H23

[1]

(1) $y\frac{dy}{dx} = e^{y^2+x}$

$y e^{-y^2} dy = e^x dx$

$-\frac{1}{2}e^{-y^2} = e^x + C$

(2) $xy' + y = x\log x$

$(xy)' = x\log x$

$xy = \int x\log x\, dx$

$= \frac{1}{2}x^2\log x - \frac{1}{4}x^2 + C$

$y = \frac{1}{2}x\log x - \frac{1}{4}x + Cx^{-1}$

$= \frac{1}{4}x(2\log x - 1) + Cx^{-1}$

(3) $y'' + y' - 2y = x$

$s^2 + s - 2 = 0$

$(s+2)(s-1) = 0$

$s = 1, -2$

$\eta = Ax + B$

$A - 2Ax - 2B = x$

$\begin{cases} -2A = 1 & A = -\frac{1}{2} \\ A - 2B = 0 & B = -\frac{1}{4} \end{cases}$

$\eta = -\frac{1}{2}x - \frac{1}{4}$

$y = C_1 e^x + C_2 e^{-2x} - \frac{1}{2}x - \frac{1}{4}$

(4) $s^2 + s - 2 = 0$

$s = 1, -2$

$\eta_1 = Ax + B$

$\eta_2 = cxe^x$

$A - 2Ax - 2B = 4x$

$c(x+2)e^x + c(x+1)e^x - 2cxe^x = 3e^x$

$\begin{cases} -2A = 4 & A = -2 \\ A - 2B = 0 & B = -1 \\ 2c + c = 3 & c = 1 \end{cases}$

$\eta_1 = -2x - 1$

$\eta_2 = xe^x$

$\eta = \eta_1 + \eta_2 = xe^x - 2x - 1$

$y = C_1 e^x + C_2 e^{-2x} + xe^x - 2x - 1$