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Research Article

Economics

Stochastic Dynamic Optimization for Forest Rotation with Uncertain Stumpage Prices

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Abstract

This study investigates the effect of uncertain stumpage prices on the optimal forest rotation decision and related profitability under various silvicultural scenarios. The study applies loblolly pine (*Pinus taeda* L.) growth and yield models with distinct silvicultural scenarios in the Piedmont and Upper Coastal Plain region of the southeastern United States. It then applies chance-constrained dynamic optimization to derive optimal rotation age and related profitability on per acre basis. The growth and yield models show that using a combination of herbicide and fertilizer silvicultural treatment leads to a higher timber yield and profit at a lower optimal rotation age than other treatment scenarios. Furthermore, the stochastic optimization model shows that low risk tolerance results in low returns. Within a particular risk tolerance level, silvicultural treatment options with higher standard deviations almost always produce higher returns. The findings of this article contribute to the literature on optimizing forest management when considering uncertain stumpage prices over time, given different risk tolerance levels.

Study Implications: This study applies a stochastic dynamic mathematical programming method to evaluate the impact of uncertain stumpage prices on the optimal forest rotation decision and related profitability under different silvicultural and risk preference scenarios. The implication of this model demonstrates a tradeoff between maximizing average returns and minimizing the uncertainty of returns. The results suggest a risk-averse landowner would likely wait longer to realize greater sawtimber yield to assure a positive profit as sawtimber is the most valuable timber relative to the other two products considered, chip-N-saw and pulpwood.

Key words: stochastic, dynamic optimization, chance constraint, southern United States, stand management

Uncertainty[1] in roundwood prices plays a crucial role in determining optimal rotation ages in forest management (Forboseh et al. 1996, Susaeta and Gong 2019). Due to unanticipated demand shifts, roundwood prices fluctuate from year to year (Brazee and Mendelsohn 1990). Ideally, the general harvest decision rule is that owners should harvest a forest stand when their bundles of roundwood products reach the maximum value given market prices. Nevertheless, forest owners may know current prices but remain uncertain about prices in a future year. In this context, data-driven adaptive rotation strategies are needed, which could account for volatility in prices over time for informed decision-making.

To date, various studies have assessed the effect of uncertain roundwood prices on forest management from different perspectives. Many of them investigate optimal rotation strategies by applying the reservation price model, where a forest owner decides whether to harvest a forest stand by comparing the market price and an age-dependent reservation price (Brazee and Mendelsohn 1988). Other studies have used this approach to investigate stochastic prices with multiple timber products (Clarke and Reed 1989, Haight and Holmes 1991, Haight and Smith 1991, Forboseh et al. 1996), optimal thinning strategies (Brazee and Bulte 2000, Lu and Gong 2003), nontimber benefits (Gong et al. 2005), catastrophic events (Susaeta and Gong 2019), and forest tax (Gong and Susaeta 2020).

Risk preferences of forest owners play a vital role in determining rotation age in light of uncertain roundwood prices relative to various forest management options. Gong (1998) incorporated stochastic stumpage prices of Scots pine (*Pinus sylvestris* L.) and forest owners' risk preferences into forestry decisions and found that risk-averse forest owners prefer to harvest earlier than risk-neutral forest owners. Gong and Löfgren (2008) investigated the effect of stochastic Scots pine and Douglas-fir (*Pseudotsuga menziesii*) stumpage prices on the rotation length. Their results showed that the optimal rotation under risk aversion might be shorter than, equal to, or longer than the corresponding optimal rotation under risk neutrality, depending on the interest rate and the actual regeneration cost. Pukkala and Kellomäki (2012) applied a stochastic adaptive optimization to investigate Norway spruce (*Picea abies*) and Scots pine and birch (*Betula pubescens* L.) rotation when price and tree growth are stochastic. They

[1]Note that the definition of uncertainty and risk in this article follows Park and Shapira (2017), which assumes uncertainty is the situation under which the parameter distribution information is unknown by a decision-maker, whereas risk is the situation under which the parameter distribution information is known by a decision-maker.

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found that risk related to timber price and tree growth tended to increase the rotation length. Pukkala (2015) studied the effect of uncertain timber prices, tree growth, and regeneration on boreal forests and showed that increased risk aversion of the forest owners leads to earlier cutting in mature forest lands. Nevertheless, to date, much less is known about the effect of uncertain timber prices on optimizing rotation strategies under different silvicultural treatments and risk preferences for loblolly pine (*Pinus taeda* L.), which is a commercially important tree species in the southern United States (Oswalt et al. 2019).

Pine plantations are vital to land uses, welfare of forest owners, and the overall economy in the southern United States. Specifically, loblolly pine, along with shortleaf pine (*Pinus echinata*), covers 64 million acres in the region (Oswalt et al. 2019). As a result, loblolly pine is a critical species in maintaining the leadership position of the southern United States as a roundwood supplier at the national level. Approximately 55% of loblolly pine is planted and the rest is natural (Oswalt et al. 2019). Private forest owners manage loblolly pine plantations for economic benefits and plan for harvest at an age when forecasted profits are highest.

To quantify risks associated with uncertainties in an optimization problem, the chance-constrained optimization modeling approach (Charnes and Cooper 1959, 1962) has been developed and used in retirement financial planning (Booth 2004), supply chain management (Wu and Olson 2008, Bilsel and Ravindran 2011), climate and energy optimization (Held et al. 2009), project planning in construction (Gurgur and Morley 2008, Wibowo and Kochendoerfer 2010), storm and flood prevention (Jacobs et al. 1997, Wang et al. 2015), and water quality management (Jia and Culver 2006, Liu et al. 2008, Zhang et al. 2011, Zhu et al., 2019). The use of chance-constrained programming-based models provides several advantages. First, they allow researchers to assess the decision problem as a stochastic condition under the prescribed levels of probability constraint (Charnes et al. 1958, Charnes and Cooper 1959). Second, they provide computational advantages by converting a nonlinear mathematical programming problem to a linear problem (Charnes and Cooper 1962). Finally, they are flexible enough where stochasticity can be introduced to different components (e.g., left-hand-side constraint, right-hand-side constraint, or objective function) of an optimization problem (Olson and Wu 2017).

In this context, this study aimed to use an established loblolly pine growth and yield model system with distinct silvicultural treatment scenarios to investigate the effect of uncertain roundwood prices on the optimal forest rotation decision, and in the process, ascertain economic values of harvest using a chance-constrained stochastic optimization approach under different risk preferences of a forest owner in the southern United States. Thus far, the application of a chance-constrained stochastic optimization model in evaluating risk management for optimal forest rotation has not been investigated. Combining a biophysical growth and yield model system with a stochastic economic optimization model for forest management can provide informative insights into a forest owner's expected profits and potential losses given a risk preference.

Methods

To investigate the effect of roundwood price risk on the optimal rotation decision, we applied stand-level growth and yield models under different silvicultural treatments (i.e., herbaceous weed control and fertilization) and various risk tolerance levels. The ensuing subsection presents the growth and yield models for loblolly pine in the Piedmont and Upper Coastal Plain. Descriptions articulating the deterministic and stochastic form of the economic optimization model follow.

