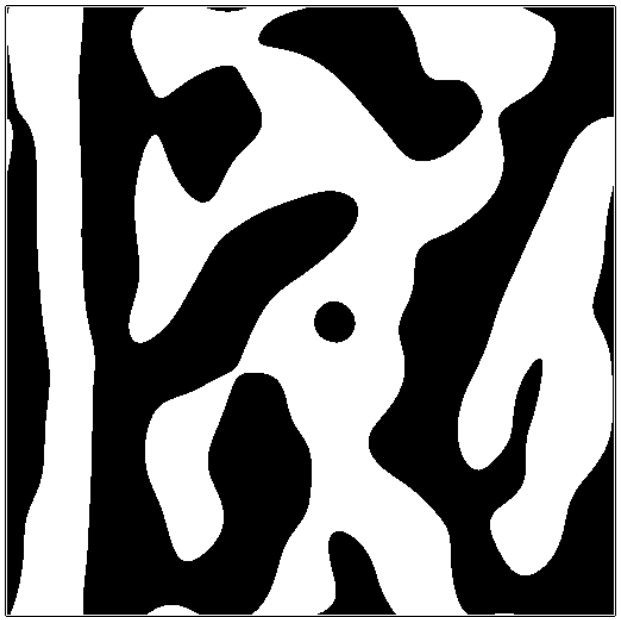
Exemple de filtre LoG ($\sigma = 10$)



Exemple de filtre LoG ($\sigma = 15$)


Exemple de filtre LoG ($\sigma = 25$)


Exemple de filtre LoG ($\sigma = 50$)