CSL7320: Digital Image Analysis

Morphological Image Processing

Morphological Image Processing

- The language of mathematical morphology is set theory.
- Morphological operations on images are defined in terms of sets.
- Morphology with two types of sets of pixels: **objects** and **structuring elements (SE’s).**
- **Objects** are defined as sets of foreground pixels.
- **Structuring elements** can be specified in terms of both foreground and background pixels
  - It also includes “don’t care” elements, denoted by ×, signifying that the value of that particular element in the SE does not matter

Morphological Image Processing

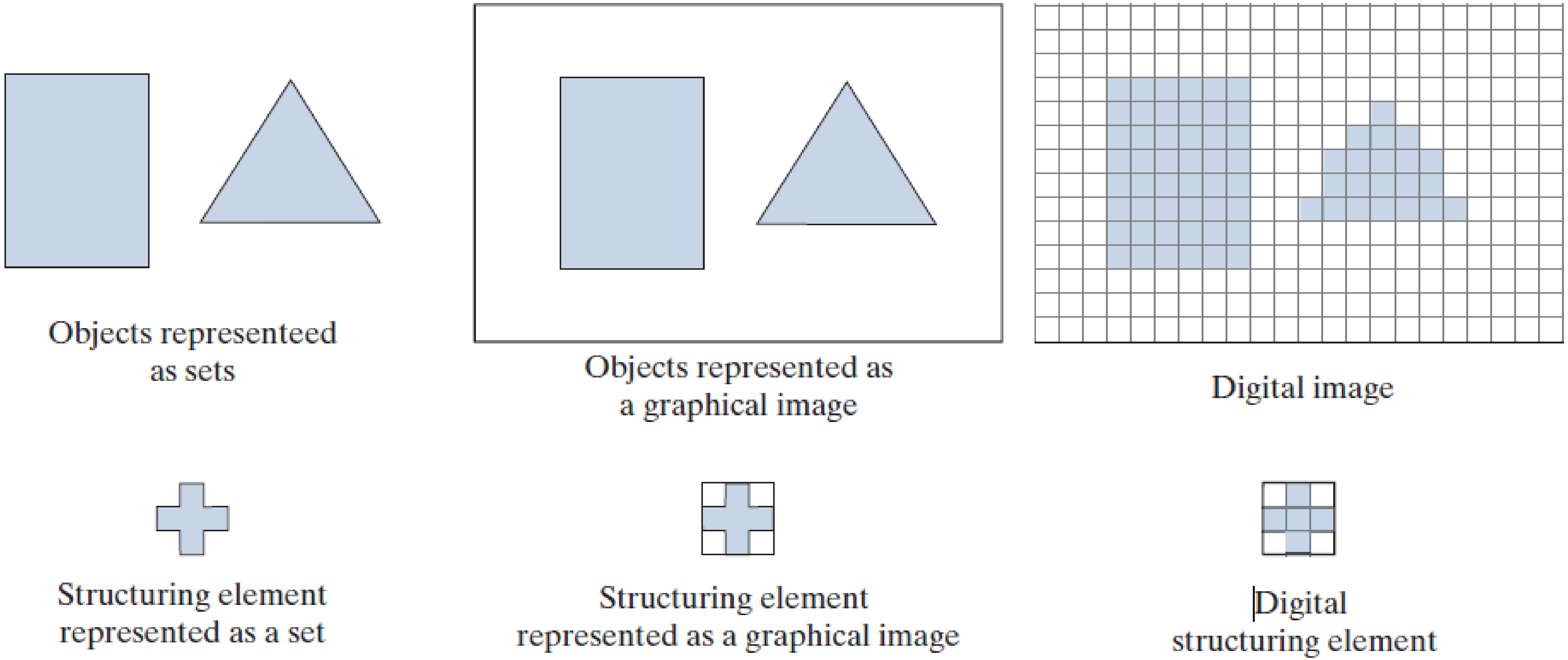


FIGURE 9.1 Top row. *Left:* Objects represented as graphical sets. *Center:* Objects embedded in a background to form a graphical image. *Right:* Object and background are digitized to form a digital image (note the grid). Second row: Example of a structuring element represented as a set, a graphical image, and finally as a digital SE.

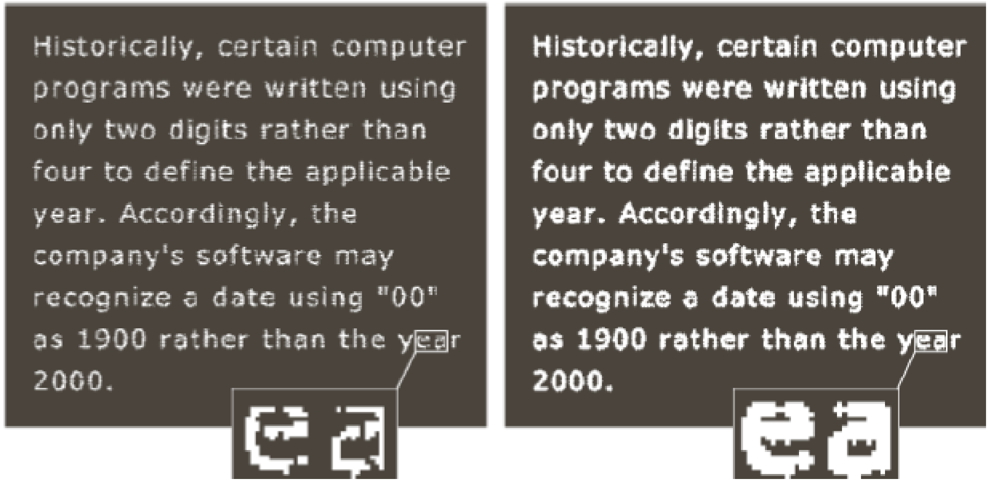
Morphological Image Processing

Objective: Extract image components for representation and description of region shape including

