CLO Portfolio Analysis Report

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Abstract

Portfolio Construction is one of the key areas in today's investment management industry. In this report, I'll first show my process of constructing high-risk and low-risk loan portfolios with Portfolio Optimization, and then make the final portfolio selection decision by looking at their performance under different default scenarios. More importantly, I'll cover the further improvements about current assumptions as well as the portfolio optimization process, which could be applied to refine the process to make it closer to reality. All code can be found here.

1 Data

In this section, I'll cover my process of data analysis and data cleaning.

1.1 Credit Data

Credit Data covers credit-level features with information from different perspectives.

- **Data Fields**: Credit, Facility, Modeled Par, Model Price, Spread, LIBOR Floor, 1 year LIBOR, 1 Year Income Rate, Maturity, Moody's CFR, Moody's Ratings Factor, S&P Industry, Recovery Rate
- **No. of Credits**: 209
- **No. of S&P Industries**: 50
- **Existing Rating Categories**: Baa2, Ba1, Ba2, Ba3, B1, B2, B3, Caa1

1.2 Constraint Data

Constraint Data sets limits when constructing high risk and low risk portfolios.

Constraint	Low Risk Portfolio	High Risk Portfolio
Minimum Weighted Average Income Rate	3.25%	4.35%
Maximum Weight per Credit	1.5%	2.5%
Maximum Weighted Average Price	100%	99.7%
Maximum Weight Per Industry	10%	15%
Maximum % in B3 or lower	15%	40%
Maximum % in Caa or lower	0%	2%
Minimum S&P WARR	43%	39%

Table 1: Portfolio Constraints

1.3 Default Rate Data

Default Rate Data is about the Base Default Rates and Default Rate Sensitivities of different rating categories. **One problem I found while cleaning the data is, there's Baa2 in the Credit Data given, but its base default rate and default rate sensitivity are not provided. After looking at the base default rates and default rate sensitivities of other rating categories,**

I assigned it a Base Default Rate of 0.36%, and a default rate sensitivity of 5 bps default rate change per 100 bps change in B2 default rate. Please see below table for the final version of Default Rate Data, where Default Rate Sensitivity is defined as bps Change in Default Rate per 100 bps Change in B2 Default Rate:

Rating	Base Default Rate	Default Rate Sensitivity
Baa2	0.36%	5
Baa3	0.61%	10
Ba1	0.94%	20
Ba2	1.35%	35
Ba3	1.77%	50
B1	2.22%	70
B2	2.72%	100
B3	3.49%	150
Caa1	4.77%	250

Table 2: Default Rate

2 Portfolio Construction

In this section, I'll cover the details of constructing portfolio with Portfolio Optimization, and then analyze the results for both high risk portfolio and low risk portfolio.

2.1 Portfolio Optimization

2.1.1 Mathematical Representation of the Portfolio Optimization Problem

$$
\begin{aligned}
\min_{w_1, w_2, \ldots w_{209}} \quad & \sum_{i=1}^{209} w_i \times RF_i \\
\text{s.t.} \quad & \sum_{i=1}^{209} w_i = 100\% \\
& 0\% \leq w_i \leq 1.5\% \quad (\forall credit : i = 1, 2, ..., 209) \\
& \sum_{i \in Sector_j} w_i \leq 10\% \quad (\forall sector : j = 1, 2, ..., 50) \\
& \sum_{i=1}^{209} w_i \times IR_i \geq 3.25\% \\
& \sum_{i=1}^{209} w_i \times P_i \leq 100\% \\
& \sum_{i=1}^{209} w_i \times RR_i \geq 43\% \\
& \sum_{i \in \{B3, Caa1\}} w_i \leq 15\% \\
& \sum_{i \in \{Caa1\}} w_i \leq 0\%
\end{aligned}
\tag{1}
$$

Mathematically, the Portfolio Optimization problem for the Low Risk Portfolio can be specified as above - please note that, for the High Risk Portfolio, the specification is the same except for the threshold numbers. As is shown above, there are one objective function and eight constraints, which I'll talk in more details below.

