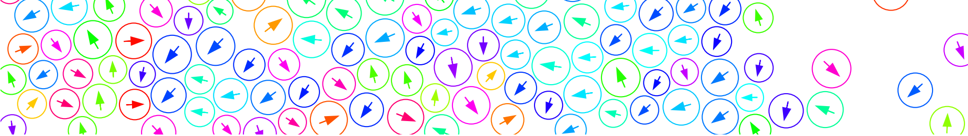


Collective motion in large deviations of active particles

Yann-Edwin Keta

in collaboration with E. Fodor, F. van Wijland, M.E. Cates, and R.L. Jack

DAMTP StatPhys & SoftMat seminars

02/03/2021

Phys. Rev. E **103**, 022603 (2021)

yketa/DAMTP_MSC_2019_Wiki

UNIVERSITY OF CAMBRIDGE

Université de Paris

Contents

Non-equilibrium dynamics breaks time-reversal symmetry and thus detailed balance.

L. Berthier and J. Kurchan, arXiv preprint arXiv:1906.04039 (2019).

M. E. Cates and J. Tailleur, Annu. Rev. Condens. Matter Phys. **6**, 219–244 (2015).

Non-equilibrium dynamics breaks time-reversal symmetry and thus detailed balance. We can identify 3 general classes:

- Systems relaxing towards equilibrium.

Example

Thermal system adapting to its thermostat, glasses.

L. Berthier and J. Kurchan, arXiv preprint arXiv:1906.04039 (2019).

M. E. Cates and J. Tailleur, Annu. Rev. Condens. Matter Phys. **6**, 219–244 (2015).

Non-equilibrium dynamics breaks time-reversal symmetry and thus detailed balance. We can identify 3 general classes:

- Systems relaxing towards equilibrium.
- Systems with boundary conditions imposing steady currents.

Example

Sheared liquid, metal rod between two thermostats.

L. Berthier and J. Kurchan, arXiv preprint arXiv:1906.04039 (2019).

M. E. Cates and J. Tailleur, Annu. Rev. Condens. Matter Phys. **6**, 219–244 (2015).

Non-equilibrium dynamics breaks time-reversal symmetry and thus detailed balance. We can identify 3 general classes:

- Systems relaxing towards equilibrium.
- Systems with boundary conditions imposing steady currents.
- Active matter.

Definition

System composed of self-driven units, *active particles*, each capable of converting stored or ambient free energy into systematic movement.

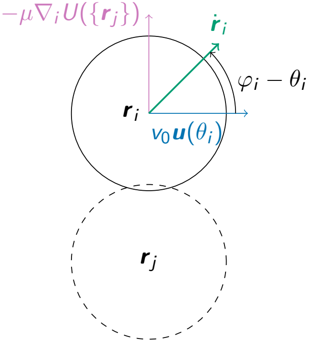
M. C. Marchetti et al., Reviews of Modern Physics **85**, 1143 (2013).

Example

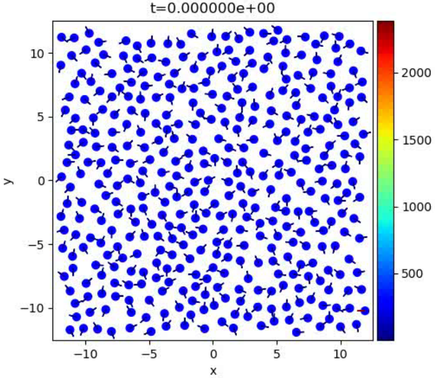
Cell tissues, swarms of bacteria, schools of fish, flocks of birds.

L. Berthier and J. Kurchan, arXiv preprint arXiv:1906.04039 (2019).

M. E. Cates and J. Tailleur, Annu. Rev. Condens. Matter Phys. **6**, 219–244 (2015).

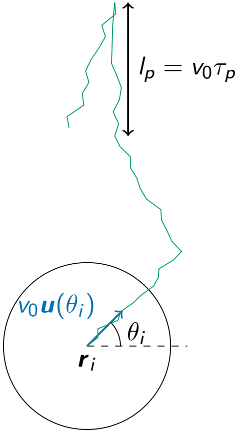


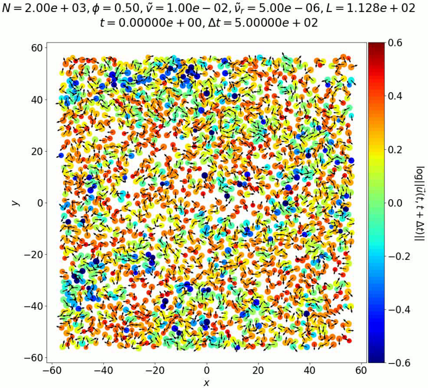
$$\dot{\theta}_i = \frac{1}{\tau_p}(\varphi_i - \theta_i) + \xi_i \quad (1)$$



T. Vicsek et al., Physical Review Letters **75**, 1226 (1995).

B. Szabo et al., Physical Review E **74**, 061908 (2006).





$$\dot{\theta}_i = \sqrt{2\tau_p^{-1}}\xi_i \quad (2)$$

$$\langle \boldsymbol{u}(\theta_i(0)) \cdot \boldsymbol{u}(\theta_i(t)) \rangle = e^{-t/\tau_p} \quad (3)$$

M. E. Cates and J. Tailleur, Annu. Rev. Condens. Matter Phys. **6**, 219–244 (2015).
Y.-E. Keta and J. Rottler, EPL (Europhysics Letters) **125**, 58004 (2019).

$$\dot{\boldsymbol{r}}_i = -\mu \nabla_i U(\{\boldsymbol{r}_j\}) + v_0 \boldsymbol{u}(\theta_i) + \sqrt{2D}\boldsymbol{\eta}_i \tag{4}$$

$$\dot{\theta}_i = \sqrt{2D_r}\xi_i \tag{5}$$

T. Nemoto et al., Physical Review E 99, 022605 (2019).

G. S. Redner et al., Physical Review Letters 110, 055701 (2013).

$$\dot{\boldsymbol{r}}_i = -\mu \nabla_i U(\{\boldsymbol{r}_j\}) + v_0 \boldsymbol{u}(\theta_i) + \sqrt{2D}\boldsymbol{\eta}_i \tag{4}$$

$$\dot{\theta}_i = \sqrt{2D_r}\xi_i \tag{5}$$

7 control parameters: N, ϕ, σ, μ, v_0, D, D_r.

T. Nemoto et al., Physical Review E **99**, 022605 (2019).

G. S. Redner et al., Physical Review Letters **110**, 055701 (2013).

$$\dot{\boldsymbol{r}}_i = -\mu \nabla_i U(\{\boldsymbol{r}_j\}) + \boldsymbol{u}(\theta_i) + \sqrt{2D}\boldsymbol{\eta}_i \tag{4}$$

$$\dot{\theta}_i = \sqrt{2D_r}\xi_i \tag{5}$$

5 control parameters: N, ϕ, μ, D, D_r.

$\rightarrow$ Units of space and time: $\sigma = 1$, $\sigma / v_0 = 1$.

T. Nemoto et al., Physical Review E 99, 022605 (2019).

G. S. Redner et al., Physical Review Letters 110, 055701 (2013).

$$\dot{\boldsymbol{r}}_i = -\mu \nabla_i U(\{\boldsymbol{r}_j\}) + \boldsymbol{u}(\theta_i) + \sqrt{2D}\boldsymbol{\eta}_i \tag{4}$$

$$\dot{\theta}_i = \sqrt{2\tilde{l}_{\mathrm{p}}^{-1}}\xi_i \tag{5}$$

5 control parameters: N, ϕ, μ, D, $\tilde{l}_{\mathrm{p}} = D_r^{-1}$.

$\rightarrow$ Units of space and time: $\sigma = 1$, $\sigma / v_0 = 1$.

T. Nemoto et al., Physical Review E 99, 022605 (2019).

G. S. Redner et al., Physical Review Letters 110, 055701 (2013).

$$\dot{\boldsymbol{r}}_i = -\mu \nabla_i U(\{\boldsymbol{r}_j\}) + \boldsymbol{u}(\theta_i) + \sqrt{\frac{2}{3}\tilde{l}_{\mathrm{p}}^{-1}}\boldsymbol{\eta}_i \tag{4}$$

$$\dot{\theta}_i = \sqrt{2\tilde{l}_{\mathrm{p}}^{-1}}\xi_i \tag{5}$$

4 control parameters: N, ϕ, μ, $\tilde{l}_{\mathrm{p}} = D_r^{-1}$.

→ Units of space and time: $\sigma = 1$, $\sigma/v_0 = 1$.

→ Stokes-Einstein relation: $D = \frac{1}{3}D_r = \frac{1}{3}\tilde{l}_{\mathrm{p}}^{-1}$.

T. Nemoto et al., Physical Review E 99, 022605 (2019).

G. S. Redner et al., Physical Review Letters 110, 055701 (2013).

$$\dot{\boldsymbol{r}}_i = -\frac{1}{3}\tilde{l}_{\mathrm{p}}^{-1}\nabla_i U(\{\boldsymbol{r}_j\}) + \boldsymbol{u}(\theta_i) + \sqrt{\frac{2}{3}\tilde{l}_{\mathrm{p}}^{-1}}\boldsymbol{\eta}_i \tag{4}$$

$$\dot{\theta}_i = \sqrt{2\tilde{l}_{\mathrm{p}}^{-1}}\xi_i \tag{5}$$

3 control parameters: N, ϕ, $\tilde{l}_{\mathrm{p}} = D_r^{-1}$.