- Boundaries
- Skeletons
- Convex hull

Applications:

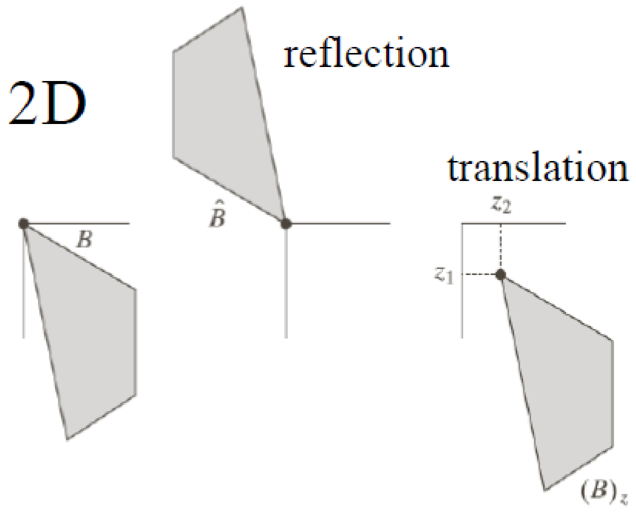
- Edge detection
- Blob/connected component detection

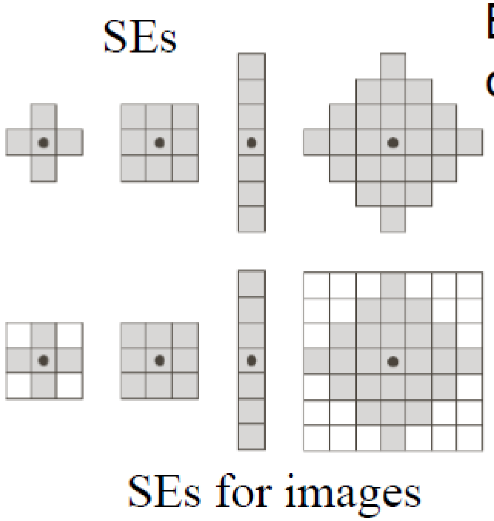


Slide credit: Yan Tong

Basic Concepts

- **2D Integer space Z^2**
- **Union, intersection, complement, difference**
- **Set reflection** $\widehat{\mathcal{B}} = \{\mathbf{w} | \mathbf{w} = -\mathbf{b}, \mathbf{b} \in \mathcal{B}\}$ $\mathcal{B}$ is a set of 2D points (x, y)
- **Set translation** $(\mathcal{B})_z = \{\mathbf{c} | \mathbf{c} = \mathbf{b} + \mathbf{z}, \mathbf{b} \in \mathcal{B}\}$ -- move the center/origin of $\mathcal{B}$ by **z**
- **Structure elements (SEs): small sets/subimages used in morphology**



