SOME FINAL EXAM REVIEW

We will not have time to go over all the questions in this document on the review day! Full answers will be posted after class (like always). The order/selection of problems gone over in class will be based on the most votes via:

`https://tinyurl.com/MTH234-final`

1. What is the distance between the plane $x+y+2z=1$ and the line parameterized by $\mathbf{r}(t) = \langle t, 2+t, 3-t\rangle$?

   A. $\frac{11}{\sqrt{6}}$
   B. $\frac{9}{\sqrt{6}}$
   C. $\frac{8}{\sqrt{6}}$
   D. $\frac{7}{\sqrt{6}}$
   E. $\frac{5}{\sqrt{6}}$

2. What is the range of $f(x,y) = 1 + 2^{xy}$?

   A. $(-\infty, \infty)$
   B. $(2, \infty)$
   C. $(1, \infty)$
   D. $(0, \infty)$
   E. $[2, \infty)$

3. Suppose $f(x, y)$ has continuous second partial derivatives everywhere on the xy-plane. Suppose also that

$$f_x(2,1) = f_y(2,1) = 0, \qquad f_{xx}(2,1) = -3, \qquad f_{yy}(2,1) = -2, \qquad \text{and} \qquad f_{xy}(2,1) = 2.$$

Which of the following statements are true?

A. f has a local minimum at the point $(2, 1)$.

B. f has a local maximum at the point $(2, 1)$.

C. f has neither a local minimum nor a local maximum at the point $(2, 1)$.

D. The second derivative test is inconclusive.

E. None of the above.

4. A thin sheet of candy lies in the region R shown to the right. It has density $\rho(x, y) = 2x - 4y^3$. Find the mass of the candy.

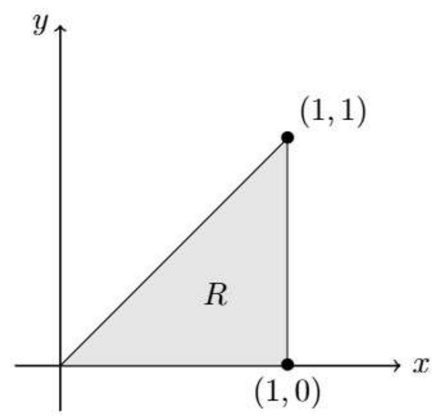
A. $\frac{7}{3}$

B. $\frac{7}{15}$

C. 23

D. $\frac{2}{3}$

E. 3



5. Identify which of the following statements are true. Take $\mathbf{F}$ and $\mathbf{G}$ to be vector fields

   I. $\text{curl}(\mathbf{F} + \mathbf{G}) = \text{curl } \mathbf{F} + \text{curl } \mathbf{G}$

   II. $\text{curl}(\mathbf{F} \cdot \mathbf{G}) = \text{curl } \mathbf{F} \cdot \text{curl } \mathbf{G}$

   III. There is a vector field $\mathbf{H}$ such that $\text{curl } \mathbf{H} = \langle x, y, z \rangle$

   A. None of the above are true

   B. Only I. is true

   C. Only I. and III. are true

   D. Only II. is true

   E. Only II. and III. are true

6. Consider the vector field $\mathbf{F}$ to the right. At which point is div $\mathbf{F} < 0$?

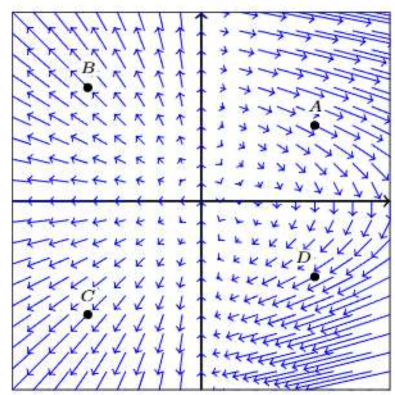
   A. A

   B. B

   C. C

   D. D

   E. None of the above.



7. Find a vector function that fully parameterizes the curve of the intersection of cone $z^2 = x^2 + y^2$ and the hemisphere $x^2 + y^2 + z^2 = 8$ with $z > 0$.

8. Find a vector function to fully parameterize the curve of the intersection of cone $z^2 = x^2 + y^2$ and the plane $z = 2 - x$.

9. Consider the set of points inside the top half ($z \geq 0$) of a sphere centered at the origin with radius a. Show that the average height of these points is $\dfrac{3a}{8}$.

10. Calculate the upward flux of $\mathbf{F} = \langle x, y, 1 \rangle$ through the part of the paraboloid of $z = x^2 + y^2$, with $z \in [0, 1]$.