Dec 30. 2025 Ruobing Zhang

Einstein Manifolds.

1. Def: M^n $\mathrm{Ric}_g = \underset{\text{(global)}}{\lambda} \cdot g$ $\lambda \in \mathbb{R}$.

2. Coordinate, harmonic system

3. Two questions : 1. Existence
2. Structure of solutions
—— moduli space

$n=1$: trivial

$n=2$: Existence : Uniformization Th

$n=3$: Moduli space : Teichmuller space.

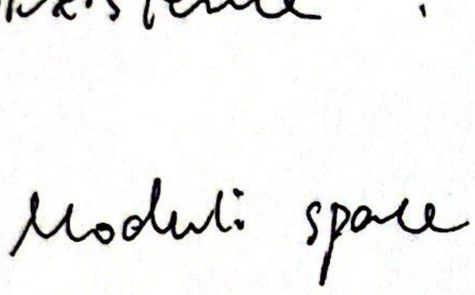
(chord) Existence : S^3, $\underset{\text{(chord)}}{\mathbb{H}^3}$ — Poincaré Conjecture.

Moduli space : easy, by Bieberbach th and some ths.

$n \geq 4$: Holonomy groups.

1. SO_n trivial.
2. $G\mathbb{C}/H$
3. U_n Kähler
4. SU_n Calabi-Yau.
5. $Sp(1)\cdot Sp(n)$ hyper-Kähler
6. Others — $\mathbb{H}$ Quaternion ——.

—— Higher topics



K3 surface

bad ex: 2-torus $\mathbb{R}^3 \times \mathbb{R}/\mathbb{Z}$. Cone $C(\frac{2\pi i}{\beta})$ $\beta \in \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}\}$. (something like this)

$\mathbb{Z}$ = rotation by irrational $\times$ transposition. $\to [0,\infty)$.