H17

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(1) $y' - \frac{1}{x}y = x\log x$

積分因子は

$$\int -\frac{1}{x}\,dx = -\log x$$

$$e^{-\log x} = \frac{1}{x}$$

両辺に $\frac{1}{x}$ をかけて

$$\left(\frac{1}{x}y\right)' = \log x$$

$$\frac{1}{x}y = x\log x - x + c$$

$$y = x^2(\log x - 1) + cx$$

(2) $s^2 + 2s + 3 = 0$

$s = 1 \pm \sqrt{2}i$

$$y = e^x\left(c_1\cos\sqrt{2}x + c_2\sin\sqrt{2}x\right)$$

(3) $s^2 + 2s + 3 = 0$

$s = 1 \pm \sqrt{2}i$

$$\eta = A\cos 2x + B\sin 2x$$

$$\eta' = -2A\sin 2x + 2B\cos 2x$$

$$\eta'' = -4A\cos 2x - 4B\sin 2x$$

$$(-4A + 4B + 3A)\cos 2x + (-4B - 4A + 3B)\sin 2x = \cos 2x$$

$$\begin{cases} 4B - A = 1 & A = -\frac{1}{17} \\ -B - 4A = 0 \rightarrow B = -4A & B = \frac{4}{17} \end{cases}$$

$$\eta = -\frac{1}{17}\cos 2x + \frac{4}{17}\sin 2x$$

$$y = e^x\left(c_1\cos\sqrt{2}x + c_2\sin\sqrt{2}x\right) - \frac{1}{17}(\cos 2x - 4\sin 2x)$$