H18

3 続き

(2)

$$\frac{f(z)}{z^2} = \frac{1}{z^3} + \frac{\frac{\pi^2}{3}}{z} + \cdots$$

$$= \frac{1}{z^3} + \frac{\pi^2}{3z} + \cdots$$

$\frac{1}{z-0} = \frac{1}{z}$ の係数より

$$Res\left[\frac{f(z)}{z^2}, 0\right] = \frac{\pi^2}{3}$$

(3)

$$\left(\pi^2 + \frac{2\pi i}{z} + \frac{3}{z^2}\right)\left(\frac{1}{z} - \frac{\pi^2}{3}z + \cdots\right)$$

$$= \left(\frac{\pi^2}{z} - \frac{\pi^4}{3}z + \cdots\right) + \left(\frac{2\pi i}{z^2} - \frac{2\pi^3 i}{3} + \cdots\right) + \left(\frac{3}{z^3} - \frac{\pi^2}{z} + \cdots\right)$$

$$= \left(\frac{\pi^2}{z} - \frac{\pi^2}{z}\right) + \cdots$$

$\frac{1}{z}$ の係数が 0 なので

$$Res\left[\left(\pi^2 + \frac{2\pi i}{z} + \frac{3}{z^2}\right) f(z), 0\right] = 0$$

よって

$$\int_C \left(\pi^2 + \frac{2\pi i}{z} + \frac{3}{z^2}\right) f(z)\, dz = 0$$