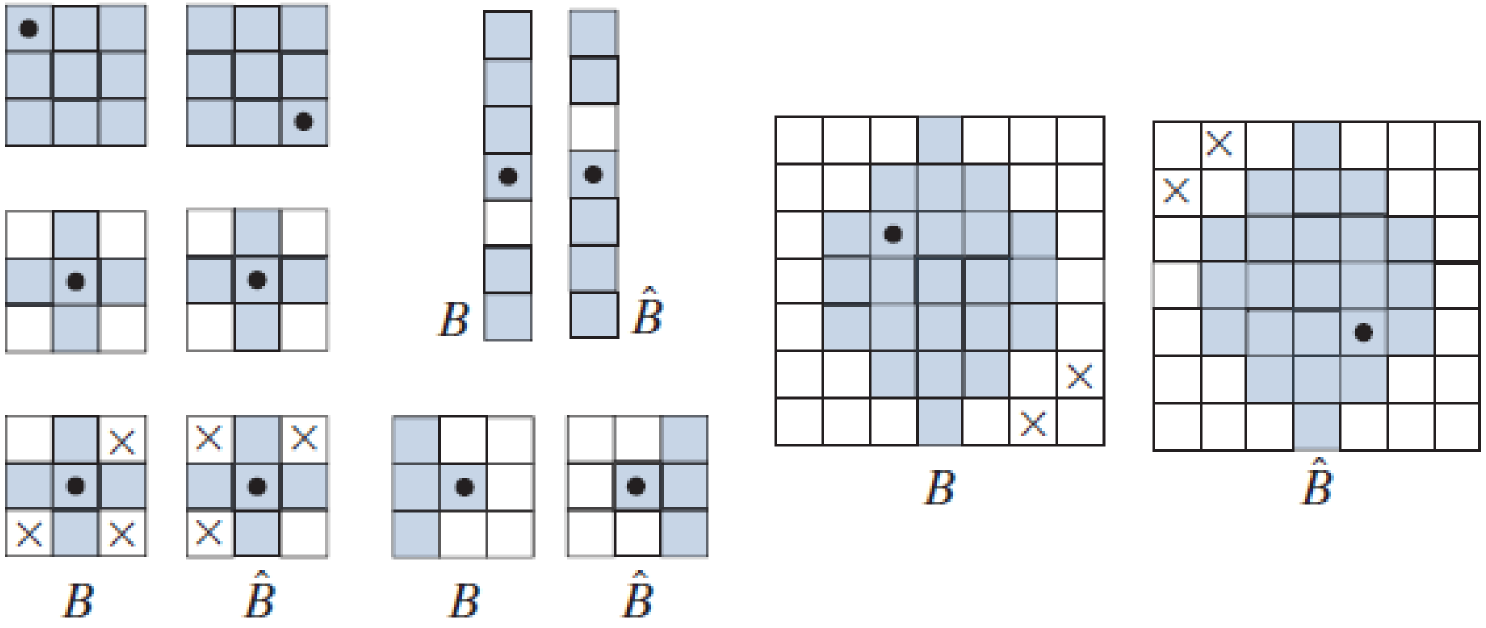


Black dots are centers/origins of SE

Slide credit: Yan Tong

Basic Concepts

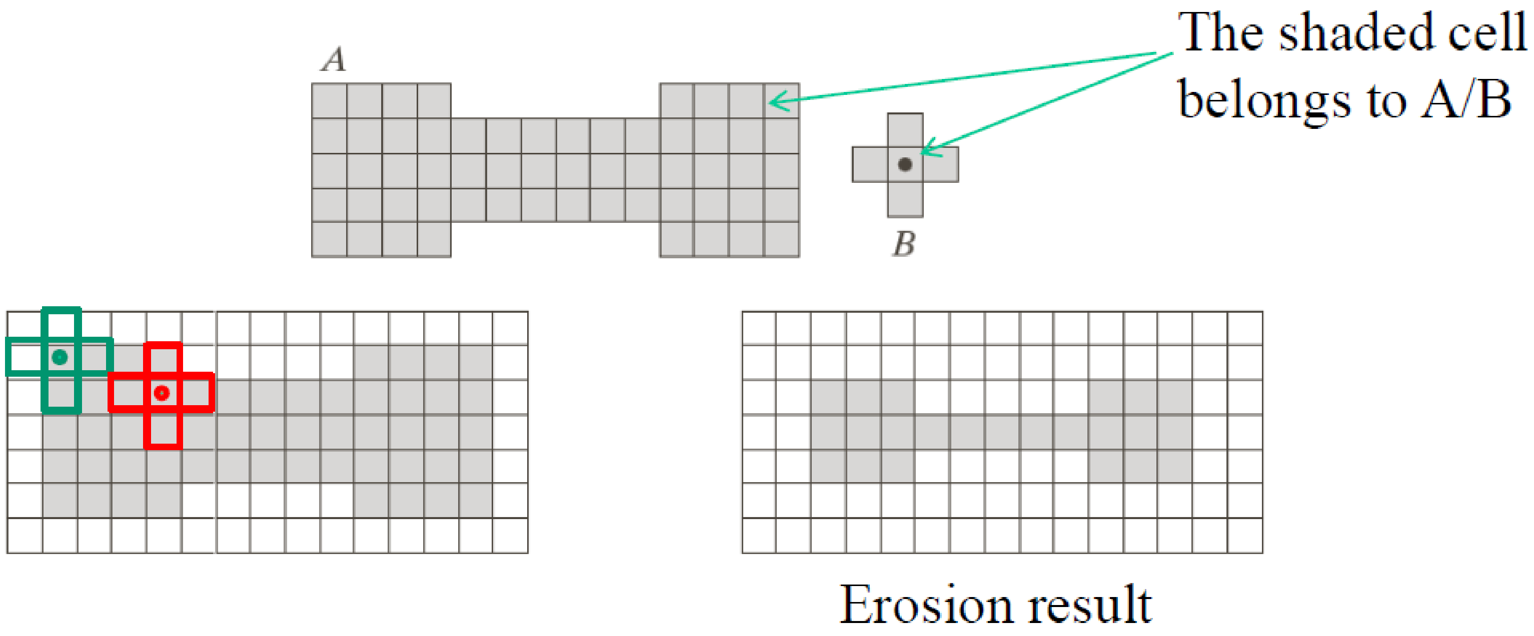
FIGURE 9.2 Structuring elements and their reflections about the origin (the $\times$'s are don't care elements, and the dots denote the origin). Reflection is rotation by 180° of an SE about its origin.



Morphological Image Processing: Example

Morphology - Create a new set by running B over A so that the origin of B visits every element of A.

An example of erosion: If B is completely contained in A for each operation, the new element is a member of the new set.



Slide credit: Yan Tong

Morphological Image Processing: Example

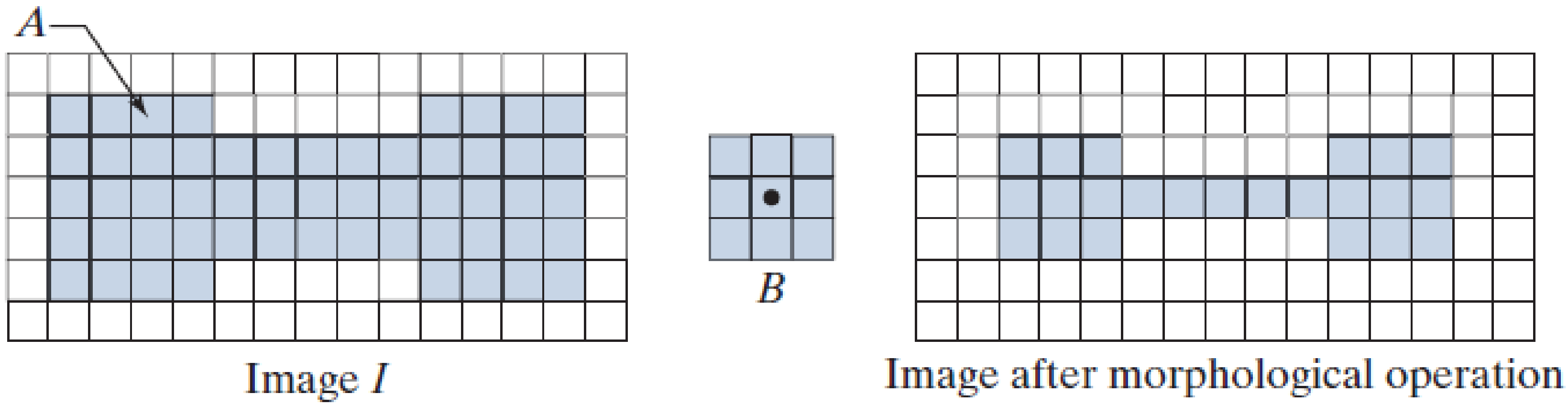


FIGURE 9.3 (a) A binary image containing one object (set), A. (b) A structuring element, B. (c) Image resulting from a morphological operation (see text).