- → Units of space and time: $\sigma = 1$, $\sigma / v_0 = 1$.
- → Stokes-Einstein relation: $D = \frac{1}{3} D_r = \frac{1}{3}\tilde{l}_{\mathrm{p}}^{-1}$.
- → Scaling of energy: $\mu = D$.

T. Nemoto et al., Physical Review E 99, 022605 (2019).

G. S. Redner et al., Physical Review Letters 110, 055701 (2013).

$$\text{EOM} \quad \dot{\boldsymbol{r}}_i = v_0 \boldsymbol{u}(\theta_i) - D\nabla_i U + \sqrt{2D}\boldsymbol{\eta}_i \tag{6}$$

$$\text{EOM} \quad \dot{\boldsymbol{r}}_i = v_0 \boldsymbol{u}(\theta_i) - D\nabla_i U + \sqrt{2D}\boldsymbol{\eta}_i \tag{6}$$

$$\text{dissipated power} \quad \dot{\mathcal{W}}_i = \sum_{i=1}^{N} \dot{\boldsymbol{r}}_i \circ \frac{1}{D}(\dot{\boldsymbol{r}}_i - \sqrt{2D}\boldsymbol{\eta}_i) \tag{7}$$

$$\text{EOM} \qquad \dot{\boldsymbol{r}}_i = v_0 \boldsymbol{u}(\theta_i) - D\nabla_i U + \sqrt{2D}\boldsymbol{\eta}_i \tag{6}$$

$$\text{dissipated power} \qquad \dot{\mathcal{W}}_i = \sum_{i=1}^{N} \dot{\boldsymbol{r}}_i \circ \frac{1}{D}(\dot{\boldsymbol{r}}_i - \sqrt{2D}\boldsymbol{\eta}_i) \tag{7}$$

$$\frac{1}{\tau}\int_0^\tau \dot{\mathcal{W}}_i(t)\,\mathrm{d}t = \frac{Nv_0^2}{D} w_\tau + \frac{1}{\tau}[U(\tau) - U(0)] \tag{8}$$

EOM
$$\dot{\boldsymbol{r}}_i = v_0 \boldsymbol{u}(\theta_i) - D\nabla_i U + \sqrt{2D}\boldsymbol{\eta}_i \tag{6}$$

dissipated power
$$\dot{\mathcal{W}}_i = \sum_{i=1}^{N} \dot{\boldsymbol{r}}_i \circ \frac{1}{D}(\dot{\boldsymbol{r}}_i - \sqrt{2D}\boldsymbol{\eta}_i) \tag{7}$$

$$\frac{1}{\tau}\int_0^\tau \dot{\mathcal{W}}_i(t)\,\mathrm{d}t = \frac{Nv_0^2}{D} w_\tau + \cancel{\frac{1}{\tau}[U(\tau) - U(0)]} \tag{8}$$

$$\text{EOM} \qquad \dot{\boldsymbol{r}}_i = v_0 \boldsymbol{u}(\theta_i) - D\nabla_i U + \sqrt{2D}\boldsymbol{\eta}_i \tag{6}$$

$$\text{dissipated power} \qquad \dot{\mathcal{W}}_i = \sum_{i=1}^{N} \dot{\boldsymbol{r}}_i \circ \frac{1}{D}(\dot{\boldsymbol{r}}_i - \sqrt{2D}\boldsymbol{\eta}_i) \tag{7}$$

$$\frac{1}{\tau}\int_0^\tau \dot{\mathcal{W}}_i(t)\,\mathrm{d}t = \frac{Nv_0^2}{D} w_\tau + \cancel{\frac{1}{\tau}[U(\tau) - U(0)]} \tag{8}$$

$$\text{active work} \qquad w_\tau = \frac{1}{v_0 N \tau} \sum_{i=1}^{N} \int_0^\tau \boldsymbol{u}(\theta_i) \circ \mathrm{d}\boldsymbol{r}_i \tag{9}$$

EOM
$$\dot{\boldsymbol{r}}_i = v_0 \boldsymbol{u}(\theta_i) - D\nabla_i U + \sqrt{2D}\boldsymbol{\eta}_i \tag{6}$$

dissipated power
$$\dot{\mathcal{W}}_i = \sum_{i=1}^{N} \dot{\boldsymbol{r}}_i \circ \frac{1}{D}(\dot{\boldsymbol{r}}_i - \sqrt{2D}\boldsymbol{\eta}_i) \tag{7}$$

$$\frac{1}{\tau}\int_0^\tau \dot{\mathcal{W}}_i(t)\,\mathrm{d}t = \frac{Nv_0^2}{D} w_\tau + \cancel{\frac{1}{\tau}[U(\tau) - U(0)]} \tag{8}$$

active work
$$w_\tau = \frac{1}{v_0 N \tau} \sum_{i=1}^{N} \int_0^\tau \boldsymbol{u}(\theta_i) \circ \mathrm{d}\boldsymbol{r}_i \tag{9}$$

$$w_\tau = 1 + w_{f,\tau} + w_{\eta,\tau} \tag{10}$$

$$\text{EOM} \qquad \dot{\boldsymbol{r}}_i = v_0 \boldsymbol{u}(\theta_i) - D\nabla_i U + \sqrt{2D}\boldsymbol{\eta}_i \tag{6}$$

$$\text{dissipated power} \qquad \dot{\mathcal{W}}_i = \sum_{i=1}^{N} \dot{\boldsymbol{r}}_i \circ \frac{1}{D}(\dot{\boldsymbol{r}}_i - \sqrt{2D}\boldsymbol{\eta}_i) \tag{7}$$

$$\frac{1}{\tau}\int_0^\tau \dot{\mathcal{W}}_i(t)\,\mathrm{d}t = \frac{N v_0^2}{D} w_\tau + \cancel{\frac{1}{\tau}[U(\tau) - U(0)]} \tag{8}$$

$$\text{active work} \qquad w_\tau = \frac{1}{v_0 N \tau} \sum_{i=1}^{N} \int_0^\tau \boldsymbol{u}(\theta_i) \circ \mathrm{d}\boldsymbol{r}_i \tag{9}$$

$$w_\tau = 1 + w_{f,\tau} + \cancel{w_{\eta,\tau}} \tag{10}$$

EOM $$\dot{\boldsymbol{r}}_i = v_0 \boldsymbol{u}(\theta_i) - D\nabla_i U + \sqrt{2D}\boldsymbol{\eta}_i \tag{6}$$

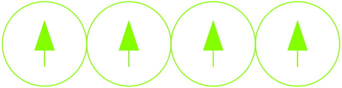
dissipated power $$\dot{\mathcal{W}}_i = \sum_{i=1}^{N} \dot{\boldsymbol{r}}_i \circ \frac{1}{D}(\dot{\boldsymbol{r}}_i - \sqrt{2D}\boldsymbol{\eta}_i) \tag{7}$$

$$\frac{1}{\tau}\int_0^\tau \dot{\mathcal{W}}_i(t)\,\mathrm{d}t = \frac{Nv_0^2}{D} w_\tau + \cancel{\frac{1}{\tau}[U(\tau) - U(0)]} \tag{8}$$

active work $$w_\tau = \frac{1}{v_0 N \tau}\sum_{i=1}^{N}\int_0^\tau \boldsymbol{u}(\theta_i) \circ \mathrm{d}\boldsymbol{r}_i \tag{9}$$

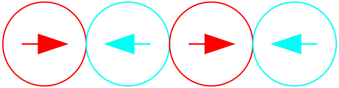
$$w_\tau = 1 + w_{f,\tau} + \cancel{w_{\eta,\tau}} \tag{10}$$

Flocking



$$\boldsymbol{u}(\theta_i)\cdot\nabla_i U = 0 \Rightarrow \langle w_\tau \rangle = 1$$

Jamming



$$\dot{\boldsymbol{r}}_i = 0 \Rightarrow \langle w_\tau \rangle = 0$$

$X_1, \ldots, X_n$ a sequence of (i.i.d.) random variables

$$\langle X_i \rangle = \mu, \ \left\langle (X_i - \mu)^2 \right\rangle = \sigma^2 \tag{11}$$

$$\text{sample average } R_n = \frac{1}{n} \sum_{i=1}^{n} X_i \tag{12}$$

H. Touchette, Physics Reports 478, 1–69 (2009).

$X_1, \dots, X_n$ a sequence of (i.i.d.) random variables

$$\langle X_i \rangle = \mu, \ \left\langle (X_i - \mu)^2 \right\rangle = \sigma^2 \tag{11}$$

$$\text{sample average } R_n = \frac{1}{n} \sum_{i=1}^{n} X_i \tag{12}$$

Law of large numbers

$$P\left(\lim_{n \to \infty} R_n = \mu \right) = 1 \tag{13}$$

H. Touchette, Physics Reports 478, 1–69 (2009).

$X_1, \ldots, X_n$ a sequence of (i.i.d.) random variables

$$\langle X_i \rangle = \mu,\ \left\langle (X_i - \mu)^2 \right\rangle = \sigma^2 \tag{11}$$

$$\text{sample average } R_n = \frac{1}{n} \sum_{i=1}^{n} X_i \tag{12}$$

Central limit theorem

$$P(R_n) \underset{n \to \infty}{\sim} \mathcal{N}\left(\mu, \frac{\sigma^2}{n}\right) \tag{13}$$

