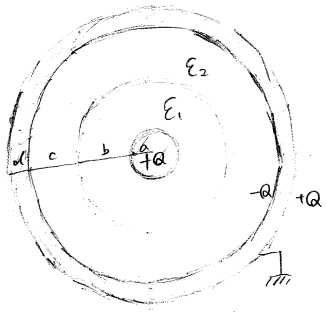
H22

問1.

(1)



内側壁面に $-Q$ [C] の電荷が誘起される

外側壁面に 0 [C]

(2)

(i) $0 \leq r \leq a$

閉曲面内に電荷は存在しないので

$$E_1 = 0$$

(ii) $a \leq r \leq b$

$$\int E_2 \, dr = \frac{Q}{\varepsilon_1}$$

$$E_2 = \frac{Q}{4\pi\varepsilon_1 r^2}$$

(iii) $b \leq r \leq c$

$$\int E_3 \, dr = \frac{Q}{\varepsilon_2}$$

$$E_3 = \frac{Q}{4\pi\varepsilon_2 r^2}$$

(iv) $c \leq r \leq d$

閉曲面内の電荷は $Q - Q$ で 0 なので

$$E_4 = 0$$

(3)

(i) $c \leq r \leq d$

積分しやすい

$$\phi_{4(r)} = -\int_{\infty}^{d} E \, dr - \int_{d}^{r} E_4 \, dr$$

$$= 0 \text{ [V]}$$

(ii) $b \leq r \leq c$

$$\phi_{3(r)} = \phi_{4(c)} - \int_{c}^{r} E_3 \, dr$$

$$= 0 + \frac{Q}{4\pi\varepsilon_2 r} - \frac{Q}{4\pi\varepsilon_2 c} = \frac{Q}{4\pi\varepsilon_2}\left(\frac{1}{r} - \frac{1}{c}\right) \text{ [V]}$$

(iii) $a \leq r \leq b$

$$\phi_{2(r)} = \phi_{3(b)} - \int_{b}^{r} E_2 \, dr$$

$$= \frac{Q}{4\pi\varepsilon_2}\left(\frac{1}{b} - \frac{1}{c}\right) + \frac{Q}{4\pi\varepsilon_1}\left(\frac{1}{r} - \frac{1}{b}\right) \text{ [V]}$$

(iv) $0 \leq r \leq a$

$$\phi_{1(r)} = \phi_{2(a)} - \int_{a}^{r} E_1 \, dr$$

$$= \frac{Q}{4\pi\varepsilon_2}\left(\frac{1}{b} - \frac{1}{c}\right) + \frac{Q}{4\pi\varepsilon_1}\left(\frac{1}{a} - \frac{1}{b}\right) \text{ [V]}$$

(4) $Q = CV$ より

$$C = \frac{Q}{V}$$

ここで導体球と同心導体球殻の間の電位差は

$$\phi_{2(a)} + \phi_{3(b)} = \frac{Q}{4\pi\varepsilon_2}\left(\frac{1}{b} - \frac{1}{c}\right) + \frac{Q}{4\pi\varepsilon_1}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$C = \frac{4\pi\varepsilon_1\varepsilon_2}{\varepsilon_1\left(\frac{1}{b} - \frac{1}{c}\right) + \varepsilon_2\left(\frac{1}{a} - \frac{1}{b}\right)} \text{ [F]}$$