Common Morphological Operations

Two basic operations

- Erosion
- Dilation

Other operations

- Opening/closing
- Hit-or-Miss transform
- Thinning/thickening
- Hole filling

Slide credit: Yan Tong

Erosion

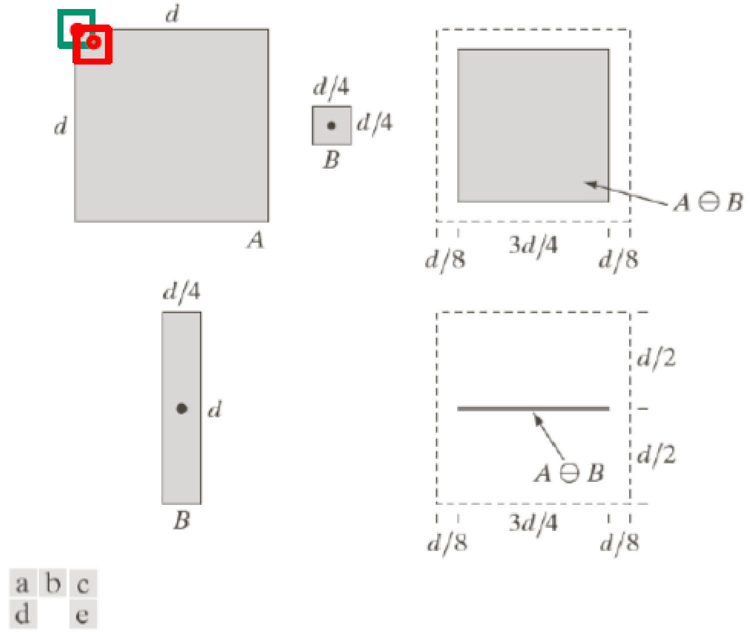


FIGURE 9.4 (a) Set A. (b) Square structuring element, B. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A by B using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference.

Recall Set Translation

$$(B)_z = \{\mathbf{c} \mid \mathbf{c} = \mathbf{b} + \mathbf{z}, \mathbf{b} \in B\}$$

$$A \ominus B = \{z \mid (B)_z \subseteq A\} \quad \text{or} \quad A \ominus B = \{z \mid (B)_z \cap A^c = \emptyset\}$$