Growth and Yield Models for Loblolly Pine in Upper Coastal Plain

The growth and yield model system for loblolly pine in the Piedmont and Upper Coastal Plain used in this study was developed at the Plantation Management Research Cooperative housed at the University of Georgia Warnell School of Forestry and Natural Resources (Harrison and Borders 1996). The details of the same are reported in this subsection for clarity.

The dominant height projection equation is

$$HD_t = SI_{25}\left[\frac{0.30323}{1-exp(-0.014452\cdot t)}\right]^{-0.8216} + RZ_t^{HD} + RH_t^{HD},\ \forall t \tag{1.1}$$

where subscript t indicates a stand age, from 0 to 70 years. HD_t denotes the average dominant height in feet at stand age t, and SI_{25} is a site index defined as the average height of dominant and codominant trees at base age 25 years (set at 75 feet in this study). RZ_t^{HD} and RH_t^{HD} are adjusted dominant height (in feet) responses to fertilizers and herbaceous weed controls at stand age t and can be expressed as follows:

$$RZ_t^{HD} = (0.00106N + 0.2506P)\cdot(t-8)\cdot exp\left[-0.1096\,(t-8)\right],\ \forall t \tag{1.2}$$

$$RH_t^{HD} = (t-1)\left(\frac{R_{max}}{T_{max}}\right) exp\left[1-0.25\cdot HW^{(-0.5833+0.01667SI_{25})}\right]\cdot exp\left[-(t-1)\,(1/T_{max})\right],\ \forall t \tag{1.3}$$

where N denotes nitrogen fertilization (in pounds of elemental nitrogen fertilization per acre) and P denotes phosphorous fertilization, an indicator variable equal to 1 if phosphorous fertilizers are applied and equal to 0 otherwise. The fertilization, following regional practice, is assumed to take place in year 8. HW is the proportion of hardwood at a basal area, and its default value is 0.1 in this study. R_{max} is the maximum growth response (in feet) to herbaceous weed control and T_{max} denotes the age (in years) to reach R_{max}. R_{max} and T_{max} assume underlying site indices extending from 50 to 80 feet, and herbaceous weed control is assumed to take place at the beginning of planting (Borders et al. 2004). If fertilization or herbaceous weed control treatment is not applied, then RZ_t^{HD} and RH_t^{HD} are dropped from equation (1.1).

We assumed a planting density of 726 seedlings/acre in year 1, which drops to 680 seedlings/acre in year 2. The survival function measuring the number of survival trees per acre at each stand age t post-year 2 (TPA_t) is given below:

$$TPA_t = 100 + \left[(TPA_{t-1}-100)^{-0.745339} + 0.0003425^2\cdot SI_{25}\cdot\left(t^{1.97472}-(t-1)^{1.97472}\right)\right]^{\frac{-1}{0.745339}},\ \forall t>2 \tag{2}$$

The prediction of the basal area in the first three stand ages is formulated as

$$BA_t = \exp\left(-0.855557 - \frac{36.050347}{t}\right)\cdot TPA_t^{0.299071+3.309212/t}\cdot HD_t^{0.980246+3.787258/t},\ \forall t\le 3 \tag{3.1}$$

where BA_t denotes the basal area in ft[2] per acre at stand age t. Equation (3.1) allows us to calculate BA_t without knowing BA_{t-1}. Once BA_{t-1} is computed, the projection equation for the basal area after stand age 3 takes the following form:

$$BA_t = BA_{t-1} \cdot exp\left[-36.050347\left(\frac{1}{t} - \frac{1}{t-1}\right)\right] \cdot TC_t \cdot exp\left(RZ_t^{BA}\right), \forall t > 3 \quad (3.2)$$

where

$$TC_t = \left(\frac{TPA_t}{TPA_{t-1}}\right)^{0.299071} \cdot \left(\frac{HD_t}{HD_{t-1}}\right)^{0.980246} \cdot \frac{TPA_t^{3.309212/t}}{TPA_{t-1}^{-3.309212/(t-1)}} \cdot \frac{HD_t^{3.787258/t}}{HD_{t-1}^{-3.787258/(t-1)}} \quad (3.3)$$

RZ_t^{BA} is an adjusted basal area (in feet[2] per acre) response to fertilizers at stand age t. If fertilization treatment is not applied, then RZ_t^{BA} is dropped from equation (3.2). RZ_t^{BA} is formulated as

$$RZ_t^{BA} = (t-8) \cdot (0.0121N + 1.3639P) \cdot exp\left[-0.2635(t-8)\right], \ \forall t \quad (3.4)$$

Whole stand yield prediction for outside bark total volume at stand age t is expressed as

$$Y_t = HD_t^{0.268552+8.934524/t} \cdot BA_t^{1.368844+3.553411/t} \cdot TPA_t^{-7.466863/t} \quad (4)$$

where Y_t denotes stem volume (in ft[3] per acre) at stand age t. Whole stand yield can be further broken down by each merchantable timber product defined by a top diameter (TD) and a diameter at breast height (DBH) threshold limit (BD) at stand age t:

$$y_{w,t} = Y_t \cdot exp\left[-0.982648 \cdot \left(\frac{TD}{D_t}\right)^{3.991140} - 0.748261 \cdot TPA_t^{-0.111206} \cdot \left(\frac{BD}{D_t}\right)^{5.784780}\right], \ \forall t \quad (5)$$

where $y_{w,t}$ is a merchantable yield for timber product w at stand age t (in ft[3]/acre), TD is a top diameter of outside bark limit (in inches), BD is a DBH limit (in inches), and D_t denotes quadratic mean DBH (in inches). Sawtimber consists of trees larger than 11.5" at DBH and to a top diameter of 8". Chip-n-Saw consists of trees at DBH of 8.5" and to a top diameter of 4". Pulpwood consists of trees at DBH of 4.5" and to a top diameter of 2". Note that the applied growth and yield model does not account for thinning.

Deterministic Economic Modeling

To estimate the maximum harvesting profit at the optimal rotation age, we first develop the following deterministic objective function:

$$\max_y \pi = \sum_w (1+r)^{-t}\bar{p}_w y_{w,t} - c^c - c^p - c^s - c^z \cdot N \cdot (1+r)^{-8} - c^h \quad (6)$$

where π refers to the net present value (NPV) of annual harvesting profit (in US$ per acre), $\bar{p}_w$ is the past 45-year (1976–2020) average timber product price for each wood product w (in US$ per ft[3]), and r is the real discount rate (set to 4% in this study[2]). We use stumpage prices for our analysis. Costs (in US$ per acre) incurred by the forest owner consist of five types: c^c is the chemical site preparation cost, c^p and c^s refer to respective planting and seedling cost, c^h is the herbicide cost after planting, and c^z is the fertilization cost. Nitrogen fertilizer application (N), in accordance with regional practice, is assumed to take place in year 8, and therefore fertilization cost is discounted at year 8. The default amount for elemental nitrogen fertilization usage is 125 lb/ac. The default values for chemical, planting, seedling, fertilization, and herbicide costs are $77.6/ac, $86.8/ac, $60.5/ac, $0.25/lb, and $57.1/ac, respectively. Note that the conventional silviculture practice in the southern United States does not sell roundwood less than age 15 in the majority of cases. Therefore, our economic model imposes an age constraint restricting harvesting before year 15.

