2016 電気回路

問1

(1) $\dot{V}_1 = \cancel{L\frac{di}{dt}} + j\omega L \dot{I}_1 + \frac{\dot{I}_1 + \dot{I}_2}{j\omega C}$

$\dot{V}_2 = \frac{\dot{I}_1 + \dot{I}_2}{j\omega C}$

より。

$\dot{Z}_{11} = j\omega L + \frac{1}{j\omega C}$

$\dot{Z}_{12} = \frac{1}{j\omega C} = \dot{Z}_{21} = \dot{Z}_{22}$

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(2) $\cancel{= R\dot{I}_1}$ $\dot{I}_2 = 0$ より。

$\dot{V}_2 = \frac{\frac{1}{j\omega C}}{(R + j\omega L) + \frac{1}{j\omega C}} E$

$\cancel{\frac{\frac{E}{\omega C}}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} e^{j(-\frac{\pi}{2} - j\tan^{-1}\frac{\omega L - \frac{1}{\omega C}}{R})}}$

$= \frac{E}{1 - \omega^2 LC + j\omega CR}$

$= \frac{E}{\sqrt{(1 - \omega^2 LC)^2 + \omega^2 C^2 R^2}} e^{-j\tan^{-1}\frac{\omega CR}{1 - \omega^2 LC}}$

$f(\omega) = (1 - \omega^2 LC)^2 + \omega^2 C^2 R^2$ とおくと。

$f'(\omega) = 2(1 - \omega^2 LC) \cdot 2\omega(-LC) + 2\omega C^2 R^2$

$= 0$

となるとき、$2LC(1 - \omega^2 LC) = C^2 R^2$

$\cancel{2 - 2\omega^2 LC = -C^2 R^2}$

$\cancel{\omega^2 = \frac{1}{2LC}(C^2R^2 + 2) = \frac{CR^2}{2L} + \frac{1}{LC}}$

$1 - \omega^2 LC = \frac{CR^2}{2L}$

$\omega^2 = \sqrt{\frac{1}{LC}\left(1 - \frac{CR^2}{2L}\right)}$, $\omega = 0$

$\cancel{f''(\omega) = (-4\omega LC(1 - \omega^2 LC) + 2\omega C^2 R^2)'}$

$\cancel{= -4LC + 12\omega^2 L^2 C^2 + 2C^2 R^2}$

$\cancel{= 0}$

$\cancel{\text{のとき、}}$

$\cancel{6\omega^2 L^2 C^2 = 2LC - C^2 R^2}$

$\cancel{\omega^2 = \sqrt{\frac{1}{6L^2C^2}(2LC - C^2R^2)}}$

$\cancel{= \sqrt{\frac{1}{6L^2C}(2L - CR^2)}}$

$\cancel{= \sqrt{\frac{1}{3LC}\left(1 - \frac{CR^2}{2L}\right)}}$

$f(0) = 1$

$f\left(\sqrt{\frac{1}{LC}\left(1 - \frac{CR^2}{2L}\right)}\right) = \left(1 - \left(1 - \frac{CR^2}{2L}\right)\right)^2 + \frac{CR^2}{L}\left(1 - \frac{CR^2}{2L}\right)$

$= \frac{C^2R^4}{4L^2} + \frac{CR^2}{L} - \frac{C^2R^4}{2L^2}$

$= \frac{CR^2}{L} - \frac{C^2R^4}{4L^2}$

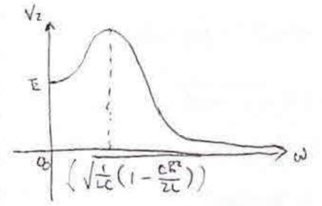
$= \frac{CR^2}{L}\left(1 - \frac{CR^2}{4L}\right)$

$= \frac{4LCR^2 - C^2R^4}{4L^2}$

$\lim_{\omega \to \infty} V_2 = 0$

$V_2(0) = E$

より。

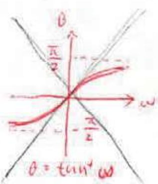


$\phi(0) = \lim_{\omega \to 0} \phi = 0$

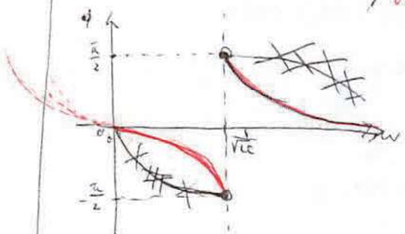
$\omega = \frac{1}{\sqrt{LC}}$ のとき、

$\lim_{\omega \to \frac{1}{\sqrt{LC}}+0} \phi = \frac{\pi}{2}$

$\lim_{\omega \to \frac{1}{\sqrt{LC}}-0} \phi = -\frac{\pi}{2}$



より。



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(3) $\frac{\dot{I}_1 + \dot{I}_2}{j\omega C} = -R\dot{I}_2 - j\omega L\dot{I}_2$

