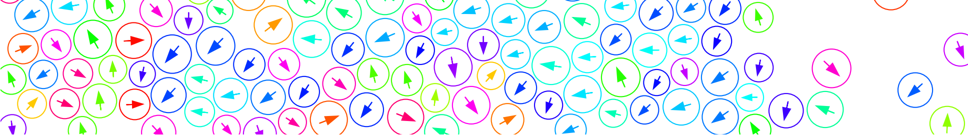


Collective motion in large deviations of active particles

Yann-Edwin Keta

in collaboration with E. Fodor, F. van Wijland, M.E. Cates, and R.L. Jack

APS March Meeting 2021

19/03/2021

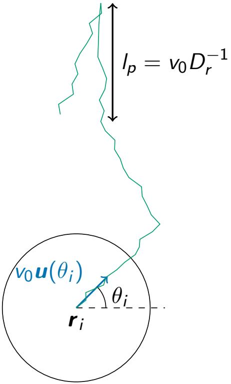
Phys. Rev. E **103**, 022603 (2021)

yketa/DAMTP_MSC_2019_Wiki

SIMONS FOUNDATION

Université de Paris

1. Model of active Brownian particles.
2. Active work and its large deviations.
3. Description of dynamical phase transition.
4. Mechanism for collective motion.



standard ABP model

$$\dot{\boldsymbol{r}}_i = -D\nabla_i U(\{\boldsymbol{r}_j\}) + v_0 \boldsymbol{u}(\theta_i) + \sqrt{2D}\boldsymbol{\eta}_i \quad (1)$$

$$\dot{\theta}_i = \sqrt{2D_r}\xi_i \quad (2)$$

$$\tilde{l}_p = v_0 D_r^{-1}/\sigma \quad (3)$$

T. Nemoto *et al.*, Physical Review E **99**, 022605 (2019).

G. S. Redner *et al.*, Physical Review Letters **110**, 055701 (2013).

How far does the active forcing translate into real motion?

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active work $$w_\tau = \frac{1}{v_0 N \tau} \sum_{i=1}^{N} \int_0^\tau \boldsymbol{u}(\theta_i) \circ \dot{\boldsymbol{r}}_i \, \mathrm{d}t \tag{4}$$

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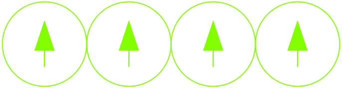
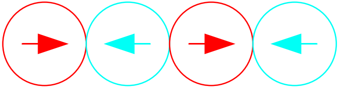


$$\dot{\boldsymbol{r}}_i \approx v_0 \boldsymbol{u}(\theta_i) \Rightarrow \langle w_\tau \rangle = 1$$

jamming



$$\dot{\boldsymbol{r}}_i \approx 0 \Rightarrow \langle w_\tau \rangle = 0$$

How does the active work control emerging behaviours?

H. Touchette, Physics Reports **478**, 1–69 (2009).

R. L. Jack, The European Physical Journal B **93**, 1–22 (2020).

T. Nemoto *et al.*, Physical Review E **93**, 062123 (2016).

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2. Singularities in ψ $\Rightarrow$ qualitative changes at dynamical transition.
3. Compute probabilities of these fluctuations.

$$P(w_\tau) \asymp \exp(-N\tau I(w_\tau)) \tag{8}$$

$$I(w_\tau) = \sup_s \{-s w_\tau - \psi(s)\} \tag{9}$$

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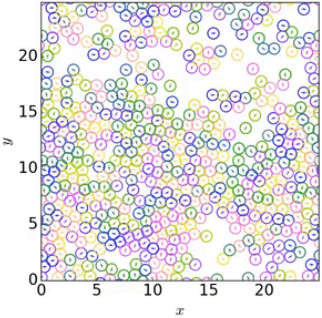
unbiased steady state

