2013
数学

1

(1) $\varepsilon^{\int \cos x\,dx} = \varepsilon^{\sin x}$ を両辺にかけて

$(\varepsilon^{\sin x} y)' = \varepsilon^{\sin x} \sin 2x$

$\Leftrightarrow \varepsilon^{\sin x} y = 2\left\{ \varepsilon^{\sin x} \sin x - \int \varepsilon^{\sin x} \cos x\,dx \right\}$

$= 2\varepsilon^{\sin x} \sin x - 2\varepsilon^{\sin x} + C_1$

$y = 2\sin x - 2 + C_1 \varepsilon^{-\sin x}$

(2) $y_t'' + 6y_t' + 11y_t = 0$

$s^2 + 6s + 11 = 0$

$s = -3 \pm i\sqrt{2}$

$y_t = \varepsilon^{-3x}(C_1 \cos\sqrt{2}x + C_2 \sin\sqrt{2}x)$

(3) $y_{s1}'' + 6y_{s1}' + 11y_{s1} = 11x$

$y_{s1} = ax + b$ とおいて

$11(ax + b) + 6a = 11x$

$a = 1,\quad b = -\frac{6}{11}$

$y_{s1} = x - \frac{6}{11}$

$y = y_t + y_{s1}$

$= \varepsilon^{-3x}(C_1 \cos\sqrt{2}x + C_2 \sin\sqrt{2}x) + x - \frac{6}{11}$

(4) $y_{s2}'' + 6y_{s2}' + 11y_{s2} = \sin x$

$y_{s2} = a\cos x + b\sin x$ とおいて

$11(a\cos x + b\sin x)$

$+6(-a\sin x + b\cos x)$

$+(-a\cos x - b\sin x) = \sin x$

$10b - 6a = 1$

$10a + 6b = 0$ $\quad a = -\frac{3}{68},\quad b = \frac{5}{68}$

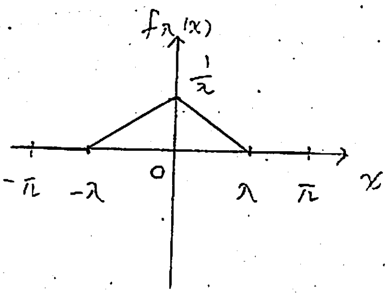
$y_{s2} = -\frac{3}{68}\cos x + \frac{5}{68}\sin x$

$y = y_t + y_{s2}$

$= \varepsilon^{-3x}(C_1 \cos\sqrt{2}x + C_2 \sin\sqrt{2}x) - \frac{3}{68}\cos x + \frac{5}{68}\sin x$

2

(1)




(2) 図より $f_\lambda(x)$ は偶関数である

$T = 2\pi,\quad w_0 = 1$ より

$a_n = \frac{4}{T}\int_0^{\frac{T}{2}} f_\lambda(x) \cos n w_0 x\,dx$

$= \frac{2}{\pi}\int_0^{\lambda}\left(-\frac{1}{\lambda^2}x + \frac{1}{\lambda}\right)\cos nx\,dx$

$= -\frac{2}{\lambda^2\pi}\left\{\left[\frac{x}{n}\sin nx\right]_0^{\lambda} - \frac{1}{n}\left[-\frac{1}{n}\cos nx\right]_0^{\lambda}\right\} + \frac{2}{\lambda\pi}\left[\frac{1}{n}\sin nx\right]_0^{\lambda}$

$= -\frac{2}{\lambda\pi n}\sin(\lambda n) - \frac{2}{\lambda^2\pi n^2}(\cos(\lambda n) - 1) + \frac{2}{\lambda\pi n}\sin(\lambda n)$

$= \frac{2}{\lambda^2\pi n^2}\{1 - \cos(\lambda n)\} \quad (n \neq 0)$

$a_0 = \frac{2}{\pi}\int_0^{\lambda}\left(-\frac{1}{\lambda^2}x + \frac{1}{\lambda}\right)dx$

$= \frac{2}{\pi}\left[-\frac{1}{2\lambda^2}x^2 + \frac{1}{\lambda}x\right]_0^{\lambda}$

$= \frac{1}{\pi}$

$f_\lambda(x) = \frac{1}{2\pi} + \sum_{n=1}^{\infty}\left\{\frac{2}{\lambda^2\pi n^2}(1 - \cos\lambda n)\cos nx\right\}$

(3) $\lim_{\lambda\to+0} a_n(\lambda) = \lim_{\lambda\to+0}\frac{1}{\pi}\cdot\frac{4}{\lambda^2 n^2}\sin^2\left(\frac{\lambda n}{2}\right)$

$= \lim_{\lambda\to+0}\frac{1}{\pi}\cdot\frac{\sin\left(\frac{\lambda n}{2}\right)}{\left(\frac{\lambda n}{2}\right)}\cdot\frac{\sin\left(\frac{\lambda n}{2}\right)}{\left(\frac{\lambda n}{2}\right)}$

$= \frac{1}{\pi}$