H21

3

(1) $f(z) = \dfrac{z}{(z+1)^2(z^2+2z+3)}$

特異点は $z = -1,\ -1 \pm \sqrt{2}i$

$z = -1$ のとき $(z+1)f(z)$ は正則

$z = -1+\sqrt{2}i$ のとき $(z+1-\sqrt{2}i)f(z)$ は正則

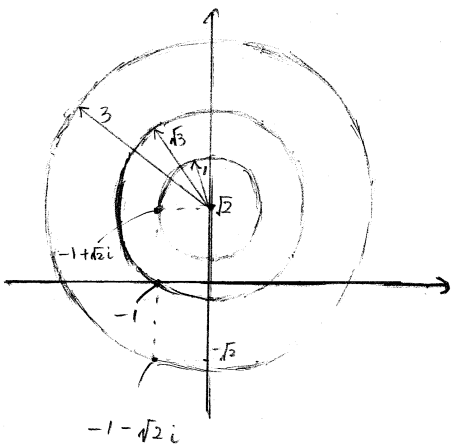
$z = -1-\sqrt{2}i$ のとき $(z+1+\sqrt{2}i)f(z)$ は正則

$$Res[-1] = \lim_{z\to -1} \frac{d}{dz}\frac{z}{z^2+2z+3} = \lim_{z\to -1} \frac{z^2+2z+3-z(2z+2)}{(z^2+2z+3)^2} = \frac{1}{2}$$

$$Res[-1+\sqrt{2}i] = \lim_{z\to -1+\sqrt{2}i} \frac{z}{(z+1)^2(z+1+\sqrt{2}i)} = -\frac{2+\sqrt{2}i}{8}$$

$$Res[-1-\sqrt{2}i] = \lim_{z\to -1-\sqrt{2}i} \frac{z}{(z+1)^2(z+1-\sqrt{2}i)} = -\frac{2-\sqrt{2}i}{8}$$

(2)



$0 < r < 1$ のとき 特異点なし

$1 < r < \sqrt{3}$ のとき $z = -1+\sqrt{2}i$

$\sqrt{3} < r < 3$ のとき $z = -1+\sqrt{2}i,\ -1$

$3 < r$ のとき $z = -1 \pm \sqrt{2}i,\ -1$

(a) $\displaystyle\int_C f(z)\,dz = 0$

(b) $\displaystyle\int_C f(z)\,dz = 2\pi i \cdot Res[-1+\sqrt{2}i] = \frac{\sqrt{2}-2i}{4}\pi$

(c) $\displaystyle\int_C f(z)\,dz = 2\pi i\left(Res[-1+\sqrt{2}i] + Res[-1]\right) = \frac{\sqrt{2}+2i}{4}\pi$

(d) $\displaystyle\int_C f(z)\,dz = 2\pi i\left(Res[-1+\sqrt{2}i] + Res[-1-\sqrt{2}i] + Res[-1]\right)$

$= 0$