An Overview of the Chern-Simons Interaction

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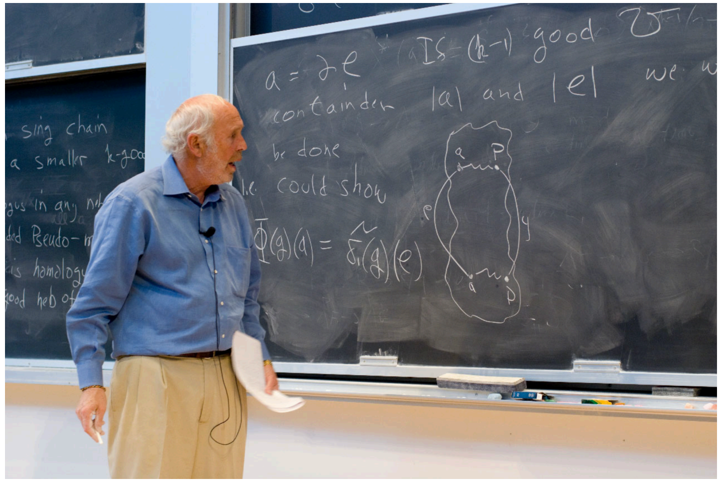
Annual Racah Memorial Lecture

SIMONS CENTER
FOR GEOMETRY AND PHYSICS

My reasons for giving this talk are fourfold:

- The recent passing of Jim Simons, who did a great deal for science both in terms of his original contributions and also in terms of philanthropy.



- The growing importance of the Chern–Simons interaction for physics. There are **multiple** physics papers **every day** mentioning the Chern–Simons interaction.

- To explain my own interest in the field and some contributions I was involved in.

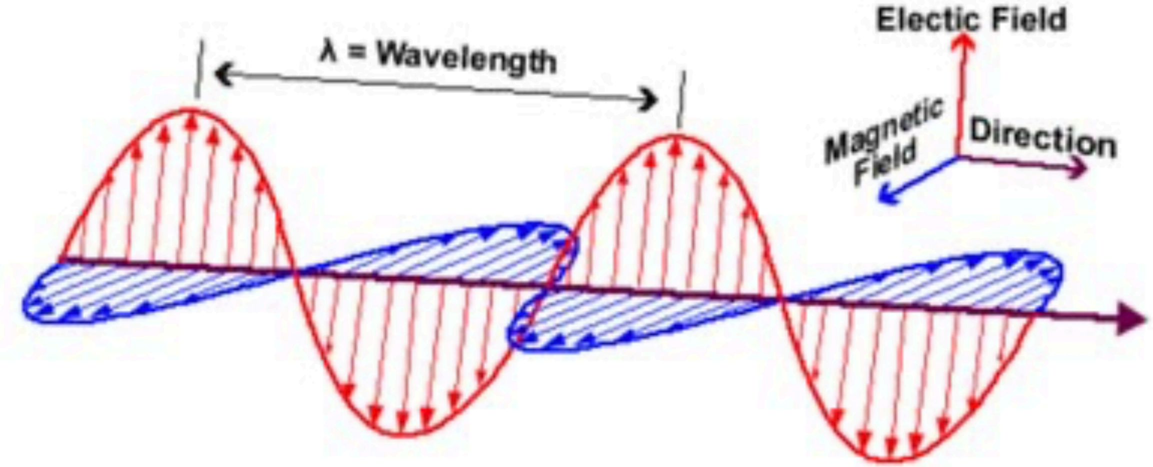
- Remarkably there is a deep connection to the Racah coefficients (but I won't have time to explain it).

$$\vec{\nabla} \cdot \vec{E} = \rho \qquad \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = \frac{-1}{c} \frac{\partial \vec{B}}{\partial t} \qquad \vec{\nabla} \times \vec{B} = \frac{1}{c} \frac{\partial \vec{E}}{\partial t} + \frac{1}{c} \vec{J}$$

- The Maxwell equations describe the electromagnetic fields produced by charges.

- There are electromagnetic waves with two distinct polarizations.



In 2 space dimensions the situation is not VERY different except that the magnetic field is a scalar i.e. has no arrow.

For any vector $\vec{v} = (v_x, v_y)$ we can define a ``dual'' vector $\tilde{\vec{v}} = (v_y, -v_x)$. We can do the same with the gradient operator and the electric field etc.

This notation is useful when doing physics in 2 space dimensions.

The Maxwell equations in 2 space dimensions are just

$$\vec{\nabla} \cdot \vec{E} = \rho \qquad \tilde{\vec{\nabla}} \cdot \vec{E} = \frac{\partial B}{\partial t} \qquad \tilde{\vec{\nabla}} B = \frac{\partial \vec{E}}{\partial t} + \vec{J}$$

We have ``lost’’ one equation.
The phenomenology of electromagnetic waves is slightly different now. We have only one polarization. It is transverse to the direction of propagation.

The surprise is that the equations admit an apparently consistent modification !!

