

[2]

(1)

$$\text{rot } u = (0, 0, 2)$$

$$\begin{aligned} \text{div } v &= (2x+1+x+1+2z-3x) \\ &= 2z \end{aligned}$$

(2)

$$\begin{aligned} \frac{1}{2} \int_C u \cdot dr &= \frac{1}{2} \int_0^{2\pi} (-b \sin t, a \cos t, 0) \frac{dr}{dt} dt \\ &= \frac{1}{2} \int_0^{2\pi} (-b \sin t, a \cos t, 0) \cdot (-a \sin t, b \cos t, 0) dt \\ &= \frac{1}{2} \int_0^{2\pi} (ab \sin^2 t + ab \cos^2 t) dt \\ &= \frac{1}{2} ab \cdot 2\pi = ab\pi \end{aligned}$$

(3)

ガウスの発散定理より

$$\begin{aligned} \iint_S v \cdot n \, d\sigma &= \iiint_V \text{div } v \, dV \\ &= \iiint_V 2z \, dV \\ &\Rightarrow ab\pi \int_0^c 2z \, dz \\ &= abc^2\pi \end{aligned}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

の面積