2.1.2 Objective Function

The objective is to maximize the credit quality of the loan portfolio, which can be reduced to minimizing the weighted average ratings factor (WARF). In the above Mathematical Representation, I specified this as $\sum_{i=1}^{209} w_i RF_i$, where RF_i is the Rating Factor for Credit i.

2.1.3 Constraints

The constraints are taken from table 1, which sets limits from different perspectives for the portfolio. **In addition, I added the "Fully Investment" and "Long Only" constraints into the portfolio optimization problems when constructing the portfolios as they are widely used by industry practitioners.** Let's go through the constraints one by one in their order of presence:

1. **Fully Invested**: All money are invested into the securities, so their weights sum up to 100%.
2. **Credit Weights Constraint & Long Only**: These two limits set thresholds for individual credit weights - each of they should be no less than 0 and no greater than "Maximum Weight per Credit".
3. **Sector Weights Constraint**: The sum of all the weights in any sector should be no greater than "Maximum Weight Per Industry" - **one thing we need to notice is that the sector weights are implicitly constrained by maximum individual weights times the number of credits in this sector.** $Sector_j$ is the set of credits within sector j.
4. **Income Rating Constraint**: The portfolio's weighted average income rate should be no less than "Minimum Weighted Average Income Rate". IR_i is the 1 Year Income Rate for credit i.
5. **Price Constraint**: The weighted average price of the portfolio should be no greater than "Maximum Weighted Average Price". P_i is the Model Price for credit i.
6. **Recovery Rate Constraint**: The weighted average recovery rate of the portfolio should be no less than "Minimum S&P WARR". RR_i is the Recovery Rate for credit i.
7. **B3 or Lower Constraint**: The sum of weights with credit rating of B3 or lower should be no greater than "Maximum % in B3 or lower".
8. **Caa or Lower Constraint**: The sum of weights with credit rating of Caa or lower should be no greater than "Maximum % in Caa or lower".

2.2 High Risk Portfolio vs. Low Risk Portfolio

2.2.1 Portfolio Metrics

Portfolio Metric	Low Risk Portfolio	High Risk Portfolio
Weighted Average Spread	2.86%	3.8%
Weighted Average Income Rate	3.25%	4.35%
Weighted Average Maturity	2026-12-06 03:29:42	2027-03-12 15:25:44
Weighted Average Ratings Factor	1785.9	2132.1
Weighted Average Recovery Rate	46.96%	41.78%
Weighted Average Price	100%	99.7%

Table 3: Portfolio Metrics

As is shown in table above, the Weighted Average Income Rate and Weighted Average Price for both portfolios are all within the pre-set threshold. Meanwhile, it's easy to see that the Low Risk Portfolio has smaller Weighted Average Spread, closer Weighted Average Maturity, smaller Weighted Average Ratings Factor (meaning higher credit quality), and higher Weighted Average Recovery Rate than the High Risk Portfolio. These are risk measurements from different perspectives, and the results are reasonable because the Low Risk Portfolio is structured to have lower risk than the High Risk Portfolio.

Low Risk Portfolio

	Credit	Modeled Par
0	AAdvantage Loyalty IP Ltd	$7,575,762
25	Asurion, LLC	$7,575,762
58	Enviva Holdings, LP	$7,575,762
103	LogMeIn, Inc.	$7,556,679
64	Froneri International Limited	$7,552,874

High Risk Portfolio

	Credit	Modeled Par
58	Enviva Holdings, LP	$12,626,261
0	AAdvantage Loyalty IP Ltd	$12,626,261
130	PetSmart LLC	$12,626,258
25	Asurion, LLC	$12,626,257
103	LogMeIn, Inc.	$12,594,454

Figure 1: Top 5 Credits by Modeled Par

Low Risk Portfolio

S&P Industry	Modeled Par
Commercial Services and Supplies	$49,859,621
Specialty Retail	$25,322,643
Food Products	$22,560,209
Insurance	$22,557,228
Capital Markets	$22,483,032

