Random sphere packing lattices

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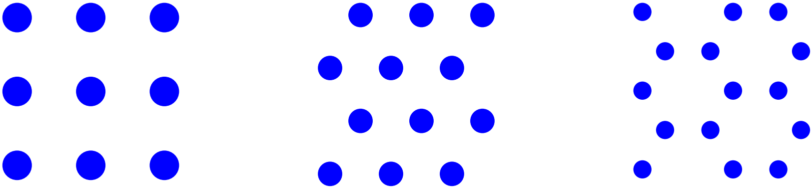
What do I mean by a lattice

$$L = A\mathbb{Z}^n = \{\sum_{i=1}^n m_i \mathbf{a}_i : m_i \in \mathbb{Z}\}$$



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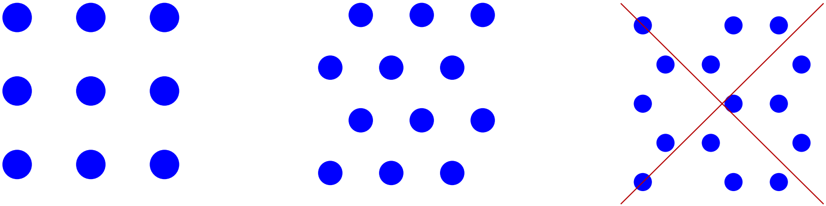

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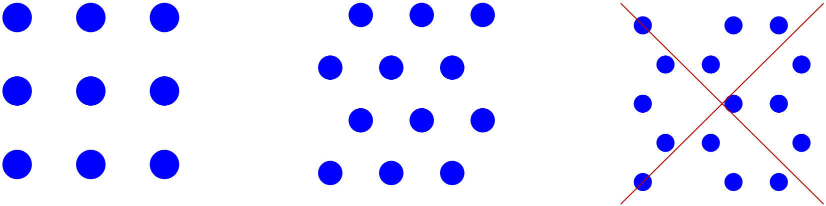
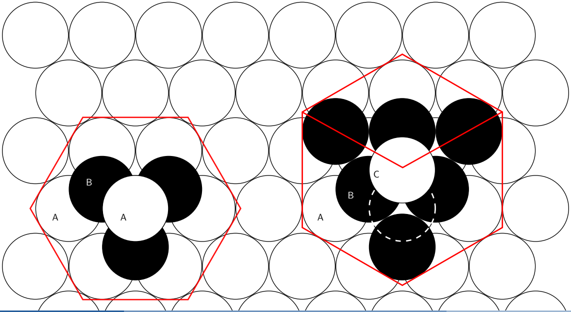
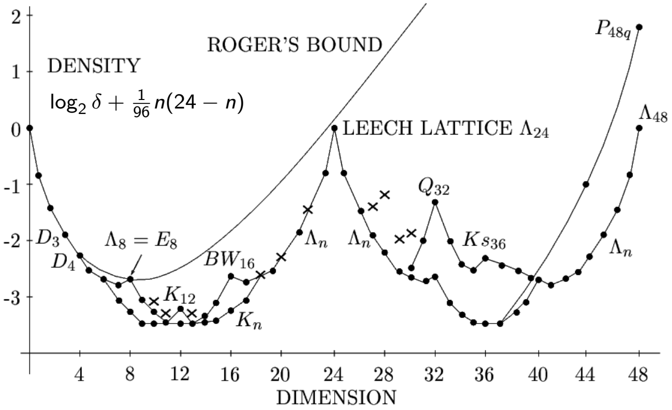




