H2.

$\boxed{1}$

(1) $\frac{dy}{dx} + 2y = xe^{-3x}$

$y' + 2y = xe^{-3x}$

両辺に e^{2x} をかけて

$(e^{2x}y)' = xe^{-x}$

積分して

$e^{2x}y = -(x+1)e^{-x} + c$

$y = ce^{-2x} - (x+1)e^{-3x}$

(2) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} = 2x^2 + x$

$s(s+4) = 0$

$s = 0, -4$

$y = C_1 + C_2e^{-4x} + \eta$

$\eta = (Ax^2 + Bx + C)x$

$(6Ax + 2B) + 4(3Ax^2 + 2Bx + C) = 2x^2 + x$

$\begin{cases} 12A = 2 & A = \frac{1}{6} \\ 6A + 8B = 1 & B = 0 \\ 2B + 4C = 0 & C = 0 \end{cases}$

$\eta = \frac{1}{6}x^3$

よって

$y = C_1 + C_2e^{-4x} + \frac{1}{6}x^3$

(3) $\frac{d^2y}{dx^2} + \frac{dy}{dx} + 2y = 3\sin x$

$s^2 + s + 2 = 0$

$s = -\frac{1}{2} \pm \frac{\sqrt{7}}{2}i$

$y = e^{-\frac{1}{2}}(C_1\cos\frac{\sqrt{7}}{2}x + C_2\sin\frac{\sqrt{7}}{2}x) + \eta$

$\eta = A\cos x + B\sin x$

$-A\cos x - B\sin x - A\sin x + B\cos x + 2A\cos x + 2B\sin x = 3\sin x$

$\begin{cases} -A + B + 2A = 0 & B = \frac{3}{2} \\ -B - A + 2B = 3 & A = -\frac{3}{2} \end{cases}$

$\eta = -\frac{3}{2}\cos x + \frac{3}{2}\sin x$

$y = e^{-\frac{1}{2}}(C_1\cos\frac{\sqrt{7}}{2}x + C_2\sin\frac{\sqrt{7}}{2}x) - \frac{3}{2}\cos x + \frac{3}{2}\sin x$

(4) $\left(\frac{x}{y^2} - \frac{y}{x^2}\right)dx + \left(\frac{1}{x} - \frac{x^2}{y^3}\right)dy = 0$

$\frac{\partial}{\partial y}\left(\frac{x}{y^2} - \frac{y}{x^2}\right) = -2\frac{x}{y^3} - \frac{1}{x^2}$

$\frac{\partial}{\partial x}\left(\frac{1}{x} - \frac{x^2}{y^3}\right) = -\frac{1}{x^2} - 2\frac{x}{y^3}$

よって完全微分形

$d\left(\frac{1}{2}\frac{x^2}{y^2} + \frac{y}{x}\right) = 0$

$\frac{1}{2}\frac{x^2}{y^2} + \frac{y}{x} = c$