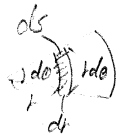
H23

問1

(1) 面上電荷分布より

$$\phi = \frac{1}{4\pi\varepsilon_0}\iint \frac{\sigma}{\sqrt{r^2+z^2}}\,dS$$



$$dS = r\,dr\,d\theta$$

$$\phi = \frac{1}{4\pi\varepsilon_0}\int_0^{2\pi}\int_a^b \frac{\sigma}{\sqrt{r^2+z^2}}\,r\,dr\,d\theta$$

$$= \frac{\sigma}{4\pi\varepsilon_0}\int_0^{2\pi}\left[\sqrt{r^2+z^2}\right]_a^b d\theta$$

$$= \frac{\sigma}{4\pi\varepsilon_0}\cdot 2\pi\left(\sqrt{b^2+z^2}-\sqrt{a^2+z^2}\right)$$

$$= \frac{\sigma}{2\varepsilon_0}\left(\sqrt{b^2+z^2}-\sqrt{a^2+z^2}\right)\ [V]$$

(2)

$$E = -\mathrm{grad}\,V$$

z方向のみしかないので

$$E = -\frac{\partial V}{\partial z}k$$

$$= \frac{\sigma}{2\varepsilon_0}\left(\frac{z}{\sqrt{a^2+z^2}}-\frac{z}{\sqrt{b^2+z^2}}\right)\ [V/m]$$

$b > a$ より $E > 0$ z軸 正の方向

(3) $F = qE$

$$= \frac{Q\sigma}{2\varepsilon_0}\left(\frac{z}{\sqrt{a^2+z^2}}-\frac{z}{\sqrt{b^2+z^2}}\right)\ [N]$$

z軸 正の方向

(4) $F = 0$

$z = 0$