2014 数学

[1]

(1) $\frac{dy}{dx} = xe^{-x}(y+1)^2$

$\frac{dy}{(y+1)^2} = xe^{-x}dx$

両辺を積分して、

~~$(y+1)^{-1} =$~~

$-\frac{1}{y+1} = \int xe^{x}dx$

$= [-xe^{-x}] + \int e^{-x}dx$

$= -xe^{-x} - e^{-x} + C$

$y+1 = \frac{1}{xe^{-x}+e^{-x}-C}$

$y = \frac{1}{xe^{-x}+e^{-x}-C} - 1$

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(2) $\frac{d^2y}{dx^2} + ay = 0$

$f(s) = s^2 + a = 0$

$s = i\sqrt{a}$

よって、

$y = C_1\cos\sqrt{a}x + C_2\sin\sqrt{a}x$

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(3) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = x^2 + x$

$f(s) = s^2 + 2s = 0$

$s = 0, -2$

$y_t = C_1 + C_2e^{-2x}$

$y = \cancel{x(x^2+x)a+}$

$ax^3 + bx^2 + cx$

とおくと、

$y' = 3ax^2 + 2bx + c.$

$y'' = 6ax + 2b$

より、

$6ax + 2b + 6ax^2 + 4bx + 2c = x^2 + x$

恒等式だから、

$$\begin{cases} 6a = 1, \\ 6a + 4b = 1 \\ 2b + 2c = 0 \end{cases}$$

$a = \frac{1}{6}, b = 0, c = 0$

より、

$y_s = \frac{1}{6}x^3$

したがって、

$y = C_1 + C_2e^{-2x} + \frac{1}{6}x^3$

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[4]

(1) $\mathcal{L}[e^{at}f(t)]$

$= \int_0^\infty e^{at}f(t)e^{-st}dt$

$= \int_0^\infty f(t)e^{(a-s)t}dt$

~~$= \left[\frac{1}{a-s}e^{(a-s)t}f(t)\right]_0^\infty - \int_0^\infty$~~

$= \int_0^\infty f(t)e^{-(s-a)t}dt$

$= \mathcal{L}[f(t)](s-a)$

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(2) $\mathcal{L}[\cos at]$

$= \int_0^\infty \cos at\, e^{-st}dt$

$= \left[-\frac{e^{-st}}{s}\cos at\right]_0^\infty - \int_0^\infty \frac{ae^{-st}}{s}\sin at\,dt$

$= \left[-\frac{e^{-st}}{s}\cos at\right]_0^\infty - \frac{a}{s}\left\{\left[-\frac{e^{-st}}{s}\sin at\right]_0^\infty + \int_0^\infty \frac{e^{-st}}{s}\cdot a\cos at\,dt\right\}$

$= \left[-\frac{e^{-st}}{s}\cos at + \frac{a}{s^2}e^{-st}\sin at\right]_0^\infty - \frac{a^2}{s^2}\int_0^\infty \cos at\cdot e^{-st}dt$

$= \frac{1}{1+\frac{a^2}{s^2}}\left(\frac{1}{s}\right)$

$= \frac{s}{s^2+a^2}.$

$\mathcal{L}[\sin at]$

$= \int_0^\infty e^{-st}\sin at\,dt$

$= \left[-\frac{e^{-st}}{s}\sin at\right]_0^\infty + \int_0^\infty \frac{e^{-st}}{s}a\cos at\,dt$

$= \left[-\frac{e^{-st}}{s}\sin at\right]_0^\infty - \left[\frac{e^{-st}}{s^2}a\cos at\right]_0^\infty - \int_0^\infty \frac{e^{-st}}{s^2}\cdot a^2\sin at\,dt$

$= \left[-\frac{e^{-st}}{s}\sin at - \frac{a}{s^2}e^{-st}\cos at\right]_0^\infty - \frac{a^2}{s^2}\int_0^\infty e^{-st}\sin at\,dt$

$= \frac{1}{1+\frac{a^2}{s^2}}\left(\frac{a}{s^2}\right)$

$= \frac{a}{s^2+a^2}$

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(3) $\frac{2s-3}{s^2+2s+5} = \frac{2s-3}{(s+1)^2+4}$

$= \frac{2(s+1)}{(s+1)^2+4} - \frac{5}{(s+1)^2+4}$

$= \frac{2(s+1)}{(s+1)^2+4} - \frac{5}{2}\frac{2}{(s+1)^2+4}$

よって、

$f(t) = 2e^{-t}\cos 2t - \frac{5}{2}e^{-t}\sin 2t$