Good packings are often lattices



Packing problem restricted to lattices

n	L	
2	A_2	Lagrange (1773)
3	$D_3 = A_3$	Gauss (1840)
4	D_4	Korkin & Zolotarev (1877)
5	D_5	Korkin & Zolotarev (1877)
6	E_6	Blichfeldt (1935)
7	E_7	Blichfeldt (1935)
8	E_8	Blichfeldt (1935)
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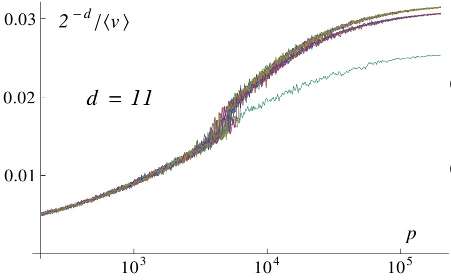
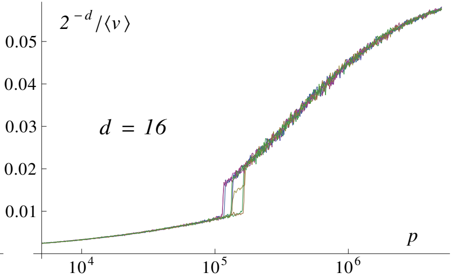
Restriction to lattices makes the packing problem much more manageable. Does it offer leverage for statistical mechanical questions?

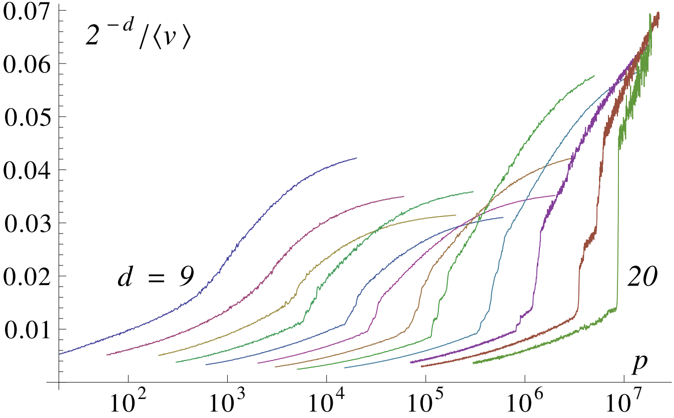
Thermodynamics of hard-sphere lattices





Kallus, Phys. Rev. E 87, 063307 (2013)

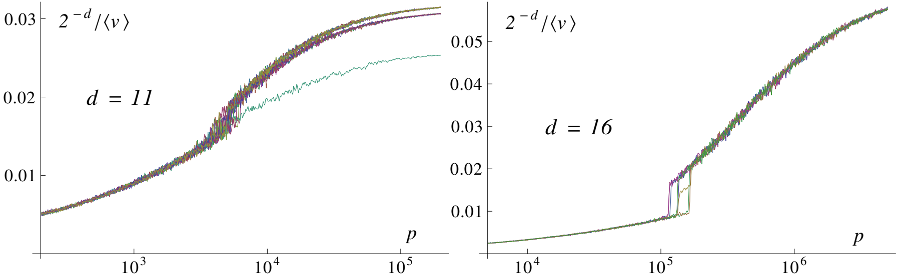
Thermodynamics of hard-sphere lattices



Densest known lattice recovered in some runs for $n \leq 20$

Kallus, Phys. Rev. E 87, 063307 (2013)

Thermodynamics of hard-sphere lattices



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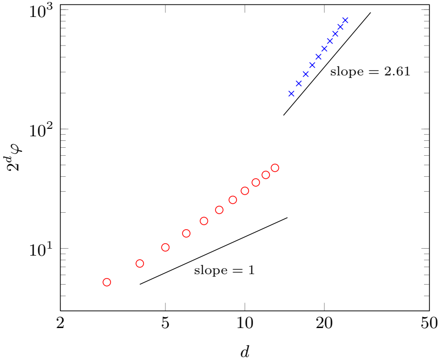
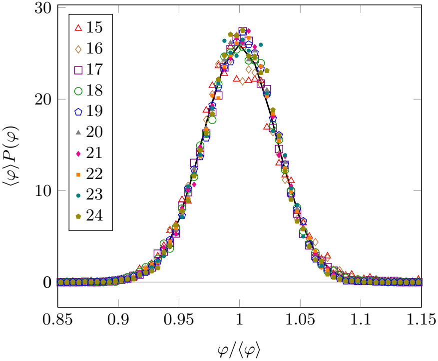
Lattice RCP





