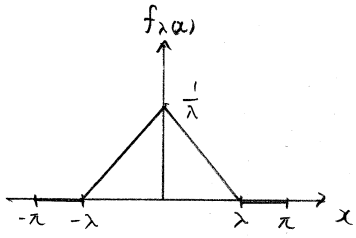
H25

④

(1)



周期 2L の周期関数

$$a_n = \frac{1}{L}\int_0^{2L} f(x)\cos\left(\frac{\pi}{L}nx\right)dx$$

$$a_0 = \frac{1}{L}\int_0^{2L} f(x)\,dx$$

$$b_n = \frac{1}{L}\int_0^{2L} f(x)\sin\left(\frac{\pi}{L}nx\right)dx$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left\{a_n\cos\left(\frac{\pi}{L}nx\right) + b_n\sin\left(\frac{\pi}{L}nx\right)\right\}$$

(2)

偶関数 なので $b_n = 0$

$$a_n(\lambda) = \frac{2}{\pi}\int_0^{\pi} f_\lambda(x)\cos nx\,dx$$

$$= \frac{2}{\pi}\int_0^{\lambda}\left(-\frac{1}{\lambda^2}x + \frac{1}{\lambda}\right)\cos nx\,dx$$

X

$$= \frac{2}{\pi}\left\{\left[\left(-\frac{1}{\lambda^2}x+\frac{1}{\lambda}\right)\sin nx\right]_0^{\lambda} - \int_0^{\lambda}-\frac{1}{\lambda^2}\sin nx\,dx\right\}$$

$$= \frac{2}{\pi}\left\{0 - \frac{1}{\lambda^2}\left[\cos nx\right]_0^{\lambda}\right\}$$

$$= \frac{2}{\pi\lambda^2}(1-\cos n\lambda)$$

$f(x)\cos nx$

$= f(x)\left(\frac{1}{n}\sin nx\right)'$

○

$$\frac{2}{\pi}\left\{\left[\left(-\frac{1}{\lambda^2}x+\frac{1}{\lambda}\right)\frac{1}{n}\sin nx\right]_0^{\lambda} - \int_0^{\lambda}\left(-\frac{1}{\lambda^2}\frac{1}{n}\sin nx\right)dx\right\}$$

$$= \frac{2}{\pi}\left\{0 + \frac{1}{\lambda^2 n}\int_0^{\lambda}\sin nx\,dx\right\}$$

$$= \frac{2}{\pi\lambda^2 n}\left[-\frac{1}{n}\cos nx\right]_0^{\lambda}$$

$$= \frac{2}{\pi\lambda^2 n^2}(1-\cos\lambda n)$$

$$a_0(\lambda) = \frac{2}{\pi}\int_0^{\lambda}\left(-\frac{1}{\lambda^2}x+\frac{1}{\lambda}\right)dx$$

$$= \frac{2}{\pi}\left[-\frac{1}{2\lambda^2}x^2+\frac{1}{\lambda}x\right]_0^{\lambda}$$

$$= \frac{2}{\pi}\left(-\frac{1}{2}+1\right) = \frac{1}{\pi}$$

X

$$f_\lambda(x) = \frac{1}{2\pi} + \sum_{n=1}^{\infty}\frac{2}{\pi\lambda^2}(1-\cos n\lambda)\cos nx$$

○

$$f_\lambda(x) \sim \frac{1}{2\pi} + \sum_{n=1}^{\infty}\frac{2}{\pi\lambda^2 n^2}(1-\cos\lambda n)\cos nx$$

(3)

$$\lim_{\lambda\to+0} a_n(\lambda) = \lim_{\lambda\to+0}\frac{2}{\pi\lambda^2}(1-\cos n\lambda)$$

?

○

$$\lim_{\lambda\to+0} a_n(\lambda) = \lim_{\lambda\to+0}\frac{2}{\pi\lambda^2 n^2}(1-\cos\lambda n)$$

$$= \frac{2}{\pi n^2}\lim_{\lambda\to+0}\frac{1-\cos\lambda n}{\lambda^2}$$

$$= \frac{2}{\pi n^2}\lim_{\lambda\to+0}\frac{n\sin\lambda n}{2\lambda} \quad \text{ロピタルの定理}$$

$$= \frac{2}{\pi n^2}\lim_{\lambda\to+0}\frac{n^2\cos\lambda n}{2}$$

$$= \frac{1}{\pi}$$