$\dot{I}_2\left(\frac{1}{j\omega C} + R + j\omega L\right) = -\frac{\dot{I}_1}{j\omega C}$

$\dot{I}_2 = -\frac{1}{j\omega C} \cdot \frac{1}{R + j(\omega L - \frac{1}{\omega C})}\dot{I}_1$

だから、(1) より。

$\dot{V}_1 = j\left(\omega L - \frac{1}{\omega C}\right)\dot{I}_1 + \frac{1}{\omega^2 C^2}\frac{\dot{I}_1}{R + j(\omega L - \frac{1}{\omega C})}$

$\dot{V}_2 = \frac{\dot{I}_1}{j\omega C} + \frac{1}{\omega^2 C^2}\frac{\dot{I}_1}{R + j(\omega L - \frac{1}{\omega C})}$

$\frac{\dot{V}_2}{\dot{V}_1} = \frac{\frac{1}{j\omega C} + \frac{1}{\omega^2 C^2}\frac{1}{R + j(\omega L - \frac{1}{\omega C})}}{j(\omega L - \frac{1}{\omega C}) + \frac{1}{\omega^2 C^2}\frac{1}{R + j(\omega L - \frac{1}{\omega C})}}$

$= \frac{\frac{1}{\omega^2 C^2} + \frac{1}{\omega C}(\omega L - \frac{1}{\omega C}) - j\frac{R}{\omega C}}{\frac{1}{\omega^2 C^2} - (\omega L - \frac{1}{\omega C})^2 + jR(\omega L - \frac{1}{\omega C})}$

$= \frac{\frac{1}{\omega C} + (\omega L - \frac{1}{\omega C}) - jR}{\frac{1}{\omega C} - \omega C(\omega L - \frac{1}{\omega C})^2 + jR(\omega^2 LC - 1)}$

$\omega = \frac{1}{\sqrt{LC}}$ のとき、

$\frac{\dot{V}_2}{\dot{V}_1} = \frac{\omega L - jR}{\cancel{\frac{1}{\omega C}} - \omega^3 CL^2 + 2\omega L + jR(\omega^2 LC - 1)}$

$= \frac{\sqrt{\frac{L}{C}} - jR}{-\frac{CL^2}{LC\sqrt{LC}} + 2\sqrt{\frac{L}{C}}}$

$= \frac{\sqrt{\frac{L}{C}} - jR}{2\sqrt{\frac{L}{C}} - \sqrt{\frac{L}{C}}}$

$= \frac{\sqrt{\frac{L}{C}} - jR}{\sqrt{\frac{L}{C}}}$

$= \sqrt{\frac{\frac{L}{C} + R^2}{\frac{L}{C}}} e^{-j\tan^{-1}\frac{R}{\sqrt{\frac{L}{C}}}}$

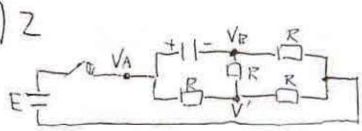
$\tan^{-1}\frac{R}{\sqrt{\frac{L}{C}}} = \frac{\pi}{4}$

$\Rightarrow R = \sqrt{\frac{L}{C}}$

$\left|\frac{\dot{V}_2}{\dot{V}_1}\right| = \sqrt{2}$ としても $R = \sqrt{\frac{L}{C}}$

問2

(1)



直流だから、コンデンサは開放し、

$V_A = \frac{R}{R + \frac{2R^2}{2R+R}}E = \frac{3}{5}E$

$V' = \frac{\frac{2}{3}R}{R + \frac{2}{3}R}E = \frac{2}{5}E$

$V_B = \frac{1}{2}V' = \frac{1}{5}E$

よって、コンデンサの両端電圧 V_{AB} は、

$V_{AB} = V_A - V_B = \frac{2}{5}E$

よって、

$Q = CV_{AB} = \frac{2}{5}CE$

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(2) $U = \frac{1}{2}CV_{AB}^2 = \frac{2}{25}CE^2$

