

③

$$(1) f(z) = \frac{1}{(z^2+1)(z-3)^2}$$

$$= \frac{1}{(z+i)(z-i)(z-3)^2}$$

特異点 は

$$z = \pm i, 3$$

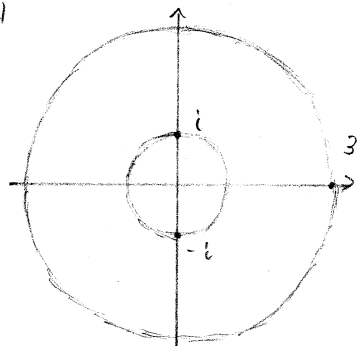
 $z = \pm i$  は 1 位,  $z = 3$  は 2 位の極

$$\text{Res}[i] = \lim_{z \rightarrow i} \frac{1}{(z+i)(z-3)^2} = \frac{1}{12+16i} = \frac{1}{4(3+4i)}$$

$$\text{Res}[-i] = \lim_{z \rightarrow -i} \frac{1}{(z-i)(z-3)^2} = \frac{1}{-12-16i} = \frac{1}{4(3-4i)}$$

$$\text{Res}[3] = \lim_{z \rightarrow 3} \frac{d}{dz} \frac{1}{z^2+1} = -\frac{3}{50}$$

(2)


 $0 < a < 1$  のとき 特異点  $z = \pm i$ 

$$\int_C f(z) dz = 0$$

 $1 < a < 3$  のとき 特異点  $z = \pm i$ 

$$\int_C f(z) dz = 2\pi i (\text{Res}[i] + \text{Res}[-i])$$

$$= \frac{3}{25} \pi i$$

 $3 < a$  のとき 特異点  $z = \pm i, 3$ 

$$\int_C f(z) dz = 2\pi i (\text{Res}[i] + \text{Res}[-i] + \text{Res}[3])$$

$$= 0$$