H26

[2]

(1)

$$V:\ x^2+y^2+\frac{z^2}{4}\le 1$$

$$x^2+y^2\le 1-\frac{z^2}{4}$$

半径が $\sqrt{1-\frac{z^2}{4}}$ の円 ← この円の面積は $\pi\left(\sqrt{1-\frac{z^2}{4}}\right)^2=\pi\left(1-\frac{z^2}{4}\right)$

$$1-\frac{z^2}{4}>0$$ ← ルートの中だから

$$-2<z<2$$

偶関数

$$V=\int_{-2}^{2}\pi\left(1-\frac{z^2}{4}\right)dz$$

$$=2\int_0^{2}\pi\left(1-\frac{z^2}{4}\right)dz$$

$$=2\pi\left[z-\frac{1}{12}z^3\right]_0^2=\frac{8}{3}\pi$$

(2)

$$S:\ x^2+y^2+z^2=1$$

$$z=2\sqrt{1-x^2-y^2}$$

$$\frac{\partial z}{\partial x}=-2x(1-x^2-y^2)^{-\frac{1}{2}}$$

$$\frac{\partial z}{\partial y}=-2y(1-x^2-y^2)^{-\frac{1}{2}}$$

$$n=\frac{\left(-\frac{\partial z}{\partial x},\ -\frac{\partial z}{\partial y},\ 1\right)}{\sqrt{\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2+1}}$$

$$=\left(2x,\ 2y,\ \sqrt{1-x^2-y^2}\right)\frac{1}{\sqrt{3x^2+3y^2+1}}$$

$$A\cdot n=\frac{1}{\sqrt{3x^2+3y^2+1}}\left(2x,\ 2y,\ \sqrt{1-x^2-y^2}\right)\left(x,\ y,\ \frac{z}{4}\right)$$

$$=\frac{1}{\sqrt{3x^2+3y^2+1}}\left\{2x^2+2y^2+\frac{1}{2}(1-x^2-y^2)\right\}$$

$$=\frac{3x^2+3y^2+1}{2\sqrt{3x^2+3y^2+1}}=\frac{1}{2}\sqrt{3x^2+3y^2+1}$$