CSL7320: Digital Image Analysis

Feature Extraction

Representation and Detection

- A segmented region can be represented by $\begin{cases} \text{boundary pixels} \\ \text{internal pixels} \end{cases}$
- When shape is important, a boundary (external) representation is used
- When colour or texture is important, an internal representation is used
- The description of a region is based on its representation, for example a boundary can be described by its length
- The features selected as descriptors are usually required to be as insensitive as possible to variations in (1) scale, (2) translation and (3) rotation, that is the features should be scale, translation and rotation invariant

Slide credit: TW793

Representation

Image data, for example a boundary, is usually represented in a more compact way so that it can be described more easily

Slide credit: TW793

Representation

- Boundary descriptors

Some simple descriptors

- **Length**
- **Diameter**
- **Major axis**
- **Minor axis**
- **Basic rectangle**
- **Eccentricity**

- **Curvature: rate of change of slope ...**

  ... vertex point p part of
  - **Convex segment**
  - **Concave segment**
  - **Straight segment**
  - **Corner point**

Slide credit: TW793

Representation

- Region descriptors

Some simple descriptors

- **Area: Number of pixels in region**
- **Perimeter: Length of boundary**
- **Compactness: Perimeter^2/Area**
- **Mean and median gray levels**
- **Min and max gray level values**
- **Number of pixels with values above or below mean**

Texture Descriptors

Slide credit: TW793

Representation: Textures

(1) Statistical approaches (2) Structural approaches (3) Spectral approaches

Statistical approaches

When $p(z_i),\ i = 0, \ldots, L-1$ represents a histogram of gray-levels, the nth moment of z about the mean is

$$\mu_n(z) = \sum_{i=0}^{L-1} (z_i - m)^n \, p(z_i)$$

where m is the mean value of z...

$$m = \sum_{i=0}^{L-1} z_i \, p(z_i)$$

- **Relative smoothness...**

$$R(z) = 1 - \frac{1}{1 + \sigma^2(z)}$$

- **The third moment...**

$$\mu_3(z) = \sum_{i=0}^{L-1} (z_i - m)^3 \, p(z_i)$$

Representation: Textures

- **The fourth moment...**

$$\mu_4(z) = \sum_{i=0}^{L-1} (z_i - m)^4 \, p(z_i)$$

... measure of histogram's flatness

- **Measure of uniformity...**

$$U(z) = \sum_{i=0}^{L-1} p^2(z_i)$$

... is maximum for an image in which all grey levels are equal

- **Average entropy measure...**

$$e(z) = -\sum_{i=0}^{L-1} p(z_i) \, \log_2 p(z_i)$$

... measure of variability and is 0 for a constant image

Measures of texture computed using only histograms suffer from the limitation that they carry no information regarding the relative position of pixels with respect to each other