The new Chern-Simons term.

$$\vec{\nabla} \cdot \vec{E} + \kappa B = \rho$$

$$\tilde{\vec{\nabla}} \cdot \vec{E} = \frac{\partial B}{\partial t}$$

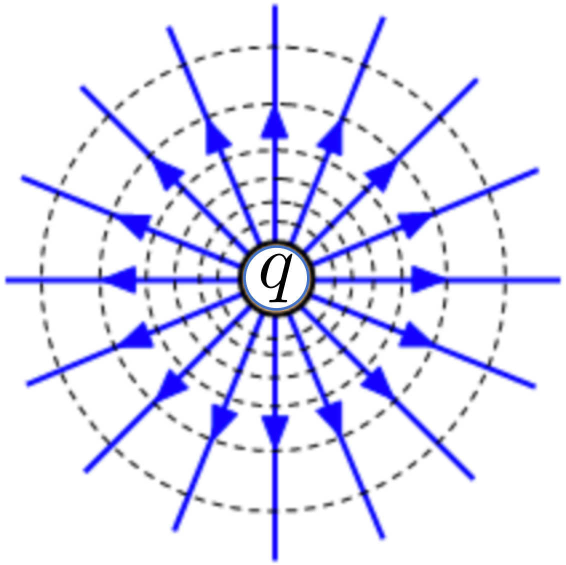
$$\tilde{\vec{\nabla}} B + \kappa \tilde{\vec{E}} = \frac{\partial \vec{E}}{\partial t} + \vec{J}$$

This kind of modification is not possible in 3 space dimensions.

In the math literature the Chern-Simons term arose from differential geometry.

In the physics literature the Chern-Simons' term effect on electromagnetism was first studied by Deser-Jackiw-Templeton.

- The waves are not moving at the speed of light any longer.
- Static charges are producing an electric field which decays exponentially away from the sources.

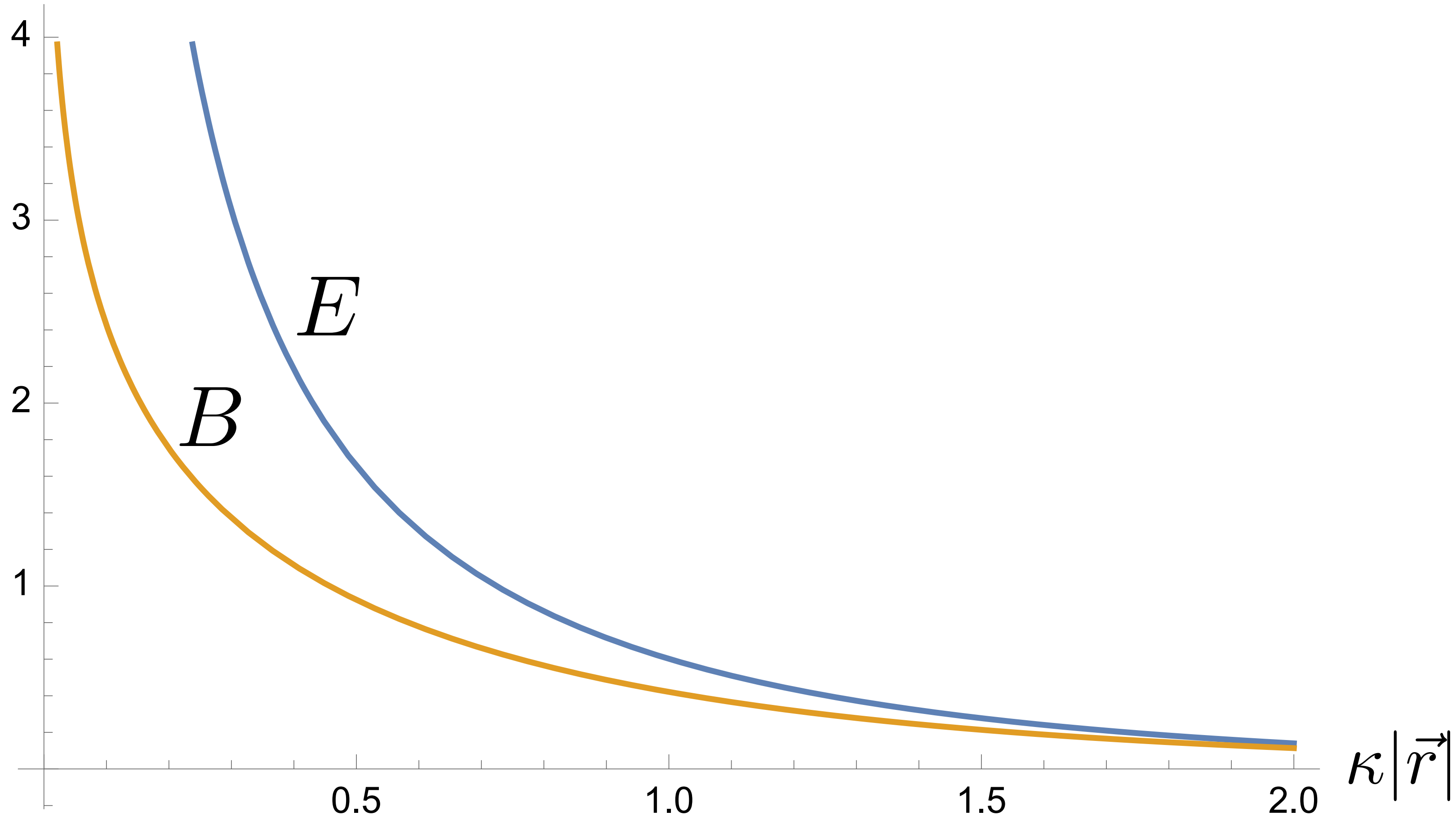


$$\vec{E} \sim e^{-\kappa|\vec{r}|}\hat{r}$$

More interestingly the localized charge produces a magnetic field as well. It decays asymptotically in the same manner as the electric field $B \sim e^{-\kappa|\vec{r}|}$.

$$\vec{E} = \frac{q\kappa}{2\pi} K_1(\kappa|\vec{r}|)\hat{r} \qquad\qquad \vec{B} = \frac{q\kappa}{2\pi} K_0(\kappa|\vec{r}|)$$

At short distances the magnetic field blows up logarithmically while the electric field is a standard Coulomb field at short distances.



