H19

[2]

(1)

$\mathrm{rot}\, u = (0,\ 0,\ 2)$

$\mathrm{div}\, v = (2x+1) + x - 1 + 2z - 3z$

$= 2x$

(2)

$\frac{1}{2}\int_C u \cdot dr = \frac{1}{2}\int_C (-b\sin t,\ a\cos t,\ 0)\frac{dr}{dt}\,dt$

$= \frac{1}{2}\int_C (-b\sin t,\ a\cos t,\ 0)\cdot(-a\sin t,\ b\cos t,\ 0)\,dt$

$= \frac{1}{2}\int_0^{2\pi} (ab\sin^2 t + ab\cos^2 t)\,dt$

$= \frac{1}{2}\, ab \cdot 2\pi = ab\pi$

(3)

ガウスの発散定理より

$\iint_S v \cdot n\, dS = \iiint_V \mathrm{div}\, v\, dV$

$= \iiint_V 2z\, dV$

$= ab\pi \int_0^c 2z\, dV$

$= abc^2\pi$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ の面積