② 2019 数学

問1

(1) $\frac{dy}{dx}+(\cos x)y=\sin 2x$

両辺に $e^{\int \cos x dx}$ をかけて,

$(e^{\sin x}y)'=e^{\sin x}\sin 2x$

$e^{\sin x}y=\int e^{\sin x}\sin 2x dx$

$\cancel{=e^{\sin x}\left(-\frac{\cos 2x}{2}\right)+\int \cos x\, e^{\sin x}\frac{\cos 2x}{2}dx}$

$\cancel{=\frac{e^{\sin x}\cos 2x}{2}+\frac{1}{2}\int}$

$\cancel{=\int e^{t}\, 2\sin x}$

$\int e^{\sin x}\sin 2x dx$

$=\int e^{t}\cdot 2t\,dt\Big|_{t=\sin x}$

$=2te^{t}-\int 2e^{t}dt$

$=2te^{t}-2e^{t}+C$

よって,

$e^{\sin x}y=2e^{t}(t-1)+C$

$y=2(\sin x-1)+\frac{C}{e^{\sin x}}$

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(2) $\frac{d^2y}{dx^2}+6\frac{dy}{dx}+11y=0$

$f(s)=s^2+6s+11=0$

$s=-3\pm\sqrt{9-11}=-3\pm\sqrt{2}i$

よって,

$y=e^{-3x}(C_1\cos\sqrt{2}x+C_2\sin\sqrt{2}x)$

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(3) $\frac{d^2y}{dx^2}+6\frac{dy}{dx}+11y=11x$

$y=ax+b$ とおいて,

$y'=a$, $y''=0$ より,

$6a+11(ax+b)=11x$

$11a=11$, $6a+11b=0$

$a=1$, $b=-\frac{6}{11}$

よって,

$y=e^{-3x}(C_1\cos\sqrt{2}x+C_2\sin\sqrt{2}x)+x-\frac{6}{11}$

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(4) $\frac{d^2y}{dx^2}+6\frac{dy}{dx}+11y=\sin x$

$y=a\cos x+b\sin x$ とおいて,

$y'=b\cos x-a\sin x$

$y''=-a\cos x-b\sin x$

より,

$(10a+6b)\cos x+(10b-6a)\sin x=\sin x$

$10a+6b=0$, $10b-6a=1$

$b=-\frac{5}{3}a$

$-\frac{50}{3}a-6a=1$

$-\frac{68}{3}a=1$

$a=-\frac{3}{68}$

$b=\frac{5}{68}$

よって,

$y=e^{-3x}(C_1\cos\sqrt{2}x+C_2\sin\sqrt{2}x)$
$-\frac{3}{68}\cos x+\frac{5}{68}\sin x$

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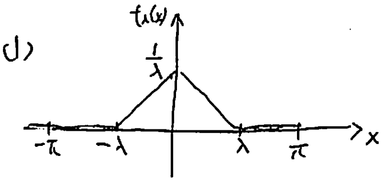
$$f_\lambda(x)=\begin{cases}0 & (-\pi\le x<-\lambda)\\ \frac{x}{\lambda^2}+\frac{1}{\lambda} & (-\lambda\le x<0)\\ -\frac{x}{\lambda^2}+\frac{1}{\lambda} & (0\le x<\lambda)\\ 0 & (\lambda\le x<\pi)\end{cases}$$

(1)



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(2) $\cancel{f_\lambda(-x)=\begin{cases}\frac{x}{\lambda^2}+\frac{1}{\lambda} & (-\lambda\le x<0)\\ \frac{x}{\lambda^2}+\frac{1}{\lambda} & (0\le x<\lambda)\end{cases}}$

~~また~~, (1) より,

$\cancel{f_\lambda(-x)=-f_\lambda(x)}$ 偶

$f_\lambda(x)$ は~~奇~~偶関数で $b_n(x)=0$

$a_0=\frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}f_\lambda(x)dx$

$=\frac{2}{\pi}\int_0^{\pi}f_\lambda(x)dx$

$=\frac{2}{\pi}\int_0^{\lambda}\left(-\frac{x}{\lambda^2}+\frac{1}{\lambda}\right)dx$

$=\frac{2}{\pi}\left[-\frac{x^2}{2\lambda^2}+\frac{x}{\lambda}\right]_0^{\lambda}$

$=\frac{2}{\pi}\left(-\frac{1}{2}+1\right)$

$=\frac{1}{\pi}$

$a_n=\frac{2}{\pi}\int_0^{\pi}f_\lambda(x)\cos\frac{2\pi nx}{T}dx$

$=\frac{2}{\pi}\int_0^{\lambda}\left(-\frac{x}{\lambda^2}+\frac{1}{\lambda}\right)\cos nx\,dx$

$=\frac{2}{\pi}\left\{\left[\frac{1}{n}\left(-\frac{x}{\lambda^2}+\frac{1}{\lambda}\right)\sin nx\right]_0^{\lambda}-\int_0^{\lambda}\frac{1}{n}\left(-\frac{1}{\lambda^2}\right)\sin nx\,dx\right\}$

$=\frac{2}{\pi}\frac{1}{n}\left[\left(-\frac{x}{\lambda^2}+\frac{1}{\lambda}\right)\sin nx-\frac{1}{n\lambda^2}\cos nx\right]_0^{\lambda}$

$=\frac{2}{n\pi}\left(-\frac{1}{n\lambda^2}\cos n\lambda+\frac{1}{n\lambda^2}\right)$

$=\frac{2}{n^2\lambda^2\pi}(1-\cos n\lambda)$

$(n\neq 0)$

よって,

$f_\lambda(x)\sim\frac{1}{2\pi}+\sum_{n=1}^{\infty}\frac{2}{\pi n^2\lambda^2}(1-\cos n\lambda)\cos nx$

(3)

$a_n(\lambda)=\frac{2}{\pi}\cdot\frac{1}{n^2\lambda^2}\cdot 2\cdot\frac{1-\cos n\lambda}{2}$

$=\frac{4}{\pi}\cdot\frac{1}{n^2\lambda^2}\cdot\sin^2\frac{n\lambda}{2}$

$=\frac{1}{\pi}\cdot\frac{\sin^2\frac{n\lambda}{2}}{\left(\frac{n\lambda}{2}\right)^2}$

$=\frac{1}{\pi}$