H. Touchette, Physics Reports **478**, 1–69 (2009).

$X_1, \ldots, X_n$ a sequence of (i.i.d.) random variables

$$\langle X_i \rangle = \mu, \; \left\langle (X_i - \mu)^2 \right\rangle = \sigma^2 \tag{11}$$

$$\text{sample average } R_n = \frac{1}{n} \sum_{i=1}^{n} X_i \tag{12}$$

Large deviation principle

$$R_n \text{ satisfies a LDP } \Leftrightarrow -\log P(R_n = r) \underset{n \to \infty}{\sim} nI(r) \tag{13}$$

$$I \equiv \text{ rate function of } R_n \Leftrightarrow P(R_n = r) \asymp \exp(-nI(r)) \tag{14}$$

H. Touchette, Physics Reports **478**, 1–69 (2009).

$X_1, \ldots, X_n$ random bits, taking value 0 or 1 with equal probability

$$\text{sample average } R_n = \frac{1}{n}\sum_{i=1}^{n} X_i \tag{15}$$

$X_1, \ldots, X_n$ random bits, taking value 0 or 1 with equal probability

$$\text{sample average } R_n = \frac{1}{n}\sum_{i=1}^{n} X_i \tag{15}$$

$$\lim_{n\to\infty} -\frac{1}{n}\log P(R_n = r) = \log 2 + r\log r + (1-r)\log(1-r) = I(r) \tag{16}$$

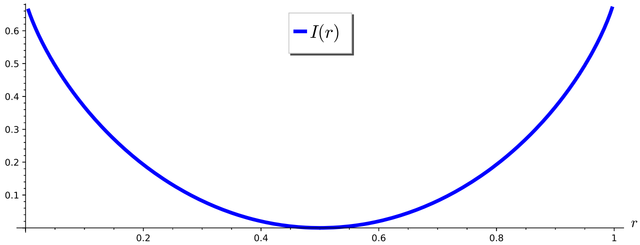
$$P(R_n = r) \asymp \exp(-nI(r)) \tag{17}$$

$X_1, \ldots, X_n$ random bits, taking value 0 or 1 with equal probability

$$\text{sample average } R_n = \frac{1}{n} \sum_{i=1}^{n} X_i \tag{15}$$

$$\lim_{n\to\infty} -\frac{1}{n} \log P(R_n = r) = \log 2 + r \log r + (1-r)\log(1-r) = I(r) \tag{16}$$

$$P(R_n = r) \asymp \exp(-nI(r)) \tag{17}$$

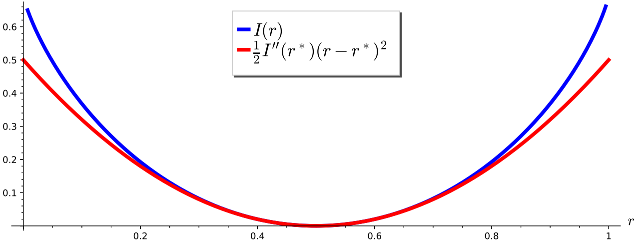


$X_1, \ldots, X_n$ random bits, taking value 0 or 1 with equal probability

$$\text{sample average } R_n = \frac{1}{n} \sum_{i=1}^{n} X_i \tag{15}$$

$$\lim_{n\to\infty} -\frac{1}{n} \log P(R_n = r) = \log 2 + r \log r + (1-r)\log(1-r) = I(r) \tag{16}$$

$$P(R_n = r) \asymp \exp(-nI(r)) \tag{17}$$



$$\text{SCGF } \psi(s) = \lim_{n\to\infty} \frac{1}{n} \log \langle \exp(-snR_n) \rangle \tag{18}$$

H. Touchette, Physics Reports 478, 1–69 (2009).

$$\text{SCGF } \psi(s) = \lim_{n\to\infty} \frac{1}{n} \log \langle \exp(-snR_n) \rangle \tag{18}$$

Gärtner-Ellis theorem

$$\psi \text{ is differentiable } \Rightarrow I(r) = \sup_s \{-sr - \psi(s)\} \tag{19}$$

H. Touchette, Physics Reports **478**, 1–69 (2009).

mean energy per particle for n particles E_n

$$\text{Boltzmann distribution } P_\beta(\omega) = \frac{e^{-\beta n E_n(\omega)}}{Z_n(\beta)} \tag{20}$$

mean energy per particle for n particles E_n

$$\text{Boltzmann distribution } P_\beta(\omega) = \frac{e^{-\beta n E_n(\omega)}}{Z_n(\beta)} \tag{20}$$

$$\text{SCGF } \psi_\beta(\Delta\beta) = \lim_{n\to\infty} \frac{1}{n} \log \int e^{-\Delta\beta n E_n(\omega)} P_\beta(\omega)\, \mathrm{d}\omega \tag{21}$$

mean energy per particle for n particles E_n

$$\text{Boltzmann distribution } P_\beta(\omega) = \frac{e^{-\beta n E_n(\omega)}}{Z_n(\beta)} \tag{20}$$

$$\begin{aligned}\text{SCGF } \psi_\beta(\Delta\beta) &= \lim_{n\to\infty} \frac{1}{n} \log \int e^{-\Delta\beta n E_n(\omega)} P_\beta(\omega)\, \mathrm{d}\omega \\ &= \lim_{n\to\infty} \frac{1}{n} \log \frac{Z_n(\beta+\Delta\beta)}{Z_n(\beta)} \\ &= \beta F(\beta) - (\beta+\Delta\beta) F(\beta+\Delta\beta)\end{aligned} \tag{21}$$

$$\text{free energy density } \beta F(\beta) = -\lim_{n\to\infty} \frac{1}{n} \log Z_n(\beta) \tag{22}$$

$$\psi_\beta \text{ is differentiable} \xRightarrow[\text{Gärtner-Ellis}]{} P_\beta(E_n) \asymp \exp(-nI_\beta(E_n)) \tag{23}$$

$$\psi_\beta \text{ is differentiable} \xRightarrow[\text{Gärtner-Ellis}]{} P_\beta(E_n) \asymp \exp(-n I_\beta(E_n)) \tag{23}$$

$$P_\beta(E_n) \asymp \exp(n(\underbrace{S(E_n)}_{\substack{\text{entropy} \\ \equiv\text{number of states}}} - \overbrace{\beta E_n}^{\substack{\text{Boltzmann} \\ \text{function}}} + \underbrace{\beta F(\beta)}_{\substack{\text{partition} \\ \text{function}}})) \tag{24}$$

$$\psi_\beta \text{ is differentiable} \xRightarrow[\text{Gärtner-Ellis}]{} P_\beta(E_n) \asymp \exp(-n I_\beta(E_n)) \tag{23}$$

$$P_\beta(E_n) \asymp \exp(n(\underbrace{S(E_n)}_{\substack{\text{entropy}\\ \equiv\text{number of states}}} - \overbrace{\beta E_n}^{\substack{\text{Boltzmann}\\ \text{function}}} + \underbrace{\beta F(\beta)}_{\substack{\text{partition}\\ \text{function}}})) \tag{24}$$

$$I_\beta(E_n) = -S(E_n) + \beta E_n - \beta F(\beta) \tag{25}$$

$$\psi_\beta \text{ is differentiable} \xRightarrow[\text{Gärtner-Ellis}]{} P_\beta(E_n) \asymp \exp(-nI_\beta(E_n)) \tag{23}$$

$$P_\beta(E_n) \asymp \exp(n(\underbrace{S(E_n)}_{\substack{\text{entropy}\\ \equiv\text{number of states}}} - \overbrace{\beta E_n}^{\substack{\text{Boltzmann}\\ \text{function}}} + \underbrace{\beta F(\beta)}_{\substack{\text{partition}\\ \text{function}}})) \tag{24}$$

$$I_\beta(E_n) = -S(E_n) + \beta E_n - \beta F(\beta) \tag{25}$$

$$I_\beta(E_n) = 0 \Leftrightarrow F(\beta) = E_n - \frac{1}{\beta}S(E_n) \tag{26}$$

d-dimensional system of size N represented by $\{\boldsymbol{X}_1(t), \ldots, \boldsymbol{X}_N(t)\}$

$$R_{N\tau} = \frac{1}{N\tau} \int_0^\tau \sum_{i=1}^N f(\boldsymbol{X}_i(t))\,\mathrm{d}t \tag{27}$$

d-dimensional system of size N represented by $\{\boldsymbol{X}_1(t), \ldots, \boldsymbol{X}_N(t)\}$

$$R_{N\tau} = \frac{1}{N\tau} \int_0^\tau \sum_{i=1}^N f(\boldsymbol{X}_i(t))\,\mathrm{d}t \tag{27}$$

$$I(r) = \lim_{\tau \to \infty} -\frac{1}{N\tau} \log P(R_{N\tau} = r), \;\; \psi(s) = \lim_{\tau \to \infty} \frac{1}{N\tau} \log \left\langle e^{-sN\tau R_{N\tau}} \right\rangle \tag{28}$$

d-dimensional system of size N represented by $\{\boldsymbol{X}_1(t), \ldots, \boldsymbol{X}_N(t)\}$

$$R_{N\tau} = \frac{1}{N\tau} \int_0^\tau \sum_{i=1}^{N} f(\boldsymbol{X}_i(t))\, \mathrm{d}t \tag{27}$$

$$I(r) = \lim_{\tau \to \infty} -\frac{1}{N\tau} \log P(R_{N\tau} = r), \;\; \psi(s) = \lim_{\tau \to \infty} \frac{1}{N\tau} \log \left\langle e^{-sN\tau R_{N\tau}} \right\rangle \tag{28}$$