High Risk Portfolio

S&P Industry	Modeled Par
Commercial Services and Supplies	$50,071,573
Specialty Retail	$25,189,162
Trading Companies and Distributors	$25,157,173
Chemicals	$25,125,561
Building Products	$25,062,788

Figure 2: Top 5 Industries by Modeled Par

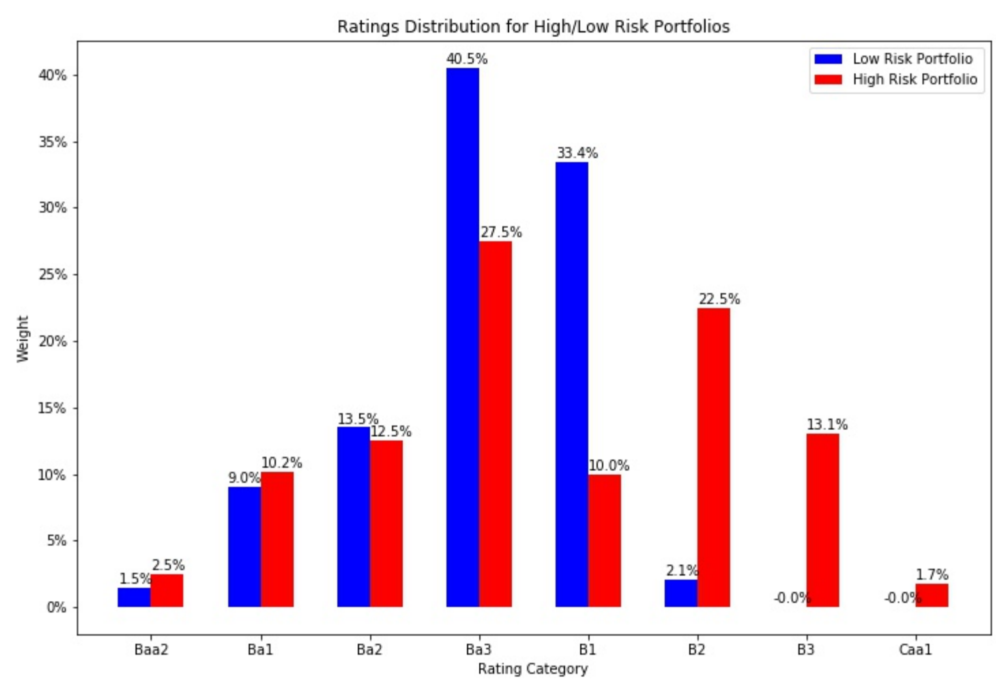

Figure 3: Rating Distribution

2.2.2 Top 5 Credits by Size

Size can refer to "Cost" or "Modeled Par", but here "Modeled Par" is used to rank the credits and generate the table below with information about the top 5 credits by size for both Low Risk Portfolio and High Risk Portfolio. As is shown in Figure 1, the top 5 credits are all very close to their weight constraint.

2.2.3 Top 5 Industries by Size

Figure 2 shows the top 5 sectors by size for both Low Risk Portfolio and High Risk Portfolio - one thing we need notice is that the sector weights are implicitly constrained by maximum individual weights times the number of credits in this sector. It's easy to see that the sector weight constraint is only reached by the "Commercial Services and Supplies" sector when constructing the Low Risk Portfolio.

2.2.4 Sector Skewness

	Low Risk Portfolio	High Risk Portfolio
Skewness	1.60	1.16

Table 4: Sector Skewness

Both the High Risk Portfolio and the Low Risk Portfolio are positively skewed, and the High Risk Portfolio has less significant skewness than the Low Risk Portfolio.