Slide credit: TW793

Representation: Textures

- **Gray-level co-occurrence matrices:**

1	1	7	5	3	2
5	1	6	1	2	5
8	8	6	8	1	2
4	3	4	5	5	1
8	7	8	7	6	2
7	8	6	2	6	2

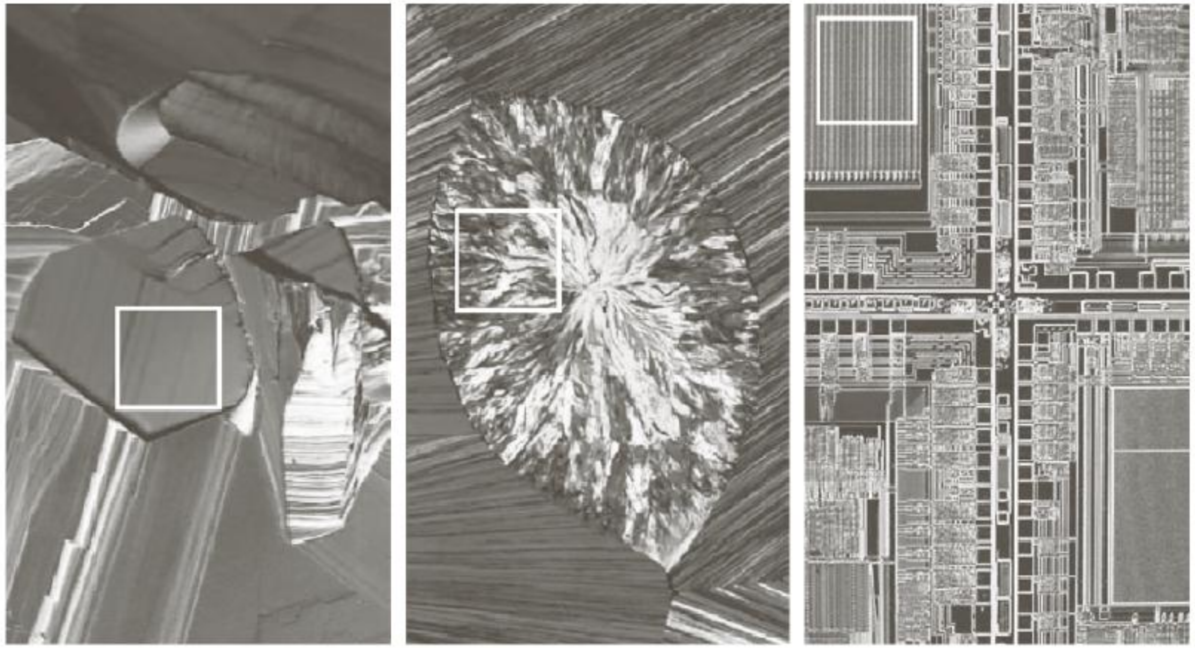
Image f

	1	2	3	4	5	6	7	8
1	1	2	0	0	0	1	1	0
2	0	0	0	0	1	1	0	0
3	0	1	0	1	0	0	0	0
4	0	0	1	0	1	0	0	0
5	2	0	1	0	1	0	0	0
6	1	3	0	0	0	0	0	1
7	0	0	0	0	1	1	0	2
8	1	0	0	0	0	2	2	1

Co-occurrence matrix **G**

Representation: Textures

Texture measures based on histograms



a b c

FIGURE 11.28 The white squares mark, from left to right, smooth, coarse, and regular textures. These are optical microscope images of a superconductor, human cholesterol, and a microprocessor. (Courtesy of Dr. Michael W. Davidson, Florida State University.)

Texture	Mean	Standard deviation	R (normalized)	Third moment	Uniformity	Entropy
Smooth	82.64	11.79	0.002	−0.105	0.026	5.434
Coarse	143.56	74.63	0.079	−0.151	0.005	7.783
Regular	99.72	33.73	0.017	0.750	0.013	6.674

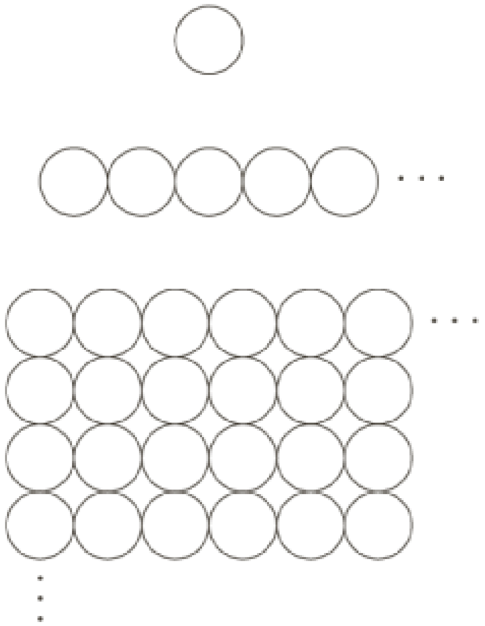
TABLE 11.2 Texture measures for the subimages shown in Fig. 11.28.

Slide credit: TW793

Representation: Textures

Structural approaches

A simple "texture primitive" can be used to form more complex texture patterns by means of some rules that limit the number of possible arrangements of the primitive(s)



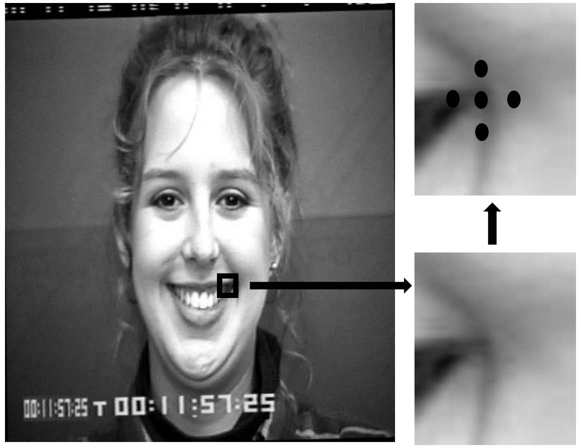
a
b
c

FIGURE 11.34
(a) Texture primitive.
(b) Pattern generated by the rule $S \rightarrow aS$.
(c) 2-D texture pattern generated by this and other rules.

Slide credit: TW793

Representation: Textures

Local Binary Pattern (LBP)



	142	
121	130	140
	218	

$q_i = 130$

	$b_3=1$	
$b_2=0$		$b_0=1$
	$b_1=1$	

$(1101)_2 = (13)_{10}$

	13	

$d(q_i) = 13$

$$b_j = \begin{cases} 1, & if\ v(q_j) > v(q_i) \\ 0, & otherwise \end{cases} \quad Where,\ q_j \in Nbd_4(q_i)$$

Non-Overlapping blocks (9 x 8)

(a)

(b)

Overlapping blocks(4x3) , overlap size = 10 pixels

Representation: Textures

Local Binary Pattern (LBP)

