H24

[2]

(1)

$$\mathrm{grad}\,\phi^2 = \left(\frac{\partial \phi^2}{\partial x}, \frac{\partial \phi^2}{\partial y}, \frac{\partial \phi^2}{\partial z}\right)$$

ここで

$$\frac{\partial}{\partial x}(\phi A) = \phi \frac{\partial A}{\partial x} + A \frac{\partial \phi}{\partial x} \quad より$$

$$\frac{\partial \phi^2}{\partial x} = \phi \frac{\partial \phi}{\partial x} + \phi \frac{\partial \phi}{\partial x} = 2\phi \frac{\partial \phi}{\partial x}$$

したがって

$$\mathrm{grad}\,\phi^2 = \left(2\phi \frac{\partial \phi}{\partial x}, 2\phi \frac{\partial \phi}{\partial y}, 2\phi \frac{\partial \phi}{\partial z}\right)$$

$$= 2\phi \left(\frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}, \frac{\partial \phi}{\partial z}\right)$$

$$= 2\phi\, \mathrm{grad}\,\phi$$

(2)

$$\phi = xy - z \rightarrow \phi^2 = x^2y^2 - 2xyz + z^2$$

$$\mathrm{grad}\,\phi^2 = \left(2xy^2 - 2yz, 2x^2y - 2xz, 2z - 2xy\right)$$

(i)

(1, 0, 0), (3, 2, 2) を通る直線のベクトル方程式は

パラメータ t を用いて

$$r(t) = (1, 0, 0) - t(2, 2, 2)$$

$$= (2t+1, 2t, 2t)$$

$$\mathrm{grad}\,\phi^2_{(t)} = \left(2(2t+1)(2t)^2 - 2\cdot 2t \cdot 2t,\ 2(2t+1)^2 2t - 2(2t+1)(2t),\ 2t - 2(2t+1)2t\right)$$

$$= \left(16t^3, 16t^3 + 8t^2, -8t^2\right) = 8\left(2t^3, 2t^3 + t^2, -t^2\right)$$

$$\int_C \phi\, \mathrm{grad}\,\phi\, dr = \frac{1}{2}\int_0^1 \mathrm{grad}\,\phi^2_{(t)} \cdot \frac{dr}{dt}\, dt$$

$$= 4\int_0^1 \left(2t^3, 2t^3 + t^2, -t^2\right)(2, 2, 2)\, dt$$

$$= 8\int_0^1 4t^3\, dt$$

$$= 8\left[t^4\right]_0^1 = 8$$