1 The Vertex-Vertex Lemma

It is possible to construct polygons that have half-length parallelograms that share vertices with the bounding polygon.

The stability condition: given a collection of points that determine an affine diameter and a half parallelogram, we give criteria on the slopes of the support lines and the movement of the vertices as the affine diameter changers.

At each vertex, there is a coupled family of directions which determine the motion for which the the area of the half-length parallelogram is preserved to first order.

We describe the slopes as follows...

2 vertices of the affine diameter

Proposition 2.1. *The affine diameter meets only one vertex, unless certain properties hold...*

Proof. □

3 vertices of the half-length parallelogram

Proposition 3.1. *A vertex of a half-parallelogram is imperturbable if a rotation of the affine diameter does not cause that vertex to move. By the stability condition, a vertex is imperturbable only when the opposite end of it's half diameter has a motion that is parallel to the motion of moving end of the affine diameter. This follows from a symbolic computation. (modulo the correct parametrization of slopes...)*

Proposition 3.2. *The vertices of the half length parallelogram meet no vertices unless there is an imperturbable vertex.*

Proposition 3.3. *The stability condition is given by a function of slopes and point positions that are required for the configuration to be stable with respect to the half parallelogram construction. This function is continuous and satisfies certain intermediate value properties. (modulo the correct parametrization)*

3.1 Pyramids

Definition 3.1. *If a polygon has an imperturbable vertex, we call it a pyramid.*

The pyramid is a stable configuration by considering it as a union of two over constrained regions of configuration space.

perturbation of quadrangle vertices

depends on analysis of the the stability criterion of slopes

sketch at a single vertex...
intermediate value theorem