Stochastic Economic Modeling

Equation (6) can be modified to a stochastic model by incorporating the chance constraint formula (Olson and Wu 2017). As a result, the objective function takes the following form:

$$\max_y \pi = (1+r)^{-t}\left(\sum_w \bar{p}_w y_{w,t} - Z_\alpha \sum_{wi \le w} \sum_w \sqrt{y_{w,t} \cdot \sigma_{w,wi} \cdot y_{wi,t}}\right) - c^c - c^p - c^s - c^z \cdot N \cdot (1+r)^{-8} - c^h \quad (7)$$

where Z_α represents the one-tailed normal quantile at $1-\alpha$ evel confidence and acts as a risk discount factor, implying the risk preference related to variations of stumpage prices. $\sigma_{w,wi}$ is element of 3 × 3 variance-covariance matrix for stumpage prices. The stochastic objective function, equation (7), can reflect a decision-maker's desire to avoid risk by specifying the confidence limit α. Namely, α represents the reverse of risk tolerance. Uncertain stumpage prices may yield a substantial risk to a forest owner, contributing to volatility in harvesting profits over time. Thus, the expected annual profit determined by the level of assurance of a positive profit can be mathematically expressed as

$$P\left[E(\pi) \ge 0\right] \ge \alpha \quad (8)$$

where candidate α values are 0.5, 0.8, 0.9, and 0.95, following Olson and Wu (2017). Varying α from 0.5, 0.8, 0.9, and 0.95 indicates that a probability of having a positive annual harvesting profit is greater than 0.5, 0.8, 0.9, and 0.95, respectively. When α = 0.95, it implies that the decision-maker could only tolerate a 5% chance of losing money. When α is 0.5, equation (7) is equivalent to equation (6) because $Z_{0.5}$ implies a forest owner is risk neutral, and there is a fifty-fifty chance that the expected profit is positive (i.e.,$Z_{0.5} = 0$). Namely, if α is 0.5, the resulting objective function is deterministic, for which no assurance for having a positive profit is specified. Note that this stochastic optimization model assumes roundwood prices are normally distributed around the average roundwood prices[3]. Table 1

[2]The assumed discount rate in our study is based on a range of 3% to 7% commonly applied in social discounting of environmental investments (Office of Management and Budget 2003).

[3]Only historical prices for pulpwood were normally distributed. Although stumpage prices for sawtimber and chip-n-saw were not found normally distributed, our sample was sufficient in size to ensure the distribution of sample means for these product types converged to a normal distribution by the central limit theorem.

shows the range of treatment and risk-preference scenarios considered in this study, whereas Table 2 summarizes all mathematical notations used in the above dynamic optimization model.

Modeling Procedure

The deterministic dynamic optimization modeling process is as below

1. Analyze the stumpage prices: The original timber roundwood price data was in nominal \$ per ton. We first converted it to \$ per cubic feet at the 2012 price level using a GDP implicit price deflator (Federal Reserve Bank 2021). Then we calculated the mean price of each roundwood product (i.e., sawtimber, chip-n-saw, and pulpwood) and their covariance (Table 3) using historical data from 1976 to 2020.
2. Formulate the dynamic optimization model: We formulated the deterministic objective function (equation 6) and seven loblolly pine growth and yield constraint functions, including dominant height function (equation 1.1), survival function (equation 2), basal area function (equation 3.1 or 3.2), whole stand yield function (equation 4), three merchantable stem volume functions for three roundwood products (equation 5).
3. Undertake the optimization modeling: We used the mean price for each roundwood product from step 1 and recursively solved the deterministic dynamic optimization modeling given four different silvicultural treatment scenarios (i.e., base, herbicide, fertilizer, and both herbicide and fertilizer) in the General Algebraic Modeling System. Namely, the value of survival trees and stem volumes at a later year is determined by its value at an earlier year. This recursive optimization process was solved for 70 years. Thus, 560 equations (one objective function and seven growth and yield constraint functions over 70 years) were generated in each scenario. After running the model, the maximum NPV, the optimal rotation age, and associated stand stem volumes (Table 4) were identified.

The stochastic (or chance-constrained) dynamic optimization modeling process is similar to the deterministic one, but instead of using equation (6) as the objective function, the stochastic dynamic optimization model uses equation (7) as the objective function and uses the timber price covariance from step 1 to capture the correlation among three timber products.

Results

Loblolly Pine Growth and Yield Simulation Results

Figure 1 illustrates the breakdown of yields of three roundwood products (i.e., sawtimber, chip-n-saw, and pulpwood) under alternate silvicultural scenarios across different stand ages. Figure 1A reveals that yield for each roundwood product is slightly higher under the herbicide scenario than the base scenario for which no treatment is applied. Pulpwood under the herbicide scenario reaches peak yield (2,819 ft^3/ac) at stand age 15, followed by chip-n-saw, which attains peak yield (4,309 ft^3/ac) at stand age 27 years. Sawtimber achieves peak yield (8,909 ft^3/ac) in the last year of our growth and yield simulation (at stand age 70 years).

Figure 1B presents the yield of roundwood products under fertilization treatment. The overall trend for each roundwood type remains similar to depictions in Figure 1A, but the gap between fertilizer and base scenarios is revealed as much larger than the gap between herbicide and base scenarios. For example, peak yield for pulpwood under the fertilizer treatment is 2,798 ft^3/ac at stand age 14 years, chip-n-saw is 5,228

Table 1. Description of scenarios selected for the study.

Treatment	Risk preference	Description
Base	0.5	No treatment applied and without assuring the profit
	0.8	No treatment applied and assure the probability of having a positive profit is > 0.8
	0.9	No treatment applied and assure the probability of having a positive profit is > 0.9
	0.95	No treatment applied and assure the probability of having a positive profit is > 0.95
Herbicide	0.5	Herbicide is applied and without assuring the profit
	0.8	Herbicide is applied and assure the probability of having a positive profit is > 0.8
	0.9	Herbicide is applied and assure the probability of having a positive profit is > 0.9
	0.95	Herbicide is applied and assure the probability of having a positive profit is > 0.95
Fertilizer	0.5	Fertilizer is applied and without assuring the profit
	0.8	Fertilizer is applied and assure the probability of having a positive profit is > 0.8
	0.9	Fertilizer is applied and assure the probability of having a positive profit is > 0.9
	0.95	Fertilizer is applied and assure the probability of having a positive profit is > 0.95
Herbicide + fertilizer	0.5	Herbicide and fertilizer are applied and without assuring the profit
	0.8	Herbicide and fertilizer are applied and assure the probability of having a positive profit is > 0.8
	0.9	Herbicide and fertilizer are applied and assure the probability of having a positive profit is > 0.9
	0.95	Herbicide and fertilizer are applied and assure the probability of having a positive profit is > 0.95

Note that risk preferences defined as α = 0.5 denote a deterministic model scenario (i.e., risk neutral scenario).

