①

(1) $y' - \frac{2}{x}y = x+1$

積分因子は

$\int -\frac{2}{x}dx = -2\log x$

$e^{-2\log x} = x^{-2}$

両辺に $\frac{1}{x^2}$ をかけて

$\left(\frac{y}{x^2}\right)' = \frac{1}{x} + \frac{1}{x^2}$

$\frac{y}{x^2} = \log x - \frac{1}{x} + c$

$y = x^2\log x - x + cx^2$

(2) $s^2 - 3s + 2 = 0$

$(s-1)(s-2) = 0$

$s = 1, 2$

$\eta = Ax^2 + Bx + C$

$2A - 3(2Ax+B) + 2(Ax^2+Bx+c) = x^2$

$\begin{cases} 2A = 1 & A = \frac{1}{2} \\ -6A + 2B = 0 & B = \frac{3}{2} \\ 2A - 3B + 2c = 0 & C = \frac{7}{4} \end{cases}$

$\eta = \frac{1}{4}(2x^2 + 6x + 7)$

$y = C_1 e^x + C_2 e^{2x} + \frac{1}{4}(2x^2+6x+7)$

(3) $s^2 - 3s + 2 = 0$

$s = 1, 2$

$\eta_1 = Ax^2 + Bx + C$

$\eta_2 = Dxe^x$

(2)より $\eta_1 = \frac{1}{4}(2x^2+6x+7)$

$D(x+2)e^x - 3D(x+1)e^x + 2Dxe^x = e^x$

$D - 3D + 2D = 0$

$2D - 3D = 1 \qquad D = -1$

$\eta = \eta_1 + \eta_2$

$= \frac{1}{4}(2x^2+6x+7) - xe^x$

$y = C_1 e^x + C_2 e^x + \frac{1}{4}(2x^2+6x+7) - xe^x$