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(3) $\begin{cases} \frac{\int i_1 dt}{C} = -Ri_1 + R(-i_1 + i_2) & \cdots ① \\ 2Ri_2 + R(i_2 - i_1) = 0 & \cdots ② \end{cases}$

② より、$i_2 = \frac{i_1}{3}$

① ~~両辺~~ のラプラス変換は、

$\frac{1}{CS}(I_1(S) + Q) = -\frac{5}{3}RI_1(S)$

$\Rightarrow I_1(S) = -\frac{Q}{CS} \cdot \frac{1}{\frac{1}{CS} + \frac{5}{3}R}$

$= -\frac{Q}{1 + \frac{5}{3}RCS}$

$= -\frac{Q}{\frac{3}{5RC} + S} \cdot \frac{3}{5RC}$

よって、

$i_1(t) = -\frac{3Q}{5RC}e^{-\frac{3}{5RC}t}$

$= -\frac{6E}{25R}e^{-\frac{3}{5RC}t}$

$i_2(t) = \frac{i_1(t)}{3}$

$= -\frac{2E}{25R}e^{-\frac{3}{5RC}t}$

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(4) 電力量 P は、

$P = \int_0^\infty (Ri_1^2 + 2Ri_2^2 + R(i_1 - i_2)^2)dt$

$= \frac{E^2}{625R}\int_0^\infty (36 + 8 + 16)e^{-\frac{3}{5RC}t}dt$

$= \frac{12E^2}{125R}\left[-\frac{5RC}{3}e^{-\frac{3}{5RC}t}\right]_0^\infty$

$= \frac{4CE^2}{25}$

? (2) と違う

2016 電気磁気学

問1

(1) 真空中のガウスの法則より，

$$\int_S \mathbb{E}\, d\mathbb{S} = \frac{1}{\varepsilon_0}\int_V \rho(r)\, dV \quad \cdots ①$$

（S：面積，V：体積）

電界と電位の関係式は，

$$\phi = -\int_C \mathbb{E}\, d\mathbb{r} \quad \cdots ②$$

②の両辺をrで微分して，

$$\nabla\phi = -\mathbb{E} \quad \cdots ③$$

さらに微分して，

$$\nabla^2\phi = -\mathrm{div}\,\mathbb{E} \quad \cdots ④$$

ここで，①の左辺は，

$$\int_S \mathbb{E}\, d\mathbb{S} = \int_V \mathrm{div}\,\mathbb{E}\, dV \quad \cdots ⑤$$

だから，①，⑤より，

$$\mathrm{div}\,\mathbb{E} = \frac{\rho(r)}{\varepsilon_0} \quad \cdots ⑥$$

以上から，④，⑥より，

$$\nabla^2\phi = -\frac{\rho(r)}{\varepsilon_0}$$

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(2)
$$\nabla^2\phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{-2Qr}{8\pi\varepsilon_0 a^3}\right)$$
$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{-2Qr^3}{8\pi\varepsilon_0 a^3}\right)$$
$$= \frac{-2Q}{8\pi\varepsilon_0 a^3 r^2}\cdot 3r^2$$
$$= -\frac{6Q}{8\pi\varepsilon_0 a^3}$$
$$= -\frac{3Q}{4\pi\varepsilon_0 a^3}$$
$$= -\frac{\rho(r)}{\varepsilon_0}$$

よって，

$$\rho(r) = \frac{3Q}{4\pi a^3}$$

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(3) 全電荷量 q は，

$$q = \int_V \rho(r)\, dV$$
$$= \rho(r)\cdot\frac{4}{3}\pi a^3$$
$$= Q$$

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(4) ガウスの法則より，

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$\phi = -\int_\infty^r E\, dr$$
$$= -\frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_\infty^r$$
$$= \frac{Q}{4\pi\varepsilon_0 r}$$

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(5) 求めるエネルギーをUとして，

$$U = \int_V \frac{1}{2}\varepsilon_0 E^2 dV$$
$$= \frac{\varepsilon_0}{2}\int_a^\infty \frac{Q^2}{16\pi^2\varepsilon_0^2 r^4}\cdot 4\pi r^2 dr$$
$$= \frac{Q^2}{8\pi\varepsilon_0}\int_a^\infty \frac{1}{r^2}dr$$
$$= -\frac{Q^2}{8\pi\varepsilon_0}\left[\frac{1}{r}\right]_a^\infty$$
$$= \frac{Q^2}{8\pi\varepsilon_0 a}$$

