H24

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(1) $f(z) = \dfrac{1}{(z^2+4)(z^2-2z+2)}$

$= \dfrac{1}{(z+2i)(z-2i)(z-1+i)(z-1-i)}$

特異点は $z = \pm 2i,\ 1 \pm i$

$z = \alpha$ $(\alpha = \pm 2i,\ 1 \pm i)$ のとき

$f(z)(z-\alpha)$ は正則である

$\mathrm{Res}[2i] = \lim_{z \to 2i} \dfrac{1}{(z+2i)(z-1+i)(z-1-i)}$

$= \dfrac{1}{4i(-1+3i)(-1+i)}$

$= \dfrac{1}{16-8i} = \dfrac{1}{8(2-i)}$

$\mathrm{Res}[-2i] = \lim_{z \to -2i} \dfrac{1}{(z-2i)(z-1+i)(z-1-i)}$

$= \dfrac{1}{-4i(-1-i)(-1-3i)}$

$= \dfrac{1}{16+8i} = \dfrac{1}{8(2+i)}$

$\mathrm{Res}[1+i] = \lim_{z \to 1+i} \dfrac{1}{(z-2i)(z+2i)(z-1+i)}$

$= \dfrac{1}{(1-i)(1+3i)2i}$

$= \dfrac{1}{-4+8i} = -\dfrac{1}{4(1-2i)}$

$\mathrm{Res}[1-i] = \lim_{z \to 1-i} \dfrac{1}{(z-2i)(z+2i)(z-1-i)}$

$= \dfrac{1}{(1-3i)(1+i)(-2i)}$

$= \dfrac{1}{-4-8i} = -\dfrac{1}{4(1+2i)}$