Abstract of Job market papers

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This document is purely for the reference of my letter writers, as a supplement to my research statement.

My applications to business schools and OR departments will have three job market papers: one on network bargaining [1], one on naive social learning [2] and one on Bayesian social learning [3]. All three papers concern agents interacting over networks. Agents possess limited local information and make decisions based on this information. Yet, we show that the networks allow for rapid aggregation of distributed information, leading to structured outcomes.

My applications to electrical engineering and computer science will also contain my paper on the deletion channel [4].

1 Bargaining Networks

The problem. Consider an exogenously specified network of possible agreements between agents with each possible partnership been ascribed a given total value. Each agent is allowed to form at most one pairwise partnership for making a profit. In order to form a partnership, the agents agents involved must reach an agreement on how to split the value. Such a model is of interest in understanding the housing market, the job market, the 'market' for social relationships, and so forth. A natural question is: Which partnerships are formed, and on what terms? In the case of two agents with a possible deal between them, this question can be modeled as a Nash bargaining problem. For general bargaining networks, *balanced outcomes*, a generalization of Nash bargaining solutions, have been proposed as a predictive solution concept. A balanced outcome satisfies two properties: (i) *Stability:* no pair of agents has an incentive to abandon their current deal, if any, and strike a deal with each other instead. (ii) *Balance:* In each deal that occurs, each of the two agents gain utility exceeding their best alternative (suitably defined) by the same amount. These postulates seem eminently reasonable. However, to justify this solution concept, there must be a local bargaining process that agents engage in, that leads rapidly to balanced outcomes. This requirement had not been addressed.

Our solution. We formulated a natural dynamical description of the bargaining process involving iterative updates of offers and counteroffers, based on agents' current estimates of best alternatives [1]. Importantly, we showed rapid convergence of the dynamics to balanced outcomes on (nearly) all networks where such solutions exist. In particular, we showed polynomial time convergence under a smoothed analysis on bipartite graphs. Thus we obtained an elegant solution to the problem of 'learning' balanced outcomes in bargaining networks.

Additional technical contributions:

- We introduced a new powerful technique from functional analysis to this set of problems, and to the computer science community in general.
- Our analysis yields a new class of fast message passing algorithms for (provably) finding the maximum weight matching, with max-product belief propagation being a special case.

In follow up work, I found an FPTAS for the case of unequal bargaining powers.

Summary of contribution. We were able to successfully resolve a question that has generated considerable recent interest. A key strength of the main result is that it applies to (almost) all weighted graphs. We hope that this work will be of interest to economists (in addition to computer scientists), since it deals with a fundamental open question in this area[1].

2 Majority dynamics on trees and the dynamic cavity method

The problem. A voter sits on each vertex of graph, and has to decide between two alternative opinions. At each time step, each voter switches to the opinion of the majority of her neighbors. This is a model of social learning with naive agents. It is significantly more challenging than alternative models of naive agents like the voter model or weighted average updates, due to the threshold nature of the decision rule.

We analyze this majority process [2] on an infinite tree of degree k, when opinions are initialized to independent and identically distributed Bernoulli($(1+\theta)/2$) random variables. Here $\theta \in [-1, 1]$ is the *bias* parameter.

Our results. In particular, we bound the threshold value $\theta_*(k)$ of the initial bias such that the process converges to consensus: we show that the threshold decays to 0 faster than any inverse polynomial in k, i.e., $\theta_*(k) = O(1/k^M)$ for any $M > 0$. In order to prove an upper bound, we introduce a new tool called the *dynamic cavity method*, which should be of general interest in the analysis of random *dynamical* processes on trees. We use this tool to characterize the process of a single node in the large k limit using this method.

We also derive a lower bound on $\theta_*(k)$ that is nontrivial for small, odd values of k. The proof involves a percolation argument.

Technical contribution. This paper is very technical, and involved several significant technical innovations. The analysis leading to the upper bound is extremely delicate. However, I think the most useful contribution may the dynamic cavity method, despite it being one of the less complicated aspects of our analysis. This method extends the widely used cavity method from statistical physics, which applies to static random processes on trees, to *dynamical* processes. This method overcomes the following challenge: Denote by $\sigma_i(t)$ the (binary) vote of node i at time t. The dynamics on subtrees formed by removing node i from G *do not decouple*, even after conditioning on s and the trajectory $(\sigma_i(0), \sigma_i(1), \ldots)$.

3 Tractable Bayesian Social Learning

The problem. Suppose there are two possible states of the world -1 or 1, with a 50-50 prior. Denote by s the true (random) state of the world. There are n players who want to correctly guess s. Each player receives an iid noisy observation of s. In round 0, all players simultaneously guess a state of the world based on their respective private signals/observations. There is an underlying graph G, such that players can observe guesses of neighbors on the graph. Based on these observations in round 0, players form a Bayesian estimate and guess again (simultaneously) in round 1. Again they observe their neighbors, form updated Bayesian estimates, and so on.

