H25

①

(1)

積分因子は

$$e^{\int \cos x\,dx} = e^{\sin x}$$

P19 $y' + P(x)y = Q(x)$ 型

両辺に $e^{\sin x}$ をかけて

$$(ye^{\sin x})' = \sin 2x\, e^{\sin x}$$

$$ye^{\sin x} = \int \sin 2x\, e^{\sin x}\,dx$$

$t = \sin x$ とおくと

$$dx = \frac{1}{\cos x}\,dt$$

$$\int \sin 2x\, e^{\sin x}\,dx = \int 2te^t\,dt$$

$$= 2(t-1)e^t + C$$

$$= 2(\sin x - 1)e^{\sin x} + C$$

$$y = 2(\sin x - 1) + Ce^{-\sin x}$$

(2)

$$s^2 + 6s + 11 = 0$$

$$s = -3 \pm \sqrt{2}\,i$$

$$y = C_1 e^{-3x}\cos\sqrt{2}t + C_2 e^{-3x}\sin\sqrt{2}t$$

(3)

基本解は (2) と同じ

$$\eta = Ax + B$$

$$\eta' = A$$

$$\eta'' = 0$$

$$6A + 11Ax + 11B = 11x$$

$$A = 1,\quad B = -\frac{6}{11}$$

$$y = C_1 e^{-3x}\cos\sqrt{2}t + C_2 e^{-3x}\sin\sqrt{2}t + x - \frac{6}{11}$$