Kallus, Marcotte, & Torquato, Phys. Rev. E 88, 062151 (2013)

Lattice isostaticity

Isostaticity:
$\#$constraints $= \#$dof's

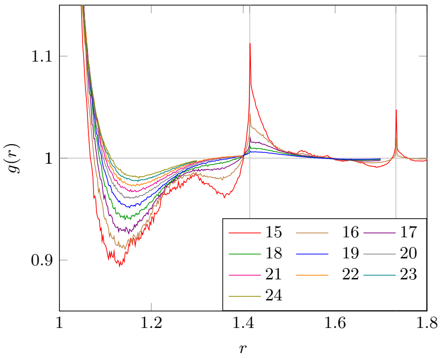
In RCP, Isostaticity $\rightarrow$
average $\#$contacts $= 2d$.

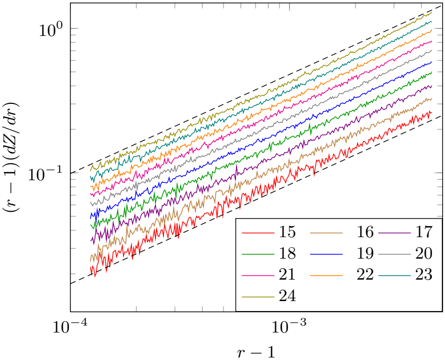
In Lattice RCP: $\#$dof's
$= \frac{1}{2}d(d+1)$.
Isostaticity $\rightarrow$
$\#$contacts $= d(d+1)$.

Kallus, Marcotte, & Torquato, Phys. Rev. E 88, 062151 (2013)

d	Runs	Isostatic
13	10,000	365
14	10,000	1,625
15	10,000	5,196
16	10,000	6,761
17	10,000	9,235
18	10,000	9,590
19	20,000	19,200
20	20,000	19,085
21	10,000	9,473
22	10,000	9,406
23	10,000	9,281
24	10,000	9,205

Pair correlations and quasicontacts

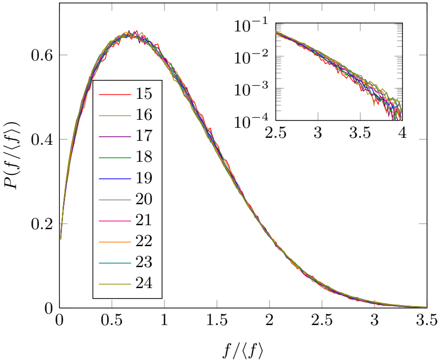


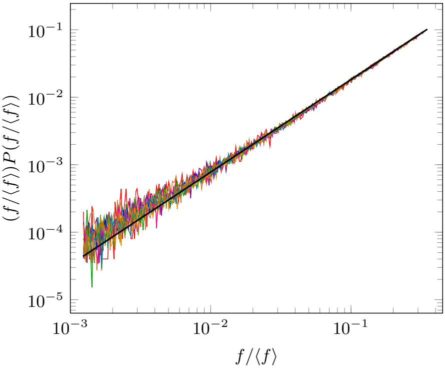


$$g(r) \sim (r-1)^{-\gamma}$$
$$Z(r) \sim d(d+1) + A_d(r-1)^{1-\gamma}$$
$$\gamma = 0.314 \pm 0.004$$

Kallus, Marcotte, & Torquato, Phys. Rev. E 88, 062151 (2013)

Contact force distribution

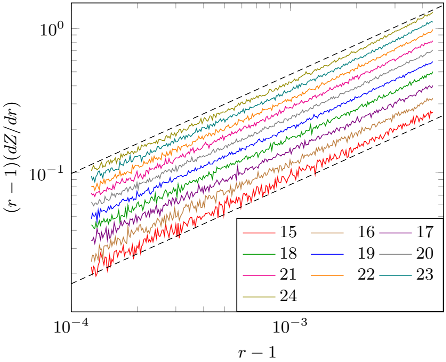


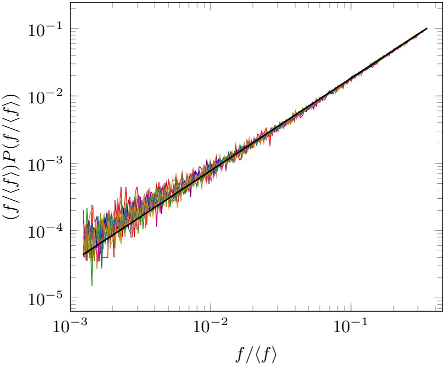


$$P(f) \sim f^{\theta}$$
$$\theta = 0.371 \pm 0.010$$

Kallus, Marcotte, & Torquato, Phys. Rev. E 88, 062151 (2013)

