2018　電気磁気学

問1

(1)

ガウスの定理より

$$E = \frac{Q}{4\pi\varepsilon r^2}$$ //

$$V = -\int_a^r \frac{Q}{4\pi\varepsilon r^2}dr$$

$$= \frac{Q}{4\pi\varepsilon r}$$ //

(2)

$$V_a = \frac{Q}{4\pi\varepsilon a}$$

$$C = \frac{Q}{V_a} = 4\pi\varepsilon a$$ //

(3)

電束密度は.

$$D_{上} = \varepsilon_1 E = \frac{\varepsilon_1 k}{r^2}$$

$$D_{下} = \varepsilon_2 E = \frac{\varepsilon_2 k}{r^2}$$

ガウスの定理より

$$2\pi r^2 D_{上} + 2\pi r^2 D_{下} = Q$$

$$2\pi(\varepsilon_1 + \varepsilon_2)k = Q$$

$$k = \frac{Q}{2\pi(\varepsilon_1 + \varepsilon_2)}$$

(4)

電荷面密度は.

$$\sigma_{上} = D_{上}|_{r=a} = \frac{\varepsilon_1 Q}{2\pi a^2(\varepsilon_1 + \varepsilon_2)}$$

$$\sigma_{下} = D_{下}|_{r=a} = \frac{\varepsilon_2 Q}{2\pi a^2(\varepsilon_1 + \varepsilon_2)}$$

よって.

$$Q_1 = \frac{\varepsilon_1}{\varepsilon_1 + \varepsilon_2}Q$$ //

$$Q_2 = \frac{\varepsilon_2}{\varepsilon_1 + \varepsilon_2}Q$$ //

(5)

球表面上の電位は.

$$V_a = \frac{Q}{2\pi(\varepsilon_1 + \varepsilon_2)a}$$

$$C = \frac{Q}{V_a} = 2\pi(\varepsilon_1 + \varepsilon_2)a$$ //

問2

(1)

$0 \leqq r \leqq a$

$$2\pi r H = \frac{r^2}{a^2} I$$

$$H = \frac{rI}{2\pi a^2}$$

$a \leqq r \leqq b$

$$H = \frac{I}{2\pi r}$$

$b \leqq r \leqq c$

$$2\pi r H = I - \frac{r^2 - b^2}{c^2 - b^2} I$$

$$H = \frac{c^2 - r^2}{c^2 - b^2} \cdot \frac{I}{2\pi r}$$

(2)

$$U = \frac{1}{2} H \cdot B$$

$$= \frac{1}{2} \mu H^2$$

$$= \frac{\mu r^2 I^2}{8\pi^2 a^4}$$

(3)

鎖交するのは．$\frac{r^2}{a^2}$の部分

$$d\phi = \frac{r^2}{a^2} \cdot \mu H dr$$

$$= \frac{\mu r^3 I}{2\pi a^4} dr$$

$$\phi = \int_0^a \frac{\mu r^3 I}{2\pi a^4} dr$$

$$= \frac{\mu I}{8\pi}$$

$$L = \frac{\phi}{I} = \frac{\mu}{8\pi}$$

(4)

$$\phi = \int_a^b \frac{\mu_0 I}{2\pi r} dr$$

$$= \frac{\mu_0 I}{2\pi} \log \frac{b}{a}$$

$$L = \frac{\phi}{I} = \frac{\mu_0}{2\pi} \log \frac{b}{a}$$

(5)

鎖交するのは．$\frac{c^2 - r^2}{c^2 - b^2}$の部分

$$\phi = \int_b^c \frac{\mu I}{2\pi r} \left(\frac{c^2 - r^2}{c^2 - b^2} \right)^2 dr$$

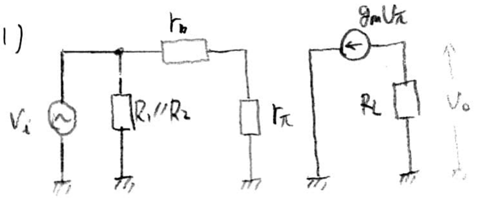
$$= \frac{\mu_0 I}{2\pi (c^2 - b^2)} \left(\frac{c^4}{c^2 - b^2} \log \frac{c}{b} - \frac{3c^2 - b^2}{4} \right)$$

$$L = \frac{\phi}{I} = \frac{\mu_0}{2\pi (c^2 - b^2)} \left(\frac{c^4}{c^2 - b^2} \log \frac{c}{b} - \frac{3c^2 - b^3}{4} \right)$$

電子回路

問1

(1)