- **Description of pixels neighbourhood**
- **Binary short code to describe neighbourhood**
- **Operates by taking difference of central pixel with neighbouring pixels**
- **Mathematically**

$$LBP_{R,P} = \sum_{p=0}^{P-1} s(g_p - g_C).2^p$$

where,
neighborhood pixels (g_p) in each block
is thresholded by its center pixel value (g_c)
$p \rightarrow$ sampling points (e.g., $p = 0, 1, \ldots, 7$ for a 3x3 cell, where P = 8)
$R \rightarrow$ radius (for 3x3 cell, it is 1).
Coordinates of "g_c" is (0,0) and of "g_p" is (x + Rcos(2π p/P), y − Rsin(2π p/P))

Binary threshold function $s(x)$ is,

$$s(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases}$$

Slide credit: BIOMISA

Representation: Textures

Local Binary Pattern (LBP)

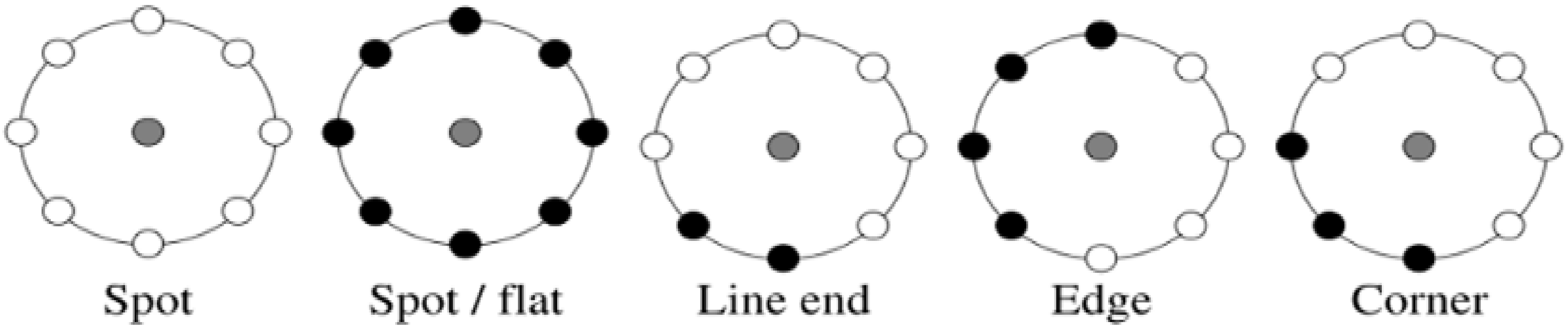


Fig. 2.3 Different texture primitives detected by the LBP

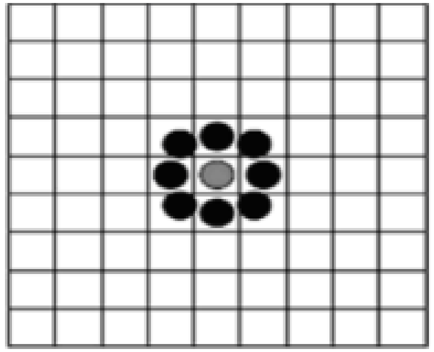
Slide credit: BIOMISA

Representation: Textures

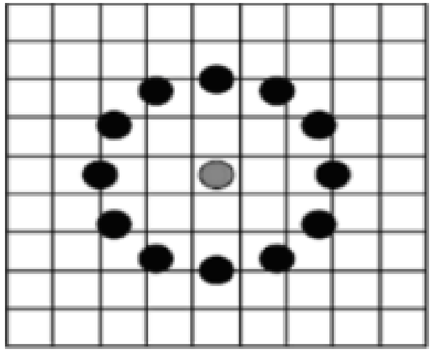
Local Binary Pattern (LBP)

Advanced LBP $_{(P,R)}$

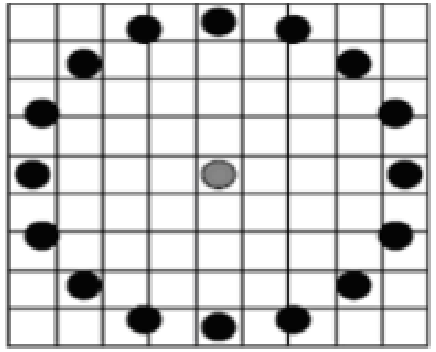
P = Pixels
R = Radius



LBP(8,1)



LBP(16,2)



LBP(20,4)

Slide credit: BIOMISA

Representation: Textures

Local Binary Pattern (LBP)

LBP Advantages and disadvantages

Advantages

- High discriminative power
- Computational simplicity
- Invariance to grayscale changes and
- Good performance.

Disadvantages

- Not invariant to rotations
- The size of the features increases exponentially with the number of neighbours which leads to an increase of computational complexity in terms of time and space
- The structural information captured by it is limited. Only pixel difference is used, magnitude information ignored.

Slide credit: BIOMISA

Representation: Principal Components As Feature Descriptors

- Applicable to boundaries and regions
- Can also describe sets of images that were registered differently, for example the three component images of a color RGB image...
- Treat the three images as a unit by expressing each group of corresponding pixels as a vector...

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

- When we have n registered images...

