H20

問1

(1) 線上分布なので

$$V = \frac{1}{4\pi\varepsilon_0}\int_0^{2\pi}\frac{\lambda}{\sqrt{a^2+z^2}}\,a\,d\varphi$$

$$= \frac{1}{{}_2\cancel{4\pi}\varepsilon_0}\frac{\lambda}{\sqrt{a^2+z^2}}\cancel{2\pi}a$$

$$= \frac{\lambda a}{2\varepsilon_0\sqrt{a^2+z^2}} \quad [\mathrm{V}]$$

$$\boldsymbol{E} = -\mathrm{grad}\,V$$

z方向のみしかないので

$$\boldsymbol{E} = -\frac{\partial V}{\partial z}\boldsymbol{k}$$

$$= \frac{\lambda a}{2\varepsilon_0}\frac{z}{(a^2+z^2)^{\frac{3}{2}}}\boldsymbol{k} \quad [\mathrm{V/m}]$$

(2) 面上分布なので

$$V = \frac{1}{4\pi\varepsilon_0}\iint\frac{\sigma}{\sqrt{r^2+z^2}}\,dS$$

$$= \frac{1}{4\pi\varepsilon_0}\int_0^{2\pi}\int_0^{a}\frac{\sigma}{\sqrt{r^2+z^2}}\,r\,dr\,d\theta$$

$$= \frac{\sigma}{2\varepsilon_0}\left[\sqrt{r^2+z^2}\right]_0^a$$

$$= \frac{\sigma}{2\varepsilon_0}\left(\sqrt{a^2+z^2}-z\right)$$

$$\boldsymbol{E} = -\mathrm{grad}\,V$$

z方向のみしかないので

$$\boldsymbol{E} = -\frac{\partial V}{\partial z}\boldsymbol{k}$$

$$= -\frac{\sigma}{2\varepsilon_0}\left(\frac{z}{\sqrt{a^2+z^2}}-1\right)$$

$$= \frac{\sigma}{2\varepsilon_0}\left(1-\frac{z}{\sqrt{a^2+z^2}}\right)$$

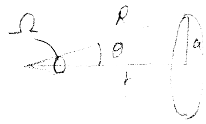
(3)

$$V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{a^2+z^2}-z\right)$$

$z=0$ なので

$$V = \frac{a}{2\varepsilon_0}\sigma$$

(4)



円板の立体角は

$\Omega = \frac{S}{R^2} = 2\pi(1-\cos\theta)$ で表わされる

$$\boldsymbol{E} = \frac{\sigma}{2\varepsilon_0}\left(1-\frac{z}{\sqrt{a^2+z^2}}\right)\boldsymbol{k}$$

$$= \frac{\sigma}{2\varepsilon_0}(1-\cos\theta)$$

$$= \frac{\sigma}{4\pi\varepsilon_0}\Omega \quad [\mathrm{V/m}]$$