H21

4

$f(x) = x\sin x$

$f(-x) = -x\sin(-x) = x\sin x$

よって偶関数

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos n\pi + b_n \cos n\pi)$

偶関数なので $b_n = 0$

$a_n = \int_{-\pi}^{\pi} f(x)\cos nx\, dx$

$= \int_{-\pi}^{\pi} x\sin x \cos nx\, dx$

$= \frac{1}{2}\int_{-\pi}^{\pi}\{x\sin(n+1)x + x\sin(n-1)x\}dx$

$= \frac{1}{2}\left\{\left[-\frac{x}{n+1}\cos(n+1)x\right]_{-\pi}^{\pi} + \int_{-\pi}^{\pi}\frac{1}{n+1}\cos(n+1)x\,dx + \left[-\frac{x}{n-1}\cos(n-1)x\right]_{-\pi}^{\pi} + \int_{-\pi}^{\pi}\frac{1}{n-1}\cos(n-1)x\,dx\right\}$

$= \frac{1}{2}\left\{-\frac{2\pi}{n+1}\cos(n+1)\pi + \left[\frac{1}{(n+1)^2}\sin(n+1)x\right]_{-\pi}^{\pi} + -\frac{2\pi}{n-1}\cos(n-1)\pi \left[\frac{1}{(n-1)^2}\sin(n-1)x\right]_{-\pi}^{\pi}\right\}$

$= -\frac{\pi}{n+1}\cos(n+1)\pi - \frac{\pi}{n-1}\cos(n-1)\pi = \begin{cases} -\frac{2\pi}{n-1} & n \neq 1,\ n\text{が奇数} \\ \frac{2\pi}{n-1} & n\text{が偶数} \end{cases}$

$= \frac{2\pi}{n-1}(-1)^n$

$a_1 = \int_{-\pi}^{\pi} x\sin x\cos x\, dx$

$= \frac{1}{2}\int_{-\pi}^{\pi} x\sin 2x\, dx$

$= \frac{1}{2}\left\{\left[-\frac{1}{2}x\sin 2x\right]_{-\pi}^{\pi} + \frac{1}{2}\int \sin 2x\, dx\right\}$

$= \frac{1}{4}\left[\frac{1}{2}\cos 2x\right]_{-\pi}^{\pi} = 0$

$a_0 = \int_{-\pi}^{\pi} x\sin x\, dx$

$= \left[-x\cos x\right]_{-\pi}^{\pi} + \left[\sin x\right]_{-\pi}^{\pi} = -2\pi$

よって

$f(x) = -\pi + \sum_{n=1}^{\infty}\frac{2\pi}{n-1}(-1)^n \cos n\pi$