$\boxed{1}$

(1) (a)

特性方程式

$$s^4 - 4s^3 + 7s^2 - 12s + 12 = 0$$

$$(s-2)^2(s^2+\sqrt{3}) = 0$$

$$s = 2 \text{ (重解)}, \pm\sqrt{3}i$$

$$\underline{y = (C_1 + C_2 x)e^{2x} + C_3\cos\sqrt{3}x + C_4\sin\sqrt{3}x} \;//$$

(b)

特性方程式

$$s^2 - 2s + 1 = 0$$

$$(s-1)^2 = 0$$

$$s = 1 \text{ (重解)}$$

余関数は. $e^x(C_1 + C_2 x)$

特殊解 $y = A\sin x + B\cos x$

$$y' = A\cos x - B\sin x$$

$$y'' = -A\sin x - B\cos x$$

与式に代入.

$$-A\sin x - B\cos x - 2A\cos x + 2B\sin x + A\sin x + B\cos x$$

$$= 2B\sin x - 2A\cos x = \sin x$$

$$2B = 1, \quad -2A = 0 \quad \therefore A = 0,\ B = \frac{1}{2}$$

$$\underline{y = (C_1 + C_2 x)e^x + \frac{1}{2}\cos x} \;//$$

(2) (a)

$x^a y^b$ をかける.

$$(x^{a-1}y^b + x^{a+1}y^{b+1})dx + x^{a+2}y^b dy = 0$$

上式が完全微分形になればよいので.

$$\frac{\partial}{\partial y}(x^{a-1}y^b + x^{a+1}y^{b+1}) = \frac{\partial}{\partial x}x^{a+2}y^b$$

$$bx^{a-1}y^{b-1} + (b+1)x^{a+1}y^b = (a+2)x^{a+1}y^b$$

$$b = 0$$

$$b+1 = a+2 \quad \therefore \underline{a = -1,\ b = 0} \;//$$

(b)

(a) より. 積分因子は. x^{-1}

$$\left(\frac{1}{x^2} + y\right)dx + x\,dy = 0$$

初期値. $(x, y) = (1, 0)$ とすると.

$$\int_1^x \left(\frac{1}{x^2} + y\right)dx + \int_0^y dy$$

$$= \left[-\frac{1}{x} + xy\right]_1^x + [y]_0^y$$

$$= -\frac{1}{x} + xy + 1 = C$$

$$\therefore \underline{xy - x = C} \;//$$

(1)

$$\int_0^{2\pi} f(x)\,dx$$

$$= \int_0^{\frac{\pi}{2}} dx + \int_{\frac{\pi}{2}}^{\pi} \left(-\frac{2}{\pi}x + 2\right) dx$$

$$+ \int_{\pi}^{\frac{3}{2}\pi} \left(\frac{2}{\pi}x - 2\right) dx + \int_{\frac{3}{2}\pi}^{2\pi} dx$$

$$= -\pi \quad //$$

(2)

$$\int x \cos ax\,dx$$

$$= \frac{x}{a}\sin ax + \frac{1}{a^2}\cos ax \quad //$$

(3)

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$$

$$= -1$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx$$

$$= -\frac{4}{n^2\pi^2}\cos n\pi$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx$$

$$= \frac{4}{n^2\pi^2}\sin\frac{n\pi}{2}$$

$$f(x) \sim -\frac{1}{2} + \sum_{n=1}^{\infty}\frac{4}{n^2\pi^2}\left(-\cos n\pi\cos nx + \sin\frac{n\pi}{2}\sin nx\right) \quad //$$

(4)

$x=0$ のとき、$f(0)=1$.

$n \Rightarrow 2n-1$ とすると.

$\cos(2n-1)\pi = -1$.

$\sin\dfrac{(2n-1)\pi}{2} = -1$

$\cos 0 = 1$

$\sin 0 = 0$

(3) に代入して.

$$1 = -\frac{1}{2} + \sum_{n=1}^{\infty} \frac{4}{(2n-1)^2\pi^2}$$

$$\Leftrightarrow \sum_{n=1}^{\infty}\frac{1}{(2n-1)^2} = \frac{3}{8}\pi^2 \quad //$$