

## mid-term test

Translate the following sentences into propositional logic or predicate logic (15)

a. Tom can access the Gate only if he is a student or he is not a graduate.

b. Every integer has an inverse.

Translate the statement into English sentences (15) :

a.  $\forall x (\exists y (C(y) \wedge F(x, y)) \vee C(x) \neg)$

where  $C(y)$  is "y has a math book," and  $F(x, y)$  is "x and y are friends," and the domain for both x and y consists of all students in our class.

b.  $\exists s \forall b \exists e (P(s, e) \wedge Q(e, b))$

where  $P(s, e)$  be "s has understand e",  $Q(e, b)$  be "e is an equation in book b"

and the domain of s is all student, the domain of e is all equations, and the domain of b is all math books.

Put  $P \wedge Q \vee R$  into PCNF and PDNF (20分)

About set: (20)

Cardinality?  $s_1 = \{3, \{1, 2, 3, 4\}, \{\}, \emptyset, \{\{\}\}\}$ ,  $s_2 = \{x \mid x \text{ is non-negative integer, } x < 4\}$

$\forall x (x \in A \rightarrow x \in B) \wedge \exists x (x \in B \wedge x \in A) \Leftrightarrow ? \Leftrightarrow ? \Leftrightarrow ?$  (three equivalent forms)

5. a.  $h \circ f \circ g(x) = ?$ ,  $f(x) = 3x + 5$ ,  $g(x) = x^2 - 1$ ,  $h = 2x$  (10)

b. prove: if f, g are bijective then f-g is bijective. (20)

1. a:  $P$ : Tom can access the Gate

$Q$ : Tom is a student

$R$ : Tom is a graduate

$$P \rightarrow (Q \wedge \neg R)$$

b:

$$\forall x \exists y (x+y=0)$$

the domain of  $x, y$  is integer.

2. a: Every students in our class has a math book or has a friend who have a math book.

b: There is a student who understand at least one equation in every math book.

3. PCNF

$$(P \wedge Q) \vee R$$

$$\equiv (P \vee R) \wedge (Q \vee R) \equiv \underline{(P \vee R \vee Q)} \wedge \underline{(P \vee \neg R \vee R)} \wedge \underline{(Q \vee R \vee P)} \wedge \underline{(Q \vee R \vee \neg P)}$$

$$\equiv (P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (\neg P \vee Q \vee R)$$

$$\equiv \Pi (0, 1, 4)$$

$$P \text{ DNF} \equiv \Sigma (2, 3, 5, 6, 7)$$

P	Q	R	$P \wedge Q \vee R$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$$4. \mathcal{C}(S_1) = 4$$

$$S_2 = 5$$

$$b \Leftrightarrow \forall x (x \notin A \vee x \in B) \wedge \exists x (x \in B \wedge x \notin A)$$

$$\Leftrightarrow A \subseteq B \wedge \exists x (x \in B \wedge x \notin A)$$

$$\Leftrightarrow A \subseteq B \wedge \neg \forall x (x \notin B \vee x \in A)$$

$$\Leftrightarrow A \subset B$$

$$5. 2x(3(x^2-1))+5)$$