H27

2

(1)

$$\mathrm{div}\, A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$= y - z + z - x + x - y = 0$$

$$\mathrm{rot}\, A = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z},\ \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x},\ \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$

$$= \left(-z + 2y - y,\ -x + 2z - z,\ -y + 2x - x \right)$$

$$= (y - z,\ z - x,\ x - y)$$

(2)

$S : x^2 + y^2 + z^2 = 1$

$$z = \sqrt{1 - x^2 - y^2}$$

$$\frac{\partial z}{\partial x} = -x (1 - x^2 - y^2)^{-\frac{1}{2}}$$

$$\frac{\partial z}{\partial y} = -y (1 - x^2 - y^2)^{-\frac{1}{2}}$$

$z > 0$ なので

$$n = \frac{\left(-\frac{\partial z}{\partial x},\ -\frac{\partial z}{\partial y},\ 1 \right)}{\sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}}$$

$$= \left(x,\ y,\ \sqrt{1 - x^2 - y^2} \right)$$

$D : x^2 + y^2 \leq 1$

$$n = (0,\ 0,\ 1)$$