H19

3 続き

(3) $C = C_1 + C_2 - C_3$ より

$I = I_1 + I_2 - I_3$

$R \to +\infty$ のとき

$$I = \int_0^{+\infty} \frac{1}{x^3+1}\,dx + 0 - \int_0^{+\infty} \frac{e^{\frac{2}{3}\pi i}}{x^3+1}\,dx$$

$$= \left(1 - e^{\frac{2}{3}\pi i}\right) \int_0^{+\infty} \frac{1}{x^3+1}\,dx$$

ここで $I = \frac{2}{3}\pi i\, e^{-\frac{2}{3}\pi i}$ より

$$\left(1 - e^{\frac{2}{3}\pi i}\right) \int_0^{+\infty} \frac{1}{x^3+1}\,dx = \frac{2}{3}\pi i\, e^{-\frac{2}{3}\pi i}$$

$$\int_0^{+\infty} \frac{1}{x^3+1}\,dx = \frac{\frac{2}{3}\pi i\, e^{-\frac{2}{3}\pi i}}{1 - e^{\frac{2}{3}\pi i}}$$

$$= \frac{2}{3}\pi i \frac{-\frac{1}{2} - \frac{\sqrt{3}}{2}i}{1 + \frac{1}{2} - \frac{\sqrt{3}}{2}i}$$

$$= -\frac{2}{3}\pi i \frac{\overset{2}{\cancel{4}}\sqrt{3}\, i}{\cancel{12}\ 3} = \frac{2\sqrt{3}}{9}\pi i$$