H25

$\boxed{3}$

(1) $\dfrac{(z^2-1)^2}{z^2(z^2-6z+1)}$

$= \dfrac{(z^2-1)^2}{z^2(z-3+2\sqrt{2})(z-3-2\sqrt{2})}$

特異点は $z=0,\ 3\pm2\sqrt{2}$

$z=0$ で $\dfrac{(z^2-1)^2}{z^2-6z+1}$

$z=3+2\sqrt{2}$ で $\dfrac{(z^2-1)^2}{z^2(z-3+2\sqrt{2})}$ 　は正則

$z=3-2\sqrt{2}$ で $\dfrac{(z^2-1)^2}{z^2(z-3-2\sqrt{2})}$

$z=0$ は 2位、$z=3\pm2\sqrt{2}$ は1位の極

$$\mathrm{Res}[0] = \lim_{z\to0}\frac{d}{dz}\frac{(z^2-1)^2}{z^2-6z+1} = \lim_{z\to0}\frac{4z(z^2-1)(z^2-6z+1)-(z^2-1)^2(2z-6)}{(z^2-6z+1)^2} = 6$$

$$\mathrm{Res}[3+2\sqrt{2}] = \lim_{z\to3+2\sqrt{2}}\frac{(z^2-1)^2}{z^2(z-3+2\sqrt{2})} = \frac{(16+12\sqrt{2})^2}{(3+2\sqrt{2})^2\cdot4\sqrt{2}} = \frac{4^2(4+3\sqrt{2})^2}{(4+3\sqrt{2})^2\cdot2\sqrt{2}} = \frac{8}{\sqrt{2}} = 4\sqrt{2}$$

$$\mathrm{Res}[3-2\sqrt{2}] = \lim_{z\to3-2\sqrt{2}}\frac{(z^2-1)^2}{z^2(z-3-2\sqrt{2})} = \frac{(16-12\sqrt{2})^2}{(3-2\sqrt{2})^2\cdot(-4\sqrt{2})} = -\frac{4^2(4-3\sqrt{2})^2}{(4-3\sqrt{2})^2\,2\sqrt{2}} = -4\sqrt{2}$$

(2) $z=e^{i\theta}$ とすると

$dz = iz\,d\theta$

$\sin^2\theta = -\dfrac{z^2+z^{-2}-2}{4}$

$\cos\theta = \dfrac{z+z^{-1}}{2}$

$$I_1 = \int_{|z|=1}\frac{-\frac{z^2+z^{-2}-2}{4}}{3-\frac{z+z^{-1}}{2}}\cdot\frac{1}{iz}\,dz$$

$$= \int_{|z|=1}\frac{(z^2-1)^2}{2iz^2(z-6z+1)}$$

$$= \frac{1}{2i}I_2$$