Shrink or thin objects and remove the details smaller than the SE

Slide credit: Yan Tong

Erosion

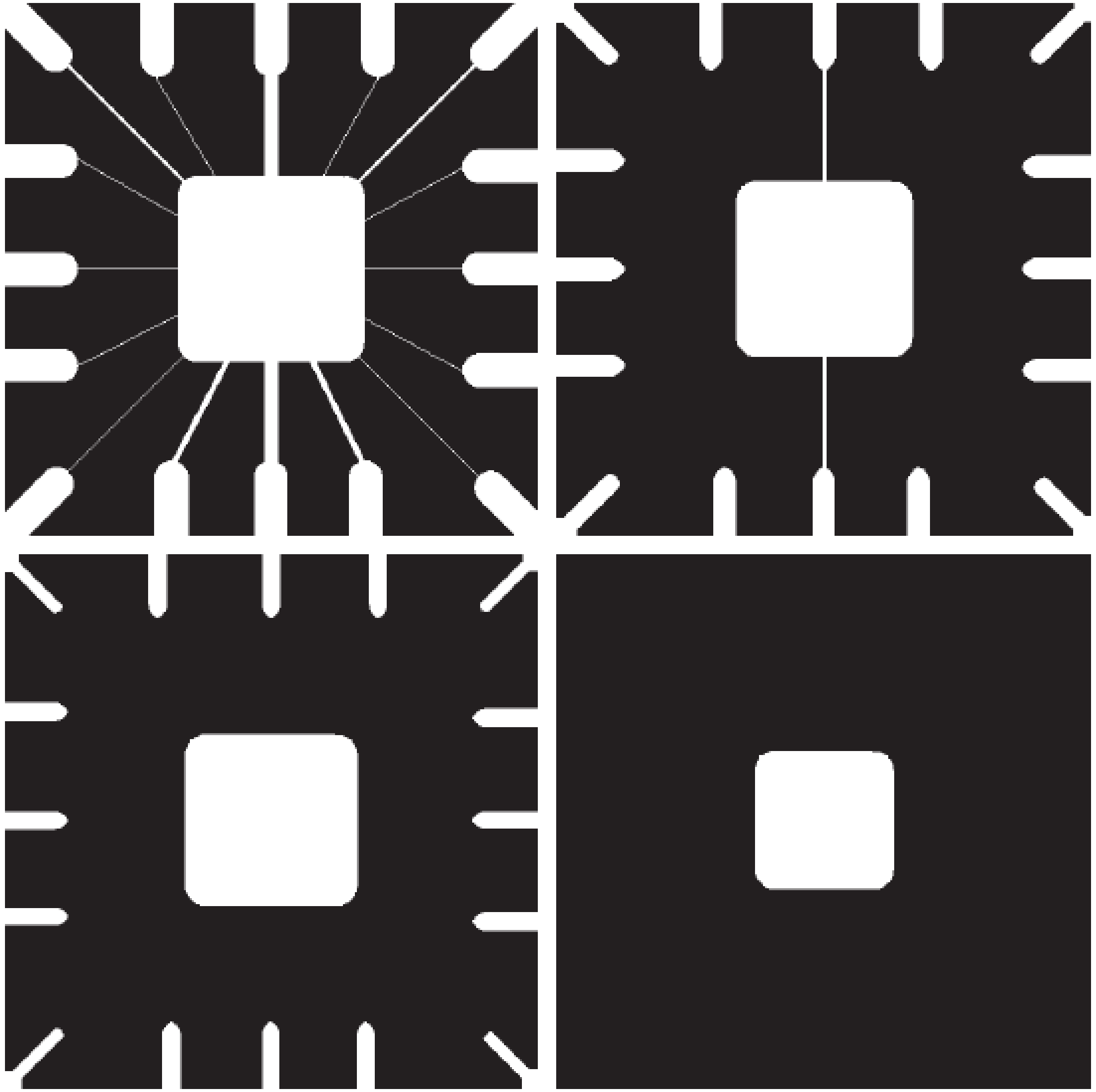


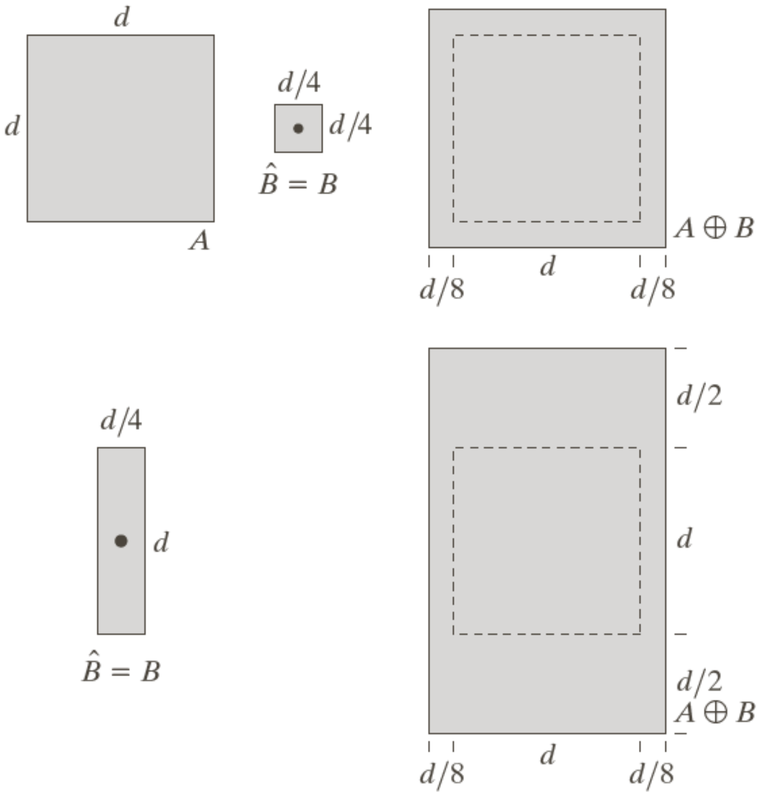
FIGURE 9.5 Using erosion to remove image components. (a) A 486 × 486 binary image of a wire-bond mask in which foreground pixels are shown in white. (b)–(d) Image eroded using square structuring elements of sizes 11 × 11, 15 × 15, and 45 × 45 elements, respectively, all valued 1.

$$A \ominus B = \{z \mid (B)_z \subseteq A\} \quad \text{or} \quad A \ominus B = \{z \mid (B)_z \cap A^c = \emptyset\}$$

Shrink or thin objects and remove the details smaller than the SE

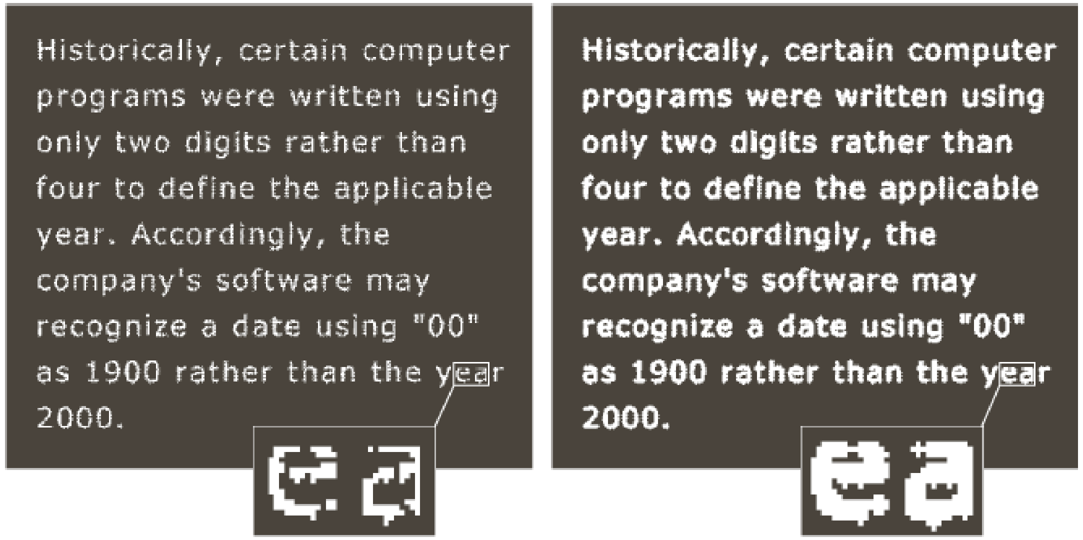
Slide credit: Yan Tong

Dilation



a b c
d e

FIGURE 9.6
(a) Set A.
(b) Square structuring element (the dot denotes the origin).
(c) Dilation of A by B, shown shaded.
(d) Elongated structuring element. (e) Dilation of A using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference



0	1	0
1	1	1
0	1	0

Recall set reflection about its origin: $\hat{B} = \{w \mid w = -b, b \in B\}$

$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq 0\} \quad \text{or} \quad A \oplus B = \{z \mid (\hat{B})_z \cap A \subseteq A\}$$