2.2.5 Rating Category Distribution

Figure 3 compares the rating distribution of the constructed Low Risk Portfolio and High Risk Portfolio side by side. We could see that, when going from Low Risk Portfolio to High Risk Portfolio, the weights show **a trend of decentralization** - the weights in medium rating categories (e.g., Ba2, Ba3 and B1) decreased significantly, the weights in low credit rating categories (e.g., B2, B3, Caa1) increased sharply, and the weights in high credit rating categories (e.g., Baa2, Ba1) increased slightly. **This is a intuitive representation of the "risk-return trade-off" if we look at the objective function and constraints - the portfolio is trying assign more weights to low rating categories to increase its weighted average income rate and meet the "Income Rate Constraint", but it still needs to minimize the rating factor as required by the objective function so it assigned more weights to the higher rating categories to try to maintain the overall credit quality.**

3 Portfolio Simulation Under Various Default Senarios

3.1 Theory

3.1.1 Profit at Year End for Credit i

$$Profit_i = PV_i \cdot IR_i$$

PV_i is the Par Value or Modeled Par for credit i, and IR_i is the Income Rate for credit i.

3.1.2 Loss Upon Default for Credit i

$$Loss_i = PD_i \cdot EAD_i \cdot LGD_i$$

Loss upon default for credit i can be decomposed into Probability of Default (PD), Exposure At Default (EAD), and Loss Given Default (LGD). In the above example, PD_i is the Probability of Default for credit i, EAD_i is the Exposure At Default for credit i, and LGD_i is the Loss Given Default (in percentage) for credit i.

$$PD_i = BasePD_i + PDSensi_i^{B2} \cdot PDShock_{B2}$$

$$PDShock_{B2} = CurPD_{B2} - BasePD_{B2}$$

PD can be further decomposed as shown above, where $BasePD_i$ is the Base Default Rate for credit i, $PDShock_{B2}$ is percent change in B2's current default rate $CurPD_{B2}$ from base default rate $BasePD_{B2}$, and $PDSensi_i^{B2}$ is the Probability Default sensitivity of credit i to B2, which measures the Change in Default Rate of credit i per 1% Change in B2 Default Rate.

$$EAD_i = w_i \cdot Cost_{ptf}$$

EAD can be further decomposed as shown above, where w_i is the percentage weight for credit i, and $Cost_{ptf}$ is the total cost of the portfolio.

$$LGD_i = 1 - RR_i$$

LGD can be further specified as above, where RR_i is the recovery rate of credit i.

3.1.3 PnL for Credit i

$$PnL_i = Profit_i - Loss_i$$

3.1.4 PnL for Portfolio

$$Ret_{ptf} = \frac{\sum_{i=1}^{209} PnL_i}{Cost_{ptf}}$$

Putting all the equations together, we have:

$$Ret_{ptf} = \frac{\sum_{i=1}^{209} PV_i \cdot IR_i - [BasePD_i + PDSensi_i^{B2} \cdot (CurPD_{B2} - BasePD_{B2})] \cdot (w_i \cdot Cost_{ptf}) \cdot (1 - RR_i)}{Cost_{ptf}}$$

In the below part, we'll use $CurPD_{B2}$ as x variable and Ret_{ptf} as y variable to depict the portfolio return under various default scenarios.

