

Q1.1

((lambda (x1 y1) (if (> x1 y1) #t #f)) 8 3)-

Assign type variables for every sub expression:

((lambda (x1 y1) (if (> x1 y1) #t #f)) 8 3)	T_0
(lambda (x1 y1) (if (> x1 y1) #t #f))	T_1
(if (> x1 y1) #t #f)	T_2
(> x1 y1)	T_4
#t	T_5
#f	T_6
>	T_7
x1	T_8
x2	T_9
8	T_{10}
3	T_{11}

Construct type equations:

((lambda (x1 y1) (if (> x1 y1) #t #f)) 8 3)	$T_1 = [T_{10} * T_{11} \rightarrow T_0]$
(lambda (x1 y1) (if (> x1 y1) #t #f))	$T_1 = [T_8 * T_9 \rightarrow T_2]$
(if (> x1 y1) #t #f)	$T_2 = T_5$
(> x1 y1)	$T_4 = \text{Boolean}$
#t	$T_5 = \text{Boolean}$
#f	$T_6 = \text{Boolean}$
>	$T_7 = [\text{Number} * \text{Number} \rightarrow \text{Boolean}]$
8	$T_{10} = \text{Number}$
3	$T_{11} = \text{Number}$

Solving the equations:

$T_1 = [T_{10} * T_{11} \rightarrow T_0]$	$T_1 = [T_{10} * T_{11} \rightarrow T_0]$
$T_1 = [T_8 * T_9 \rightarrow T_2]$	
$T_2 = T_5$	
$T_4 = \text{Boolean}$	
$T_5 = \text{Boolean}$	
$T_6 = \text{Boolean}$	
$T_7 = [\text{Number} * \text{Number} \rightarrow \text{Boolean}]$	
$T_{10} = \text{Number}$	
$T_{11} = \text{Number}$	

T₄ = [T ₈ *T ₉ →T ₂]	T ₁ = [T ₁₀ *T ₁₁ →T ₀]
T ₂ = T ₅	
T ₄ = Boolean	
T ₅ = Boolean	
T ₆ = Boolean	
T ₇ = [Number*Number→Boolean]	
T ₁₀ = Number	
T ₁₁ = Number	
T ₈ = T ₁₀	
T ₉ = T ₁₁	
T ₂ = T ₀	

T₂ = T ₅	T ₁ = [T ₁₀ *T ₁₁ →T ₀] T ₂ = T ₅ T ₄ = Boolean
T₄ = Boolean	
T ₅ = Boolean	
T ₆ = Boolean	
T ₇ = [Number*Number→Boolean]	
T ₁₀ = Number	
T ₁₁ = Number	
T ₂ = T ₀	

T₅ = Boolean	T ₁ = [T ₁₀ *T ₁₁ →T ₀] T ₂ = Boolean T ₄ = Boolean T ₅ = Boolean T ₆ = Boolean
T₆ = Boolean	
T ₇ = [Number*Number→Boolean]	
T ₁₀ = Number	
T ₁₁ = Number	
T ₂ = T ₀	

T₇ = [Number*Number→Boolean]	T ₁ = [T ₁₀ *T ₁₁ →T ₀] T ₂ = Boolean T ₄ = Boolean T ₅ = Boolean T ₆ = Boolean T ₇ = [Number*Number→Boolean]
T ₁₀ = Number	
T ₁₁ = Number	
T ₂ = T ₀	

T₁₀ = Number	T ₁ = [Number*Number→T ₀] T ₂ = Boolean T ₄ = Boolean T ₅ = Boolean T ₆ = Boolean T ₇ = [Number*Number→Boolean] T ₁₀ = Number T ₁₁ = Number
T₁₁ = Number	
T ₂ = T ₀	

T₂ = T ₀	T ₁ = [Number*Number→ Boolean] T ₂ = Boolean T ₄ = Boolean T ₅ = Boolean T ₆ = Boolean T ₇ = [Number*Number→Boolean] T ₁₀ = Number T ₁₁ = Number T ₀ = Boolean
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T₀ = Boolean => ((lambda (x1 y1) (if (> x1 y1) #t #f)) 8 3) is of type Boolean.

Q1.2

a. $\{f:[T1 \rightarrow T2], x: T1\} \vdash (f\ x): T2$

True.

In the given environment; x is from type T1, and f is a function from T1 to T2. Hence, the output of provoking f on x, is from type T2.

b. $\{f:[T1 \rightarrow T2], g: [T2 \rightarrow T3]\}, x: T2\} \vdash (f\ g\ x): T3$

False.

(f g x) is not a valid expression because f accept 1 operand of type T1, hence False

c. $\{f:[T2 \rightarrow T1], g: [T1 \rightarrow T2], x: T1\} \vdash (f\ (g\ x)): T1$

True.

In the given environment x is from type T1, and g operates on T1 and output T2 $\Rightarrow g(x) : T2$

f is a function from T2 to T1 $\Rightarrow g(x)$ is in the range of f so the operation is valid, and the output will be from type T1

$\Rightarrow f\ (g(x))$

$x:T1$

$\Rightarrow g(x) : T2$

$\Rightarrow f(g(x)) : T1$

d. $\{f:[T2 \rightarrow \text{Number}], x: \text{Number}\} \vdash (f\ x\ x): \text{Number}$

False.

f doesn't operate on 2 operands, hence the expression is invalid.

3.

a. $\text{cons} :: [T1 * T2 \rightarrow \text{Pair} \langle T1, T2 \rangle]$

b. $\text{car} :: [\text{Pair} \langle T1, T2 \rangle \rightarrow T1]$

c. $\text{cdr} :: [\text{Pair} \langle T1, T2 \rangle \rightarrow T2]$

4.

(Define f (lambda (x) (values x x x)))

$[T1 \rightarrow (T1 * T1 * T1)]$

If $x:T1 \Rightarrow (\text{values } x\ x\ x) = (x * x * x)$

5. Write the MGU of the following expressions, or state that there is no such MGU.

a. $T1, T2 \Rightarrow \{T1 = T2\}$

b. $\text{Number}, \text{Number} \Rightarrow \{\}$

c. $[T1 * [T1 \rightarrow T2] \rightarrow \text{Number}], [[T3 \rightarrow \text{Number}] * [T4 \rightarrow \text{Number}] \rightarrow \text{Number}] \Rightarrow \{T1 = T4 = [T3 \rightarrow \text{Number}], T2 = \text{Number}\}$

d. $[T1 \rightarrow T1], [T1 \rightarrow [\text{Number} \rightarrow \text{Number}]] \Rightarrow$

$\{T1 = [\text{Number} \rightarrow \text{Number}]\}$

There is no unifier that will satisfies those expressions.

Q2.3

```
(define f (number -> (number * number))
  (lambda ((x: number)): (number * number)
    (values x (+ x 1))))

(define g (T -> (string * T))
  (lambda (x: T):(string * T)
    (values "x" x)))
```

Q4.b

The use of promises helps us write a cleaner code, using chained promises. Also we have the ability to handle errors of few promises with one catch.