Quantity	Equilibrium counterpart	
$\{\boldsymbol{X}_i(t)\}$	ω	Microstate in $(d+1)$-dim.
$R_{N\tau}$	E_n	Mean energy
s	$\Delta\beta$	Inverse temperature
ψ	$\beta F(\beta) - (\beta + \Delta\beta) F(\beta + \Delta\beta)$	Free energy
I	$-S(E_n) + \beta E_n - \beta F(\beta)$	Entropy

equilibrium **canonical ensemble** $\rightarrow$ **biased ensemble** of trajectories

$$P_s[\{\boldsymbol{X}_i(t)\}] \propto P_0[\{\boldsymbol{X}_i(t)\}] e^{-sN\tau R_{N\tau}} \tag{29}$$

$$\langle \mathcal{A} \rangle_s = \frac{\left\langle \mathcal{A} e^{-s\,N\,\tau\,R_{N\tau}} \right\rangle}{\left\langle e^{-s\,N\,\tau\,R_{N\tau}} \right\rangle} \tag{30}$$

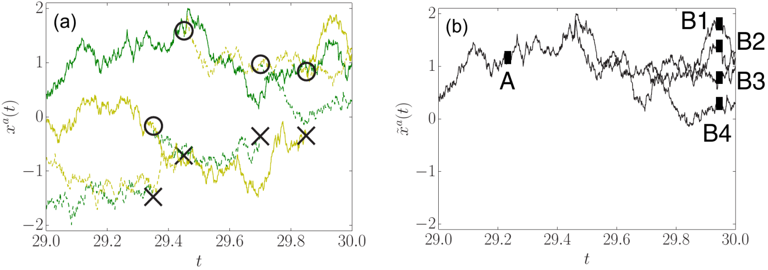
R. L. Jack, The European Physical Journal B 93, 1–22 (2020).

T. Nemoto et al., Physical Review E 93, 062123 (2016).

equilibrium **canonical ensemble** → **biased ensemble** of trajectories

$$P_s[\{\boldsymbol{X}_i(t)\}] \propto P_0[\{\boldsymbol{X}_i(t)\}]e^{-sN\tau R_{N\tau}} \tag{29}$$

$$\langle \mathcal{A} \rangle_s = \frac{\left\langle \mathcal{A} e^{-s\,N\,\tau\,R_{N\tau}} \right\rangle}{\left\langle e^{-s\,N\,\tau\,R_{N\tau}} \right\rangle} \tag{30}$$



$$\psi(s) = \lim_{\tau\to\infty} \frac{1}{\tau} \log \left\langle e^{s \int_0^\tau x(t)(1+x(t))\,dt} \right\rangle \tag{31}$$

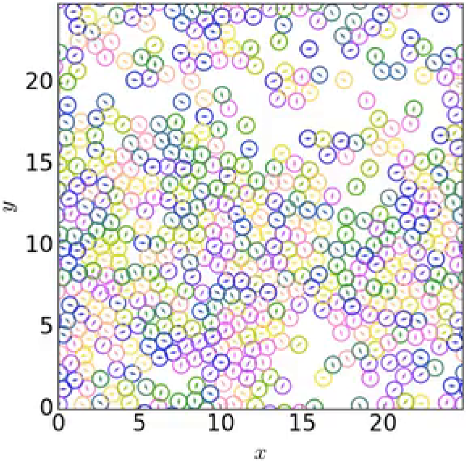
R. L. Jack, The European Physical Journal B 93, 1–22 (2020).

T. Nemoto et al., Physical Review E 93, 062123 (2016).

1. Cloning algoritm $\Rightarrow$ trajectories biased with respect to w_τ.

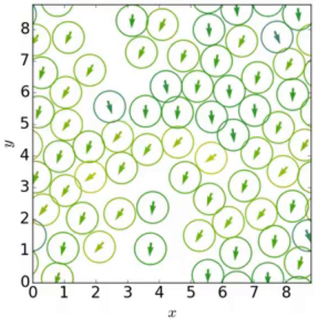
1. Cloning algoritm $\Rightarrow$ trajectories biased with respect to w_τ.
2. Singularities in ψ.

1. Cloning algoritm $\Rightarrow$ trajectories biased with respect to w_τ.
2. Singularities in ψ.
3. Description of qualitative changes at dynamical transition.

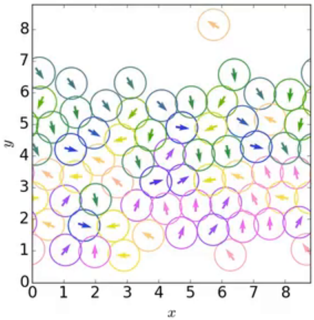


T. Nemoto et al., Physical Review E 99, 022605 (2019).

CM ($s < 0$)



PSA ($s > 0$)

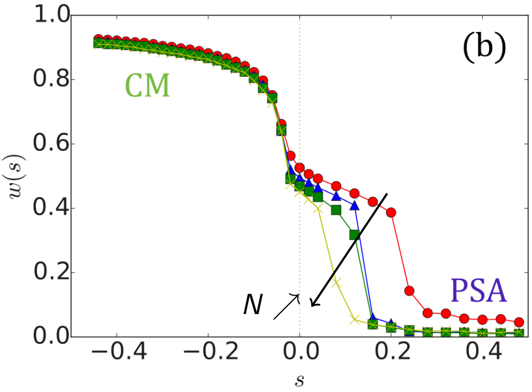


CM $\equiv$ collective motion, PSA $\equiv$ phase-separated arrest

T. Nemoto et al., Physical Review E 99, 022605 (2019).

$$\langle w_\tau \rangle_s = w(s) = -\psi'(s) \tag{32}$$

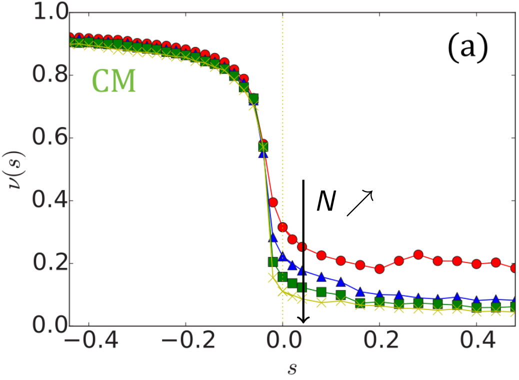
$$-w'(s) = \psi''(s) = \lim_{\tau \to \infty} N\tau \mathrm{Var}(w_\tau)_s \tag{33}$$



T. Nemoto et al., Physical Review E 99, 022605 (2019).

$$\text{polarisation } \hat{\nu} = \left| \frac{1}{N} \sum_{i=1}^{N} \boldsymbol{u}(\theta_i) \right|, \quad \bar{\nu}_\tau = \frac{1}{\tau} \int_0^\tau \hat{\nu}(t)\, \mathrm{d}t \tag{34}$$

$$\langle \bar{\nu}_\tau \rangle = \langle \bar{\nu}_\tau \rangle_{s=0} = \propto N^{-1/2} \tag{35}$$



T. Nemoto et al., Physical Review E 99, 022605 (2019).

1. Location s^* of the transition.

$$s* \neq 0 \Rightarrow I \neq 0 \tag{36}$$

1. Location s^* of the transition.

$$s* \neq 0 \Rightarrow I \neq 0 \tag{36}$$

2. Analytical illustration of collective motion.

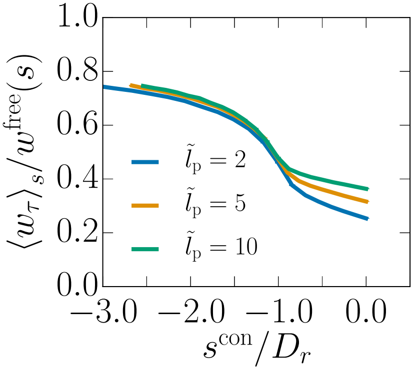
1. Location s^* of the transition.

