H23

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(1) $\int_0^{\pi} |\cos t| e^{-st} dt$

$= \int_0^{\frac{\pi}{2}} \cos t\, e^{-st} dt - \int_{\frac{\pi}{2}}^{\pi} \cos t\, e^{-st} dt$

$\int_0^{\frac{\pi}{2}} \cos t\, e^{-st} dt = \left[-\frac{1}{s}\cos t\, e^{-st}\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \frac{1}{s} \sin t\, e^{-st} dt$

$= \frac{1}{s} - \left[-\frac{1}{s^2}\sin t\, e^{-st}\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \frac{1}{s^2} \cos t\, e^{-st} dt$

$\int_0^{\frac{\pi}{2}} \cos t\, e^{-st} dt \left(1 + \frac{1}{s^2}\right) = \frac{1}{s} + \frac{1}{s^2} e^{-\frac{\pi}{2}s}$

$\int_0^{\frac{\pi}{2}} \cos t\, e^{-st} dt = \frac{s^2}{1+s^2} \left(\frac{1}{s} + \frac{1}{s^2} e^{-\frac{\pi}{2}s}\right)$

同様に

$\int_{\frac{\pi}{2}}^{\pi} \cos t\, e^{-st} dt = \frac{s^2}{1+s^2} \left(\frac{1}{s} e^{-\pi s} - \frac{1}{s^2} e^{-\frac{\pi}{2}s}\right)$

よって

$\int_0^{\pi} |\cos t| e^{-st} dt = \frac{s^2}{1+s^2} \left(\frac{1}{s} + \frac{1}{s^2} e^{-\frac{\pi}{2}s} - \frac{1}{s} e^{-\pi s} + \frac{1}{s^2} e^{-\frac{\pi}{2}s}\right)$

$= \frac{s(1-e^{-\pi s}) + 2e^{-\frac{\pi}{2}s}}{s^2+1}$

(2) $|\cos t| = |\cos(t+\pi)|$ より $|\cos(t+n\pi)| = |\cos t|$

$\int_{n\pi}^{(n+1)\pi} |\cos t| e^{-st} dt = \int_0^{\pi} |\cos(t+n\pi)| e^{-s(t+n\pi)} dt$

$= e^{-sn\pi} \int_0^{\pi} |\cos t| e^{-st} dt$

$\int_0^{\infty} |\cos t| e^{-st} dt = \int_0^{\pi} |\cos t| e^{-st} dt + \int_{\pi}^{2\pi} |\cos t| e^{-st} dt + \cdots$

$= (1 + e^{-\pi s} + e^{-2\pi s} + \cdots) \int_0^{\pi} |\cos t| e^{-st} dt$

$= \frac{1}{1-e^{-\pi s}} \cdot \frac{s(1-e^{-\pi s}) + 2e^{-\frac{\pi}{2}s}}{s^2+1}$

$= \frac{s(1-e^{-\pi s}) + 2e^{-\frac{\pi}{2}s}}{(1-e^{-\pi s})(s^2+1)}$