①

(1) $xy'' + y' = 0$

$(xy')' = 0$

$xy' = C_1$

$dy = \frac{C_1}{x} dx$

$y = C_1 \log x + C_2$

(2) $s^2 + 1 = 0$

$s = \pm i$

$a \neq \pm 1 \alpha z$

$\eta = A \cos \alpha x + B \sin \alpha x$

$(A - a^2 A) \cos \alpha x + (B - a^2 B) \sin \alpha x = \cos \alpha x$

$A = \frac{1}{1 - a^2}, \quad B = 0$

$\eta = \frac{1}{1 - a^2} \cos \alpha x$

$y = C_1 \cos x + C_2 \sin x + \frac{1}{1 - a^2} \cos \alpha x$

$a = 1 \alpha z$

$\eta = x (A \cos x + B \sin x)$

$2B \cos x - 2A \sin x = \cos x$

$B = \frac{1}{2}, \quad A = 0$

$\eta = x (\frac{1}{2} \sin x)$

$y = C_1 \cos x + C_2 \sin x + \frac{1}{2} x \sin x$

$a = -1 \alpha z$

$2B \cos x - 2A \sin x = -\cos x$

$B = -\frac{1}{2}, \quad A = 0$

$\eta = -\frac{1}{2} x \sin x$

$y = C_1 \cos x + C_2 \sin x - \frac{1}{2} x \sin x$

(3) $(2y - \sin x) \varphi_{(x)} dx + x \varphi_{(x)} dy = 0$

$\frac{d}{dy} (2y - \sin x) \varphi_{(x)} = 2 \varphi_{(x)}$

$\frac{d}{dx} x \varphi_{(x)} = \varphi_{(x)} + x \varphi'_{(x)}$

$2 \varphi_{(x)} = \varphi_{(x)} + x \varphi'_{(x)}$

$\varphi'_{(x)} - \frac{1}{x} \varphi_{(x)} = 0$

$(\frac{1}{x} \varphi_{(x)})' = 0$

$\frac{1}{x} \varphi_{(x)} = C$

$\varphi_{(x)} = Cx$

$(2y - \sin x) Cx \, dx + Cx^2 dy = 0$

$d(Cx^2 y + C(x \cos x - \sin x)) = 0$

$C\{x^2 y + x \cos x - \sin x\} = C_2$

$x^2 y + x \cos x - \sin x = C$

$y = -x^{-1} \cos x + x^{-2} \sin x + C x^{-2}$