

Variations of active junctions

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I. GENERAL EQUATIONS

We consider a junction between two vertices μ and ν , with a tension $T_{\mu\nu}$ and a length $\ell_{\mu\nu}$, which separates cells i and j . This junction contributes the following term to the force applied on μ

$$\mathbf{F}_{\mu\nu} = T_{\mu\nu} \hat{\mathbf{r}}_{\mu\nu} \quad (1)$$

with $\hat{\mathbf{r}}_{\mu\nu} = (\mathbf{r}_\nu - \mathbf{r}_\mu)/|\mathbf{r}_\nu - \mathbf{r}_\mu|$.

We will denote an ‘‘Ornstein-Uhlenbeck process’’

$$p = \text{OU}(m, \sigma, \tau) \Leftrightarrow \tau \dot{p} = -(p - m) + \sqrt{2\sigma^2\tau} \eta \quad (2)$$

where η is a zero-mean unit-variance Gaussian white noise. With this notation m may itself be a stochastic process. It is then important to distinguish

$$p^{(1)} = m + \Delta p^{(1)}, \quad (3a)$$

$$\Delta p^{(1)} = \text{OU}(0, \dots) \Leftrightarrow \Delta \dot{p}^{(1)} = -\Delta p^{(1)} + \eta, \quad (3b)$$

for which we obtain

$$\dot{p}^{(1)} = -(p^{(1)} - m) + \eta + \dot{m}, \quad (4)$$

from the similarly defined

$$p^{(2)} = \text{OU}(m, \dots) \Leftrightarrow \dot{p}^{(2)} = -(p^{(2)} - m) + \eta, \quad (5)$$

such that Eqs. 4, 5 differ by a term \dot{m} .

II. PERIMETER ELASTICITY

We define P_i and P_i^0 the perimeter and target perimeter of cell i , and Γ an elastic constant. We denote

$$T_{\mu\nu}^{\text{per}} = \Gamma[P_i - P_i^0] + \Gamma[P_j - P_j^0] \quad (6)$$

the tension deriving from perimeter elasticity. Target perimeters P_i^0 are constant.

A. Model 0

$$T_{\mu\nu} = T_{\mu\nu}^{\text{per}} + \Delta T_{\mu\nu}, \quad (7a)$$

$$\Delta T_{\mu\nu} = \text{OU}(0, \sigma, \tau_p). \quad (7b)$$

This corresponds to the classical vertex model in the limit $\sigma \rightarrow 0$.

B. Model 1

$$T_{\mu\nu} = \text{OU}(T_{\mu\nu}^{\text{per}}, \sigma, \tau_p). \quad (8)$$

In the $\sigma \rightarrow 0$ limit the tension relaxes to $T_{\mu\nu}^{\text{per}}$ over the timescale τ_p , which amounts to an effective memory kernel.

III. JUNCTION VISCOELASTICITY

We define $\ell_{\mu\nu}^0$ the rest length of junction $\mu \leftrightarrow \nu$ and k an elastic constant. We denote

$$T_{\mu\nu}^{\text{el}} = \Gamma[\ell_{\mu\nu} - \ell_{\mu\nu}^0] \quad (9)$$

the tension deriving from junction elasticity. Rest lengths $\ell_{\mu\nu}^0$ are stochastic variables which are initialised with the values of the junction lengths $\ell_{\mu\nu}$.

A. Model 2

$$T_{\mu\nu} = T_{\mu\nu}^{\text{el}} + \Delta T_{\mu\nu}, \quad (10a)$$

$$\Delta T_{\mu\nu} = \text{OU}(0, \sigma, \tau_p), \quad (10b)$$

$$\tau_r \dot{\ell}_{\mu\nu}^0 = -(\ell_{\mu\nu}^0 - \ell_{\mu\nu}) \Leftrightarrow \dot{\ell}_{\mu\nu}^0 = \text{OU}(\ell_{\mu\nu}, 0, \tau_r). \quad (10c)$$

This corresponds to the Maxwell model in the limit $\sigma \rightarrow 0$.

B. Model 3

$$T_{\mu\nu} = T_{\mu\nu}^{\text{el}}, \quad (11a)$$

$$\ell_{\mu\nu}^0 = \text{OU}(\ell_{\mu\nu}, \sigma, \tau_p). \quad (11b)$$

This corresponds to the Maxwell model in the limit $\sigma \rightarrow 0$.

C. Model 4

$$T_{\mu\nu} = \text{OU}(T_{\mu\nu}^{\text{el}}, \sigma, \tau_p), \quad (12a)$$

$$\tau_r \dot{\ell}_{\mu\nu}^0 = -(\ell_{\mu\nu}^0 - \ell_{\mu\nu}) \Leftrightarrow \dot{\ell}_{\mu\nu}^0 = \text{OU}(\ell_{\mu\nu}, 0, \tau_r). \quad (12b)$$