

II

$$(1) (1+x^2) \frac{dy}{dx} + y = 0$$

$$\frac{1}{y} dy = -\frac{1}{1+x^2} dx$$

兩邊積分

$$\log y = -\int \frac{1}{1+x^2} dx$$

$$x = \tan \theta \quad x \neq \pm \frac{\pi}{2}$$

$$dx = \frac{1}{\cos^2 \theta} d\theta$$

$$1+x^2 = \frac{1}{\cos^2 \theta}$$

$$\int \frac{1}{1+x^2} dx = \int d\theta$$

$$= \theta = \tan^{-1} x$$

$$\log y = -\tan^{-1} x + c$$

$$y = c e^{-\tan^{-1} x}$$

$$(2) \frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + y = e^{-x}$$

$$s^2 + 2s + 1 = 0$$

$$(s+1)^2 = 0$$

$$s = -1$$

$$y = (c_1 + c_2 x) e^{-x} + \eta$$

$$\begin{vmatrix} e^{-x} x e^{-x} \\ -e^{-x} (1-x) e^{-x} \end{vmatrix} = e^{-2x}$$

$$\eta = -e^{-x} \int \frac{x e^{-x} e^{-x}}{e^{-2x}} dx + x e^{-x} \int \frac{e^{-x} e^{-x}}{e^{-2x}} dx$$

$$= -e^{-x} \cdot \frac{1}{2} x^2 + x^2 e^{-x} = \frac{1}{2} x^2 e^{-x}$$

$$y = (c_1 + c_2 x) e^{-x} + \frac{1}{2} x^2 e^{-x}$$

$$(3) \frac{d^2 y}{dx^2} + 5 \frac{dy}{dx} + 6y = x^2$$

$$s^2 + 5s + 6 = 0$$

$$(s+2)(s+3) = 0$$

$$s = -2, -3$$

$$y = c_1 e^{-2x} + c_2 e^{-3x} + \eta$$

$$\eta = Ax^2 + Bx + C$$

$$2A + 5(2Ax + B) + 6(Ax^2 + Bx + C) = x^2$$

$$\begin{cases} 6A = 1 & A = \frac{1}{6} \\ 10A + 6B = 0 & B = -\frac{5}{18} \\ 2A + 5B + 6C = 0 & C = \frac{19}{108} \end{cases}$$

$$\eta = \frac{1}{6} x^2 - \frac{5}{18} x + \frac{19}{108}$$

$$y = c_1 e^{-2x} + c_2 e^{-3x} + \frac{1}{6} x^2 - \frac{5}{18} x + \frac{19}{108}$$