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

- When $K = M$(rows) $\times$ N(columns), the <u>mean vector</u> of the population is defined as

$$\mathbf{m}_\mathbf{x} = E\{\mathbf{x}\} = \frac{1}{K}\sum_{k=1}^{K} \mathbf{x}_k$$

Slide credit: TW793

Principal Components As Feature Descriptors

- When $K = M$(rows)$\times N$(columns), the covariance matrix of the population is defined as

$$\begin{aligned} \mathbf{C}_{\mathbf{x}} &= E\{(\mathbf{x}-\mathbf{m}_{\mathbf{x}})(\mathbf{x}-\mathbf{m}_{\mathbf{x}})^T\} \\ &= \frac{1}{K}\sum_{k=1}^{K}(\mathbf{x}_k-\mathbf{m}_{\mathbf{x}})(\mathbf{x}_k-\mathbf{m}_{\mathbf{x}})^T \\ &= \frac{1}{K}\sum_{k=1}^{K}\mathbf{x}_k\mathbf{x}_k^T-\mathbf{m}_{\mathbf{x}}\mathbf{m}_{\mathbf{x}}^T \end{aligned}$$

- Note that $\mathbf{C}_{\mathbf{x}}$ is an $n \times n$ matrix
- Element c_{ii} of $\mathbf{C}_{\mathbf{x}}$ is the variance of x_i
- Element c_{ij} of $\mathbf{C}_{\mathbf{x}}$ is the covariance between x_i and x_j
- The matrix $\mathbf{C}_{\mathbf{x}}$ is real and symmetric
- If x_i and x_j are uncorrelated, their covariance is zero, that is $c_{ij} = c_{ji} = 0$

Example 11.14: Mean vector and covariance matrix

Consider the four vectors $\mathbf{x}_1 = (0,0,0)^T$, $\mathbf{x}_2 = (1,0,0)^T$, $\mathbf{x}_3 = (1,1,0)^T$, and $\mathbf{x}_4 = (1,0,1)^T$, then

$$\mathbf{m}_{\mathbf{x}} = \frac{1}{4}\begin{bmatrix}3\\1\\1\end{bmatrix}$$

Slide credit: TW793

Principal Components As Feature Descriptors

$$\mathbf{C_x} = \frac{1}{16}\begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

The three components have the same variance
Elements x_1 and x_2, and x_1 and x_3 are positively correlated
Elements x_2 and x_3 are negatively correlated

- **Since $\mathrm{C_x}$ is real and symmetric, we can always find a set of n orthonormal eigenvectors**
- **Let e_i and λ_i, $i = 1, \ldots, n$ be the eigenvectors and corresponding eigenvalues of $\mathrm{C_x}$ arranged in descending order so that $\lambda_j \geq \lambda_{j+1}$ for $j = 1, 2, \ldots, n-1$**
- **Let A be a matrix whose rows are formed from the eigenvectors of $\mathrm{C_x}$, ordered so that the first row is the eigenvector corresponding to the largest eigenvalue and the last row is the eigenvector corresponding to the smallest eigenvalue**
- **Suppose that A is a transformation matrix that maps the x's into vectors denoted by y's as follows:** $\boxed{\mathrm{y} = \mathrm{A}(\mathrm{x} - \mathrm{m_x})}$
- **This expression is called the <u>Hotelling transform</u> and has some interesting and useful properties...**

Slide credit: TW793

Principal Components As Feature Descriptors

- It is possible to prove the following:

$$\mathbf{m_y} = E\{\mathbf{y}\} = 0$$

$$\mathbf{C_y} = \mathbf{A}\mathbf{C_x}\mathbf{A}^T$$

- Furthermore, $\mathbf{C_y}$ is a diagonal matrix whose elements along the main diagonal are the eigenvalues of $\mathbf{C_x}$, that is

$$\mathbf{C_y} = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix}$$

- Note that the elements of the $\mathbf{y}$ vectors are uncorrelated
- Also, $\mathbf{C_x}$ and $\mathbf{C_y}$ have the same eigenvalues; the eigenvectors of $\mathbf{C_y}$ are in the direction of the main axes

Inverse Hotelling transform: $\boxed{\mathbf{x} = \mathbf{A}^T\mathbf{y} + \mathbf{m_x}}$

- Suppose that instead of using all the eigenvectors of $\mathbf{C_x}$ we form matrix $\mathbf{A}_k$ from the k eigenvectors corresponding to the k largest eigenvalues, yielding a transformation matrix of order $k \times n$
- The $\mathbf{y}$ vectors will then be k dimensional and the reconstruction will no longer be exact

Slide credit: TW793

Principal Components As Feature Descriptors

- **The vector reconstructed by using $\mathbf{A}_k$ is**

$$\hat{\mathbf{x}} = \mathbf{A}_k^T \mathbf{y} + \mathbf{m}_{\mathbf{x}}$$

- **It can be shown that the mean square error between $\mathbf{x}$ and $\hat{\mathbf{x}}$ is given by the expression**

$$\begin{aligned} e_{\mathbf{ms}} &= \sum_{j=1}^{n} \lambda_j - \sum_{j=1}^{k} \lambda_j \\ &= \sum_{j=k+1}^{n} \lambda_j \end{aligned}$$

