H27 数学

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(1)

(a) $s^2+3s-10=0$

$(s+5)(s-2)=0$

$s=2,\ -5$

$\eta = Ax+B$

$3A-10(Ax+B)$

$=-10Ax+3A-10B=10x$

$\begin{cases} -10A=10 & A=-1 \\ 3A-10B=0 & B=-\frac{3}{10} \end{cases}$

$\eta = -x-\frac{3}{10}$

$y=C_1e^{2x}+C_2e^{-5x}-x-\frac{3}{10}$

(b) $(x-1)(y^2-9)\,dx-(x^2+1)(y+3)\,dy=0$

$\frac{x-1}{x^2+1}\,dx=\frac{y+3}{y^2-9}\,dy$

$\left(\frac{x}{x^2+1}-\frac{1}{x^2+1}\right)dx=\left(\frac{3}{y^2-9}+\frac{2}{y^2-9}\right)dy$

$x=\tan\theta$ とおく

$\int\frac{x}{x^2+1}\,dx-\int 1\,d\theta=\int\frac{y}{y^2-9}dy+\frac{1}{3}\int\left(\frac{1}{y-3}-\frac{1}{y+3}\right)dy$

$\frac{1}{2}\log(x^2+1)-\tan^{-1}x=\frac{1}{2}\log(y^2-9)+\frac{1}{3}\left\{\log(y-3)-\log(y+3)\right\}$

$3\log(x^2+1)-6\tan^{-1}x=\log\frac{(y^2-9)^3(y-3)^2}{(y+3)^2}=\log(y+3)(y-3)^5$

$e^{\log(y+3)(y-3)^5}=e^{3\log(x^2+1)-6\tan^{-1}x+C}$

$(y+3)(y-3)^5=(x^2+1)^3\cdot Ce^{-6\tan^{-1}x}$