H23

[2] 続き

(4)

$$\iint_S \mathrm{rot}\,A \cdot n\, dS = \int_0^1 \int_0^{2\pi} \mathrm{rot}\,A \cdot n \left| \frac{\partial H}{\partial \theta} \times \frac{\partial H}{\partial t} \right| dt\, d\theta$$

$$= \int_0^1 \int_0^{2\pi} \mathrm{rot}\,A \cdot \left( \frac{\partial H}{\partial \theta} \times \frac{\partial H}{\partial t} \right) dt\, d\theta$$

$$= \int_0^1 \int_0^{2\pi} (2.2.2) \left\{ b(1-t)\cos\theta \cdot a(1-t)\sin\theta \right\} \cdot ab(1-t) \} \, d\theta dt$$

$$= 2 \int_0^1 (1-t)\, dt \int_0^{2\pi} (b\cos\theta + a\sin\theta + ab)\, d\theta$$

$$= 2 \left[ t - \frac{1}{2}t^2 \right]_0^1 \cdot \left[ b\sin\theta - a\cos\theta + ab\theta \right]_0^{2\pi}$$

$$= 2 \left( 1 - \frac{1}{2} \right) (-a + 2\pi ab + a)$$

$$= 2\pi ab$$

↖ ストークスの定理でも解ける。