- **The first line indicates that the error is zero if $k = n$, that is if all the eigenvectors are used in the transformation**
- **Note that the error can be minimized by selecting the k eigenvectors associated with the largest eigenvalues**
- **The Hotelling transform is optimal in the sense that it minimizes the mean square error between $\mathbf{x}$ and $\hat{\mathbf{x}}$**
- **Due to this idea of using the eigenvectors corresponding with the largest eigenvalues, the Hotelling transform also is known as the principal components transform**

Slide credit: TW793

Principal Components As Feature Descriptors

Using principal components for image description

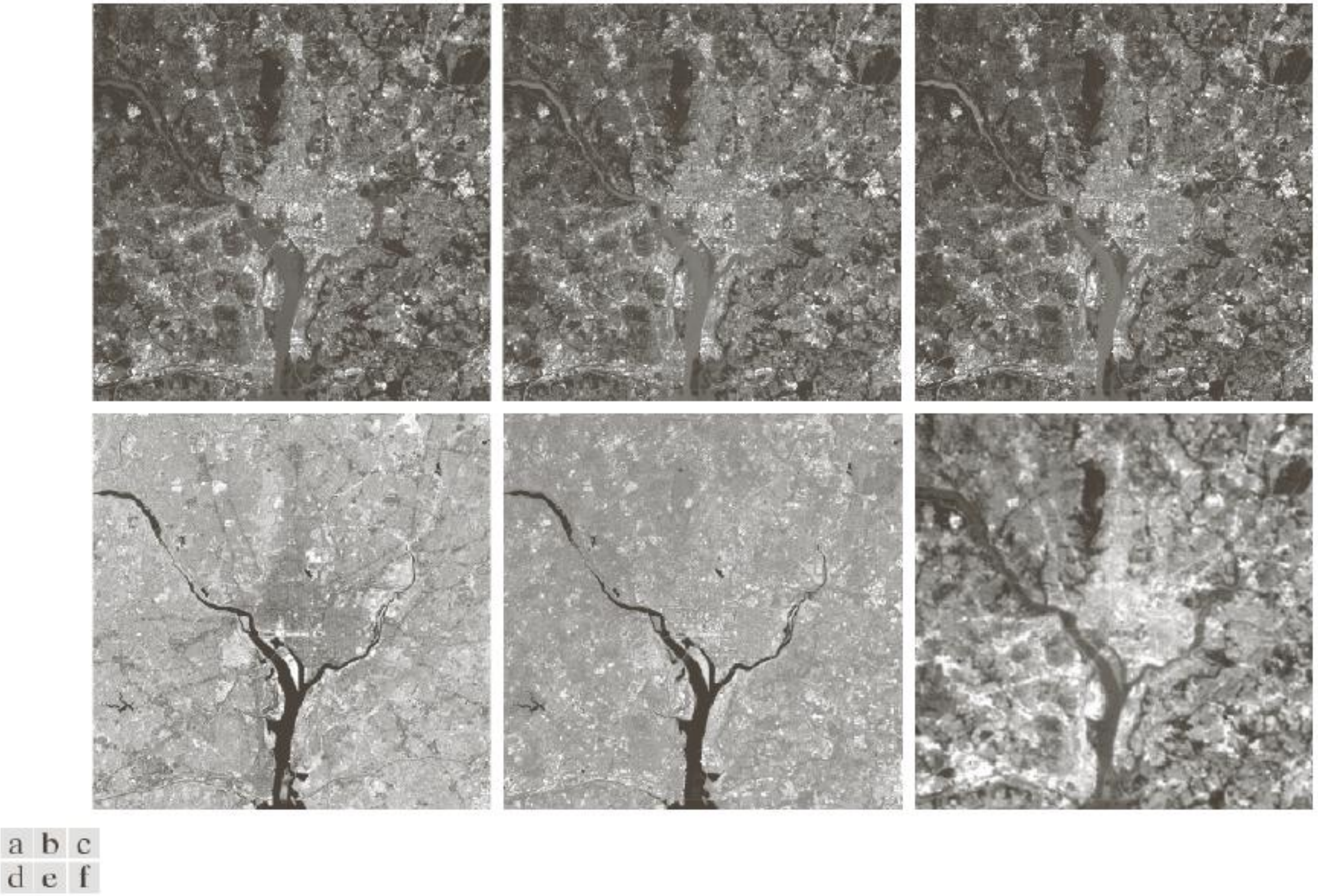


FIGURE 11.38 Multispectral images in the (a) visible blue, (b) visible green, (c) visible red, (d) near infrared, (e) middle infrared, and (f) thermal infrared bands. (Images courtesy of NASA.)

