H25

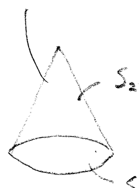
[2]

(1)

$$S_1 = \pi \cdot 1^2$$
$$= \pi$$

$$V: x^2 + y^2 - (1-z)^2 \leq 0$$

$$x^2 + y^2 \leq (1-z)^2$$



$$V = \int_0^1 2\pi (1-z)^2 \, dz$$

$$= \pi \left[-\frac{1}{3} (1-z)^2 \right]_0^1 = \frac{1}{3} \pi$$

ガウスの発散定理より

$$\iint_S (x.y.z) \cdot n \, dS = \iiint_V \mathrm{div}(x.y.z) \, dv$$

$$\iiint_V dv = \frac{1}{3} \pi$$

$$\mathrm{div}(x.y.z) = 3$$

よって

$$\iint_S (x.y.z) \cdot n \, dS = \pi$$

(2)

$$n_{S_1} = (0.0.1)$$

$$S_2: x^2 + y^2 - (1-z)^2 = 0$$

$$z = -\sqrt{x^2+y^2} + 1$$

$$\frac{\partial z}{\partial x} = -x (x^2+y^2)^{-\frac{1}{2}}$$

$$\frac{\partial z}{\partial y} = -y (x^2+y^2)^{-\frac{1}{2}}$$

$$n_2 = \frac{\left(-\frac{\partial z}{\partial x}, -\frac{\partial z}{\partial y}, 1\right)}{\sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}}$$

$$= \frac{1}{\sqrt{2}} \left(\frac{x}{\sqrt{x^2+y^2}}, \frac{y}{\sqrt{x^2+y^2}}, 1 \right)$$

よって z成分は $\frac{1}{\sqrt{2}}$ で定数