Grows or thickens objects and remove the gaps smaller than the SE

Slide credit: Yan Tong

Properties of Dilation

- **Dilation is commutative** $A \oplus B = B \oplus A$

- **Dilation is associative** $A \oplus B \oplus C = A \oplus \left(B \oplus C\right)$

- **Dilation is distributive over the union operation**

$$A \oplus (B \cup C) = (A \oplus B) \cup (A \oplus C)$$

Slide credit: Yan Tong

Properties of Erosion and Dilation

- **Erosion** $A \ominus B \ominus C = A \ominus (B \oplus C)$

- **Erosion and dilation are duals of each other**

$$A \oplus B = \left(A^c \ominus \hat{B}\right)^c$$

- $A \subseteq (C \ominus B)$ **if and only if** $(A \oplus B) \subseteq C$

- **If** $A \subseteq C$, $A \oplus B \subseteq C \oplus B$ **and** $A \ominus B \subseteq C \ominus B$

Slide credit: Yan Tong

Duality

Erosion and dilation are *duals* of each other with respect to set complementation and reflection. That is,

$$(A \ominus B)^c = A^c \oplus \hat{B} \tag{9-8}$$

and

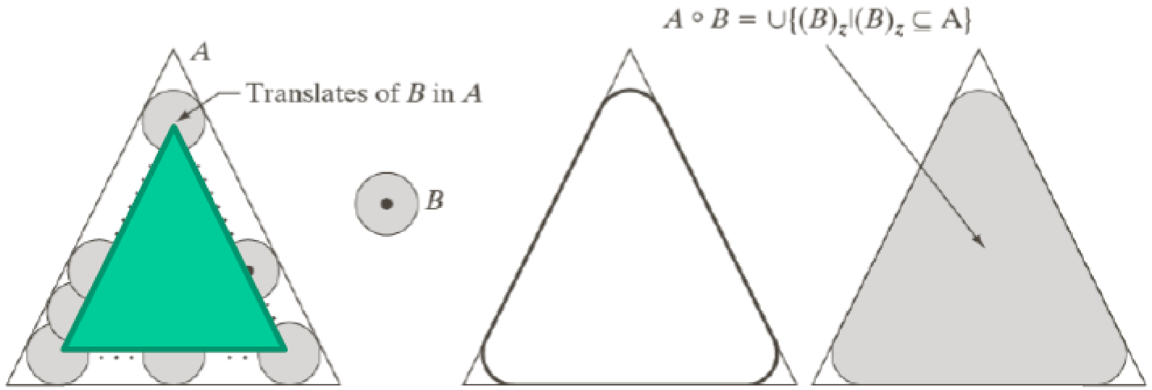
$$(A \oplus B)^c = A^c \ominus \hat{B} \tag{9-9}$$

Opening

- **Smooth the contour of an object**
- **Break narrow bridges**
- **Eliminate thin protrusions**

$$A \circ B = (A \ominus B) \oplus B$$
$$(A \circ B) \circ B = A \circ B$$
$$(A \circ B) \subseteq A$$
$$if\, A \subseteq C,\, A \circ B \subseteq C \circ B$$

