4

(1)

$$\frac{1}{s^3+1} = \frac{1}{(s+1)(s^2-s+1)}$$

ここで、

$$\frac{1}{(s+1)(s^2-s+1)} = \frac{a}{s+1} + \frac{bs+c}{s^2-s+1} \quad \text{とおくと}$$

$$= \frac{(a+b)s^2+(-a+b+c)s+a+c}{(s+1)(s^2-s+1)}$$

$$\begin{cases} a+b=0 \\ -a+b+c=0 \\ a+c=1 \end{cases} \qquad a=\frac{1}{3},\ b=-\frac{1}{3},\ c=\frac{2}{3}$$

よって、

$$\frac{1}{s^3+1} = \frac{1}{3}\cdot\frac{1}{s+1} + \frac{-\frac{1}{3}s+\frac{2}{3}}{s^2-s+1}$$

また、

$$\frac{-\frac{1}{3}s+\frac{2}{3}}{s^2-s+1} = -\frac{1}{3}\cdot\frac{s-2}{\left(s-\frac{1}{2}\right)^2+\frac{3}{4}}$$

$$= -\frac{1}{3}\cdot\frac{\left(s-\frac{1}{2}\right)+\frac{1}{2}-2}{\left(s-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= -\frac{1}{3}\left\{\frac{s-\frac{1}{2}}{\left(s-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} + \frac{-\frac{3}{2}}{\left(s-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right\}$$

$$= -\frac{1}{3}\left\{\frac{s-\frac{1}{2}}{\left(s-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} - \frac{3}{2}\cdot\frac{2}{\sqrt{3}}\cdot\frac{\frac{\sqrt{3}}{2}}{\left(s-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right\}$$

$$= -\frac{1}{3}\left\{\frac{s-\frac{1}{2}}{\left(s-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} - \sqrt{3}\,\frac{\frac{\sqrt{3}}{2}}{\left(s-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right\}$$

以上より、

$$\mathcal{L}^{-1}\left[\frac{1}{s^3+1}\right] = \frac{1}{3}e^{-t} - \frac{1}{3}e^{\frac{1}{2}t}\cos\frac{\sqrt{3}}{2}t + \frac{\sqrt{3}}{3}e^{\frac{1}{2}t}\sin\frac{\sqrt{3}}{2}t$$

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$$\frac{1}{(s+1)(s^2-1)} = \frac{1}{(s+1)^2(s-1)}$$

ここで、

$$\frac{1}{(s+1)^2(s-1)} = \frac{a}{(s+1)^2} + \frac{b}{s+1} + \frac{c}{s-1} \quad \text{とおくと}$$

$$= \frac{(b+c)s^2+(a+2c)s-a-b+c}{(s+1)^2(s-1)}$$

$$\begin{cases} b+c=0 \\ a+2c=0 \\ -a-b+c=1 \end{cases} \qquad a=-\frac{1}{2},\ b=-\frac{1}{4},\ c=\frac{1}{4}$$

よって、

$$\frac{1}{(s+1)^2(s-1)} = -\frac{1}{2}\frac{1}{(s+1)^2} - \frac{1}{4}\frac{1}{s+1} + \frac{1}{4}\cdot\frac{1}{s-1}$$

$$\mathcal{L}^{-1}\left[\frac{1}{(s+1)^2(s-1)}\right] = -\frac{1}{2}te^{-t} - \frac{1}{4}e^{-t} + \frac{1}{4}e^{t}$$

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部分分数分解の定理

- $\dfrac{bx+c}{(x+a)^2} = \dfrac{A}{(x+a)^2} + \dfrac{B}{x+a}$

- $\dfrac{cx^2+dx+e}{(x+a)^2(x+b)} = \dfrac{A}{(x+a)^2} + \dfrac{B}{x+a} + \dfrac{C}{x+b}$

(2) $\frac{d}{dx}x(t) + x(t) = t^2$, $x(0) = 1$

両辺をラプラス変換すると、($\mathcal{L}[x(t)] = X(s)$)

$$sX(s) - x(0) + X(s) = \frac{2}{s^3}$$

$$(s+1)X(s) = \frac{s^3+2}{s^3}$$

$$X(s) = \frac{s^3+2}{s^3(s+1)}$$

ここで、

$$\frac{s^3+2}{s^3(s+1)} = \frac{A}{s^3} + \frac{B}{s^2} + \frac{C}{s} + \frac{D}{s+1}$$ とおくと

$$= \frac{(C+D)s^3 + (B+C)s^2 + (A+B)s + A}{s^3(s+1)}$$

$$\begin{cases} C+D = 1 \\ B+C = 0 \\ A+B = 0 \\ A = 2 \end{cases}$$ $A=2,\ B=-2,\ C=2,\ D=-1$

よって、

$$X(s) = \frac{2}{s^3} - 2\cdot\frac{1}{s^2} + 2\cdot\frac{1}{s} - \frac{1}{s+1}$$

両辺をラプラス逆変換して、

$$x(t) = t^2 - 2t + 2 - e^{-t}$$