This model (rather, a generalization of it) was introduced by Gale and Kariv. They showed that on any network G, either the guesses of all players converge or the 'beliefs' of outwardly disagreeing neighbors converge. This left open two important questions:

[1]Several other papers in this area have rediscovered previous results by economists.

- What are the dynamics of such interactions, e.g., what is the rate of agreement/convergence?
- Is iterative Bayesian inference computationally tractable?

Let us focus on the question of tractability. Gale and Kariv, and several subsequent works argue/state (without proof) that exact Bayesian computation is very expensive/infeasible since agents need to "perform deductions about the information of every other individual in the network, while only observing the evolution of opinions of their neighbors". Indeed, using a naive dynamic program, computational effort required of each individual to compute their action at time t is $t2^{O(\min(n,d^t))}$, on a graph of degree bounded by d.

Our contribution. Suppose G is a tree. We show that a non-trivial generalization of the 'dynamic cavity method' (introduced by Andrea and me in previous work [2]) allows us to substantially simplify computation, while retaining (exact) correctness [3]. We provide a new 'dynamic cavity algorithm' for the agents' calculations, with computational effort $2^{O(td)}$, which is exponentially lower than the naive dynamic program.

We conjecture that on d-regular trees, myopic Bayesian agents learn s as quickly as agents who practice 'majority dynamics', i.e., at each round adopt the majority opinion of their neighbors. Under mild conditions, we show that with majority dynamics, agents learn s with probability $1-\epsilon$ in $O(\log\log(1/\epsilon))$ rounds. Using our algorithm, the conjecture implies that the computational effort required of Bayesian agents to learn s is only polylogarithmic in $1/\epsilon$ on d-regular trees.

We use our algorithm to perform the first numerical simulations of interacting Bayesian agents on networks with hundreds of nodes. Our numerical results support our conjecture, and indicate rapid learning of s on regular trees.

Technical contributions:

- We introduce the 'dynamic cavity method', to the algorithms community, using it to efficiently compute an implicitly specified decision rule (Bayesian updates). Our algorithmic approach should generalize beyond the model considered here.
- We prove doubly exponentially fast convergence of majority updates on regular trees. This result could be of independent interest.

One nice aspect of this project is that the result is quite surprising. It surprised us, as well as Elchanan and Andrea. The main non-trivial idea involves the extension of the dynamic cavity approach to an implicitly specified decision rule (Bayesian updates).

4 Optimal coding for the deletion channel with small deletion probabilities

The problem. The deletion channel is the simplest point-to-point communication channel that models lack of synchronization. Input bits are deleted independently with probability d, and when they are not deleted, they are not affected by the channel. Despite significant effort, little is known about the capacity of this channel, and even less about optimal coding schemes.

Our contribution. In this paper [4], we develop a new systematic approach to this problem, by demonstrating that capacity can be computed in a series expansion for small deletion probability. We compute three leading terms of this expansion, and find an input distribution that achieves capacity up to this order. This constitutes the first optimal coding result for the deletion channel. The key idea employed is the following: We understand perfectly the deletion channel with deletion probability $d = 0$. It has capacity 1 and the optimal input distribution is i.i.d. Bernoulli(1/2). It is natural to expect that the channel with small deletion probabilities has a capacity that varies smoothly with d, and that the optimal input distribution is obtained by smoothly perturbing the i.i.d. Bernoulli(1/2) process. Our results show that this is indeed the case. We think that this general strategy can be useful in a number of capacity calculations.

Importance. I believe this work significantly advances our understanding of the deletion channel, with hope for further progress using the framework established here. This also represents progress in the general class of channels without synchronization. For example, some of our results (the first two terms of the expansion) have been used and extended by Tse, Ramchandran and Ma in the context of file synchronization when there are bursty deletion edits. Further, we expect this expansion technique to be useful in analyzing other 'hard' channels. The simplicity of the idea means that it's more likely to be broadly useful.

The conference version won a **Student Paper award** at ISIT 2010.

References

[1] Y. Kanoria, M. Bayati, C. Borgs, J. Chayes, and A. Montanari, "Fast Convergence of Natural Bargaining Dynamics in Exchange Networks," Proc. ACM-SIAM Symposium on Discrete Algorithms, 2011, invited to Journal of Economic Theory.

[2] Y. Kanoria, A. Montanari, "Majority dynamics on trees and the dynamic cavity method," to appear in Annals of Applied Probability.

[3] Y. Kanoria, O. Tamuz, "Tractable Bayesian Social Learning," submitted to Games and Economic Behavior.

[4] Y. Kanoria and A. Montanari, "Optimal coding for the deletion channel with small deletion probability," submitted to IEEE Transaction on Information Theory.