H20

$\boxed{4}$

(1)

$$\mathcal{L}[\sin \lambda t] = \int_0^{\infty} \sin \lambda t \, e^{-st} \, dt$$

$$= \left[-\frac{1}{s} e^{-st} \sin \lambda t \right]_0^{\infty} + \int \frac{\lambda}{s} e^{-st} \cos \lambda t \, dt$$

$$-\frac{\lambda^2}{s^2} \mathcal{L}[\sin \lambda t]$$

$$= 0 + \left[-\frac{\lambda}{s^2} e^{-st} \cos \lambda t \right]_0^{\infty} - \int \frac{\lambda^2}{s^2} e^{-st} \sin \lambda t \, dt$$

$$= \frac{s^2}{s^2 + \lambda^2} \cdot \frac{\lambda}{s^2}$$

$$= \frac{\lambda}{s^2 + \lambda^2}$$

(2)

$$\mathcal{L}[f'(t)] = \int_0^{\infty} f'(t) \, e^{-st} \, dt$$

$$= \left[f(t) \, e^{-st} \right]_0^{\infty} + s \int_0^{\infty} f(t) \, e^{-st} \, dt$$

$$= -f(0) + s F(s)$$

(3)

(2) より

$$\mathcal{L}\left[\frac{d}{dt} \left(\int_0^t f(z) \, dz \right) \right] = s \mathcal{L}\left[\int_0^t f(z) \, dz \right] - \int_0^0 f(z) \, dz$$

$$\mathcal{L}[f(t)] = \mathcal{L}\left[\frac{d}{dt} \int_0^t f(z) \, dz \right]$$

$$= s \mathcal{L}\left[\int_0^t f(z) \, dz \right]$$

$$\rightarrow \mathcal{L}\left[\int_0^t f(z) \, dz \right] = \frac{1}{s} \mathcal{L}[f(t)]$$

$$= \frac{1}{s} F(s)$$