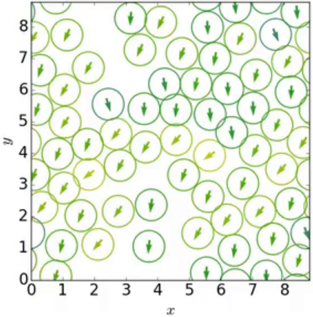
$s = 0$



MIPS

biased to large w_τ

$s < 0$



CM

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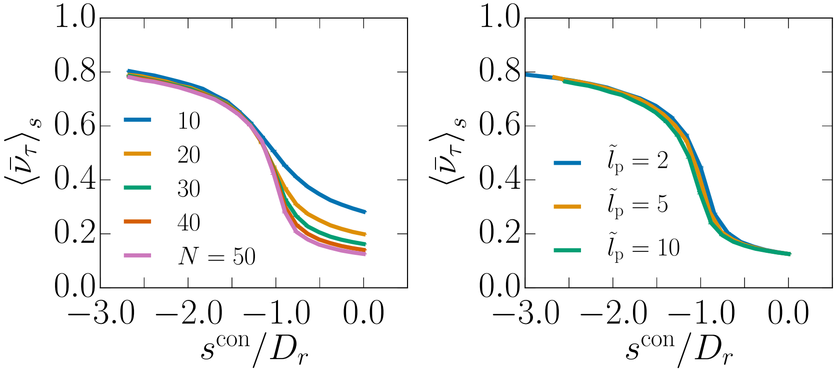

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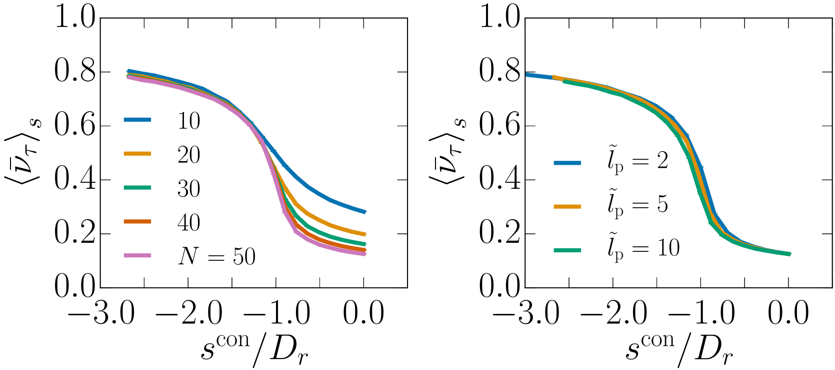

$$s^{\mathrm{con}*} \approx -D_r \qquad (11)$$

separation between steady state physics and symmetry breaking physics

Y.-E. Keta *et al.*, Physical Review E 103, 022603 (2021).

"Large deviations occur according to the least unlikely mechanism."

F. den Hollander, *Large deviations*, Vol. 14 (American Mathematical Soc., 2008).

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$$I(w) \leq \lim_{\tau\to\infty} \mathcal{D}_{\mathrm{KL}}(P_{g(w)}^{\mathrm{mod}}||P) \tag{15}$$

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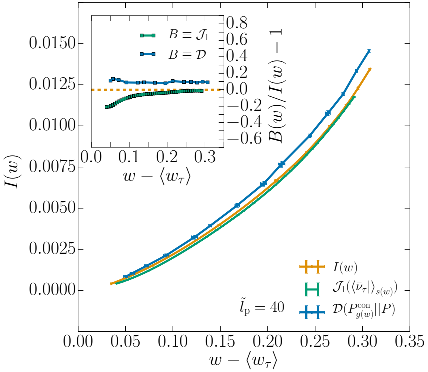