問2

(1) アンペアの法則より，

$$\mathbb{H} = \frac{I_1}{2\pi(d + r\cos\theta)}\,\mathbf{j}$$

（$\mathbf{j}$：y方向単位ベクトル）

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(2) 円形コイルの微小面積dSは，

$$dS = dr\cdot r d\theta = r dr d\theta$$

だから，微小磁束 $d\Phi$ は，

$$d\Phi = \mu_0|\mathbb{H}|\, dS$$
$$= \frac{\mu_0 I_1 r dr d\theta}{2\pi(d + r\cos\theta)}$$

よって，

$$\Phi = \int_{r=0}^{a}\int_{\theta=0}^{2\pi}\frac{\mu_0 I_1 r dr d\theta}{2\pi(d + r\cos\theta)}$$
$$= \int_{r=0}^{a}\frac{\mu_0 I_1 r}{2\pi}\cdot\frac{2\pi}{\sqrt{d^2 - r^2}}dr$$
$$= \int_0^a \frac{\mu_0 I_1 r}{\sqrt{d^2 - r^2}}dr$$
$$= \int_0^{a^2}\frac{\mu_0 I_1}{2}\cdot\frac{dt}{\sqrt{d^2 - t}}$$
$$= \frac{\mu_0 I_1}{2}\cdot 2\left[(d^2 - t)^{\frac{1}{2}}(-1)\right]_0^{a^2}$$
$$= -\mu_0 I_1\left(\sqrt{d^2 - a^2} - d\right)$$
$$= \mu_0 I_1\left(d - \sqrt{d^2 - a^2}\right)$$

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(3) 相互インダクタンスMは，

$$M = \frac{\Phi}{I_1}$$
$$= \mu_0\left(d - \sqrt{d^2 - a^2}\right)$$

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(4) 求める力をF，dが大きくなる方向を正とおくと，

$$F = I_1 I_2 \frac{\partial M}{\partial d}$$
$$= I_1 I_2 \mu_0\left(1 - \frac{1}{2}(d^2 - a^2)^{-\frac{1}{2}}\cdot 2d\right)$$
$$= \mu_0 I_1 I_2\left(1 - \frac{d}{\sqrt{d^2 - a^2}}\right)$$

ここで，

$$1 < \frac{d}{\sqrt{d^2 - a^2}}$$

より，

$$F < 0$$

よって，

$$|F| = \mu_0 I_1 I_2\left(\frac{d}{\sqrt{d^2 - a^2}} - 1\right),$$

dが縮まる方向である。

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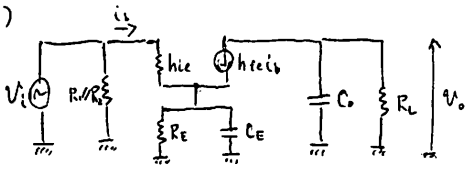
(5) 求める電圧Vは，

$$V = -M\frac{dI_1}{dt}$$
$$= \mu_0\left(\sqrt{d^2 - a^2} - d\right)\cdot\omega I_0\cos\omega t$$
$$= \omega\mu_0 I_0\left(\sqrt{d^2 - a^2} - d\right)\cos\omega t$$

2016 電子回路

問1

(1)



(2) $v_i = h_{ie} i_b + \dfrac{\frac{R_E}{j\omega C_E}}{R_E + \frac{1}{j\omega C_E}}(1+h_{fe}) i_b$

$= \left\{ h_{ie} + \dfrac{R_E}{1+j\omega C_E R_E}(1+h_{fe}) \right\} i_b$

$v_o = -h_{fe} i_b \cdot \dfrac{\frac{R_L}{j\omega C_o}}{R_L + \frac{1}{j\omega C_o}}$

$= -\dfrac{h_{fe} R_L}{1+j\omega C_o R_L} i_b$

だから、

$G(\omega) = -\dfrac{h_{fe} R_L}{1+j\omega C_o R_L} \cdot \dfrac{1}{h_{ie} + \dfrac{R_E(1+h_{fe})}{1+j\omega C_E R_E}}$

$= -\dfrac{h_{fe} R_L}{1+j\omega C_o R_L} \cdot \dfrac{1+j\omega C_E R_E}{(1+j\omega C_E R_E)h_{ie} + R_E(1+h_{fe})}$