Table 2. Mathematical description of the optimization model.

Subscripts	
w	Timber products, including sawtimber, chip-n-saw, and pulpwood
wi	Alias of w used for the calculation with the stumpage price variance-covariance matrix
t	Stand age (in years, from 0 to 70)
Variables	
$y_{w,t}$ (ft^3/acre)	Harvesting yield for timber product$w$ at stand age$t$ (decision variable)
HD_t (feet)	Average dominant height in feet at stand age$t$
SI_{25} (feet)	Site index at base age 25 years(the default value is 75 feet)
RZ_t^{HD} (feet)	Adjusted dominant height responses to fertilizers at stand age$t$
RH_t^{HD} (feet)	Adjusted dominant height responses to herbaceous weed controls at stand age$t$
N (lb/acre)	Amount of elemental nitrogen fertilization used (the default amount is 125 kg/acre)
P (0 or 1)	An indicator variable equals to 1 if phosphorous fertilizers are applied and equals to 0 otherwise.
R_{max} (feet)	The maximum growth response
T_{max} (years)	The age to reach to the maximum growth response
HW (percentage)	Proportion of hardwood at a basal area (the default value is 0.1)
TPA_t (trees/acre)	Number of survival trees per acre at each stand age$t$
BA_t (sq. feet/acre)	Basal area at stand age$t$
Y_t (ft^3/acre)	Whole stand stem volumes at stand age$t$
$y_{w,t}$ (ft^3/acre)	Merchantable yield for timber product w at stand age$t$
RZ_t^{BA} (square feet/acre)	Adjusted basal area response to fertilizers at stand age$t$
TD (inches)	Top diameter limit for wood product class
BD (inches)	Diameter at breast height limit for wood product class
D_t (inches)	Quadratic mean diameter at breast height at stand age$t$
π_t (USD/acre)	Net present value of annual harvesting profit at stand age$t$
$\bar{p}_w$ (USD/ft^3)	Past 45-year (1976–2020) average timber product price for each wood product$w$
r (percentage)	Discount rate (the default value is 0.04)
n (years)	Number of years discounted
c^c (USD/acre)	Chemical site preparation cost ($77.6/acre)
c^p (USD/acre)	Planting site preparation cost ($86.8/acre)
c^s (USD/acre)	Seedling site preparation cost ($60.5/acre)
c^z (USD/kg)	Fertilization cost depending on the amount of elemental nitrogen fertilization used($0.25/kg)
c^h (USD/acre)	Herbicide cost ($57.1/acre)
$\sigma_{w,wi}$ (USD/ft^3)	3 by 3 variance-covariance matrix for stumpage prices
Z_α	One-tailed z score, which is 0, 0.842, 1.282, and 1.645 when α is 0.5, 0.8, 0.9, and 0.95, respectively.

Note: The cost (in 2012 US$ [USD]) for the silvicultural preparations and treatments was collected from Paudel and Dwivedi (2021). Stumpage prices (in 2012 US$) were retrieved from TimberMart-South (2021).

Table 3. Variance-covariance matrix for roundwood prices (1976–2020).

	Sawtimber	Chip-N-Saw	Pulpwood
Sawtimber	0.109	0.081	0.014
Chip-N-Saw	0.081	0.130	0.011
Pulpwood	0.014	0.011	0.012

Note: Roundwood prices were obtained from TimberMart-South (2021).

ft^3/ac at stand age 23 years, and sawtimber reaches 13,828 ft^3/ac at stand age 70 years. Figure 1C, showing both silvicultural treatments, looks nearly identical to Figure 1B, but with yields for each roundwood product slightly higher in Figure 1C. Under combined herbicide and fertilizer treatment, peak yields for pulpwood, chip-N-saw, and sawtimber are 2,969 ft^3/ac (at stand age 14), 5,476 ft^3/ac (at stand age 22), and 13,518 ft^3/ac (at stand age 70), respectively.

Economic Modeling Results

Integrating the economic analysis and loblolly pine growth and yield model allows us to investigate how stochastic stumpage prices affect the economic value of harvesting timber under different silvicultural treatments. Figure 2 shows the historical stumpage prices for each timber product from 1976 to 2020. Prices for the three roundwood products appear to have similar trends. Sawtimber has the highest price among the three roundwood products, followed by chip-n-saw and pulpwood. The average price per cubic foot for sawtimber, chip-n-saw, and pulpwood are $1.10, $0.78, and $0.36, respectively. The variance-covariance matrix for stumpage prices is presented in

Table 4. Summary of the results across selected scenarios.

Model	$P[E(\pi) \geq 0]$	Age	Height	Density	Basal Area	Volume	Sawtimber	Chip-N-Saw	Pulpwood	NPV
		years	*feet*	*trees/acre*	*ft²/acre*	*ft²/acre*	*ft³/acre*	*ft³/acre*	*ft³/acre*	*$/acre*
Base	0.5	32	88.38	337.9	197.34	7459.26	2550.79	3884.71	989.81	1561.26
	0.8	43	106.16	253.52	215.12	8578.29	5638.22	2485.26	439.63	471.47
	0.9	55	122.03	198.27	228.01	9276.35	7603.73	1448.25	216.64	107.78
	0.95	67	135.03	165.69	238.15	9738.62	8671.92	934.12	127.86	-71.36
Herbicide	0.5	30	87.03	357.88	198.72	7605.46	2249.25	4175.2	1141.7	1637.97
	0.8	41	104.65	265.99	215.73	8668.51	5422.25	2730.51	498.6	448.74
	0.9	54	121.61	201.83	228.62	9346.88	7605.32	1506.62	226.86	63.39
	0.95	67	135.42	165.69	238.88	9792.21	8727.11	932.86	127.54	-121.46
Fertilizer	0.5	29	83.67	368.45	253.24	10,774.65	5349.64	4462.39	930.74	2886.24
	0.8	38	98.94	286.92	275.81	12,333.91	9101.46	2767.55	449.15	1109.25
	0.9	47	112.02	231.7	290.74	13,261.39	11,314.13	1695.91	242.55	469.21
	0.95	57	124.46	191.61	303.52	13,947.57	12,735.75	1066.44	140.11	119.56
Herbicide+Fertilizer	0.5	28	84.27	379.41	253.7	10,967.52	5181.7	4735.83	1015.2	2978.54
	0.8	36	97.58	302.49	272.43	12,226.25	8557.25	3123.8	526.83	1073.97
	0.9	46	111.83	236.79	287.37	13,114.59	11,040.1	1803.5	261.55	407.03
	0.95	57	125.13	191.61	299.86	13,738.96	12,511.33	1079.79	142.49	55.54

Note: $P[E(\pi) \geq 0]$ denotes the level of reverse risk tolerance. It also implies the risk preference related to variations of roundwood prices. The assurance level of 0.5 denotes the deterministic model scenario (i.e., risk neutral scenario). Age denotes the optimal rotation age. Height denotes the average dominant height in feet at the optimal age. Density denotes the number of survival trees per acre at the optimal age. Basal denotes basal area at the optimal age. Volume denotes whole stand stem volumes at the optimal age. Sawtimber, Chip-n-Saw, and Pulpwood denote merchantable volumes for each timber type. NPV denotes the net present value of the resulting profit at the optimal age.