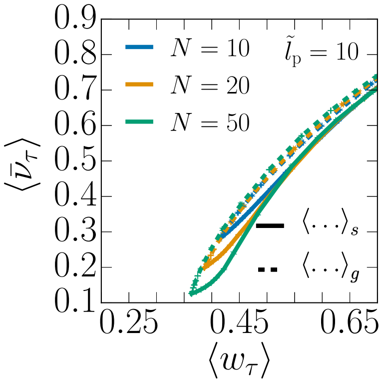

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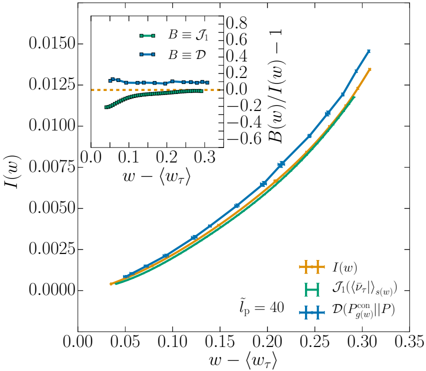

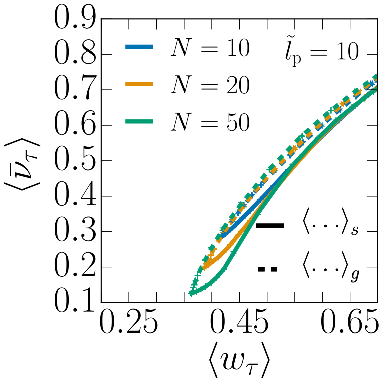

CM: fluctuations of w_τ strongly coupled to those of $\bar{\nu}_\tau$

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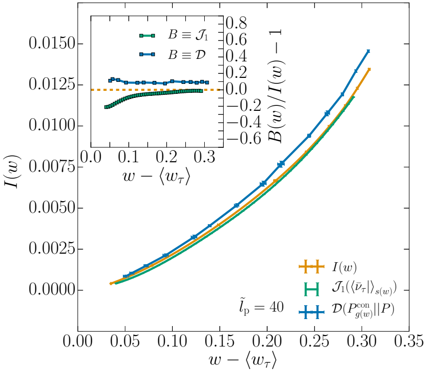

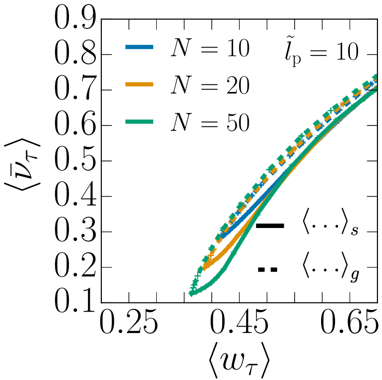

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CM: response to biasing $s \leftrightarrow$ response to aligning interaction g

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PHYSICAL REVIEW E **103**, 022603 (2021)

Collective motion in large deviations of active particles

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Conclusion

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- We compute exactly for a system of **2 particles on a ring** the relation between w_τ and particle alignment.
- We propose a **fluctuating hydrodynamic theory** which captures the emergence of polar order in the biased state.

Thank you!

- At $\boldsymbol{P} = 0$, biasing w.r.t. w_τ is equivalent to biasing w.r.t. $|\tilde{\rho}_{\boldsymbol{q}}|^2$.

$$S_s(\boldsymbol{q}) = \left\langle |\tilde{\rho}_{\boldsymbol{q}}|^2 \right\rangle_s = \begin{cases} \chi_0, & s = 0 \\ b_s q, & s < 0 \end{cases} \tag{16}$$

$\Rightarrow$ We expect hyperuniformity in the isotropic $s < 0$ phase for $N \gg 1$.

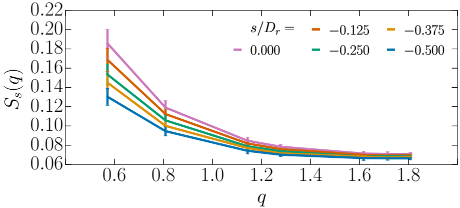

Figure: Biased structure factor S_s.

$\rightarrow$ Finite system shows suppression of density fluctuations for $s < 0$.

J. Dolezal and R. L. Jack, Journal of Statistical Mechanics: Theory and Experiment 2019, 123208 (2019).