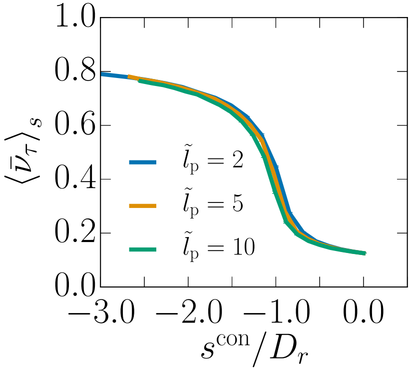
$$s* \neq 0 \Rightarrow I \neq 0 \tag{36}$$

2. Analytical illustration of collective motion.
3. Top-down hydrodynamic description.

$$w^{\text{free}}(s) = 1 - \frac{2sD}{v_0^2} \tag{37}$$

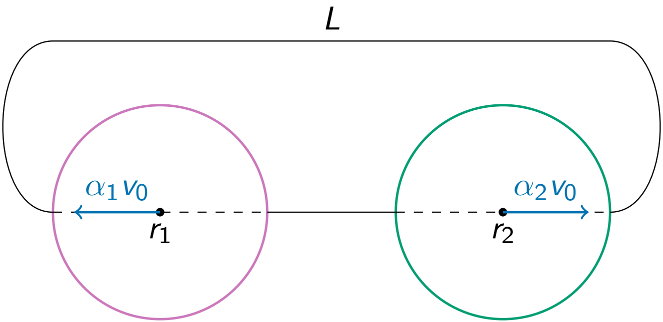
$$s^{\text{con}} = s\left(1 - \frac{2sD}{v_0^2}\right) \tag{38}$$





$$s^{\text{con}*} \approx -D_r \tag{39}$$

Y.-E. Keta et al., Physical Review E 103, 022603 (2021).



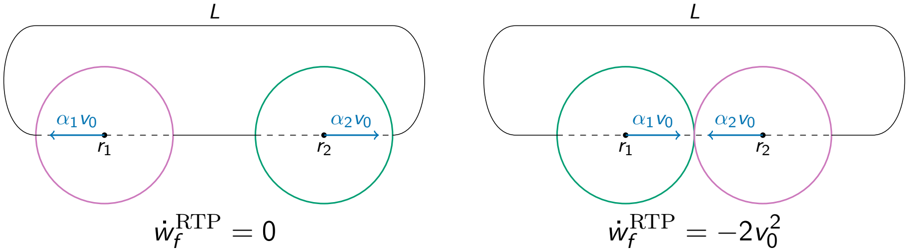
$$\alpha_i \in \{-1, +1\} \tag{40}$$

$$l = \tau_p(\alpha_i \to -\alpha_i) v_0 \tag{41}$$

A. Slowman et al., Physical Review Letters 116, 218101 (2016).

F. Cagnetta and E. Mallmin, Physical Review E 101, 022130 (2020).

$$\text{active work } \dot{w}_f^{\mathrm{RTP}} = v_0(\alpha_1 - \alpha_2)\partial_{r_1} V(r_{12}) \tag{42}$$



$$\text{SCGF} \qquad \psi^{\mathrm{RTP}}(\lambda) = \lim_{\tau\to\infty} \frac{1}{\tau} \log \left\langle e^{-\lambda \int_0^\tau \dot{w}_f^{\mathrm{RTP}}(t)\,\mathrm{d}t} \right\rangle \tag{43}$$

$$\text{polarisation} \qquad \nu^{\mathrm{RTP}} = \frac{1+\alpha_1\alpha_2}{2} \tag{44}$$

$$\psi^{\mathrm{RTP}}(\lambda)\boldsymbol{P}_\lambda = (\mathcal{L} - \lambda \dot{w}_f^{\mathrm{RTP}})\boldsymbol{P}_\lambda \tag{45}$$

$$\boldsymbol{P}_\lambda(r \equiv r_2 - r_1) \equiv (P_\lambda^{++}(r), P_\lambda^{--}(r), P_\lambda^{+-}(r), P_\lambda^{-+}(r)) \tag{46}$$

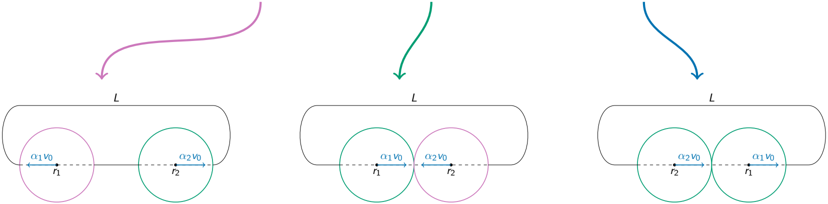
H. Touchette, Physica A: Statistical Mechanics and its Applications **504**, 5–19 (2018).

T. Arnoulx de Pirey et al., Physical Review Letters **123**, 260602 (2019).

$$\psi^{\mathrm{RTP}}(\lambda)\boldsymbol{P}_\lambda = (\mathcal{L} - \lambda \dot{w}_f^{\mathrm{RTP}})\boldsymbol{P}_\lambda \tag{45}$$

$$\boldsymbol{P}_\lambda(r \equiv r_2 - r_1) \equiv (P_\lambda^{++}(r), P_\lambda^{--}(r), P_\lambda^{+-}(r), P_\lambda^{-+}(r)) \tag{46}$$

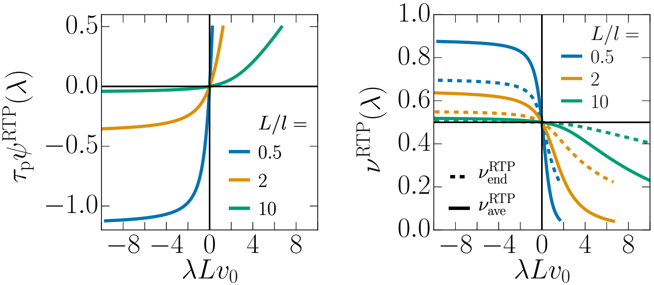
$$P_\lambda^{\alpha_1\alpha_2}(r) = \varepsilon_\lambda^{\alpha_1\alpha_2}(r) + \gamma_\lambda^{\alpha_1\alpha_2,\mathrm{l}}\delta(r) + \gamma_\lambda^{\alpha_1\alpha_2,\mathrm{r}}\delta(L-r) \tag{47}$$



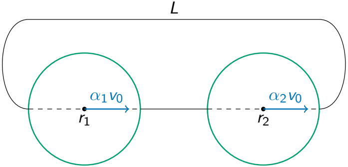
H. Touchette, Physica A: Statistical Mechanics and its Applications **504**, 5–19 (2018).

T. Arnoulx de Pirey et al., Physical Review Letters **123**, 260602 (2019).

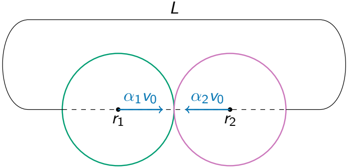
Polarisation averages



$$\left\langle w_f^{\mathrm{RTP}} \right\rangle_\lambda = -\partial_\lambda \psi^{\mathrm{RTP}}, \ \nu^{\mathrm{RTP}} = \frac{1+\alpha_1\alpha_2}{2} \tag{48}$$



$\lambda \to -\infty$



$\lambda \to \infty$

Y.-E. Keta et al., Physical Review E 103, 022603 (2021).

M. E. Cates and J. Tailleur, EPL (Europhysics Letters) 101, 20010 (2013).

M. E. Cates, arXiv preprint arXiv:1904.01330 (2019).

$$\text{continuity} \qquad \dot{\rho} = -\nabla \cdot \boldsymbol{J} \tag{49}$$

M. E. Cates and J. Tailleur, EPL (Europhysics Letters) 101, 20010 (2013).

M. E. Cates, arXiv preprint arXiv:1904.01330 (2019).

$$\text{continuity} \qquad \dot{\rho} = -\nabla \cdot \boldsymbol{J} \tag{49}$$

$$\text{flux} \qquad \boldsymbol{J} = \boldsymbol{J}_d + \sqrt{2\sigma(\rho)}\eta \tag{50}$$

M. E. Cates and J. Tailleur, EPL (Europhysics Letters) 101, 20010 (2013).

M. E. Cates, arXiv preprint arXiv:1904.01330 (2019).

$$\text{continuity} \qquad \dot{\rho} = -\nabla \cdot \boldsymbol{J} \tag{49}$$

$$\text{flux} \quad \boldsymbol{J} = \boldsymbol{J}_d + \sqrt{2\sigma(\rho)}\eta \tag{50}$$

$$\boldsymbol{J}_d = -D_c(\rho)\nabla\rho \tag{51}$$

M. E. Cates and J. Tailleur, EPL (Europhysics Letters) 101, 20010 (2013).

M. E. Cates, arXiv preprint arXiv:1904.01330 (2019).

$$\text{continuity} \qquad \dot{\rho} = -\nabla \cdot \boldsymbol{J} \tag{49}$$

$$\text{flux} \qquad \boldsymbol{J} = \boldsymbol{J}_d + \sqrt{2\sigma(\rho)}\eta \tag{50}$$

$$\boldsymbol{J}_d = -D_c(\rho)\nabla\rho + v_0\rho\boldsymbol{P} \tag{51}$$

M. E. Cates and J. Tailleur, EPL (Europhysics Letters) 101, 20010 (2013).

M. E. Cates, arXiv preprint arXiv:1904.01330 (2019).

$$\begin{aligned}
\text{continuity} \quad & \dot{\rho} = -\nabla \cdot \boldsymbol{J} & (49)\\
\text{flux} \quad & \boldsymbol{J} = \boldsymbol{J}_d + \sqrt{2\sigma(\rho)}\eta & (50)\\
& \boldsymbol{J}_d = -D_c(\rho)\nabla\rho + v_0\rho\boldsymbol{P} & (51)\\
\text{polarisation} \quad & \dot{\boldsymbol{P}} = -\gamma(\rho, \boldsymbol{P})\boldsymbol{f}(\boldsymbol{P}) & (52)
\end{aligned}$$