4
3
2
1
E
B
0.5
1.0
1.5
2.0
$\kappa|\vec{r}|$

An important question is how much total magnetic flux there is. Using the usual Gauss law, from $\vec{\nabla} \cdot \vec{E} + \kappa B = \rho$ it follows that

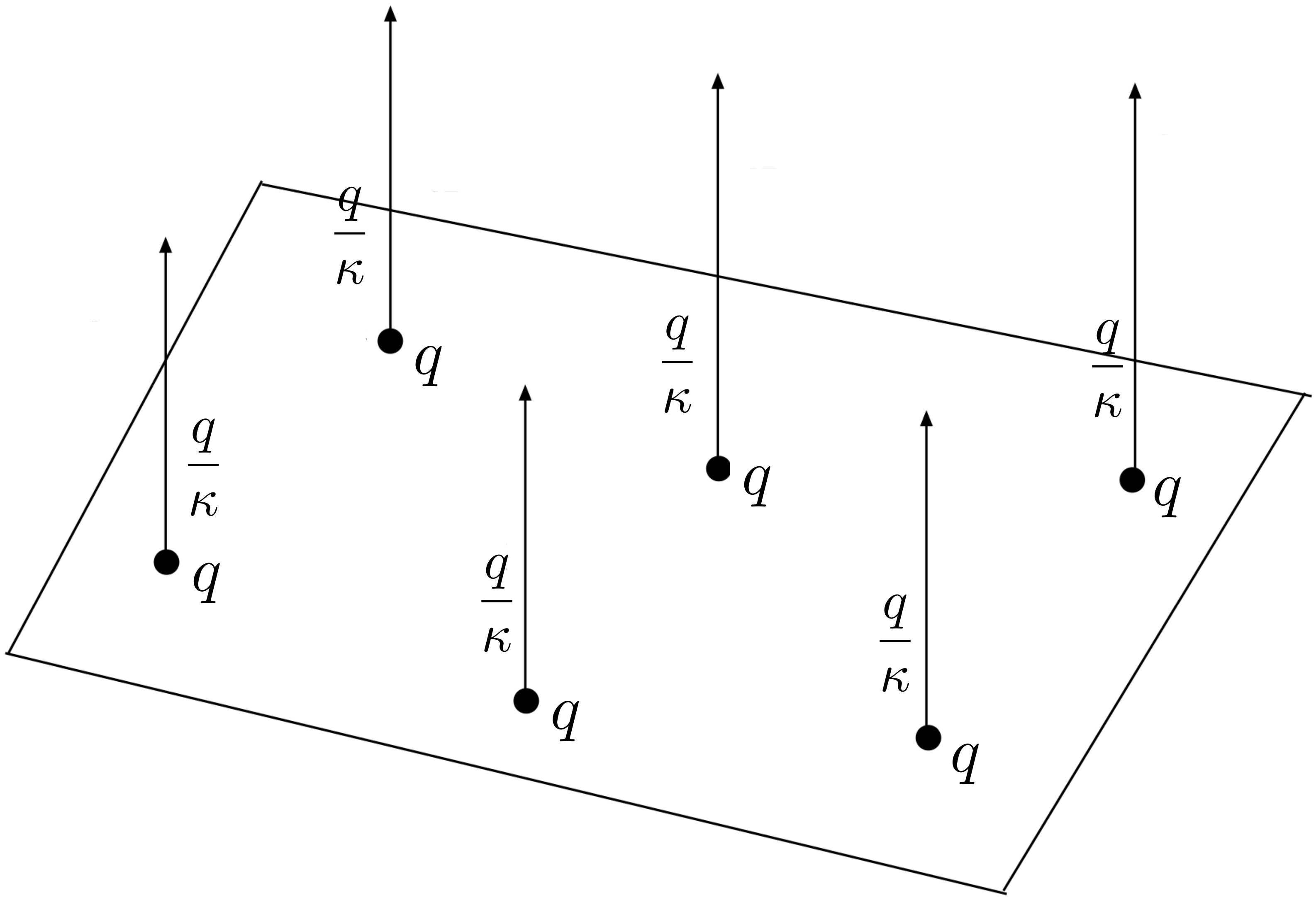
$$\int_{\mathbb{R}^2} B = \frac{q}{\kappa}$$

In the classical theory, κ is arbitrary and there is not much beyond what we described here.