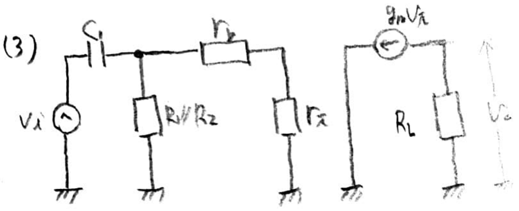
(2) $$v_\pi = \frac{r_\pi}{r_b + r_\pi} v_i$$

$$v_o = -g_m R_L v_\pi$$

$$= -g_m R_L \frac{r_\pi}{r_b + r_\pi} v_i$$

$$G_0 = -g_m R_L \frac{r_\pi}{r_b + r_\pi} \; //$$

(3)



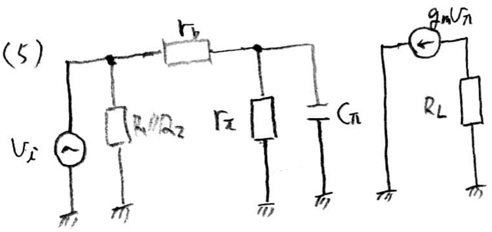
(4) $$v_\pi = \frac{(r_b + r_\pi)//(R_1//R_2)}{(r_b + r_\pi)//(R_1//R_2) + \frac{1}{j\omega C_1}} \cdot \frac{r_\pi}{r_b + r_\pi} v_i$$

$$v_o = -g_m R_L v_\pi$$

$$= -g_m R_L \cdot \frac{(r_b + r_\pi)//(R_1//R_2)}{(r_b + r_\pi)//(R_1//R_2) + \frac{1}{j\omega C_1}} \cdot \frac{r_\pi}{r_b + r_\pi} v_i$$

$$G_L(\omega) = -g_m R_L \frac{(r_b + r_\pi)//(R_1//R_2)}{(r_b + r_\pi)//(R_1//R_2) + \frac{1}{j\omega C_1}} \cdot \frac{r_\pi}{r_b + r_\pi} \; //$$

(5)



(6) $$v_\pi = \frac{r_\pi // \frac{1}{j\omega C_\pi}}{r_b + (r_\pi // \frac{1}{j\omega C_\pi})} v_i$$

$$v_o = -g_m R_L v_\pi$$

$$= -g_m R_L \frac{r_\pi // \frac{1}{j\omega C_\pi}}{r_b + (r_\pi // \frac{1}{j\omega C_\pi})} v_i$$

$$G_H(\omega) = -g_m R_L \frac{r_\pi // \frac{1}{j\omega C_\pi}}{r_b + (r_\pi // \frac{1}{j\omega C_\pi})} \; //$$

(7) $$G_L(\omega) = -g_m R_L \frac{(r_b + r_\pi)//(R_1//R_2)}{(r_b + r_\pi)//(R_1//R_2) + \frac{1}{j\omega C_1}} \cdot \frac{r_\pi}{r_b + r_\pi}$$

A_0は中域における利得なので

$$G_L(\omega) = A_0 \cdot \frac{(r_b + r_\pi)//(R_1//R_2)}{(r_b + r_\pi)//(R_1//R_2) + \frac{1}{j\omega C_1}}$$

$$= \frac{A_0}{1 + \frac{1}{j\omega C_1} \frac{1}{(r_b + r_\pi)//(R_1//R_2)}}$$

電圧利得$G_L(\omega)$の大きさが$\frac{|A_0|}{\sqrt{2}}$となる角周波数を求めればよいので

$$\frac{1}{\omega_{cl} C_1} \cdot \frac{1}{(r_b + r_\pi)//(R_1//R_2)} = 1$$

$$\therefore \omega_{cl} = \frac{1}{C_1 (r_b + r_\pi)//(R_1//R_2)} \; //$$

同様にして

$$G_H(\omega) = A_0 \cdot \frac{1}{1 + \frac{r_b}{r_\pi // \frac{1}{j\omega C_\pi}}} \cdot \frac{r_b + r_\pi}{r_\pi}$$

$$= A_0 \cdot \frac{1}{1 + \frac{r_b}{\frac{\frac{r_\pi}{j\omega C_\pi}}{r_\pi + \frac{1}{j\omega C_\pi}}}} \cdot \left(1 + \frac{r_b}{r_\pi}\right)$$

$$= A_0 \frac{1}{1 + j\omega C_\pi \frac{r_b}{r_\pi}\left(r_\pi + \frac{1}{j\omega C_\pi}\right)} \left(1 + \frac{r_b}{r_\pi}\right)$$

$$= A_0 \frac{1}{\left(1 + \frac{r_b}{r_\pi}\right) + j\omega C_\pi r_b} \left(1 + \frac{r_b}{r_\pi}\right)$$

