H 26

$\boxed{4}$

(1)

$$\mathcal{L}[e^{at} f(t)]_{(s)} = \int_0^\infty e^{(a-s)t} f(t)\, dt$$

$$\mathcal{L}[f(t)]_{(s-a)} = \int_0^\infty f(t)\, e^{-(s-a)t}\, dt = \int_0^\infty e^{(a-s)t} f(t)\, dt$$

よって

$$\mathcal{L}[e^{at} f(t)]_{(s)} = \mathcal{L}[f(t)]_{(s-a)}$$

(2)

$$\mathcal{L}[\cos at]_{(s)} = \int_0^\infty e^{-st} \cos at\, dt$$

$$= \left[-\frac{1}{s} e^{-st} \cos at\right]_0^\infty - \int_0^\infty \frac{a}{s} e^{-st} \sin at\, dt$$

$$= \frac{1}{s} + \left[\frac{a}{s^2} e^{-st} \sin at\right]_0^\infty - \int_0^\infty \frac{a^2}{s^2} e^{-st} \cos at\, dt$$

$-\frac{a^2}{s^2}\mathcal{L}[\cos at]_{(s)}$

$$\left(1 + \frac{a^2}{s^2}\right) \mathcal{L}[\cos at]_{(s)} = \frac{1}{s}$$

$$\mathcal{L}[\cos at]_{(s)} = \frac{s}{s^2 + a^2}$$

$$\mathcal{L}[\sin at]_{(s)} = \int_0^\infty e^{-st} \sin at\, dt$$

$$= \left[-\frac{1}{s} e^{-st} \sin at\right]_0^\infty + \int_0^\infty \frac{a}{s} e^{-st} \cos at\, dt$$

$$= 0 + \left[-\frac{a}{s^2} e^{-st} \cos at\right]_0^\infty - \int_0^\infty \frac{a^2}{s^2} e^{-st} \sin at\, dt$$

$$\left(1 + \frac{a^2}{s^2}\right) \mathcal{L}[\sin at] = \frac{a}{s^2}$$

$$\mathcal{L}[\sin at] = \frac{a}{s^2 + a^2}$$