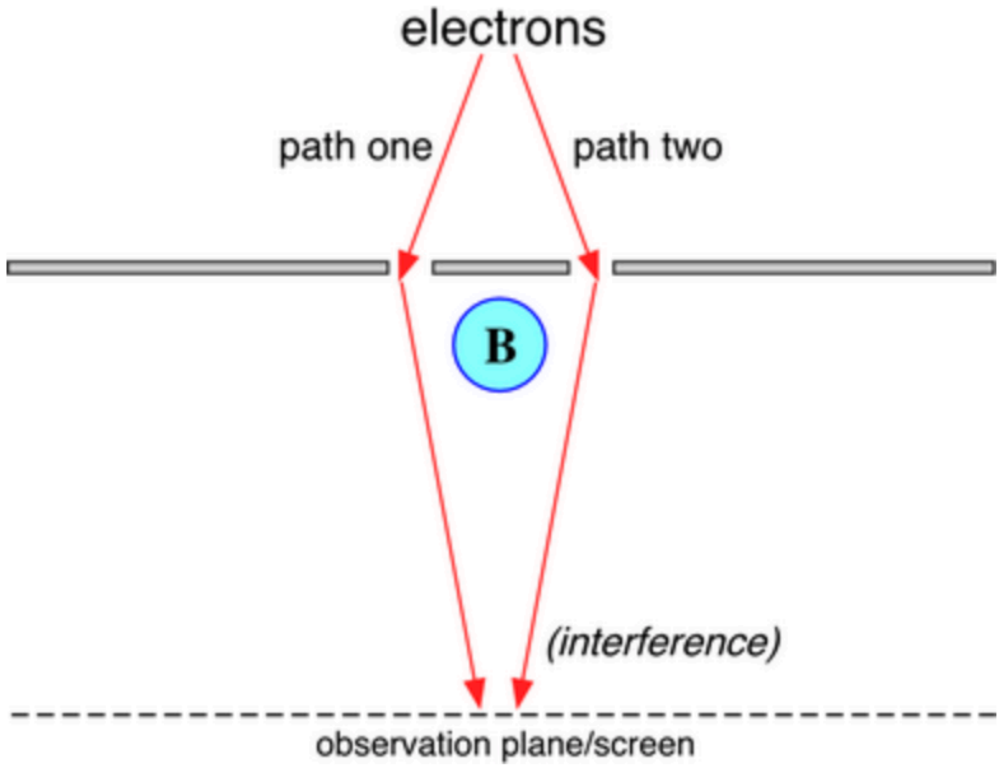
However in the quantum theory the hero is the localized magnetic flux through the charged particle

It physically means that every particle becomes effectively a fat solenoid, and solenoids lead to long-range effects in the quantum theory .

The arrows are pointing ''outside'' of the two-dimensional space.



Quantum mechanically, solenoids are very important, and they are visible from afar due to the (Dirac-) Aharonov-Bohm effect.



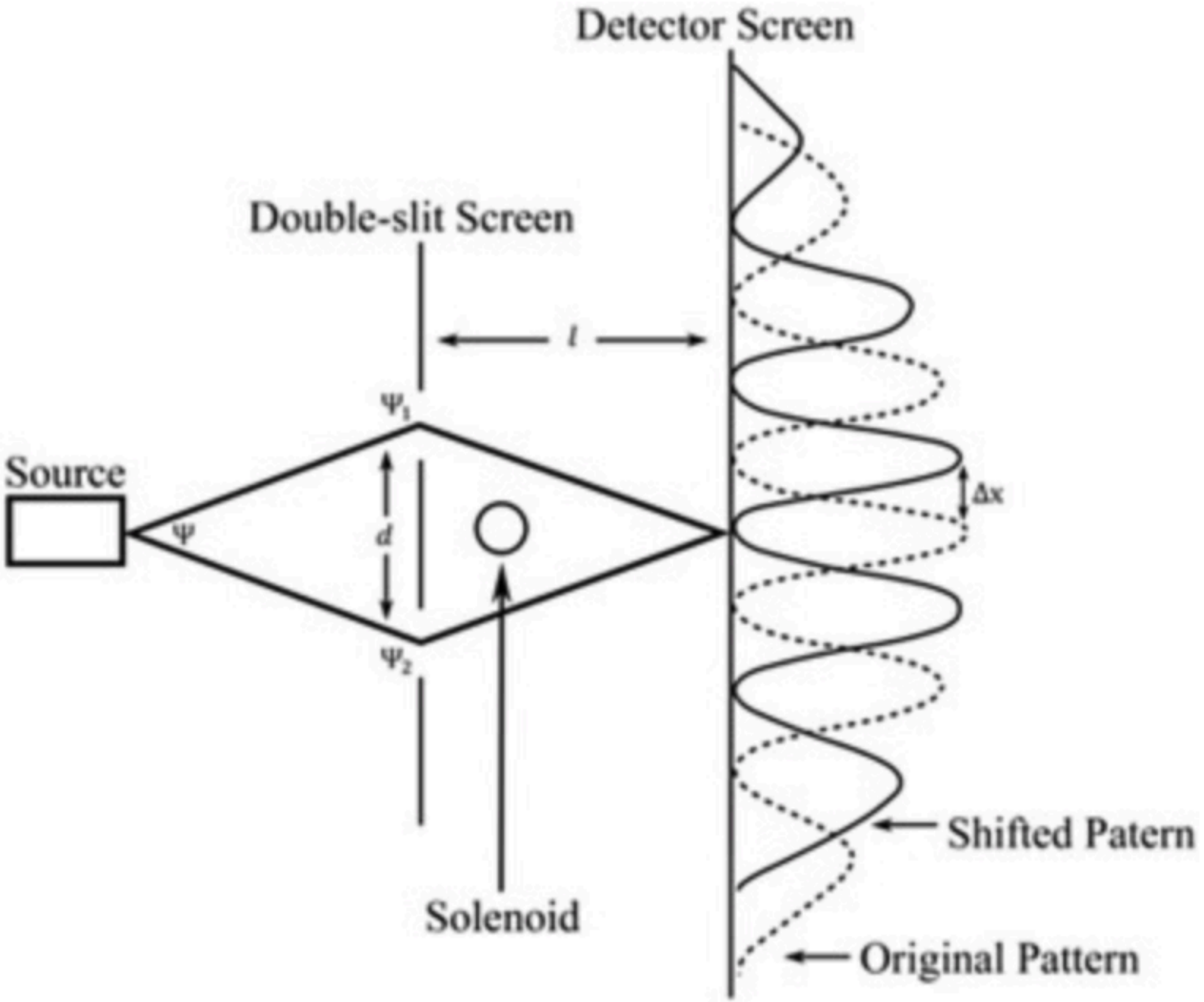
The solenoid's effect is related to the total magnetic flux but it trivializes for

$$\int B = \frac{2\pi}{e} N \ , \quad N \in \mathbb{Z}$$

With e the elementary electric charge.

