H24

4

(1)

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{\pi}{L} nx + b_n \sin \frac{\pi}{L} nx \right)$$

$f(x) = f(-x)$ より 偶関数

よって $b_n = 0$

$$a_n = \int_{-1}^{1} (|\cos \pi x| - \cos \pi x) \cos n\pi x \, dx$$

$$= 2 \int_0^1 (|\cos \pi x| - \cos \pi x) \cos n\pi x \, dx$$

$$= 2 \int_0^{\frac{1}{2}} (|\cos \pi x| - \cos \pi x) \cos n\pi x \, dx + 2 \int_{\frac{1}{2}}^{1} (|\cos \pi x| - \cos \pi x) \cos n\pi x \, dx$$

$$= 2 \int_{\frac{1}{2}}^{1} -2 \cos \pi x \cdot \cos n\pi x \, dx$$

$$= -2 \int_{\frac{1}{2}}^{1} \left\{ \cos (n+1) \pi x + \cos (n-1) \pi x \right\} dx$$

$$= -2 \left[\frac{1}{(n+1)\pi} \sin (n+1) \pi x + \frac{1}{(n-1)\pi} \sin (n-1) \pi x \right]_{\frac{1}{2}}^{1}$$

$$= 2 \left(\frac{1}{(n+1)\pi} \sin \frac{n+1}{2} \pi + \frac{1}{(n-1)\pi} \sin \frac{n-1}{2} \pi \right)$$

$$= \begin{cases} \frac{2}{(n+1)\pi} (-1)^{\frac{n}{2}} + \frac{2}{(n-1)\pi} (-1)^{\frac{n}{2}+1} & n \text{ が偶数} \\ 0 & n \text{ が奇数 } n \neq 1 \end{cases}$$

$$a_1 = -4 \int_{\frac{1}{2}}^{1} \cos^2 \pi x \, dx$$

$$= -2 \int_{\frac{1}{2}}^{1} (1 + 2\cos 2\pi x) \, dx = -1$$

$$a_0 = -4 \int_{\frac{1}{2}}^{1} \cos \pi x \, dx = \frac{4}{\pi}$$

ここで $n = 2m$ とすると

$$f(x) = \frac{2}{\pi} - \cos \pi x + \sum_{m=1}^{\infty} \left(\frac{2}{(2m+1)\pi} (-1)^m + \frac{2}{(2m-1)\pi} (-1)^{m+1} \right) \cos 2m\pi x$$