# Variations of active junctions

Yann-Edwin Keta
(Dated: 23/01/2024, 12:21)

## I. GENERAL EQUATIONS

We consider a junction between two vertices $\mu$ and $\nu$, with a tension $T_{\mu\nu}$ and a length $\ell_{\mu\nu}$, which separates cells $i$ and $j$. This junction contributes the following term to the force applied on $\mu$

$$\boldsymbol{F}_{\mu\nu} = T_{\mu\nu}\hat{\boldsymbol{r}}_{\mu\nu} \tag{1}$$

with $\hat{\boldsymbol{r}}_{\mu\nu} = (\boldsymbol{r}_\nu - \boldsymbol{r}_\mu)/|\boldsymbol{r}_\nu - \boldsymbol{r}_\mu|$.

We will denote an "Ornstein-Uhlenbeck process"

$$p = \mathrm{OU}(m,\sigma,\tau) \Leftrightarrow \tau\dot{p} = -(p-m) + \sqrt{2\sigma^2\tau}\,\eta \tag{2}$$

where $\eta$ is a zero-mean unit-variance Gaussian white noise. With this notation $m$ may itself be a stochastic process. It is then important to distinguish

$$p^{(1)} = m + \Delta p^{(1)}, \tag{3a}$$

$$\Delta p^{(1)} = \mathrm{OU}(0,\ldots) \Leftrightarrow \Delta\dot{p}^{(1)} = -\Delta p^{(1)} + \eta, \tag{3b}$$

for which we obtain

$$\dot{p}^{(1)} = -(p^{(1)} - m) + \eta + \dot{m}, \tag{4}$$

from the similarly defined

$$p^{(2)} = \mathrm{OU}(m,\ldots) \Leftrightarrow \dot{p}^{(2)} = -(p^{(2)} - m) + \eta, \tag{5}$$

such that Eqs. 4, 5 differ by a term $\dot{m}$.

## II. PERIMETER ELASTICITY

We define $P_i$ and $P_i^0$ the perimeter and target perimeter of cell $i$, and $\Gamma$ an elastic constant. We denote

$$T_{\mu\nu}^{\mathrm{per}} = \Gamma[P_i - P_i^0] + \Gamma[P_j - P_j^0] \tag{6}$$

the tension deriving from perimeter elasticity. Target perimeters $P_i^0$ are constant.

### A. Model 0

$$T_{\mu\nu} = T_{\mu\nu}^{\mathrm{per}} + \Delta T_{\mu\nu}, \tag{7a}$$

$$\Delta T_{\mu\nu} = \mathrm{OU}(0,\sigma,\tau_p). \tag{7b}$$

This corresponds to the classical vertex model in the limit $\sigma \to 0$.

### B. Model 1

$$T_{\mu\nu} = \mathrm{OU}(T_{\mu\nu}^{\mathrm{per}},\sigma,\tau_p). \tag{8}$$

In the $\sigma \to 0$ limit the tension relaxes to $T_{\mu\nu}^{\mathrm{per}}$ over the timescale $\tau_p$, which amounts to an effective memory kernel.

## III. JUNCTION VISCOELASTICITY

We define $\ell^0_{\mu\nu}$ the rest length of junction $\mu \leftrightarrow \nu$ and $k$ an elastic constant. We denote

$$T^{\text{el}}_{\mu\nu} = \Gamma[\ell_{\mu\nu} - \ell^0_{\mu\nu}] \tag{9}$$

the tension deriving from junction elasticity. Rest lengths $\ell^0_{\mu\nu}$ are stochastic variables which are initialised with the values of the junction lengths $\ell_{\mu\nu}$.

### A. Model 2

$$T_{\mu\nu} = T^{\text{el}}_{\mu\nu} + \Delta T_{\mu\nu}, \tag{10a}$$

$$\Delta T_{\mu\nu} = \text{OU}(0, \sigma, \tau_p), \tag{10b}$$

$$\tau_r \dot{\ell}^0_{\mu\nu} = -(\ell^0_{\mu\nu} - \ell_{\mu\nu}) \Leftrightarrow \dot{\ell}^0_{\mu\nu} = \text{OU}(\ell_{\mu\nu}, 0, \tau_r). \tag{10c}$$

This corresponds to the Maxwell model in the limit $\sigma \to 0$.

### B. Model 3

$$T_{\mu\nu} = T^{\text{el}}_{\mu\nu}, \tag{11a}$$

$$\ell^0_{\mu\nu} = \text{OU}(\ell_{\mu\nu}, \sigma, \tau_p). \tag{11b}$$

This corresponds to the Maxwell model in the limit $\sigma \to 0$.

### C. Model 4

$$T_{\mu\nu} = \text{OU}(T^{\text{el}}_{\mu\nu}, \sigma, \tau_p), \tag{12a}$$

$$\tau_r \dot{\ell}^0_{\mu\nu} = -(\ell^0_{\mu\nu} - \ell_{\mu\nu}) \Leftrightarrow \dot{\ell}^0_{\mu\nu} = \text{OU}(\ell_{\mu\nu}, 0, \tau_r). \tag{12b}$$