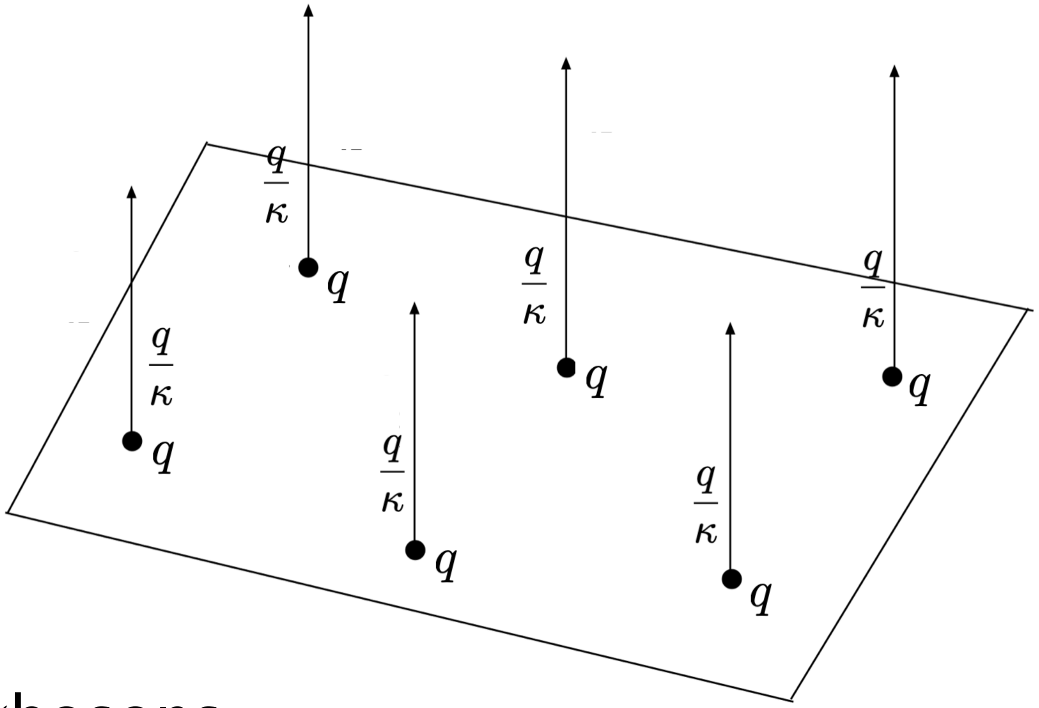
That is because the phase from going around the solenoid with a particle of charge e is

$$e^{ie \int B}$$



Let us take $q = e$, the fundamental charge.
Then the wave function is multi-valued.
When we take a particle around any other one we find a phase

$$e^{\frac{1}{\hbar} ie \int B} = e^{\frac{ie^2}{\hbar\kappa}}$$



$\Psi(x_1, ..., x_N)$ is then a wave function of ``anyons.'' The anyons have fractional spin

$$s = \frac{e^2}{4\pi\hbar\kappa}$$

Generalizing the usual statistic of fermions/bosons.

$$s = \frac{e^2}{4\pi\hbar\kappa}$$

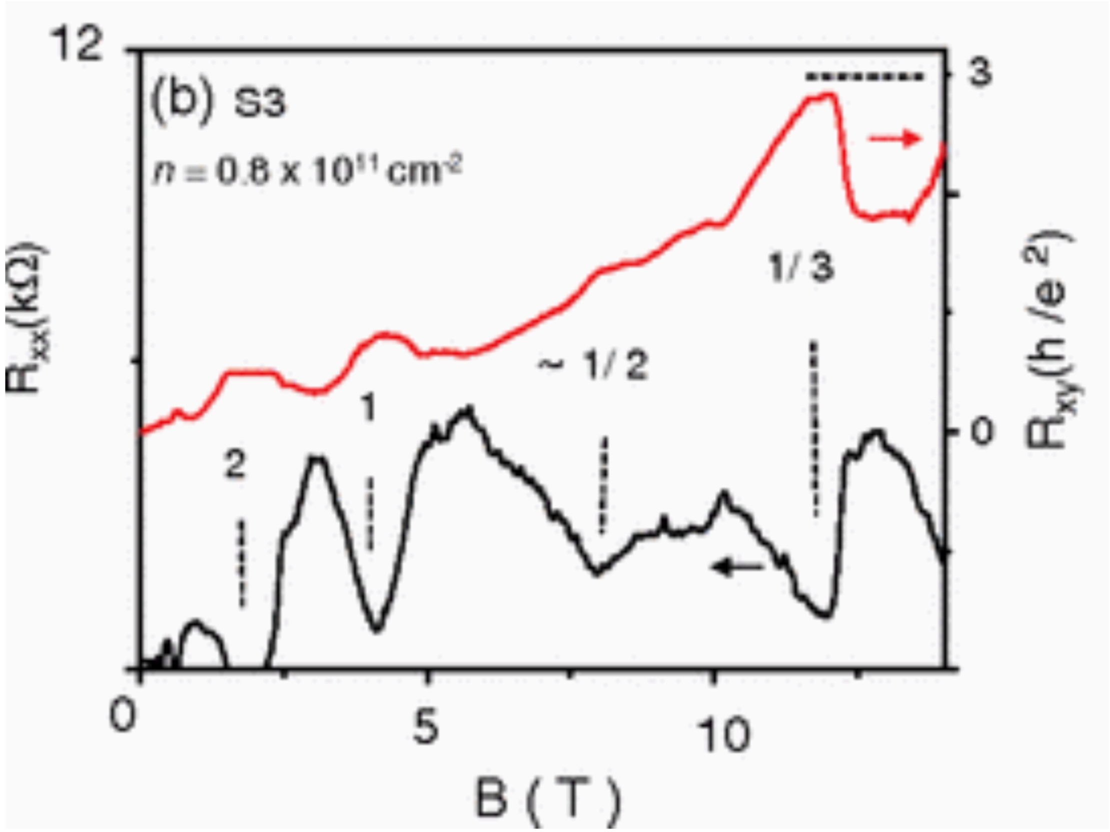
By studying the behavior of the theory in compact spaces, such as a donut, it is possible to prove that