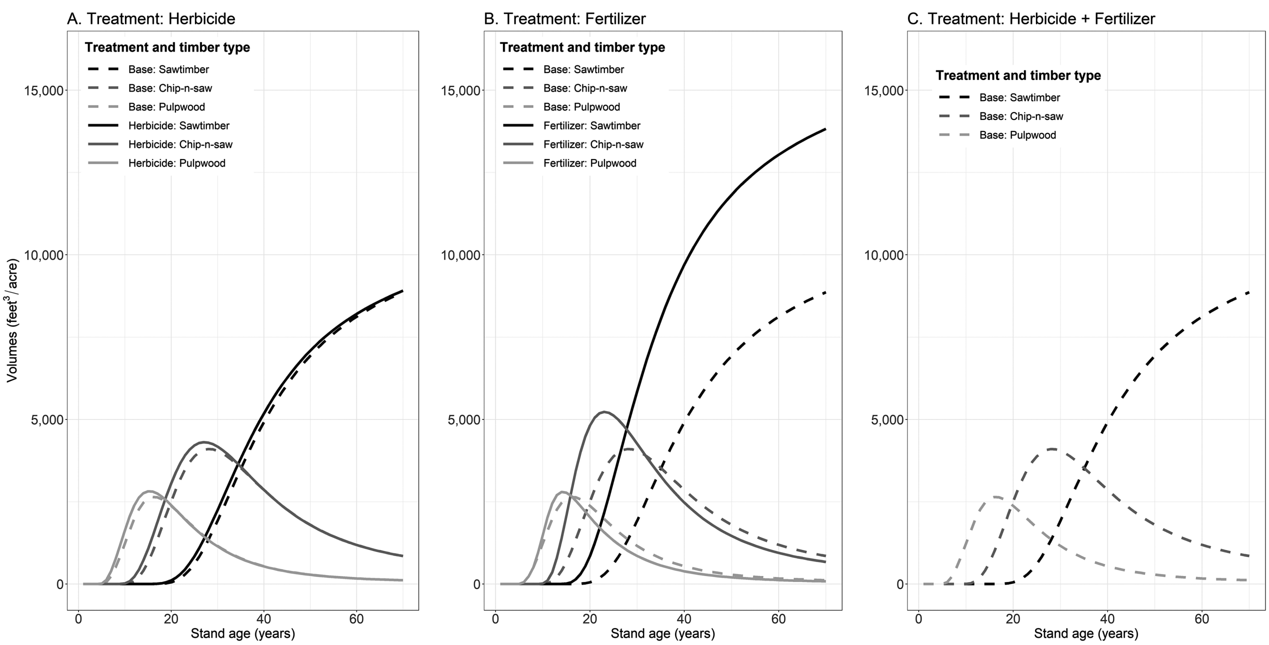

Figure 1. Yield of roundwood products under different silvicultural treatments across stand age. The y-axis denotes the harvested volume for each roundwood product (in *feet*³ *per acre*) and is the same across the three panels.

Table 3. Correlations among the three roundwood prices are as follows: *Corr*(*sawtimber, chip-n-saw*)=0.94, *Corr*(*sawtimber, pulpwood*)=0.39, *Corr*(*chip-n-saw, pulpwood*)=0.57. The Pearson correlation coefficient test suggests the correlations of the three price pairs are significant at the 1% significance level, highlighting the importance of including covariance of stumpage prices in the optimization model.

Figure 3 illustrates the NPV of annual profits under different silvicultural treatments and degrees of risk preferences. Intuitively, the NPV of the optimal profit at each stand age

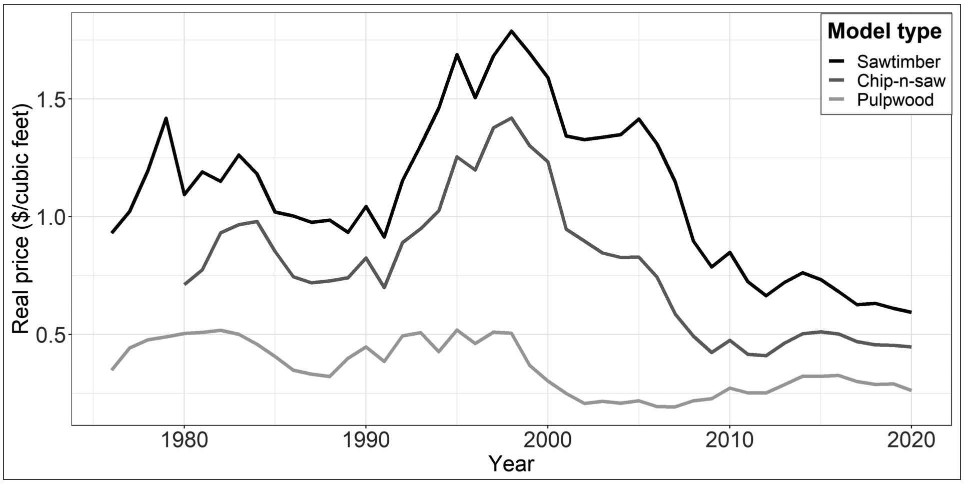

Figure 2. Historical stumpage prices (1976–2020). Stumpage prices (in 2012 US$) were obtained from TimberMart-South (2021), housed at the University of Georgia Warnell School of Forestry and Natural Resources.

