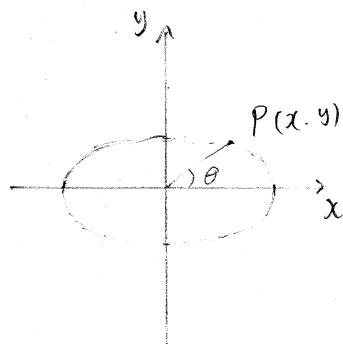


(1)



$$x = 2 \cos \theta$$

$$y = \sin \theta$$

$$P(x, y) = P(2 \cos \theta, \sin \theta)$$

$$\begin{aligned} |z| &= \sqrt{4 \cos^2 \theta + \sin^2 \theta} \\ &= \sqrt{3 \cos^2 \theta + 1} \end{aligned}$$

$$(2) \int_0^{2\pi} \frac{1}{|z|^2} d\theta$$

$$= \int_0^{2\pi} \frac{1}{3 \cos^2 \theta + 1} d\theta$$

$$z = e^{i\theta} \quad z dz = i d\theta \quad d\theta = \frac{1}{z} dz$$

$$3 \cos^2 \theta + 1 = \frac{5 + 3 \cos 2\theta}{2}$$

$$\cos 2\theta = \frac{z^2 + z^{-2}}{2}$$

$$\int_0^{2\pi} \frac{1}{|z|^2} d\theta$$

$$= \int_{|z|=1} \frac{2}{5 + \frac{3}{2}(z^2 + z^{-2})} \cdot \frac{1}{z} dz$$

$$= \frac{1}{i} \int_{|z|=1} \frac{4z}{3z^4 + 10z^2 + 3} dz$$

$$= \frac{4}{i} \int_{|z|=1} \frac{z}{(3z^2+1)(z^2+3)} dz$$

$|z|=1$  内にある特異点は

$$z = \pm \frac{1}{\sqrt{3}} i$$

$$z = \pm \frac{1}{\sqrt{3}} i \text{ だけ}$$

$$f(z) = \frac{4z}{(z + \frac{1}{\sqrt{3}}i)(z - \frac{1}{\sqrt{3}}i)(z^2+3)}$$

$$\text{Res} \left[ \frac{1}{\sqrt{3}} i \right] = \lim_{z \rightarrow \frac{1}{\sqrt{3}} i} \frac{4z}{3(z - \frac{1}{\sqrt{3}}i)(z^2+3)} = \frac{1}{4i}$$

$$\text{Res} \left[ -\frac{1}{\sqrt{3}} i \right] = \lim_{z \rightarrow -\frac{1}{\sqrt{3}} i} \frac{4z}{3(z + \frac{1}{\sqrt{3}}i)(z^2+3)} = \frac{1}{4i}$$

$$\int_{|z|=1} \frac{4}{i} \frac{z}{(3z^2+1)(z^2+3)} dz$$

$$= 2\pi i (\text{Res} \left[ \frac{1}{\sqrt{3}} i \right] + \text{Res} \left[ -\frac{1}{\sqrt{3}} i \right])$$

$$= \pi$$