$$\kappa = \frac{e^2}{2\pi\hbar}n \ , \quad n \in \mathbb{Z}$$

In particular, if bosons are coupled to a Chern-Simons term with $n = 1$ they essentially become fermions and vice-versa.

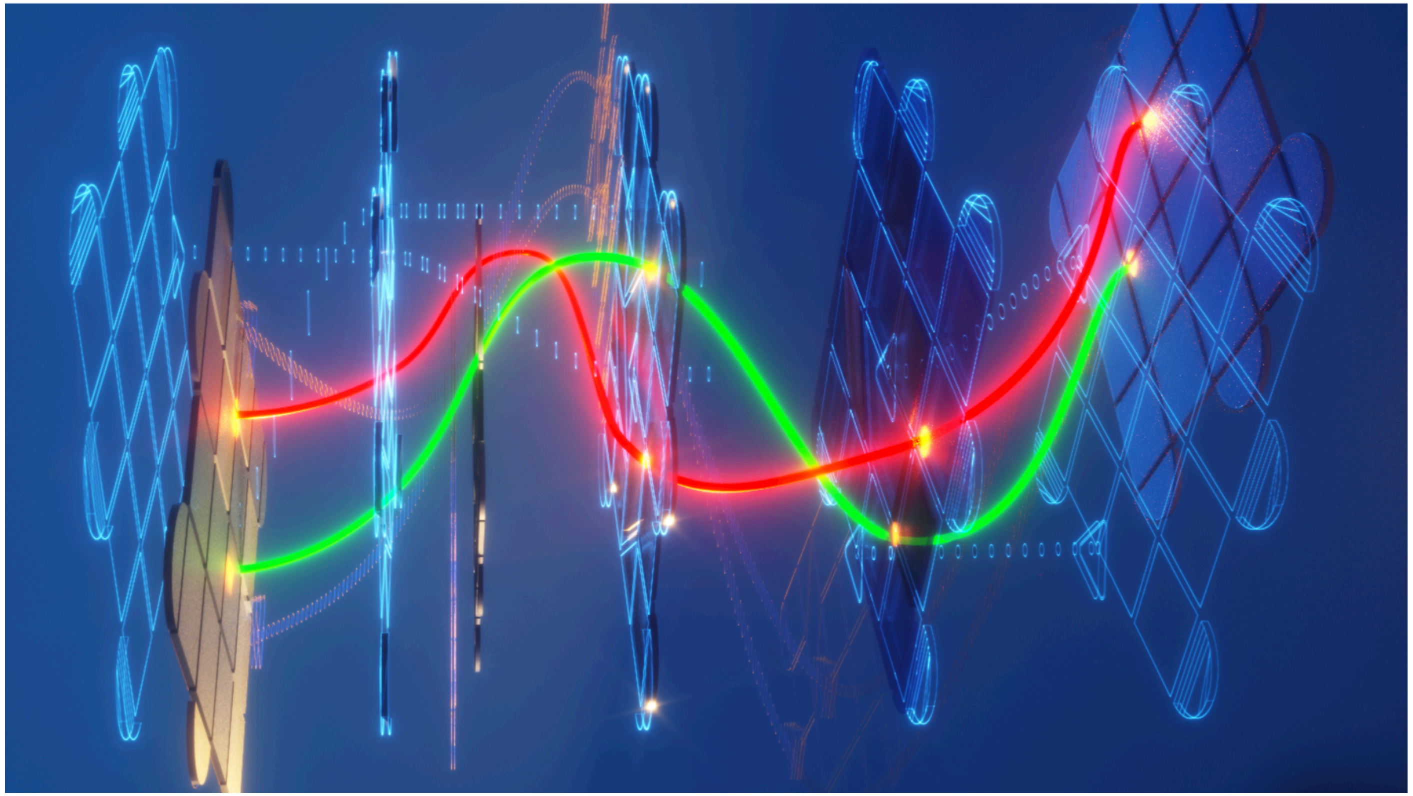
Before we go on there are two crucial remarks to put our discussion in context.