M. E. Cates and J. Tailleur, EPL (Europhysics Letters) 101, 20010 (2013).

M. E. Cates, arXiv preprint arXiv:1904.01330 (2019).

$$\text{continuity} \qquad \dot{\rho} = -\nabla \cdot \boldsymbol{J} \tag{49}$$

$$\text{flux} \qquad \boldsymbol{J} = \boldsymbol{J}_d + \sqrt{2\sigma(\rho)}\eta \tag{50}$$

$$\boldsymbol{J}_d = -D_c(\rho)\nabla\rho + v_0\rho\boldsymbol{P} \tag{51}$$

$$\text{polarisation} \qquad \dot{\boldsymbol{P}} = -\gamma(\rho, \boldsymbol{P})\boldsymbol{f}(\boldsymbol{P}) + b(\rho, \boldsymbol{P})\nabla\rho \tag{52}$$

M. E. Cates and J. Tailleur, EPL (Europhysics Letters) 101, 20010 (2013).

M. E. Cates, arXiv preprint arXiv:1904.01330 (2019).

continuity $$\dot{\rho} = -\nabla \cdot \boldsymbol{J} \tag{49}$$

flux $$\boldsymbol{J} = \boldsymbol{J}_d + \sqrt{2\sigma(\rho)}\eta \tag{50}$$

$$\boldsymbol{J}_d = -D_c(\rho)\nabla\rho + v_0\rho\boldsymbol{P} \tag{51}$$

polarisation $$\dot{\boldsymbol{P}} = -\gamma(\rho, \boldsymbol{P})\boldsymbol{f}(\boldsymbol{P}) + b(\rho, \boldsymbol{P})\nabla\rho + \sqrt{2\gamma(\rho, \boldsymbol{P})}\xi \tag{52}$$

M. E. Cates and J. Tailleur, EPL (Europhysics Letters) 101, 20010 (2013).

M. E. Cates, arXiv preprint arXiv:1904.01330 (2019).

continuity $$\dot{\rho} = -\nabla \cdot \boldsymbol{J} \tag{49}$$

flux $$\boldsymbol{J} = \boldsymbol{J}_d + \sqrt{2\sigma(\rho)}\eta \tag{50}$$

$$\boldsymbol{J}_d = -D_c(\rho)\nabla\rho + v_0\rho\boldsymbol{P} \tag{51}$$

polarisation $$\dot{\boldsymbol{P}} = -\gamma(\rho, \boldsymbol{P})\boldsymbol{f}(\boldsymbol{P}) + b(\rho, \boldsymbol{P})\nabla\rho + \sqrt{2\gamma(\rho, \boldsymbol{P})}\xi \tag{52}$$

bias $$w_\tau N\tau = \int_0^\tau \int_{[0,L]^2} \omega(\rho, \boldsymbol{P})\,\mathrm{d}^2\boldsymbol{r}\,\mathrm{d}t \tag{53}$$

M. E. Cates and J. Tailleur, EPL (Europhysics Letters) 101, 20010 (2013).
M. E. Cates, arXiv preprint arXiv:1904.01330 (2019).

$$\dot{\rho} = -\nabla \cdot \left(\boldsymbol{J}_d + \sqrt{2\sigma(\rho)}\eta \right) \tag{54}$$

$$\dot{\boldsymbol{P}} = -\gamma(\rho, \boldsymbol{P})\boldsymbol{f}(\boldsymbol{P}) + b(\rho, \boldsymbol{P})\nabla\rho + \sqrt{2\gamma(\rho, \boldsymbol{P})}\xi \tag{55}$$

$$w_\tau N\tau = \int_0^\tau \int_{[0,L]^2} \omega(\rho, \boldsymbol{P})\,\mathrm{d}^2\boldsymbol{r}\,\mathrm{d}t \tag{56}$$

L. Bertini et al., Reviews of Modern Physics **87**, 593 (2015).

$$\dot{\rho} = -\nabla \cdot \left(\boldsymbol{J}_d + \sqrt{2\sigma(\rho)}\eta \right) \tag{54}$$

$$\dot{\boldsymbol{P}} = -\gamma(\rho, \boldsymbol{P})\boldsymbol{f}(\boldsymbol{P}) + b(\rho, \boldsymbol{P})\nabla\rho + \sqrt{2\gamma(\rho, \boldsymbol{P})}\xi \tag{55}$$

$$w_\tau N\tau = \int_0^\tau \int_{[0,L]^2} \omega(\rho, \boldsymbol{P})\, \mathrm{d}^2\boldsymbol{r}\, \mathrm{d}t \tag{56}$$

hydrodynamic variables

$$\boldsymbol{r} \rightarrow \boldsymbol{r}/L, \;\; t \rightarrow Dt/L^2 \tag{57}$$

L. Bertini et al., Reviews of Modern Physics **87**, 593 (2015).

$$\dot{\rho} = -\nabla \cdot \left(\boldsymbol{J}_d + \sqrt{2\sigma(\rho)}\eta \right) \tag{54}$$

$$\dot{\boldsymbol{P}} = -\gamma(\rho, \boldsymbol{P})\boldsymbol{f}(\boldsymbol{P}) + b(\rho, \boldsymbol{P})\nabla\rho + \sqrt{2\gamma(\rho, \boldsymbol{P})}\xi \tag{55}$$

$$w_\tau N\tau = \int_0^\tau \int_{[0,L]^2} \omega(\rho, \boldsymbol{P}) \, \mathrm{d}^2\boldsymbol{r} \, \mathrm{d}t \tag{56}$$

hydrodynamic variables

$$\boldsymbol{r} \rightarrow \boldsymbol{r}/L, \;\; t \rightarrow Dt/L^2 \tag{57}$$

$$P[\rho, \boldsymbol{J}, \boldsymbol{P}] \propto \exp\left(-L^2 \int_0^\tau \int_{[0,1]^2} S(\rho, \boldsymbol{J}, \boldsymbol{P}) \, \mathrm{d}^2\boldsymbol{r} \, \mathrm{d}t \right) \tag{58}$$

L. Bertini et al., Reviews of Modern Physics **87**, 593 (2015).

$$\dot{\rho} = -\nabla \cdot \left(\boldsymbol{J}_d + \sqrt{2\sigma(\rho)}\eta \right) \tag{54}$$

$$\dot{\boldsymbol{P}} = -\gamma(\rho, \boldsymbol{P})\boldsymbol{f}(\boldsymbol{P}) + b(\rho, \boldsymbol{P})\nabla\rho + \sqrt{2\gamma(\rho, \boldsymbol{P})}\xi \tag{55}$$

$$w_\tau N\tau = \int_0^\tau \int_{[0,L]^2} \omega(\rho, \boldsymbol{P})\, \mathrm{d}^2\boldsymbol{r}\, \mathrm{d}t \tag{56}$$

hydrodynamic variables

$$\boldsymbol{r} \to \boldsymbol{r}/L, \;\; t \to Dt/L^2 \tag{57}$$

$$P[\rho, \boldsymbol{J}, \boldsymbol{P}] \propto \exp\left(-L^2 \int_0^\tau \int_{[0,1]^2} S(\rho, \boldsymbol{J}, \boldsymbol{P})\, \mathrm{d}^2\boldsymbol{r}\, \mathrm{d}t \right) \tag{58}$$

$$S = \frac{D}{4\sigma} \left| \boldsymbol{J} - \boldsymbol{J}_d \right|^2 + \frac{1}{4D\gamma} \left| \frac{D}{L}\dot{\boldsymbol{P}} + \gamma \boldsymbol{f} L - b\nabla\rho \right|^2 + \frac{sL^2}{D}\omega \tag{59}$$

L. Bertini et al., Reviews of Modern Physics **87**, 593 (2015).

$$S = \frac{D}{4\sigma}\left|\boldsymbol{J} - \boldsymbol{J}_d\right|^2 + \frac{1}{4D\gamma}\left|\frac{D}{L}\dot{\boldsymbol{P}} + \gamma \boldsymbol{f} L - b\nabla\rho\right|^2 + \frac{sL^2}{D}\omega \tag{60}$$

Y.-E. Keta et al., Physical Review E 103, 022603 (2021).

$$S = \frac{D}{4\sigma}\left|\boldsymbol{J} - \boldsymbol{J}_d\right|^2 + \frac{1}{4D\gamma}\left|\cancel{\frac{D}{L}\dot{\boldsymbol{P}}} + \gamma \boldsymbol{f} L \cancel{- b\nabla\rho}\right|^2 + \frac{sL^2}{D}\omega \tag{60}$$

$(\rho, \boldsymbol{J}, \boldsymbol{P})$ independent of space and time

Y.-E. Keta et al., Physical Review E 103, 022603 (2021).

$$S = \frac{D}{4\sigma}\cancel{|\boldsymbol{J} - \boldsymbol{J}_d|^2} + \frac{1}{4D\gamma}|\gamma \boldsymbol{f} L|^2 + \frac{sL^2}{D}\omega \tag{60}$$

