

H15

12

(1)

$$\text{rot } u = (1+1 \cdot 1+1 \cdot 1+1) \\ = (2 \quad 2 \quad 2)$$

(2)

$$x^2 + y^2 + z^2 = \frac{1}{4}$$

$$z = \sqrt{\frac{1}{4} - x^2 - y^2}$$

$$\frac{\partial z}{\partial x} = -\frac{x}{\sqrt{\frac{1}{4} - x^2 - y^2}}$$

$$\frac{\partial z}{\partial y} = -\frac{y}{\sqrt{\frac{1}{4} - x^2 - y^2}}$$

$$n = \frac{\left(-\frac{\partial z}{\partial x}, -\frac{\partial z}{\partial y}, 1\right)}{\sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}} = 2(x, y, \sqrt{\frac{1}{4} - x^2 - y^2}) \\ = 2(x, y, z)$$

$$n \times u = 2(y^2 + z^2 - xy - zx, x^2 + z^2 - yz - xy, x^2 + y^2 - xz - yz)$$

(3)

$$\iint_{x^2 + y^2 + z^2 = \frac{1}{4}} v \, dS$$

$$= \frac{1}{2} \iint_{x^2 + y^2 + z^2 = \frac{1}{4}} n \times u \, dS$$