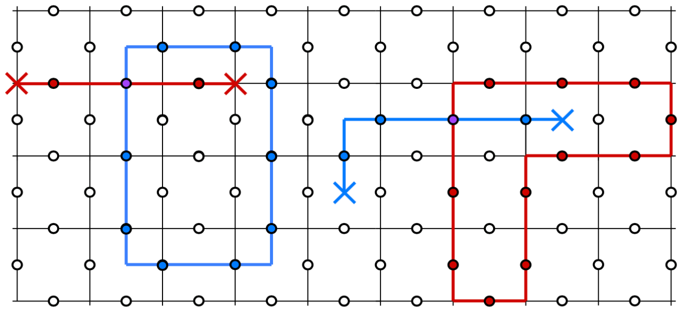
- Famously, the fractional quantum Hall effect leads to particles with fractional spin and charge. This is because a Chern-Simons gauge field emerges from strongly interacting electrons in a magnetic field.



From [Ghahari et al.]

- Anyons are very important for quantum computers because their long range statistical interaction is very hard to destroy. One can construct quantum gates by manipulating anyons.

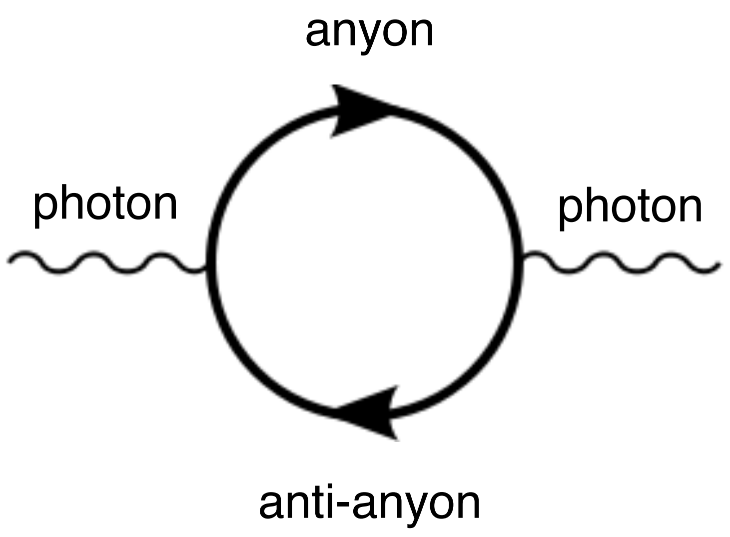




Kitaev's code

So far our anyons were heavy external charges that we could manipulate. Can we make the theory of anyons relativistically invariant?

As usual the price to pay is that we should allow anyons to be created and destroyed from the vacuum.

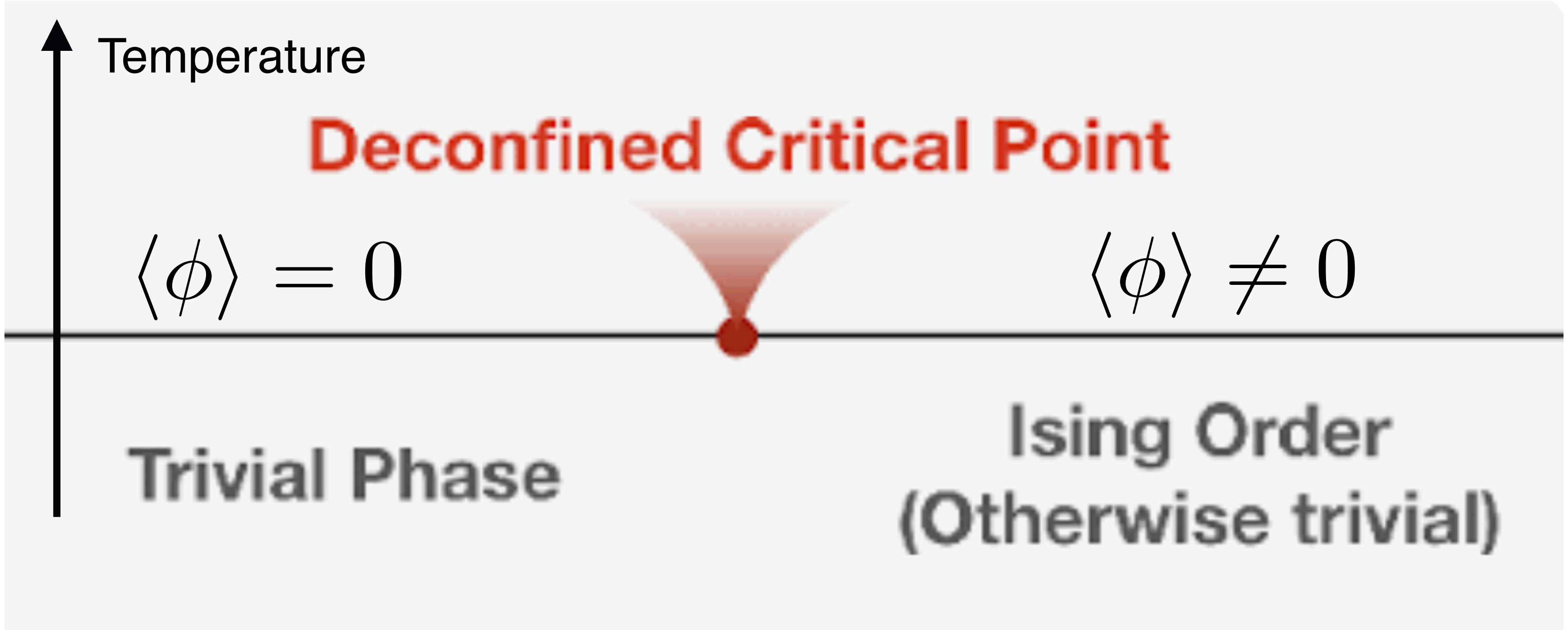


This leads to a vast generalization of the relativistic quantum theories in $2+1$ dimensions — anyonic particles with various mutual interactions. They are sometimes called ``Chern-Simons-Matter'' theories.

