$f(0) + f'(0)z + \frac{f''(0)}{2!}z^2 + \frac{f'''(0)}{3!}z^3$

$\cos z = 1 - 0 + \frac{z^2}{2!} - 0 + \frac{z^4}{4!}$

$= 1 + \frac{z^2}{2!} + \frac{z^4}{4!}$

NO.

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$\boxed{3}$ (2) $f(z) = \frac{\cos z}{\sin z} - \frac{1}{z} = \frac{z\cos z - \sin z}{z \sin z}$

（特異点 $z = 0, n\pi$）

$z = n\pi$ は 1位の極

$$\mathrm{Res}[n\pi] = \frac{n\pi \cos n\pi - \sin n\pi}{\sin n\pi + z\cos n\pi} = \frac{n\pi}{n\pi} = 1$$

$\sin z$ を $z = 0$ まわりで展開

$$\sin z = z - \frac{z^3}{3!} + \frac{z^5}{5!} - \cdots$$

$$f(z) = \frac{z\cos z - \sin z}{z^2\left(1 - \frac{z^2}{3!} + \frac{z^4}{5!} - \cdots\right)}$$

$z = 0$ は 2位の極

$$\mathrm{Res}[0] = \lim_{z\to 0} \frac{d}{dz} z^2 f(z)$$

$$= \lim_{z\to 0} \frac{d}{dz}\left(\frac{z^2\cos z}{\sin z} - z\right)$$

$$= \lim_{z\to 0}\left\{\frac{(z^2\cos z)'\sin z - z^2\cos z(\sin z)'}{\sin^2 z} - 1\right\}$$

$$= \lim_{z\to 0}\left\{\frac{(2z\cos z + z^2(-\sin z))\sin z - z^2\cos z \cdot \cos z}{\sin^2 z} - 1\right\}$$

$$= \lim_{z\to 0}\left\{\frac{(2\cos z - z\sin z)z\sin z - z^2\cos^2 z}{\sin^2 z} - 1\right\}$$

$$= \lim_{z\to 0}\left\{\frac{(2\cos z - z\sin z)\frac{\sin z}{z} - \cos^2 z}{\left(\frac{\sin z}{z}\right)^2} - 1\right\}$$

$$= \frac{(2-0)\cdot 1 - 1^2}{1^2} - 1 \quad \left(\because \lim_{z\to 0}\frac{\sin z}{z} = 1\right)$$

$$= \frac{2-1}{1} - 1$$

$$= 0$$

(3) $\int_C f(z)\,dz$

$= 2\pi i\,(\mathrm{Res}[\pm 2\pi, \pm\pi, 0\ の和])$

$= 8\pi i$