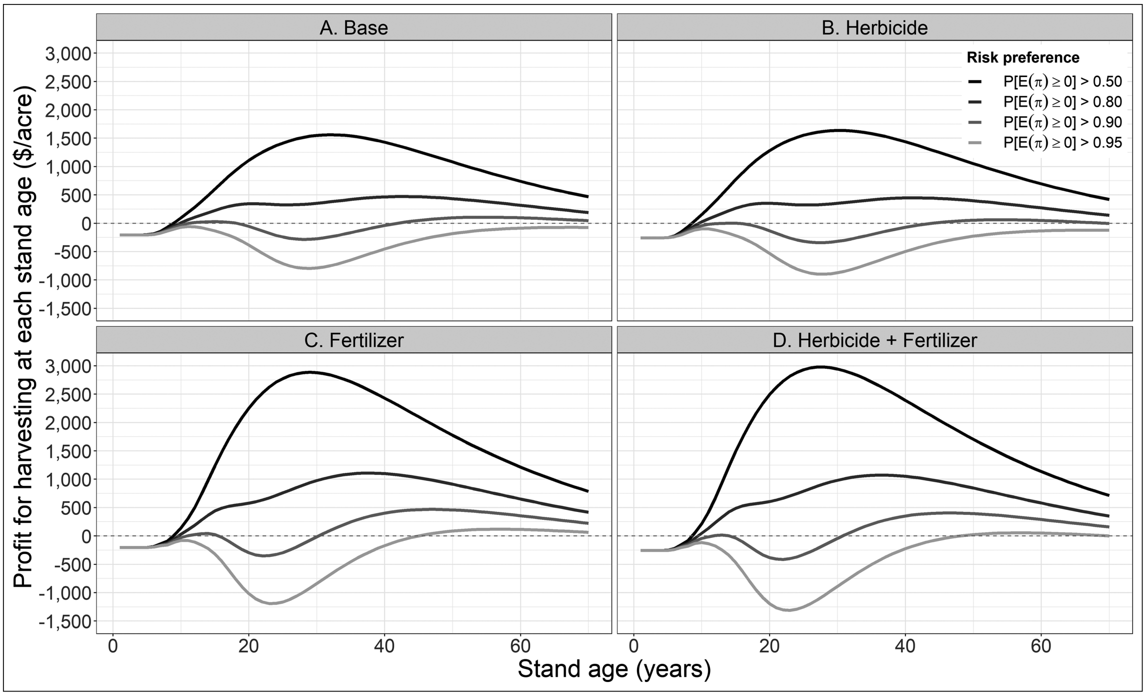

Figure 3. Net present value of profitability at each stand age (in 2012 U.S. dollars). $P[E(\pi) \geq 0]$ denotes the level of reverse risk tolerance. It implies the risk preference related to variations of stumpage prices. The assurance level of 0.5 denotes the deterministic model scenario.

is highly correlated with roundwood yields. Namely, the higher the growth in whole stand volume, the higher the annual profit incurred. Volumes of sawtimber and chip-n-saw are particularly influential in generating higher profits because these two timber products have higher prices than pulpwood. Therefore, the annual profit under the fertilizer scenarios, in which volumes of sawtimber and chip-n-saw are produced more and relatively faster, exhibits higher profits among all treatment scenarios, as shown in Figure 3C and D. Nevertheless, the sawtimber price also has a relatively higher variance among the three timber products. To find the optimal harvesting profit incorporated the price uncertainty, the chance-constrained optimization model maximizes the revenue of harvesting timbers while minimizing the variance of stumpage prices. Thus, there are some curvilinear patterns showing fluctuations of profits caused by varying shares of roundwood availability.

Comparing different levels of risk preferences shows that the optimal rotation age goes up with decreasing optimal annual profits when α increases. Specifically, without any silvicultural treatment, as α increases from 0.50 to 0.95, the optimal rotation age rises from year 32 to year 67, and the resulting optimal profit decreases from \$1,561/ac to –\$71/ac. Similarly, as α increases from 0.5 to 0.95 in the herbicide model, the optimal rotation age increases from year 30 to year 67; associated profits decrease from \$1,638/ac to –\$121/ac. The optimal rotation age increases from year 29 to year 57 for the fertilizer model and from year 28 to 57 for fertilizer/herbicide combined, respectively. Optimal profit decreases from \$2,886/ac to \$120/ac in the fertilizer model and \$2,979/

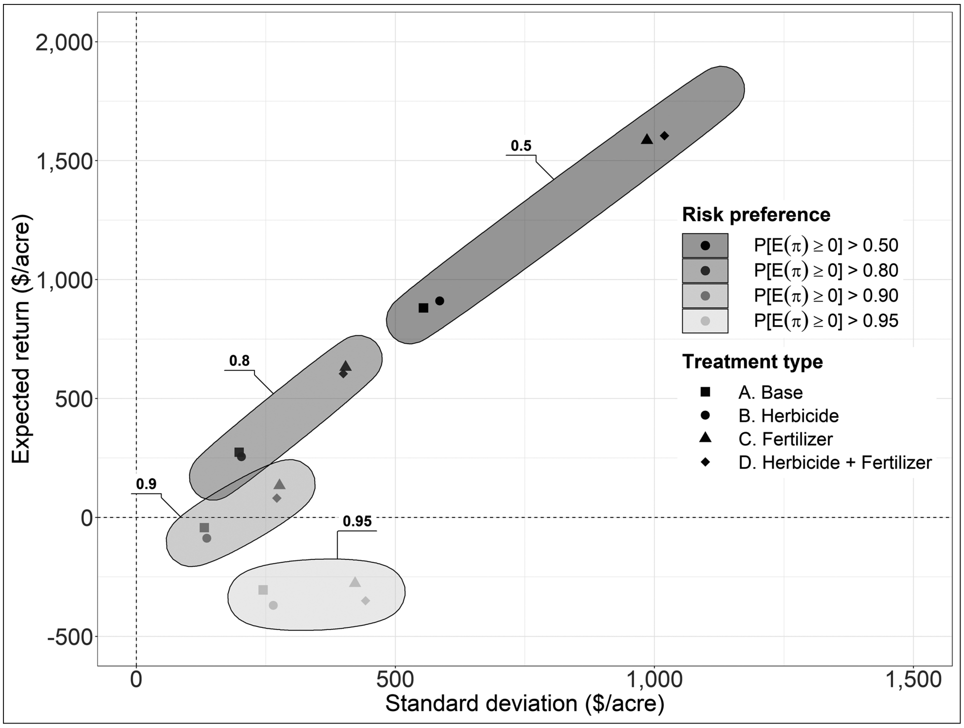

Figure 4. Mean-variance analysis diagram. The graph summarizes the expected return and standard deviation (both in 2012 U.S. dollars) of Figure 3. $P[E(\pi) \geq 0]$ denotes the level of reverse risk tolerance. It implies the risk preference related to variations of stumpage prices. The assurance level of 0.5 denotes the deterministic model scenario (i.e., risk neutral scenario).

ac to \$56/ac in the combined fertilizer/herbicide model. The above optimal roundwood yield values associated with the optimal rotation age and profit are presented in Table 4.

Discussion and Conclusion

The implications of the findings can be summarized in a mean-variance analysis. Mean and standard deviation under each treatment and risk preference scenarios from Figure 3 were calculated and presented in a mean-variance analysis graph (Figure 4). The top right of the diagram indicates a high return and high risk (i.e., high variation), whereas the bottom left of the graph indicates a low return and low risk (i.e., low variation). To compare the effect of risk preferences on forest management outcomes, four different silvicultural treatments are clustered in respective risk preference scenarios (i.e., $\alpha = 0.5$, 0.8, 0.9, and 0.95).

Within each risk preference cluster, scenarios involved fertilizer applications (i.e., triangle and diamond shapes) appear to have a higher return and standard deviation. The reason for this outcome is that because sawtimber is the most valuable timber product (Figure 2) but has a relatively higher variance (Table 3), forest owners tend to harvest more sawtimber to maximize their expected return. However, it also causes a higher variation of their return due to increased dependency on sawtimber.