Slide credit: TW793

Principal Components As Feature Descriptors

Using principal components for image description

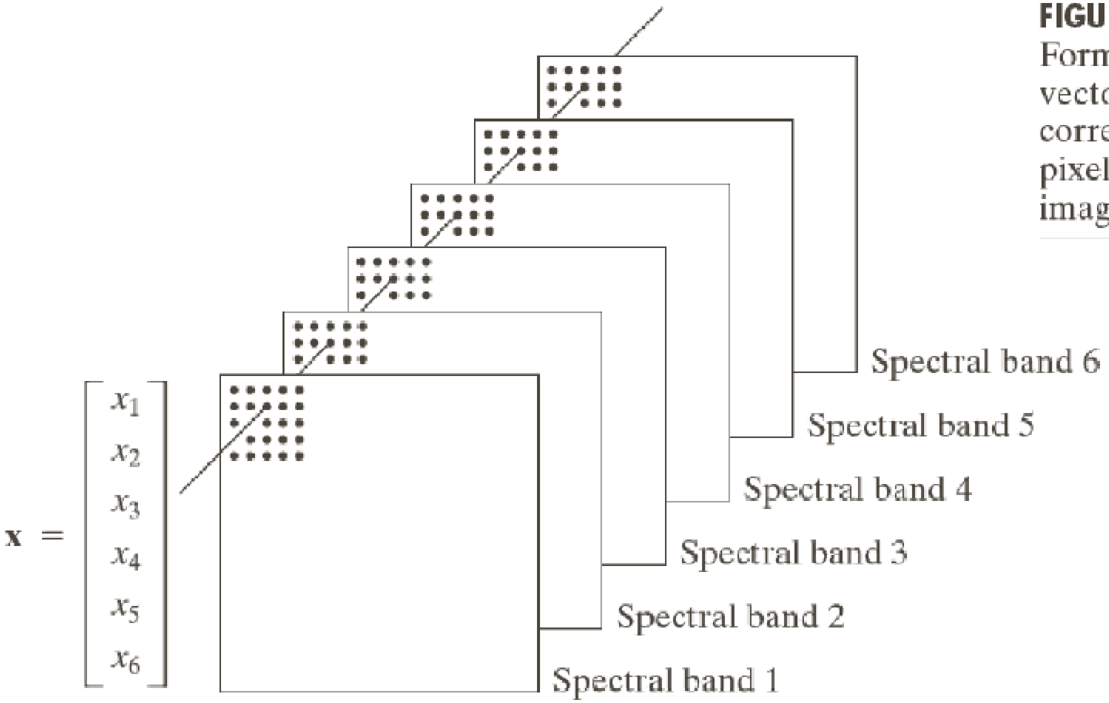


FIGURE 11.39 Formation of a vector from corresponding pixels in six images.

λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
10344	2966	1401	203	94	31

TABLE 11.6 Eigenvalues of the covariance matrices obtained from the images in Fig. 11.38.

