H18

[1]

(1) $\frac{1}{(2-y)y}\,dy = dx$

$\frac{1}{2}\left(\frac{1}{2-y}+\frac{1}{y}\right)dy = dx$

$\frac{1}{2}\left(-\log|2-y|+\log y\right) = x + C$

$\log\frac{y}{2-y} = 2x + C$

$\frac{y}{2-y} = Ce^{2x}$

$y = \frac{2Ce^{2x}}{1+Ce^{2x}} = \frac{2}{1+Ce^{-2x}}$

(2) $s^2+2s+5=0$

$s = 1 \pm 2i$

$\eta = Ax + B$

$2A + 5Ax + 5B = 5x$

$\begin{cases} 5A = 5 & A = 1 \\ 2A+5B = 0 & B = -\frac{2}{5} \end{cases}$

$\eta = x - \frac{2}{5}$

$y = e^x\left(C_1\cos 2x + C_2\sin 2x\right) + x - \frac{2}{5}$

(3) $s^2+2s+5=0$

$s = 1 \pm 2i$

$\eta = A\cos x + B\sin x$

$(-A+2B+5A)\cos x + (-B-2A+5B)\sin x = 10\sin x$

$\begin{cases} 4A+2B=0 & \rightarrow B=-2A \\ -2A+4B=10 & A=-1,\ B=2 \end{cases}$

$\eta = -\cos x + 2\sin x$

$y = e^x(C_1\cos 2x + C_2\sin 2x) - \cos x + 2\sin x$

(4) $s^2+2s+5=0$

$s = 1 \pm 2i$

$\eta_1 = A_1x + B_1$

$\eta_2 = A_2\cos x + B_2\sin x$

$\begin{cases} 2A_1 + 5A_1x + 5B_1 = 10x \\ (4A_2+2B_2)\cos x + (4B_2-2A_2)\sin x = 10\sin x \end{cases}$

$\begin{cases} 5A_1 = 10 & A_1 = 2 \\ 2A_1+5B_1 = 0 & B_1 = -\frac{4}{5} \\ 4A_2+2B_2 = 0 & A_2 = -1 \\ 4B_2-2A_2 = 10 & B_2 = 2 \end{cases}$

$\eta = \eta_1 + \eta_2 = 2x - \frac{4}{5} + 2\sin x - \cos x$

$y = e^x(C_1\cos x + C_2\sin x) + 2x - \frac{4}{5} + 2\sin x - \cos x$