

[1]

$$(1) \frac{dy}{dx} - \frac{2}{x}y = -\frac{\sin 2x}{x}y^2$$

$$\frac{1}{y^2} \frac{dy}{dx} - \frac{2}{x} \frac{1}{y} = -\frac{\sin 2x}{x}$$

$$u = y^{-1} \text{ とおくと } u' = -y^{-2} \cdot y'$$

$$\text{与式に } -1 \text{ 倍すると } -\frac{1}{y^2} y' + \frac{2}{xy} = \frac{\sin 2x}{x}$$

$$u' + \frac{2}{x}u = \frac{\sin 2x}{x}$$

1階微分方程式1点2"

$$u = e^{-\int \frac{2}{x} dx} \int e^{\int \frac{2}{x} dx} \left(\frac{\sin 2x}{x} \right) dx \quad \int \frac{2}{x} dx = \log x^2$$

$$= x^{-2} \left(\int x^2 \frac{\sin 2x}{x} dx + C \right)$$

$$= x^{-2} \left(-\frac{1}{2} x \cos 2x + \int \frac{1}{2} \cos 2x dx + C \right)$$

$$= x^{-2} \left(-\frac{1}{2} x \cos 2x + \frac{1}{4} \sin 2x + C \right)$$

$$\frac{x^2}{y} = -\frac{1}{2} x \cos 2x + \frac{1}{4} \sin 2x + C \quad (C \text{ は定数})$$

$$(2) \frac{d^2 y}{dx^2} - 6 \frac{dy}{dx} + 9y = \frac{x}{2}$$

$$\text{特性方程式 } \lambda^2 - 6\lambda + 9 = 0 \quad \lambda = 3 \text{ (2重解)}$$

$$\text{一つの解 } y_0 = C_1 e^{3x} + C_2 x e^{3x}$$

$$\text{特殊解 } y = Ax + B \text{ とおす}$$

$$-6A + 9Ax + 9B = \frac{x}{2} \text{ とおす}$$

$$\begin{cases} 9A = \frac{1}{2} & A = \frac{1}{18} \\ 9B - 6A = 0 & B = \frac{1}{27} \end{cases}$$

$$\text{よって一般解は } y = \frac{C_1 e^{3x} + C_2 x e^{3x} + \frac{1}{18} x + \frac{1}{27}}{(C_1, C_2 \text{ は定数})}$$

$$(3) \frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + 10y = \sin x$$

$$\text{特性方程式 } \lambda^2 + 2\lambda + 10 = 0 \text{ より } \lambda = -1 \pm 3i$$

$$\text{一つの解は } y_0 = C_1 e^{-x} \cos 3x + C_2 e^{-x} \sin 3x$$

$$\text{特殊解 } y = A \sin x + B \cos x \text{ とおす}$$

$$y' = A \cos x - B \sin x$$

$$y'' = -A \sin x - B \cos x$$

$$y'' + 2y' + 10y = \sin x \text{ に代入すると}$$

$$-A \sin x - B \cos x + 2A \cos x - 2B \sin x + 10A \sin x + 10B \cos x = \sin x$$

$$(9A - 2B) \sin x + (9B + 2A) \cos x = \sin x$$

$$\begin{cases} 9A - 2B = 1 \\ 9B + 2A = 0 \end{cases}$$

$$A = -\frac{9}{2}B \quad -\frac{81}{2}B - 2B = 1$$

$$B = -\frac{2}{85}$$

$$A = \frac{9}{85}$$

$$\text{よって一般解 } y = \frac{C_1 e^{-x} \cos 3x + C_2 e^{-x} \sin 3x + \frac{9}{85} \sin x - \frac{2}{85} \cos x}{(C_1, C_2 \text{ は定数})}$$

$$\lambda = \frac{-2 \pm \sqrt{4 - 40}}{2} = -1 \pm 3i$$