$(\rho, \boldsymbol{J}, \boldsymbol{P})$ independent of space and time

Y.-E. Keta et al., Physical Review E 103, 022603 (2021).

$$S = \frac{1}{4D\gamma} |\gamma \boldsymbol{f} L|^2 + \frac{sL^2}{D}\omega \tag{60}$$

$(\rho, \boldsymbol{J}, \boldsymbol{P})$ independent of space and time

Y.-E. Keta et al., Physical Review E 103, 022603 (2021).

$$S = \frac{1}{4D\gamma} \left|\gamma \boldsymbol{f} L\right|^2 + \frac{sL^2}{D}\omega \tag{60}$$

$(\rho, \boldsymbol{J}, \boldsymbol{P})$ independent of space and time

$$\text{LDP on polarisation } P_{s=0}[\bar{\rho}, \boldsymbol{J}_d, \boldsymbol{P}] \propto \exp\left(-\frac{L^2 t}{D}\bar{\rho}\mathcal{J}(\boldsymbol{P})\right) \tag{61}$$

Y.-E. Keta et al., Physical Review E 103, 022603 (2021).

$$S = \frac{\bar{\rho}L^2}{D}\mathcal{J}(\boldsymbol{P}) + \frac{sL^2}{D}\omega \tag{60}$$

$(\rho, \boldsymbol{J}, \boldsymbol{P})$ independent of space and time

$$\text{LDP on polarisation } P_{s=0}[\bar{\rho}, \boldsymbol{J}_d, \boldsymbol{P}] \propto \exp\left(-\frac{L^2 t}{D}\bar{\rho}\mathcal{J}(\boldsymbol{P})\right) \tag{61}$$

Y.-E. Keta et al., Physical Review E 103, 022603 (2021).

$$S = \frac{\bar{\rho}L^2}{D}\mathcal{J}(\boldsymbol{P}) + \frac{sL^2}{D}\omega \tag{60}$$

$(\rho, \boldsymbol{J}, \boldsymbol{P})$ independent of space and time

LDP on polarisation $P_{s=0}[\bar{\rho}, \boldsymbol{J}_d, \boldsymbol{P}] \propto \exp\left(-\frac{L^2 t}{D}\bar{\rho}\mathcal{J}(\boldsymbol{P})\right)$ (61)

analytical derivation $\mathcal{J}(\boldsymbol{P}) = \frac{1}{2}D_r|\boldsymbol{P}|^2 + \mathcal{O}(|\boldsymbol{P}|^4)$ (62)

Taylor expansion $\omega(\bar{\rho}, \boldsymbol{P}) = \bar{\rho}\left[\langle w_\tau\rangle + \frac{c_\omega}{2}|\boldsymbol{P}|^2 + \mathcal{O}(|\boldsymbol{P}|^4)\right]$ (63)

Y.-E. Keta et al., Physical Review E 103, 022603 (2021).

$$S = \frac{\bar{\rho}L^2}{D}\left[s\left\langle w_\tau\right\rangle + \frac{1}{2}(D_r + sc_\omega)|\boldsymbol{P}|^2 + \mathcal{O}(|\boldsymbol{P}|^4)\right] \tag{60}$$

$(\rho, \boldsymbol{J}, \boldsymbol{P})$ independent of space and time

LDP on polarisation $P_{s=0}[\bar{\rho}, \boldsymbol{J}_d, \boldsymbol{P}] \propto \exp\left(-\frac{L^2 t}{D}\bar{\rho}\mathcal{J}(\boldsymbol{P})\right)$ (61)

analytical derivation $\mathcal{J}(\boldsymbol{P}) = \frac{1}{2}D_r|\boldsymbol{P}|^2 + \mathcal{O}(|\boldsymbol{P}|^4)$ (62)

Taylor expansion $\omega(\bar{\rho}, \boldsymbol{P}) = \bar{\rho}\left[\left\langle w_\tau\right\rangle + \frac{c_\omega}{2}|\boldsymbol{P}|^2 + \mathcal{O}(|\boldsymbol{P}|^4)\right]$ (63)

Y.-E. Keta et al., Physical Review E 103, 022603 (2021).

$$S = \frac{\bar{\rho}L^2}{D}\left[s\left\langle w_\tau\right\rangle + \frac{1}{2}(D_r + sc_\omega)|\boldsymbol{P}|^2 + \mathcal{O}(|\boldsymbol{P}|^4)\right] \tag{64}$$

N. Goldenfeld, (CRC Press, 2018).

Y.-E. Keta et al., Physical Review E 103, 022603 (2021).

$$S = \frac{\bar{\rho} L^2}{D} \left[s \left\langle w_\tau \right\rangle + \frac{1}{2}(D_r + s c_\omega)|\boldsymbol{P}|^2 + \mathcal{O}(|\boldsymbol{P}|^4) \right] \tag{64}$$

analogy with a **ferromagnet**

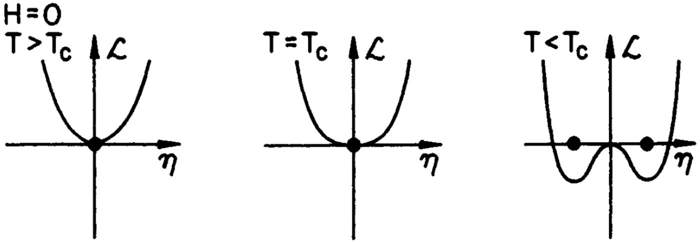
$$\mathcal{L}(H=0) = a(T_c)(T - T_c)\eta^2 + \mathcal{O}(\eta^4), \ \ T_c \propto J \tag{65}$$

N. Goldenfeld, (CRC Press, 2018).
Y.-E. Keta et al., Physical Review E 103, 022603 (2021).

$$S = \frac{\bar{\rho}L^2}{D}\left[s\left\langle w_\tau\right\rangle + \frac{1}{2}(D_r + sc_\omega)|\boldsymbol{P}|^2 + \mathcal{O}(|\boldsymbol{P}|^4)\right] \tag{64}$$

analogy with a **ferromagnet**

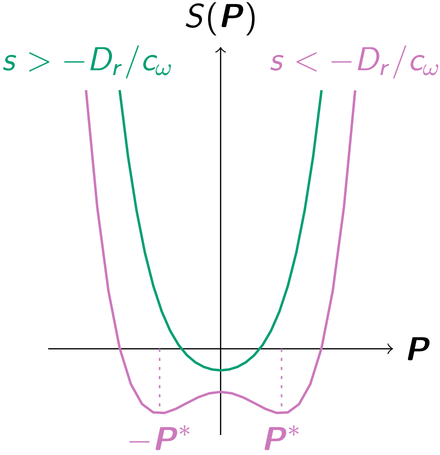
$$\mathcal{L}(H=0) = a(T_c)(T - T_c)\eta^2 + \mathcal{O}(\eta^4), \;\; T_c \propto J \tag{65}$$



N. Goldenfeld, (CRC Press, 2018).

Y.-E. Keta et al., Physical Review E 103, 022603 (2021).

$$S = \frac{\bar{\rho} L^2}{D} \left[s \langle w_T \rangle + \frac{1}{2}(D_r + s c_\omega) |\boldsymbol{P}|^2 + \mathcal{O}(|\boldsymbol{P}|^4) \right] \tag{64}$$



N. Goldenfeld, (CRC Press, 2018).

Y.-E. Keta et al., Physical Review E 103, 022603 (2021).

- We have introduced a model of **active Brownian particles** biased with respect to a measure of the **averaged dissipated power** (active work w_τ).

- We have introduced a model of **active Brownian particles** biased with respect to a measure of the **averaged dissipated power** (active work w_τ).
- This model of isotropic active matter displays **phase-separated arrest** (PSA) when w_τ is decreased ($s > 0$) and **collective motion** (CM) when w_τ is increased ($s < 0$).

- We have introduced a model of **active Brownian particles** biased with respect to a measure of the **averaged dissipated power** (active work w_τ).
- This model of isotropic active matter displays **phase-separated arrest** (PSA) when w_τ is decreased ($s > 0$) and **collective motion** (CM) when w_τ is increased ($s < 0$).
- This transition to CM happens at **finite biasing** $s^* \sim -D_r$ implying that these fluctuations are very distinct from the unbiased behaviour.

- We have introduced a model of **active Brownian particles** biased with respect to a measure of the **averaged dissipated power** (active work w_τ).
- This model of isotropic active matter displays **phase-separated arrest** (PSA) when w_τ is decreased ($s > 0$) and **collective motion** (CM) when w_τ is increased ($s < 0$).
- This transition to CM happens at **finite biasing** $s^* \sim -D_r$ implying that these fluctuations are very distinct from the unbiased behaviour.
- We have computed particle alignment exactly for a system of 2 particles, and showed that increased w_τ is accompanied by **effective aligning interactions**.

- We have introduced a model of **active Brownian particles** biased with respect to a measure of the **averaged dissipated power** (active work w_τ).
- This model of isotropic active matter displays **phase-separated arrest** (PSA) when w_τ is decreased ($s > 0$) and **collective motion** (CM) when w_τ is increased ($s < 0$).
- This transition to CM happens at **finite biasing** $s^* \sim -D_r$ implying that these fluctuations are very distinct from the unbiased behaviour.
- We have computed particle alignment exactly for a system of 2 particles, and showed that increased w_τ is accompanied by **effective aligning interactions**.
- We have proposed a **fluctuating hydrodynamic theory** which captures the emergence of polar order in the biased state.