Across the risk preference clusters, when a forest owner is risk-neutral (i.e., $\alpha = 0.5$), the cluster's range is wider and located higher than other clusters. On the other hand, when a forest owner is risk-averse (e.g., $\alpha = 0.95$), the cluster's range is narrower and located lower in the graph. The implication of this result indicates that a risk-neutral forest owner is likely to experience a higher expected return while bearing a higher risk than a risk-averse forest owner. Note that when α is 0.9 or 0.95, a risk-averse forest owner has a chance at realizing a negative return. That is because the risk-averse forest owner discounts risk related to stumpage prices (dislike the price variation) much more than the risk-neutral one. This characteristic of the risk preference is reflected by the risk discount factor Z_α in the chance-constrained model. Consequently, the risk-averse forest owner's expected return is lower than the risk-neutral forest owner's expected return. This finding is aligned with Gong (1994), which showed that deterministic price models tend to yield overestimated expected present value of the forest compared to the stochastic price model.

Prior studies have noted the importance of uncertain timber prices in determining the optimal rotation age from forest stands (Clarke and Reed 1989, Haight and Holmes 1991, Haight and Smith 1991, Forboseh et al. 1996), yet little was found in the literature concerning roundwood price uncertainty and its effect on landowners' harvesting decisions using a chance-constrained model under different silvicultural treatments and risk preferences. This study applied a chance-constrained dynamic optimization model to investigate the effect of uncertain stumpage prices on the optimal forest rotation decision and harvest economic values under different silvicultural treatment scenarios.

Our chance-constrained dynamic optimization model shows that using a combination of herbicide and fertilizer treatments leads to the highest total roundwood yield and lowest optimal rotation age among all treatment scenarios, implying that a landowner is expected to realize the highest profit under the combined herbicide and fertilizer treatment scenario. This result is in sync with Yin and Sedjo (2001), who studied the effect of intensive management on loblolly pine in Georgia and found that intensified regimes tend to have shorter optimal rotation age, higher productivity, and more profitability. Furthermore, the inclusion of the risk discount factor induces a lower expected profit and lower variance. In other words, this study shows that low risk tolerance results in low returns. Within a particular risk tolerance level, silvicultural treatment options with higher standard deviations always produce higher returns.

In addition, existing studies have shown that changes in roundwood prices affect optimal rotation lengths (Newman et al. 1985, Yin and Newman 1995, Gong 1998, Gong and Löfgren 2008, Pukkala and Kellomäki 2012, Pukkala 2015). Particularly, our chance-constrained model finds a similar result to Pukkala and Kellomäki (2012), indicating that when the risk tolerance reduces (i.e., α increases from 0.5 to 0.95), the optimal rotation age increases. This is attributed to the fact that a landowner is inclined to have a longer rotation length to produce more sawtimber, the most valuable among all timber products, to assure a higher profit. Nevertheless, sawtimber price is also the most volatile among the three timber products (Table 3 and Figure 2). Hence, when α rises to 0.9 or 0.95, the chance-constrained model result reveals a tradeoff between maximizing the revenue term by increasing sawtimber harvest versus minimizing the loss caused by increasing variance from rising sawtimber shares. As shown in Figure 3, this tradeoff relationship is especially noticeable when α is 0.9 or 0.95 during year 14 to year 40, when the sawtimber yield starts to increase and offset yields from the other timber types. In addition, the profit becomes negative for the base and herbicide models when α is 0.9 or 0.95 for some periods, and the optimal rotation age turns to be longer. This outcome indicates that a risk-averse landowner would likely wait longer to assure a positive profit. Note that this study did not consider stochastic input costs due to the relatively trivial change in real costs, only 0.02% from 1982 to 2016, for southern forestry practices (Callaghan et al. 2019).

A few related aspects that future studies could investigate further are offered. First, the growth and yield model used in this study does not include thinning parameters. Previous studies have shown the effect of thinning on optimal rotation ages (Brazee and Bulte 2000, Lu and Gong 2003). It would be worth exploring how the thinning changes the effect of uncertain stumpage prices on forest operation. Second, a comparison of the effect of uncertain prices on optimal rotation strategies among different varieties of trees using a similar risk modeling approach would be a fruitful area for further work. Third, other stochastic modeling methods, such as two-stage or multistage stochastic models, could also be used to examine the effect of uncertain stumpage prices on harvesting decisions. Fourth, stumpage prices have shown strong levels of autocorrelation (Tu and Zhou 2004, Lu and Gong 2005, Zhou and Buongiorno 2006, Pukkala and Kellomäki 2012, Pukkala 2015). The proposed chance-constrained model only accounts for covariance among different timber products but not autocorrelation within the same roundwood product. Despite our result showing a similar finding (i.e., increasing timber-price volatility increases the rotation length) as a study considering autocorrelation (Pukkala and Kellomäki 2012), further studies regarding the effect of autocorrelation in chance-constrained modeling would be worthwhile. Fifth, our study only considers one rotation due to the limitation of the static parameters in the growth and yield models. Further studies could assess whether multiple rotations lead to different forest management outcomes. Lastly, several uncertain elements can affect optimal forest rotation decisions. This study only focuses on price uncertainty. Future studies could explore the effect of natural disasters (e.g., wildfire) and related adaptation strategies on rotation decisions.

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Conflict of Interest

The authors declare no conflicts of interest.

Literature Cited

Bilsel, R.U., and A. Ravindran. 2011. A multiobjective chance constrained programming model for supplier selection under uncertainty. *Transport. Res. B-Meth.* 45(8):1284–1300.

Booth, L. 2004. Formulating retirement targets and the impact of time horizon on asset allocation. *Financ. Serv. Rev.* 13:1–17.

Borders, B.E., W.M. Harrison, Y. Zhang, B.D. Shiver, M. Clutter, C. Cieszewski, and R.F. Daniels. 2004. Growth and yield models for second rotation loblolly pine plantations in the piedmont/upper coastal plain and lower coastal plain of the Southeastern U.S. PMRC Technical Report. 2004-4, 1–67. Plantation Management Research Cooperative, Warnell School of Forestry and Natural Resources, University of Georgia, Athens, Georgia.

Brazee, R.J., and E. Bulte. 2000. Optimal harvesting and thinning with stochastic prices. *For. Sci.* 46(1):23–31.

Brazee, R., and R. Mendelsohn. 1988. Timber harvesting with fluctuating prices. *For. Sci.* 34(2):359–372.

Brazee, R., and R. Mendelsohn. 1990. A dynamic model of timber markets. *For. Sci.* 36(2):255–264.

Callaghan, D.W., P.N. Khanal, and T.J. Straka. 2019. An analysis of costs and cost trends for southern forestry practices. *J. For.* 117(1):21–29.

Charnes, A., and W.W. Cooper. 1959. Chance-constrained programming. *Manage. Sci.* 6(1):73–79.

Charnes, A., and W.W. Cooper. 1962. Chance constraints and normal deviates. *J. Am. Stat. Assoc.* 57(297):134–148.

Charnes, A., W.W. Cooper, and G.H. Symonds. 1958. Cost horizons and certainty equivalents: An approach to stochastic programming of heating oil. *Manage. Sci.* 4(3):235–263.