Quasicontacts and weak contacts





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Uniform sampling of jammed lattices

Extreme lattices can be exhaustively enumerated.

d	2	3	4	5	6	7	8	9
perfect lattices	1	1	2	3	7	33	10916	$>$50000
extreme lattices	1	1	2	3	6	30	2408	$\cdots$

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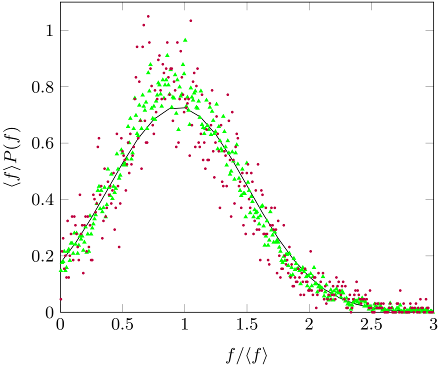
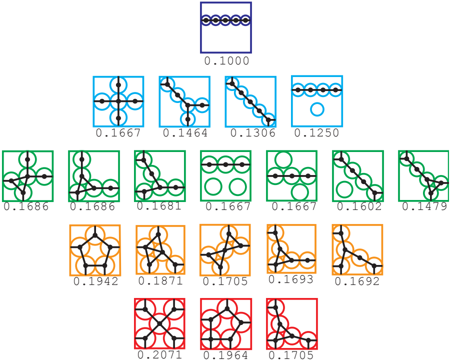
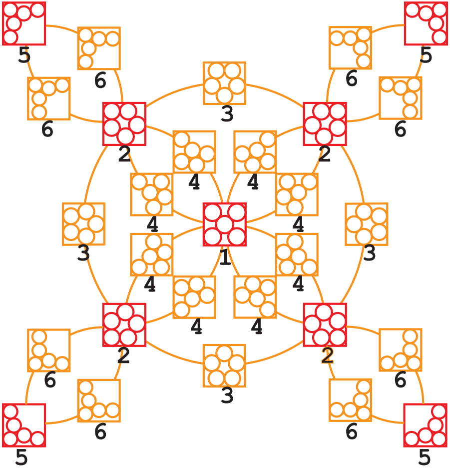


Kallus & Torquato, Phys. Rev. E 90, 022114 (2014)

Computational topology of config'n space





Carlsson, Gorham, Kahle, & Mason, Phys. Rev. E 85, 011303 (2012)

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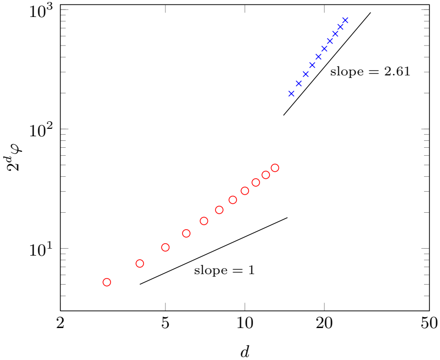

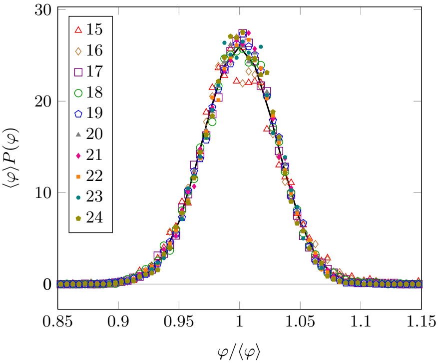

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