Thank you!

→ In the s-ensemble of trajectories $x(0 \leq t \leq \tau)$, $t = 0$ and $t = \tau$ are boundaries ⇒ dynamical analogue of boundary effects.

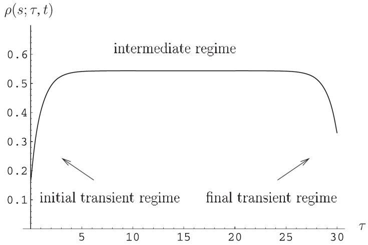


Figure: [from: J. P. Garrahan et al., Journal of Physics A: Mathematical and Theoretical **42**, 075007 (2009) (Fig. 7)]

$$P_{\mathrm{end}}(x) = \lim_{\tau\to\infty} \langle \delta(x(\tau) - x) \rangle_s \qquad (65)$$

$$P_{\mathrm{ave}}(x) = \lim_{\tau\to\infty} \left\langle \tau^{-1} \int_0^\tau \delta(x(t) - x) \mathrm{d}t \right\rangle_s \qquad (66)$$

T. Nemoto et al., Physical Review E 93, 062123 (2016).

SDE	$\dot{\boldsymbol{X}} = F(\boldsymbol{X}) + \sqrt{2D}\boldsymbol{\eta}$	(67)
Fokker-Planck equation	$\frac{\partial}{\partial t}P(\boldsymbol{x},t) = \mathcal{L}P(\boldsymbol{x},t)$	(68)
	$P(\boldsymbol{x},t) = P(\boldsymbol{X}(t) = \boldsymbol{x})$	(69)
observable	$A_\tau = \frac{1}{\tau}\int_0^\tau f(\boldsymbol{X}(t))\,\mathrm{d}t$	(70)
SCGF	$\psi(s) = \lim_{\tau\to\infty}\frac{1}{\tau}\log\langle\exp(-s\tau A_\tau)\rangle$	(71)

→ $\psi(s)$ is the largest eigenvalue of $\mathcal{L} - sf$.

$$\lambda(s)l(\boldsymbol{x}) = (\mathcal{L} - sf(\boldsymbol{x}))l(\boldsymbol{x}) \tag{72}$$

$$P_{\mathrm{end}}(\boldsymbol{x}) = l(\boldsymbol{x}) \tag{73}$$

$$\lambda(s)r(\boldsymbol{x}) = (\mathcal{L}^\dagger - sf(\boldsymbol{x}))r(\boldsymbol{x}) \tag{74}$$

$$P_{\mathrm{ave}}(\boldsymbol{x}) = l(\boldsymbol{x})r(\boldsymbol{x}) \tag{75}$$

H. Touchette, Physica A: Statistical Mechanics and its Applications **504**, 5–19 (2018).

"Large deviations occur according to the least unlikely mechanism."

$$I_A(w) \leq I(w) \leq I_B(w) \text{ the closer the better} \tag{76}$$

R. L. Jack, The European Physical Journal B 93, 1–22 (2020).

$$\dot{\boldsymbol{r}}_i = v_s^{\mathrm{con}} \boldsymbol{u}(\theta_i) - D\nabla_i U + \sqrt{2D}\boldsymbol{\eta}_i \tag{77}$$

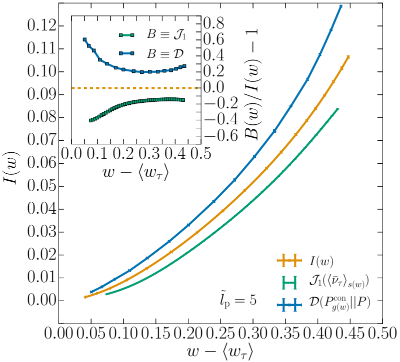
$$\dot{\theta}_i = -D_r \frac{\partial}{\partial \theta_i} U_g^{\mathrm{con}} + \sqrt{2D_r}\xi_i \tag{78}$$

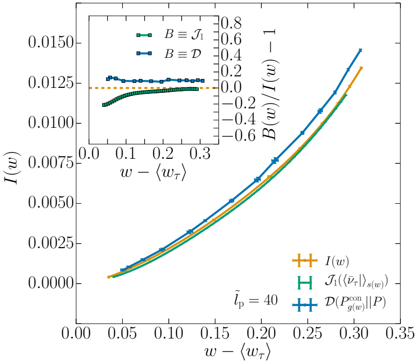
$$I(w) \leq \lim_{\tau \to \infty} \mathcal{D}_{\mathrm{KL}}(P_{g(w)}^{\mathrm{mod}} || P) \tag{79}$$

$$\mathcal{D}_{\mathrm{KL}}(P_{g(w)}^{\mathrm{mod}} || P) = \left\langle \log P_{g(w)}^{\mathrm{mod}} / P \right\rangle_{\mathrm{mod}} = f[\{\theta_i(t)\}, g(w)] \tag{80}$$

$$I_{h(A)}(b) = \inf_{a:h(a)=b} I_A(a) \tag{81}$$

$$I(w) \underset{\mathrm{CP}}{=} \inf_{\nu} I_2(w,\nu) = I_2(w,\nu(w)) \geq \inf_{w'} I_2(w',\nu(w)) \underset{\mathrm{CP}}{=} \mathcal{J}_1(\nu(w)) \tag{82}$$


$I(w)$
$w - \langle w_\tau \rangle$
$B(w)/I(w) - 1$
$B \equiv \mathcal{J}_1$
$B \equiv \mathcal{D}$
$I(w)$
$\mathcal{J}_1(\langle \bar{\nu}_\tau \rangle_{s(w)})$
$\mathcal{D}(P^{\mathrm{con}}_{g(w)} || P)$
$\tilde{l}_\mathrm{p} = 5$

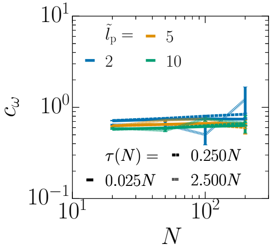

$I(w)$
$w - \langle w_\tau \rangle$
$B(w)/I(w) - 1$
$B \equiv \mathcal{J}_1$
$B \equiv \mathcal{D}$
$I(w)$
$\mathcal{J}_1(\langle |\bar{\nu}_\tau| \rangle_{s(w)})$
$\mathcal{D}(P^{\mathrm{con}}_{g(w)} || P)$
$\tilde{l}_\mathrm{p} = 40$

$$\omega(\bar{\rho}, \boldsymbol{P}) = \langle \bar{\rho} w_\tau \rangle_{\boldsymbol{h}(\boldsymbol{P})} = \frac{\langle \bar{\rho} w_\tau \exp(-\tau N \boldsymbol{h}(\boldsymbol{P}) \cdot \bar{\boldsymbol{\nu}}_\tau) \rangle}{\langle \exp(-\tau N \boldsymbol{h}(\boldsymbol{P}) \cdot \bar{\boldsymbol{\nu}}_\tau) \rangle} \tag{83}$$

$$\bar{\boldsymbol{\nu}}_\tau = \frac{1}{\tau} \int_0^\tau \frac{1}{N} \sum_{i=1}^N \boldsymbol{u}(\theta_i(t)) \, \mathrm{d}t, \ \langle \bar{\boldsymbol{\nu}}_\tau \rangle_{\boldsymbol{h}(\boldsymbol{P})} = \boldsymbol{P} \tag{84}$$

$$\omega(\bar{\rho}, \boldsymbol{P}) = \bar{\rho} \left[\langle w_\tau \rangle + \frac{c_\omega}{2} |\boldsymbol{P}|^2 + \mathcal{O}(|\boldsymbol{P}|^4) \right] \tag{85}$$

$$c_\omega = \frac{\bar{\rho} \tau^2 L^4 D_r^2}{2} \mathrm{Cov}(w_\tau, |\bar{\boldsymbol{\nu}}_\tau|^2) \tag{86}$$



- At $\boldsymbol{P} = 0$, biasing w.r.t. w_τ is equivalent to biasing w.r.t. $|\tilde{\rho}_{\boldsymbol{q}}|^2$.

$$S_s(\boldsymbol{q}) = \left\langle |\tilde{\rho}_{\boldsymbol{q}}|^2 \right\rangle_s = \begin{cases} \chi_0, & s = 0 \\ b_s q, & s < 0 \end{cases} \tag{87}$$

$\Rightarrow$ We expect hyperuniformity in the isotropic $s < 0$ phase for $N \gg 1$.

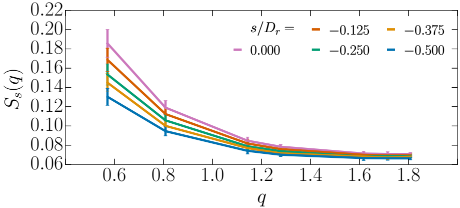


Figure: Biased structure factor S_s.

$\rightarrow$ Finite system shows suppression of density fluctuations for $s < 0$.

J. Dolezal and R. L. Jack, Journal of Statistical Mechanics: Theory and Experiment 2019, 123208 (2019).