

H21

①

$$(1) \quad x^3 \frac{dy}{dx} = -y^2$$

$$-\frac{1}{y^2} dy = \frac{1}{x^3} dx$$

$$\int -y^{-2} dy = \int x^{-3} dx$$

$$y^{-1} = -\frac{1}{2} x^{-2} + C$$

$$(2) \quad y' + xy = x$$

$$\text{Ansatz: } e^{\frac{1}{2}x^2} \text{ ist Lösung}$$

$$(y e^{\frac{1}{2}x^2})' = x e^{\frac{1}{2}x^2}$$

$$y e^{\frac{1}{2}x^2} = e^{\frac{1}{2}x^2} + C$$

$$y = 1 + C e^{-\frac{1}{2}x^2}$$

$$(3) \quad y'' + 2y' + 2y = 0$$

$$s^2 + 2s + 2 = 0$$

$$s = 1 \pm i$$

$$y = e^x (C_1 \cos x + C_2 \sin x)$$

$$(4) \quad y'' + 2y' + 2y = e^x + 5 \cos x$$

$$s = 1 \pm i$$

$$\eta_1 = A e^x$$

$$\eta_2 = B \cos x + C \sin x$$

$$A e^x + 2A e^x + 2A e^x = e^x$$

$$(-B + 2C + 2B) \cos x + (-C - 2B + 2C) \sin x = 5 \cos x$$

$$\begin{cases} 5A = 1 & A = \frac{1}{5} \\ B + 2C = 5 & B = 1 \\ C - 2B = 0 & C = 2 \end{cases}$$

$$\eta = \eta_1 + \eta_2 = \frac{1}{5} e^x + \cos x + 2 \sin x$$

$$y = e^x (C_1 \cos x + C_2 \sin x) + \frac{1}{5} e^x + \cos x + 2 \sin x$$