H19

II

(1) $\frac{dy}{dx} = (2-y)y$

$\frac{1}{(2-y)y}\,dy = dx$

$\frac{1}{2}\left(\frac{1}{2-y} + \frac{1}{y}\right)dy = dx$

$\frac{1}{2}\{-\log(2-y) + \log y\} = x + C$

$\log\frac{y}{2-y} = 2x + C$

$\frac{y}{2-y} = Ce^{2x}$

$(1+Ce^{2x})y = 2Ce^{2x}$

$y = \frac{2Ce^{2x}}{1+Ce^{2x}} = \frac{2C}{C+e^{-2x}} = \frac{2}{1+Ce^{-2x}}$

$y = 1$ のとき $x = 0$ より

$C = 1$

$y = \frac{2e^{2x}}{1+e^{2x}} = \frac{2}{1+e^{-2x}}$

(2) $s^2 - 2s + 5 = 0$

$s = -1 \pm 2i$

$y = e^{-x}(C_1\cos 2x + C_2\sin 2x)$

$y' = e^{-x}\{(C_1+2C_2)\cos 2x + (C_2-2C_1)\sin 2x\}$

$x = 0$ のとき $y = 1$, $y' = 2$ より

$\begin{cases} C_1 = 1 \\ C_1 + 2C_2 = 2 \end{cases}$ $C_2 = \frac{1}{2}$

$y = e^{-x}\left(\cos 2x + \frac{1}{2}\sin 2x\right)$

(3) $s^2 - 2s + 5 = 0$

$s = -1 \pm 2i$

$\eta = Ae^{x}$

$Ae^{x} - 2Ae^{x} + 5Ae^{x} = e^{x}$

$4A = 1 \quad A = \frac{1}{4}$

$\eta = \frac{1}{4}e^{x}$

$y = e^{-x}(C_1\cos 2x + C_2\sin 2x) + \frac{1}{4}e^{x}$

$y' = e^{-x}\{(C_1+2C_2)\cos 2x + (C_2-2C_1)\sin 2x\} + \frac{1}{4}e^{x}$

$x = 0$ のとき $y = 2$, $y' = 3$ より

$\begin{cases} C_1 + \frac{1}{4} = 2 \rightarrow C_1 = \frac{7}{4} \\ C_1 + 2C_2 + \frac{1}{4} = 3 \rightarrow C_2 = \frac{1}{2} \end{cases}$

$y = e^{-x}\left(\frac{7}{4}\cos 2x + \frac{1}{2}\sin 2x\right) + \frac{1}{4}e^{x}$

$= \frac{1}{4}e^{-x}(7\cos 2x + 2\sin 2x + 1)$