$$= \frac{A_0}{1 + j\omega \frac{C_\pi r_b}{1 + \frac{r_b}{r_\pi}}}$$

$$\omega_{ch} \frac{C_\pi r_b}{1 + \frac{r_b}{r_\pi}} = 1$$

$$\omega_{ch} \frac{C_\pi r_\pi r_b}{r_b + r_\pi} = 1$$

$$\omega_{ch} C_\pi (r_b // r_\pi) = 1$$

$$\omega_{ch} = \frac{1}{C_\pi (r_b // r_\pi)} \; //$$

問2

(1) R_1に流れる電流をI_1とすると

$$V_i = R_1 I_1 \quad \therefore I_1 = \frac{V_i}{R_1}$$

$$R_2 I_1 + V_o = 0$$

$$V_o = -\frac{R_2}{R_1} V_i$$ //

(2) $$V_i = R_1 I_1 - R_b I_B^+$$

$$\therefore I_1 = \frac{V_i + R_b I_B^+}{R_1}$$

$$R_b I_B^+ + R_2 (I_1 - I_B^-) + V_o = 0$$

$$V_o = -R_b I_B^+ - R_2 \left(\frac{V_i + R_b I_B^+}{R_1} - I_B^- \right)$$

$$= -\left\{ \frac{R_2}{R_1} V_i + \left(1 + \frac{R_2}{R_1}\right) R_b I_B^+ + R_2 I_B^- \right\}$$ //

(3) $$R_b = R_1 /\!/ R_2$$

$$= \frac{R_1 R_2}{R_1 + R_2}$$

$$V_o = -\left(\frac{R_2}{R_1} V_i + \frac{R_1 + R_2}{R_1} \cdot \frac{R_1 R_2}{R_1 + R_2} I_B^+ + R_2 I_B^- \right)$$

$$= -\left\{ \frac{R_2}{R_1} V_i + R_2 (I_B^+ + I_B^-) \right\}$$ //

問1

(1) $i(t) = 5\sqrt{2}\sin\left(100t + \frac{\pi}{3}\right)$ [A]

$\dot{I} = 5\angle 60$ [A]

よって負荷インピーダンス $\dot{Z}$ は

$\dot{Z} = \frac{\dot{V}}{\dot{I}} = 2\angle -60$ [Ω]

(2) $\dot{Z} = 2\angle -60$

$= 2(\cos(-60°) + j\sin(-60°))$

$= 2\left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)$

$= 1 - j\sqrt{3}$

(3) 素子 キャパシタ

$\frac{1}{\omega C} = \sqrt{3}$

$C = \frac{1}{100\sqrt{3}}$ [F]

問2

(1) $q_c(t) = CV_0 - \int i(t)dt$

(2) $v(t) = V_0 - \frac{1}{C}\int i(t)dt$

$L\frac{di(t)}{dt} + Ri(t)$

$V_0 - \frac{1}{C}\int i(t)dt = L\frac{di(t)}{dt} + Ri(t)$ …①

(3) $-\frac{1}{C}i(t)dt = L\frac{d^2i(t)}{dt^2} + R\frac{di(t)}{dt}$

$L\frac{d^2i(t)}{dt^2} + R\frac{di(t)}{dt} + \frac{1}{C}i(t) = 0$ …②

①において $t=0$ とすると

$V_0 = L\frac{di(t)}{dt}\Big|_{t=0}$

$\therefore \frac{di(t)}{dt}\Big|_{t=0} = \frac{V_0}{L}$

(4) ②の特性方程式をとくと

$LS^2 + RS + \frac{1}{C} = 0$

$S = \frac{-R \pm \sqrt{R^2 - \frac{4L}{C}}}{2L}$

ここで $R^2C = 4L$ より

$S = -\frac{R}{2L}$

よって $i(t)$ の解は

$i(t) = (C_1 + C_2t)e^{-\frac{R}{2L}t}$

ここで $t=0$ のとき $i(0)=0$ なので

$C_1 = 0$

よって

$i(t) = C_2te^{-\frac{R}{2L}t}$

$\frac{di(t)}{dt} = C_2\left(1 - \frac{R}{2L}t\right)e^{-\frac{R}{2L}t}$

ここで $t=0$ のとき $\frac{di(t)}{dt}\Big|_{t=0} = \frac{V_0}{L}$ より

$C_2 = \frac{V_0}{L}$

よって

$i(t) = \frac{V_0}{L}te^{-\frac{R}{2L}t}$ [A]

(5) $\frac{di(t)}{dt} = 0$ のとき $i(t)$ は最大となる

$1 - \frac{R}{2L}t_{max} = 0$

$t_{max} = \frac{2L}{R}$

このとき

$i(t_{max}) = \frac{V_0}{L}\cdot\frac{2L}{R}e^{-\frac{R}{2L}\cdot\frac{2L}{R}}$

$= \frac{2V_0}{eR}$ [A]