$$A \circ B = \cup \{(B)_z \mid (B)_z \subseteq A\}$$



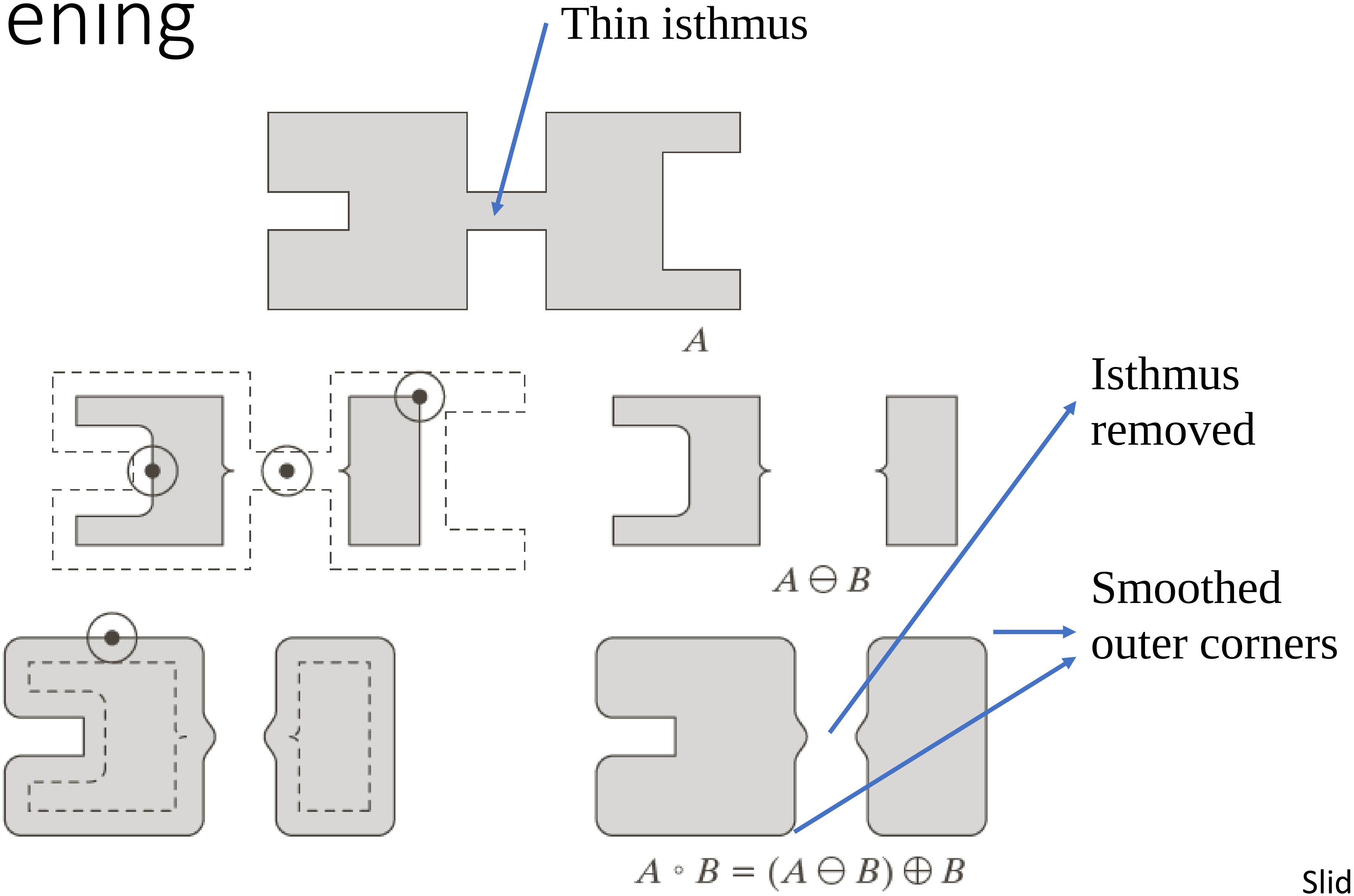
a b c d

FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (b) Structuring element. (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded). We did not shade A in (a) for clarity.

The SE rolls within the boundary of A.

Slide credit: Yan Tong

Opening



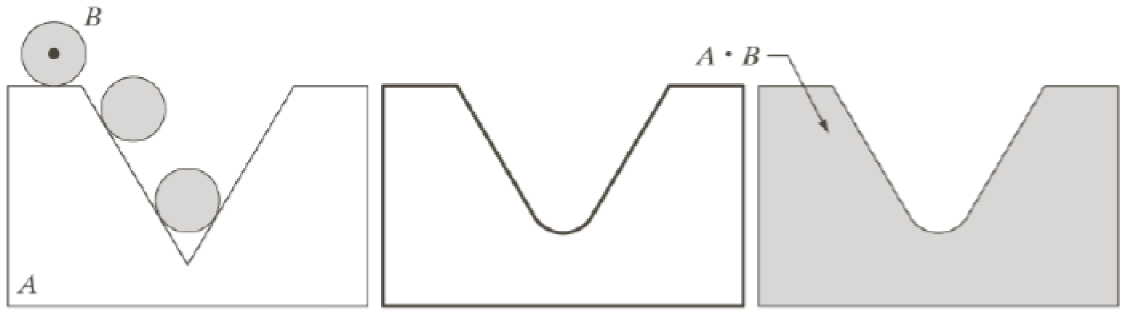
Slide credit: Yan Tong

Closing

- **Smooth the contour of an object**
- **Fill narrow breaks and gaps**
- **Eliminate long and thin gulfs**
- **Eliminate small holes**

$$A \bullet B = (A \oplus B) \ominus B$$
$$(A \bullet B) \bullet B = A \bullet B$$
$$A \subseteq (A \bullet B)$$
$$if\, A \subseteq C, A \bullet B \subseteq C \bullet B$$

Opening and closing are duals of each other $A \bullet B = (A^c \circ \hat{B})^c$



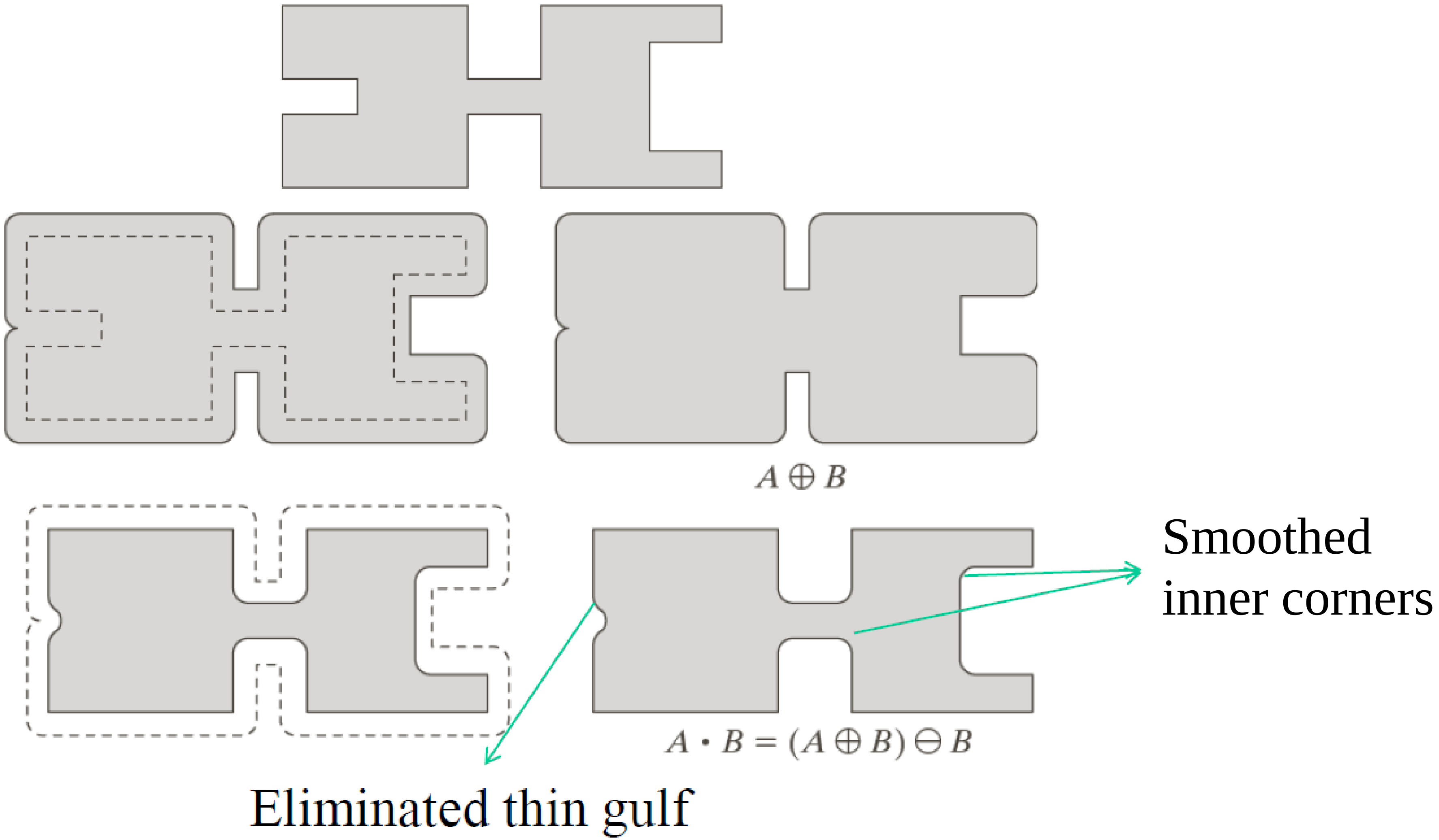
a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) The heavy line is the outer boundary of the closing. (c) Complete closing (shaded). We did not shade A in (a) for clarity.