3.2 Result

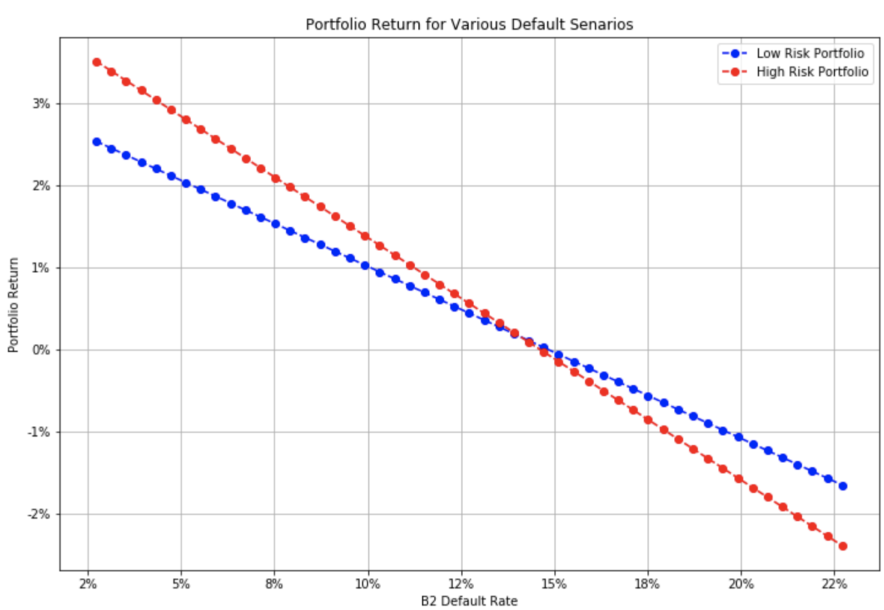

Figure 4: Portfolio Return under Various Default Scenarios

Scenario	B2 Default Rate	Low Risk Portfolio Return	High Risk Portfolio Return
Base Scenario	2.72%	2.54%	3.51%
High > Low	[2.72%, 14.10%)	[2.54%,0.15%)	[3.51%, 0.15%)
High = Low	14.10%	0.15%	0.15%
High < Low	>14.10%	<0.15%	<0.15%

Table 5: Portfolio Performance under Various Default Scenarios

3.3 Analysis

The performance of each portfolio under various default scenarios is summarized in Figure 4 and Table 5. When B2 default rate is less than 14.10%, High Risk Portfolio has better performance than the Low Risk Portfolio. During the break-even scenario where B2 default rate is 14.10%, High Risk Portfolio has the same performance as the Low Risk Portfolio, which are both 0.15%. When B2 default rate is less than 14.10%, High Risk Portfolio has worse performance than the Low Risk Portfolio.

Compared with the High Risk Portfolio, the Low Risk Portfolio has a lower base return and is subject to less risk from default as measured by the absolute value of the slope in Figure 4. **So when trying to determine which portfolio makes the most sense, one factor we should consider is the expected default rate in the near future.** If default rate is low, the High Risk Portfolio might be a better choice because it has higher return. On the contrary, if the default rate is high, the Low Risk Portfolio might be a better choice because it has lower risk, but in this situation the portfolio's return based on our analysis is almost always below 0%.

3.4 Other Analyses and Factors

We should also conduct other analyses and consider other risk factors when making the final decision, for example:

- Non-linear relationship between default rates of different rating categories
- Different recovery rate upon default for different credits, even within the same rating category
- Consider tax rate and transaction cost

4 Future Improvements

In this report, we try to use model to solve the loan portfolio construction problem under various assumptions, explicit or implicit. However, there are many parts in the process where the assumptions and the model are not accurate enough to depict the reality. As a result, we can use this as a base version and continue to refine the process by:

- Break the assumptions: examine each of our assumptions and found the mismatches with the real situation, then refine the process to make it closer to reality.
- Improve the Portfolio Optimization process: try to think of other risk factors to include into the Portfolio Optimization problem, and add them into the objective function or as constraints.

4.1 Break the Assumptions

4.1.1 The "What-Why-How" Format

This part will be structured using the "What-Why-How" format, where:

- **What**: What's the current assumption?
- **Why**: Why do you think that it doesn't depict the reality perfectly?
- **How**: How to improve the assumption and the process to make it closer to reality?

4.1.2 "What-Why-How" of Current Assumptions

1. **What**: Recovery rate upon default is 60% of par for all credits.
   Why: Recovery rate upon default can be different for different credits depending on e.g., their fundamentals, sectors, country of origin, etc.
   How: We can use historical recovery rate data to approximate the current recovery rate. Potentially, we could also apply machine learning model to historical data and learn a relationship between companies' recovery rate upon default and some features (e.g., fundamentals, sector).

2. **What**: All credits in the same ratings category have the same credit quality (e.g., Ratings Factor, Default Rate).
   Why: Different credits in the same rating category can have different credit qualities.
   How: We can increase the granularity of the credit ratings factor data, i.e. from rating-category-level to credit-level. Potentially, we could also apply machine learning model to historical data and learn a relationship between companies' rating factor and some features (e.g., fundamentals, sector, country).