$= -\dfrac{1+j\omega C_E R_E}{1+j\omega C_o R_L} \cdot \dfrac{h_{fe} R_L}{h_{ie} + R_E(1+h_{fe}) + j\omega C_E R_E h_{ie}}$

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(3) $G(0) = -\dfrac{h_{fe} R_L}{h_{ie} + R_E(1+h_{fe})}$ より、

$|G(0)| = \dfrac{h_{fe} R_L}{h_{ie} + R_E(1+h_{fe})}$

$G(\omega) = -\dfrac{\frac{1}{\omega} + jC_E R_E}{\frac{1}{\omega} + jC_o R_L} \cdot \dfrac{\frac{1}{\omega} R_L h_{fe}}{\frac{h_{ie}}{\omega} + \frac{R_E}{\omega}(1+h_{fe}) + jC_E R_E h_{ie}}$ より、

$|G(\infty)| = 0$

ここで、

$|G(\omega)| = \sqrt{\dfrac{1+\omega^2 C_E^2 R_E^2}{1+\omega^2 C_o^2 R_L^2}} \cdot \dfrac{h_{fe} R_L}{\sqrt{(h_{ie} + R_E(1+h_{fe}))^2 + \omega^2 C_E^2 R_E^2 h_{ie}^2}}$

$\simeq h_{fe} R_L \sqrt{\dfrac{1+\omega^2 C_E^2 R_E^2}{(h_{ie}+(1+h_{fe})R_E)^2 + \omega^2 C_E^2 R_E^2 h_{ie}^2}}$

$\cancel{f(\omega)} \equiv h_{fe} R_L \sqrt{f(\omega)}$

$f'(\omega) = \dfrac{2\omega C_E^2 R_E^2\{(h_{ie}+(1+h_{fe})R_E)^2 + \omega^2 C_E^2 R_E^2 h_{ie}^2\} - (1+\omega^2 C_E^2 R_E^2)\cdot 2\omega C_E^2 R_E^2 h_{ie}^2}{\{(h_{ie}+(1+h_{fe})R_E)^2 + \omega^2 C_E^2 R_E^2 h_{ie}^2\}^2}$

$= 0$

とおくと、$\omega = 0$

$\cancel{(h_{ie}+(1+h_{fe})R_E)^2 + \omega^2 C_E^2 R_E^2 h_{ie}^2 = (1+\omega^2 C_E^2 R_E^2) h_{ie}^2}$

$\cancel{\omega^2 C_E^2 R_E^2 h_{ie}^2}$

よって、

$G_{max} = |G(0)| = \dfrac{h_{fe} R_L}{h_{ie} + R_E(1+h_{fe})}$

問2

(1) $V_o = -R I_1$

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(2) $I_1 = 1[\mu A]$ のとき、$V_o = -2$ [V] より、

$-2 = -R \cdot 10^{-6}$

$R = 2 \times 10^6 = 2[M\Omega]$

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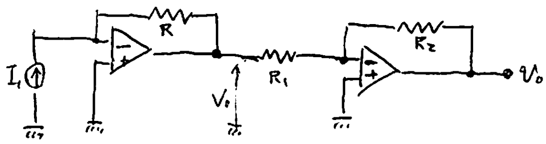
(3) V_oを反転して半分にすれば良いから、



$R_1 = 2[M\Omega]$,

$R_2 = 1[M\Omega]$

$\left(\because v_o = -\dfrac{R_2}{R_1} V_o\right)$

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(4) $v_o(\omega) = -i_1(\omega) \cdot \dfrac{1}{\frac{1}{R} + j\omega C}$

$= -\dfrac{R}{1+j\omega CR} i_1(\omega)$

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(5) $|v_o(\omega)| = \dfrac{R}{\sqrt{1+\omega^2 C^2 R^2}} |i_1(\omega)|$

$= \dfrac{10^3}{\sqrt{1+\omega^2 \cdot 10^{-12} \cdot 10^6}} \cdot 10^{-6}$

$= \dfrac{10^{-3}}{\sqrt{1+10^{-6}\omega^2}}$

$20\log_{10}|v_o(\omega)| = -60 - 20\log_{10}\sqrt{1+10^{-6}\omega^2}$

$1 = 10^{-6}\omega_c^2$ として、

$\omega_c = 10^3$ [rad/s]

だから、下図のようになる。