Error in the cloning algorithm

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We consider the following modified equation of rotational motion,

$$\dot{\theta}_i = -g\, N\, \frac{\partial}{\partial \theta_i} |\underline{\nu}(t)|^2 + \sqrt{\frac{2}{\alpha\, \mathrm{Pe}}} \xi_i, \tag{1}$$

with g a free parameter.

According to notes by Takahiro, summarised in this tiddler, we should have that

$$s\, w_{\mathrm{mod}}(0,\tau)|_{s=0} = \frac{g}{N} - g \underbrace{\frac{1}{\tau}\int_0^\tau |\underline{\nu}(t)|^2 \,\mathrm{d}t}_{\mathcal{I}_1(0,\tau)} - g^2\, \alpha\, \mathrm{Pe} \underbrace{\frac{1}{N\,\tau}\int_0^\tau |\underline{\nu}(t)|^2 \sum_{i=1}^{N} \sin^2(\theta_i(t) - \varphi(t))\,\mathrm{d}t}_{\mathcal{I}_2(0,\tau)}, \tag{2}$$

with

$$\lim_{\tau\to\infty} \mathcal{I}_1(0,\tau) = \left\langle |\underline{\nu}(t)|^2 \right\rangle_{\mathrm{mod}} = \frac{1}{N}\frac{1}{1+g\,\alpha\,\mathrm{Pe}}, \tag{3}$$

which we check in figure 1.

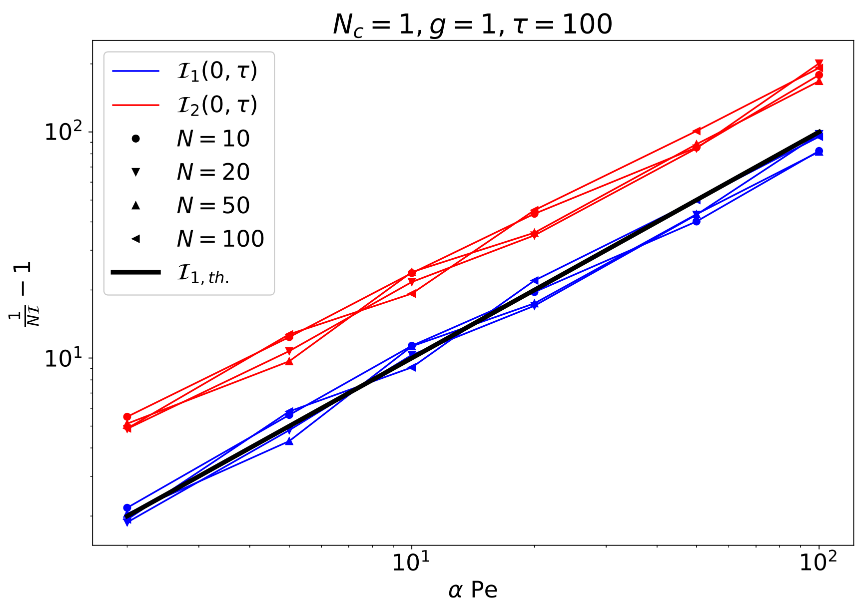

Figure 1: Output from our cloning algorithm.

Considering a single clone, $N_c = 1$, we approximate the scaled cumulant generating function with

$$\psi(s=0,\tau) = -s\, w_{\mathrm{mod}}(0,\tau)|_{s=0}\,, \tag{4}$$

and we note that

$$\psi(s=0,\tau) = \frac{1}{N\,\tau} \log \left\langle e^{-s\,N\,\tau\, w(0,\tau)} \right\rangle_0 \Big|_{s=0} = 0, \tag{5}$$

which with equations 2 and 3 should lead to

$$\mathcal{I}_2(0,\tau)|_{s=0} = \frac{1}{N}\frac{1}{1+g\,\alpha\,\mathrm{Pe}}, \tag{6}$$

and thus

$$\mathcal{I}_2(0,\tau)\big|_{s=0} = \mathcal{I}_1(0,\tau)\big|_{s=0}\,, \tag{7}$$

irrespective of g, which we see from figure 1 is not satisfied for $g = 1$.

We can see this discrepancy from the fact that

$$0 \leq \frac{1}{N}\sum_{i=1}^{N} \sin^2(\theta_i(t) - \varphi(t)) \leq 1, \tag{8}$$

and thus

$$\mathcal{I}_2(0,\tau) \leq \mathcal{I}_1(0,\tau). \tag{9}$$

Moreover, we have that the unbiased dynamics, $s = 0$, should not display orientational order, $|\underline{\nu}(t)| \approx 0$ for $N \to \infty$. Heuristically, considering that the θ_i are randomly distributed, we should get that

$$\left\langle \frac{1}{N}\sum_{i=1}^{N} \sin^2(\theta_i(t) - \varphi(t)) \right\rangle \approx \langle \sin^2 \rangle = \frac{1}{2}, \tag{10}$$

which may explain the approximate relation

$$\mathcal{I}_2(0,\tau) \approx \frac{1}{2}\mathcal{I}_1(0,\tau), \tag{11}$$

we observe numerically.