Slide credit: TW793

Principal Components As Feature Descriptors

Using principal components for image description

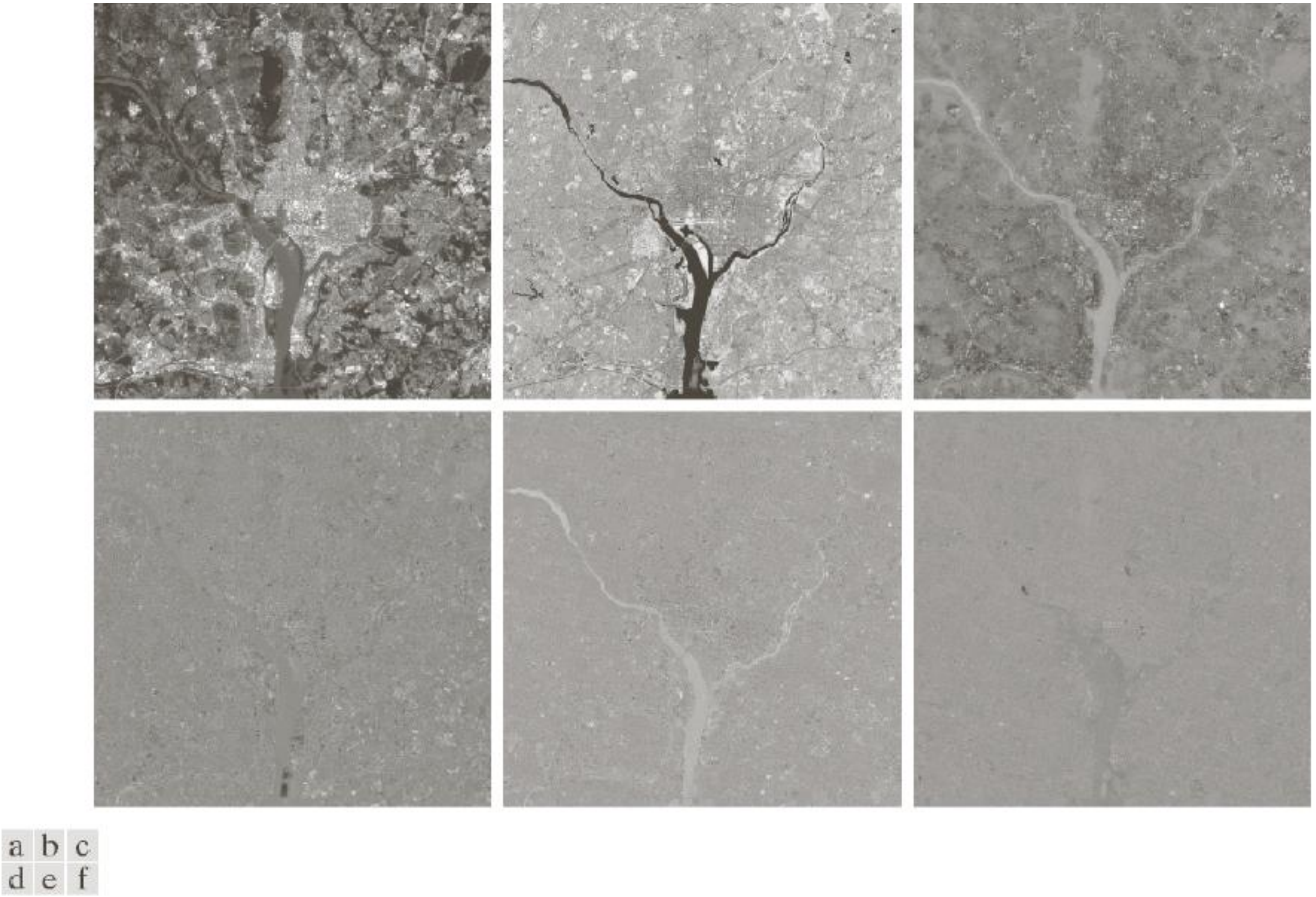
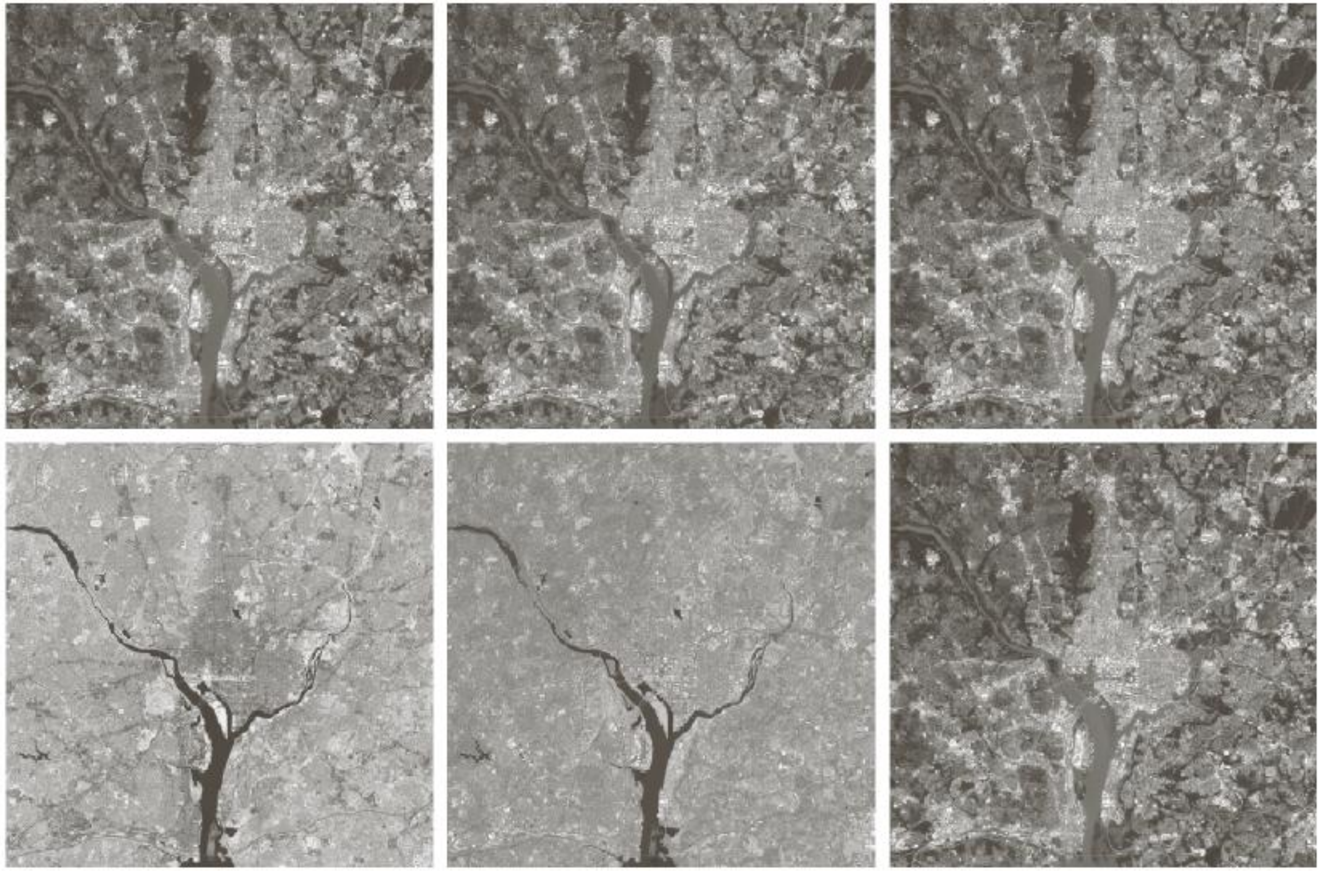


FIGURE 11.40 The six principal component images obtained from vectors computed using Eq. (11.4-6). Vectors are converted to images by applying Fig. 11.39 in reverse.