Clarke, H.R., and W.J. Reed. 1989. The tree-cutting problem in a stochastic environment: The case of age-dependent growth. *J. Econ. Dyn. Control.* 13(4):569–595.

Federal Reserve Bank. 2021. *Gross domestic product: Implicit price deflator*. Federal Reserve Bank of St. Louis. July 29, 2021. https://fred.stlouisfed.org/series/A191RD3A086NBEA.

Forboseh, P.F., R.J. Brazee, and J.B. Pickens. 1996. A strategy for multiproduct stand management with uncertain future prices. *For. Sci.* 42(1):58–66. doi: 10.1093/forestscience/42.1.58.

Gong, P. 1994. Adaptive optimization for forest-level timber harvest decision analysis. *J. Environ. Manage.* 40(1):65–90.

Gong, P. 1998. Risk preferences and adaptive harvest policies for even-aged stand management. *For. Sci.* 44(4):496–506.

Gong, P., M. Boman, and L. Mattsson. 2005. Non-timber benefits, price uncertainty and optimal harvest of an even-aged stand. *For. Policy Econ.* 7(3):283–295.

Gong, P., and K.G. Löfgren. 2008. Impact of risk aversion on the optimal rotation with stochastic price. *Nat. Resour. Model.* 21(3):385–415.

Gong, P., and A. Susaeta. 2020. Impacts of forest tax under timber price uncertainty. *For. Policy Econ.* 111(February):102030.

Gurgur, C.Z., and C.T. Morley. 2008. Lockheed Martin Space Systems Company optimizes infrastructure project-portfolio selection. *Interfaces*. 38(4):251–262.
Haight, R.G., and T.P. Holmes. 1991. Stochastic price models and optimal tree cutting: Results for loblolly pine. *Nat. Resour. Model.* 5(4):423–443.
Haight, R.G., and W.D. Smith. 1991. Harvesting loblolly pine plantations with hardwood competition and stochastic prices. *For. Sci.* 37(5):1266–1282.
Harrison, W.M., and B.E. Borders. 1996. Yield prediction and growth projection for site-prepared loblolly pine plantations in the Carolinas, Georgia, Alabama and Florida. Warnell School PMRC Technical Report, 1996-1. Plantation Management Research Cooperative, Warnell School of Forestry and Natural Resources, University of Georgia, Athens, Georgia, 1–66.
Held, H., E. Kriegler, K. Lessmann, and O. Edenhofer. 2009. Efficient climate policies under technology and climate uncertainty. *Energy Econ.* 31(SUPPL. 1):S50–S61.
Jacobs, T.L., M.A. Medina, and J.T. Ho. 1997. Chance constrained model for storm-water system design and rehabilitation. *J. Water Resour. Plann. Manage.* 123(3):163–168.
Jia, Y., and T.B. Culver. 2006. Robust optimization for total maximum daily load allocations. *Water Resour. Res.* 42(2):2412.
Liu, Y., H. Guo, F. Zhou, X. Qin, K. Huang, and Y. Yu. 2008. Inexact chance-constrained linear programming model for optimal water pollution management at the watershed scale. *J. Water Resour. Plann. Manage.* 134(4):347–356.
Lu, F., and P. Gong. 2003. Optimal stocking level and final harvest age with stochastic prices. *J. For. Econ.* 9(2):119–136.
Lu, F., and P. Gong. 2005. Adaptive thinning strategies for mixed-species stand management with stochastic prices. *J. For. Econ.* 11(1):53–71.
Newman, D.H., C.B. Gilbert, and W.F. Hyde. 1985. The optimal forest rotation with evolving prices. *Land Econ.* 61(4):347–353.
Office of Management and Budget. 2003. Circular No. A4 – regulatory analysis. Available online at https://obamawhitehouse.archives.gov/omb/circulars_a004_a-4/. Date accessed 8 June 2022.
Olson, D.L., and D. Wu. 2017. *Enterprise risk management models*. 3rd ed. Springer, Berlin, Germany.
Oswalt, S.N., W.B. Smith, P.D. Miles, and S.A. Pugh. 2019. Forest resources of the United States, 2017. A Technical Document Supporting the Forest Service 2020 RPA Assessment. doi:10.2737/WO-GTR-97.
Park, K.F., and Z. Shapira. 2017. Risk and uncertainty. *in The Palgrave encyclopedia of strategic management*, edited by Augier M. and Teece D, 1–7. London: Palgrave Macmillan.
Paudel, K., and P. Dwivedi. 2021. Economics of Southern Pines With and Without Payments for Environmental Amenities in the US South. *Frontiers in Forests and Global Change* 35.
Pukkala, T. 2015. Optimizing continuous cover management of boreal forest when timber prices and tree growth are stochastic. *For. Ecosyst.* 2(1):1–13.
Pukkala, T., and S. Kellomäki. 2012. Anticipatory vs adaptive optimization of stand management when tree growth and timber prices are stochastic. *Forestry.* 85(4):463–472.
Susaeta, A., and P. Gong. 2019. Optimal harvest strategy for even-aged stands with price uncertainty and risk of natural disturbances. *Nat. Resour. Model.* 32(3):e12211.
TimberMart-South. 2021. TimberMart-South Price Report. Available online at http://www.timbermart-south.com/. Date accessed 8 June 2022.
Tu, J., and G. Zhou. 2004. Data-generating process uncertainty: What difference does it make in portfolio decisions? *J. Finan. Econ.* 72:385–421.
Wang, S., G.H. Huang, and Y. Zhou. 2015. Inexact probabilistic optimization model and its application to flood diversion planning in a dynamic and uncertain environment. *J. Water Resour. Plann. Manage.* 141(8):04014093.
Wibowo, A., and B. Kochendoerfer. 2010. Selecting BOT/PPP infrastructure projects for government guarantee portfolio under conditions of budget and risk in the Indonesian context. *J. Constr. Eng. Manag.* 137(7):512–522.
Wu, D., and D.L. Olson. 2008. Supply chain risk, simulation, and vendor selection. *Int. J. Prod. Econ.* 114(2):646–655.
Yin, R., and D. Newman. 1995. Optimal timber rotations with evolving prices and costs revisited forest tenure reform and institutional change view project. *For. Sci.* 41(3):477–490.
Yin, R., and R.A. Sedjo. 2001. Is this the age of intensive management? A study of loblolly pine on Georgia's Piedmont. *J. For.* 99(12):10–17. doi: 10.1093/JOF/99.12.10.
Zhang, X., G.H. Huang, and X. Nie. 2011. Possibilistic stochastic water management model for agricultural nonpoint source pollution. *J. Water Resour. Plann. Manage.* 137(1):101–112.
Zhou, M., and J. Buongiorno. 2006. Space-time modeling of timber prices. *J. Agr. Resource Econ.* 31(1):40–56.
Zhu, L.-J., C.-Z. Qin, A.-X. Zhu, J. Liu, and H. Wu. 2019. Effects of different spatial configuration units for the spatial optimization of watershed best management practice scenarios. *Water.* 11(2):262.