H18

②

(1)

$a \times r = (2z - 3y,\ 3x - z,\ y - 2x)$

$\mathrm{rot}(a \times r) = (1+1,\ 2+2,\ 3+3)$

$= (2,\ 4,\ 6)$

(2)

$\int_C (a \times r)\, dr = \int_0^{2\pi} (a \times r) \frac{dr}{dt} dt$

ここで

$(a \times r) = (2c - 3\sqrt{2-c}\sin t,\ 3\sqrt{2-c}\cos t - c,\ 3\sqrt{2-c}(\sin t - 2\cos t))$

$\frac{dr}{dt} = (-\sqrt{2-c}\sin t,\ \sqrt{2-c}\cos t,\ 0)$

より

$\int_0^{2\pi} (a \times r) \frac{dr}{dt} dt = \int_0^{2\pi} \left\{ 3(2-c) - c\sqrt{2-c}(2\sin t + \cos t) \right\} dt$

$= \left[3(2-c)t - c\sqrt{2-c}(-2\cos t + \sin t) \right]_0^{2\pi}$

$= 6(2-c)\pi + 2c\sqrt{2-c} - (2c\sqrt{2-c})$

$= 6(2-c)\pi$

(3)

ストークスの定理より

$\iint_S \mathrm{rot}(a \times r) \cdot n\, dS = \oint (a \times r)\, dr$

$= 6(2-c)\pi$