Slide credit: TW793

Principal Components As Feature Descriptors

Using principal components for image description



a	b	c
d	e	f

FIGURE 11.41 Multispectral images reconstructed using only the two principal component images corresponding to the two principal component images with the largest eigenvalues (variance). Compare these images with the originals in Fig. 11.38.

Slide credit: TW793

Principal Components As Feature Descriptors

Using principal components for image description

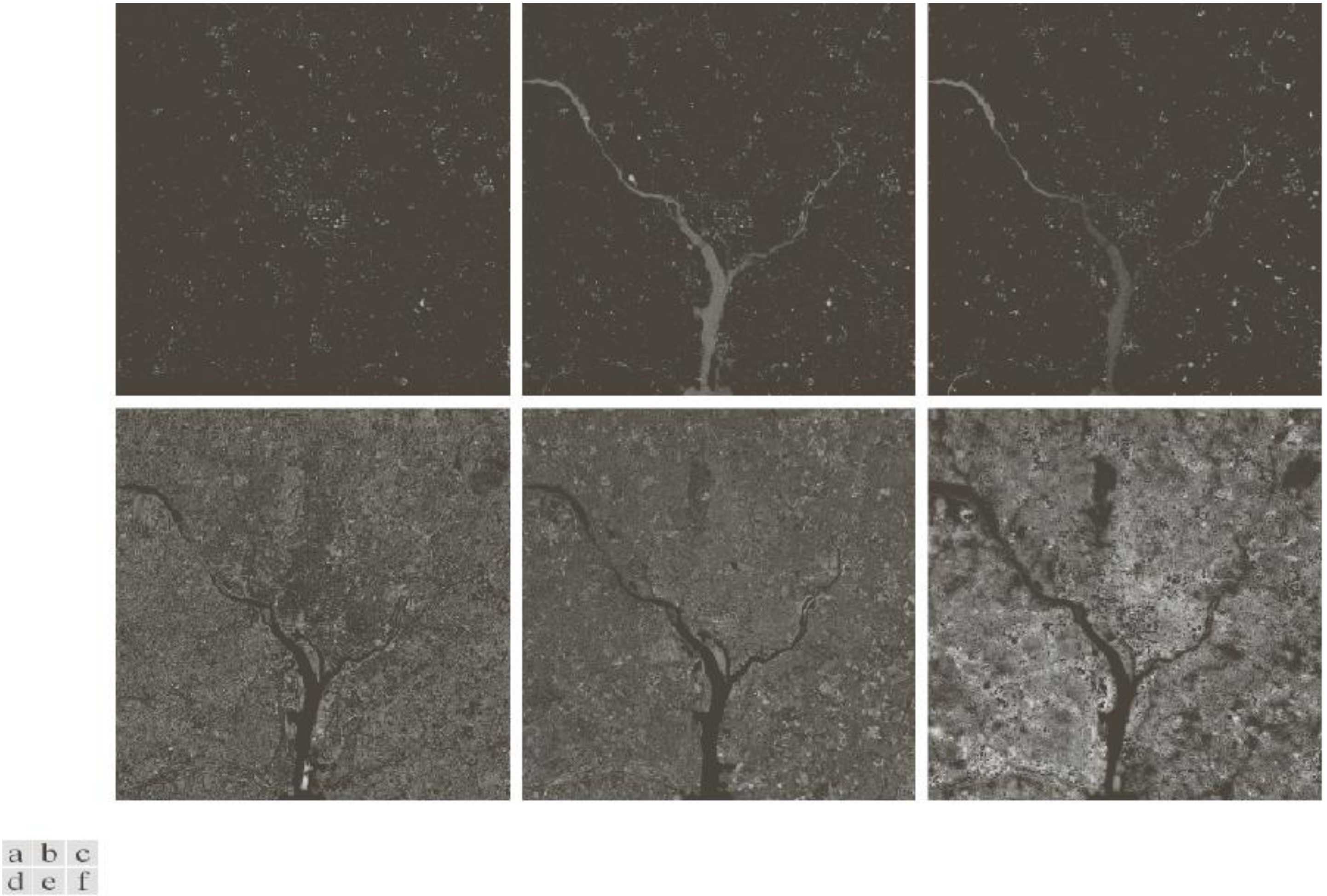


FIGURE 11.42 Differences between the original and reconstructed images. All difference images were enhanced by scaling them to the full [0, 255] range to facilitate visual analysis.

Slide credit: TW793