問2

(1)

$$\begin{cases} B = \nabla \times A \\ B = \mu_0 H \end{cases}$$

$$\mu_0 H = \nabla \times A$$

$$H = \frac{1}{\mu_0}(\nabla \times A)$$

$$dH = \frac{1}{\mu_0}(\nabla \times dA)$$

$$= \frac{I}{4\pi}\left(\nabla \times \frac{1}{r}\, ds\right)$$

与えられたベクトル公式より

$$dH = \frac{I}{4\pi}\left\{\nabla \frac{1}{r} \times ds + \frac{1}{r}(\nabla \times ds)\right\}$$

∇はP点での微分なので

dsとは関係ない

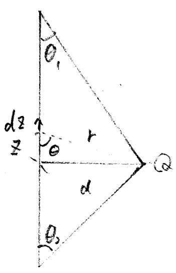
よって $\nabla \times ds = 0$

$$\nabla \frac{1}{r} = -\frac{1}{r^2} = \frac{r}{r^3}$$

$$dH = \frac{I}{4\pi} \cdot \frac{r}{r^3} \times ds$$

$$= \frac{I\, ds \times r}{4\pi r^3}$$

(2)



ビオ・サバールの法則より

$$dH = \frac{I_1 \sin\theta\, dz}{4\pi r^2}$$

$$r = \frac{d}{\sin\theta} \quad z = \frac{d}{\tan\theta}$$

$$\frac{dz}{d\theta} = -\frac{d}{\sin^2\theta}$$

$$dH = -\frac{I_1 d}{4\pi \frac{d^2}{\sin^2\theta}} \cdot \frac{\sin\theta}{\sin^2\theta}\, d\theta$$

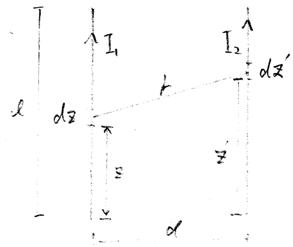
$$H = -\frac{I_1}{4\pi d}\int_{\pi-\theta_2}^{\theta_1} \sin\theta\, d\theta$$

$$= \frac{I_1}{4\pi d}(\cos\theta_1 + \cos\theta_2)$$

紙面の表から裏へと向かう方向

(3)

魔本 P.297, 256



$$r = \sqrt{d^2 + (z'-z)^2}$$

ノイマンの公式より

$$\int \frac{1}{\sqrt{a^2+x^2}}dx = \log\left(x + \sqrt{a^2+x^2}\right)$$

$$M = \frac{\mu_0}{4\pi}\int_0^l\int_0^l \frac{dz\,dz'}{r}$$

$$= \frac{\mu_0}{4\pi}\int_0^l\int_0^l \frac{1}{\sqrt{d^2+(z'-z)^2}}\, dz\, dz'$$

$$= \frac{\mu_0}{4\pi}\int_0^l \left[-\log\left\{z'-z+\sqrt{d^2+(z'-z)^2}\right\}\right]_0^l dz'$$

$$= \frac{\mu_0}{4\pi}\int_0^l \left[\log\left\{z'+\sqrt{d^2+z'^2}\right\} - \log\left\{z'-l+\sqrt{d^2+(z'-l)^2}\right\}\right] dz'$$

$$= \frac{\mu_0}{2\pi}\left\{l \log\frac{\sqrt{d^2+l^2}+l}{d} - \sqrt{d^2+l^2} + d\right\}$$

$$F = I_1 I_2 \frac{\partial M}{\partial d}$$

$$= -\frac{\mu_0 I_1 I_2}{2\pi d}\left(\sqrt{d^2+l^2} - d\right)$$

(4)

$l = \infty$ のとき (2)で $\theta_1 = \theta_2 = 0$

$$H = \frac{I_1}{4\pi d}(1+1) = \frac{I_1}{2\pi d}$$

$$F = \mu_0 H I_2$$

$$= \frac{\mu_0}{2\pi d} I_1 I_2$$