The SE rolls outside the boundary of A.

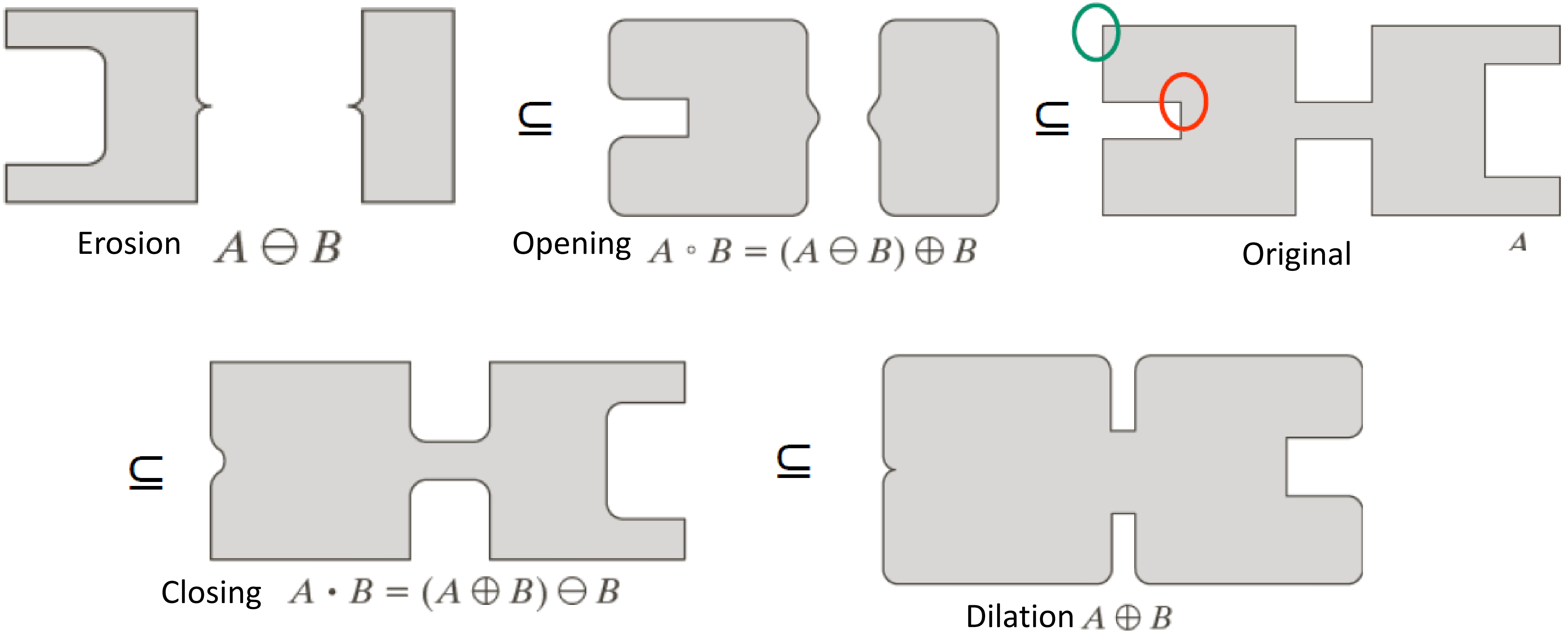
Slide credit: Yan Tong

Closing



Slide credit: Yan Tong

Opening and Closing



$$A \ominus B \subseteq A \circ B \subseteq A \subseteq \quad A \bullet B \subseteq A \oplus B$$

Slide credit: Yan Tong

An Example of Opening and Closing

- An opening removes all noise
  - removing the white noise by erosion
  - removing the black noise by dilation
- An additional closing fills the gaps

FIGURE 9.11 (a) Noisy image. (b) Structuring element. (c) Eroded image. (d) Opening of A. (e) Dilation of the opening. (f) Closing of the opening. (Original image courtesy of the National Institute of Standards and Technology.)

Slide credit: Yan Tong

Applications of Morphological Operations

- **Boundary extraction**
- **Hole filing**
- **Connected component analysis**
- **Convex hull extraction**
- **Skeleton analysis**

Slide credit: Yan Tong