3. **What**: Linear relationship between changes in default rates for B2 and that for other rating categories.
   Why: The relationship can be non-linear in reality.
   How: We can use historical data to calibrate the non-linear relationship between default rates with statistical methods.

4. **What**: Single period (one year horizon), all defaults occur at the end of the one year and we would have received a full year of income prior to the default.
   Why: In reality it's continuous time, defaults can occur at any time, and we may not receive income before default.
   How: We can model it as multiple period problem and use small time intervals to approximate continuous time. We can model the default event as a stochastic process that could happen any time, and determine whether the income is paid accordingly. Potentially, We could use Monte Carlo Simulation to model the process and get the average PnL, which is a widely used method when pricing financial derivatives.

5. **What**: No tax and transaction cost
   Why: In reality tax and transaction cost both exist.
   How: We can add tax component to adjust returns for the portfolio, and use transaction cost models to approximate real-world transaction costs.

6. **What**: All loans are fully divisible.
   Why: In reality, you cannot find a loan with par value less than 1 cent.
   How: We can only consider integer par value when constructing the loan portfolio - that's a good enough approximation.

7. **What**: Fully invested - invest all money to construct the loan portfolio.
   Why: Investors could hold cash based on their perception of the market.
   How: We can also add cash into our investment universe.

8. **What**: Long only - all the loans have positive weights.
   Why: It's possible to have short position in loans, e.g., through credit derivatives.
   How: We can consider short position in loans, but should also consider the cost associated with shorting loans.

4.2 Improve the Portfolio Optimization Process

The Portfolio Optimization Process can be decomposed into:

- Specification of the Portfolio Optimization problem (Objective Function and Constraints)
- Solving the Portfolio Optimization problem (Optimization Process)

In the below part, I'll cover how each of them can be refined to better approximate the reality.

4.2.1 Objective Function

Currently the objective function is defined as the weighted average ratings factor, but some other components can be added into the objective function. For example, we can add the portfolio's volatility as measured statistically by standard deviation into the objective function and make it into risk-adjusted ratings factor:

$$\max_{w_1, w_2, \ldots w_{209}} \quad \frac{-\sum_{i=1}^{209} w_i \cdot RF_i}{\sqrt{\sum_{i=1}^{209} \sum_{j=1}^{209} w_i \cdot w_j \cdot \sigma_{ij}}}$$

where σ_{ij} is the co-variance of credit i and credit j.

4.2.2 Constraints

Currently there are 8 constraints in our Portfolio Optimization problem as shown in Section 2.1.3 - we can add more constraints about other risk factors to provide information about how we want to structure the portfolio. For example, if we want to manage the interest risk of the portfolio, we can add a "Maturity Constraint" where the weighted average maturity of the portfolio cannot be later than a specific date. Also, current we only have linear constraints, but non-linear constraints about higher order characteristics (e.g., variance, skewness) are also likely to be used

4.2.3 Optimization Process

After finishing the set-up of the Portfolio Optimization problem, the next step is to solve the optimization problem and get weights. This process is normally done using an optimizer developed by Operations Research PhDs. Currently we model the problem as a **Linear Programming problem**, meaning the objective function and constraints are all linear functions of weights. However, we could model the problem as other types of optimization problems (e.g., Integer Programming, Combinatorial Optimization) when considering some practical questions before using a optimizer to get weights.

1. **Integer Programming**: Since we have minimum currency unit in reality, our feasible set is not any arbitrary real number - it should be discrete. Meanwhile, our constraints are all linear. As a result, we should specify the optimization problem as an Integer Programming problem, which is more difficult to solve than a Convex Optimization problem. By definition, Integer Programming problems have linear objective function and linear constraints, and some or all variables are constrained to take integer values.

2. **Combinatorial Optimization**: If we add non-linear constraints into the optimization problem and consider the fact that the feasible set is discrete (meaning loan par value can only be integer), the problem could be modeled as a Combinatorial Optimization problem. By definition, its objective function and constraints can be linear or non-linear, and its feasible solution set is always finite and discrete.

5 GitHub

All data and code can be found in my Github.