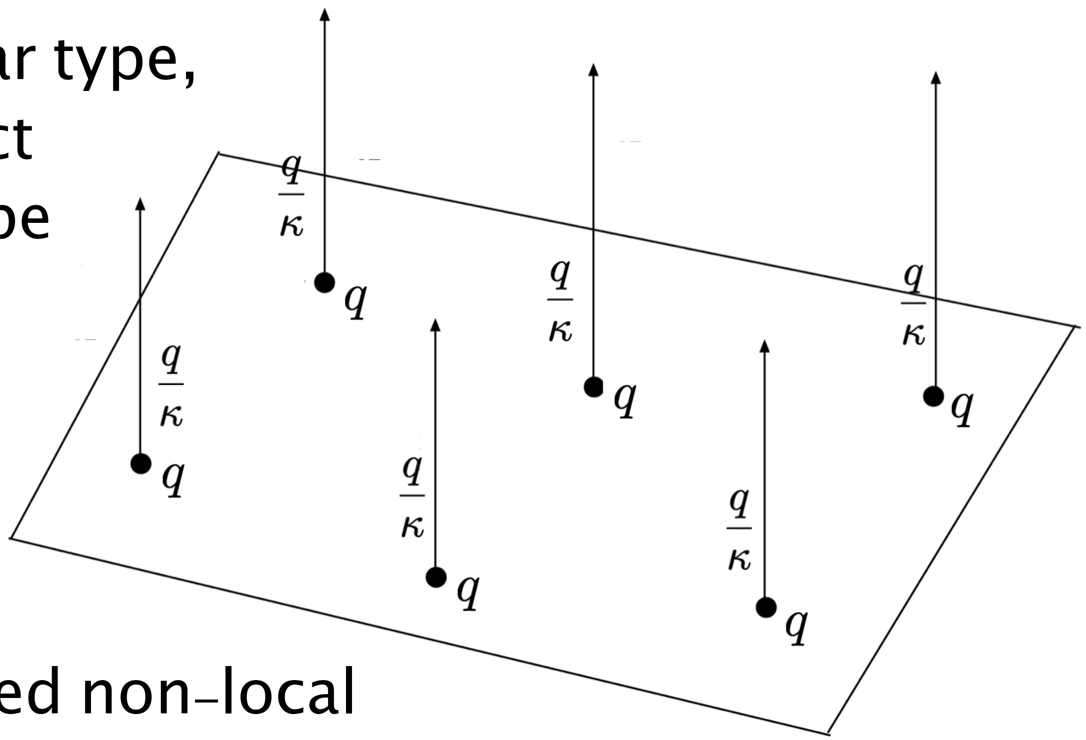
The regime where the anyons are very heavy goes back to our previous discussion. But in addition,

- The anyons can condense,
- They can become gapless (massless),
- They can confine,
- etc.

In the understanding of possible phases of matter (or possible phases of QFT) an important concept is the Landau-Ginzburg paradigm.



The phase with anyons is not of a familiar type, since no local order parameter can detect the existence of this Aharonov-Bohm type order.



In modern literature people have discussed non-local order parameters extending the Landau-Ginzburg paradigm. The Chern-Simons phase is an ordered phase for a non-local order parameter. It is called **topological order**.

An important example is QED3 — namely N_f species of fermions coupled to a gauge field with Chern-Simons terms.

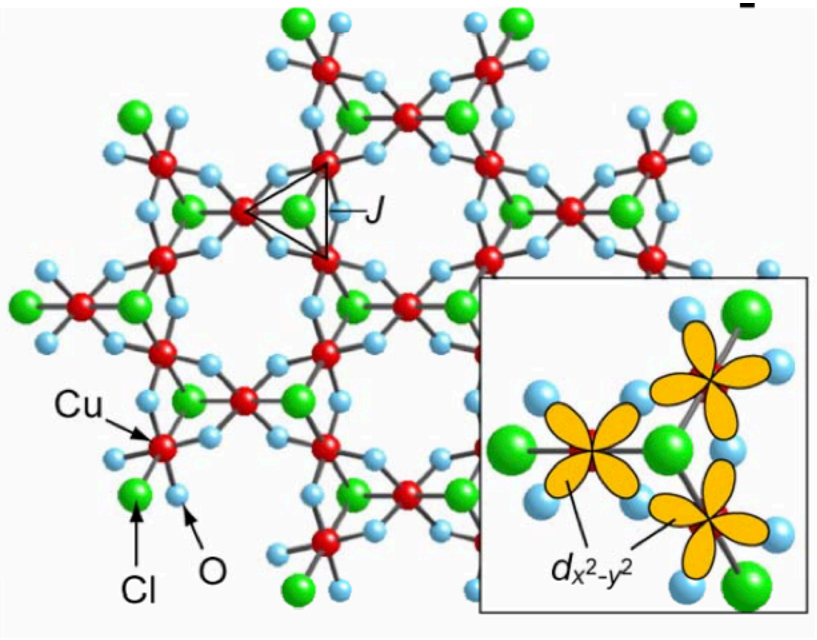
$$\mathcal{L} = L_{Maxwell-CS} + L_{Kinetic} + iM\psi^{\dagger i}\psi_i$$

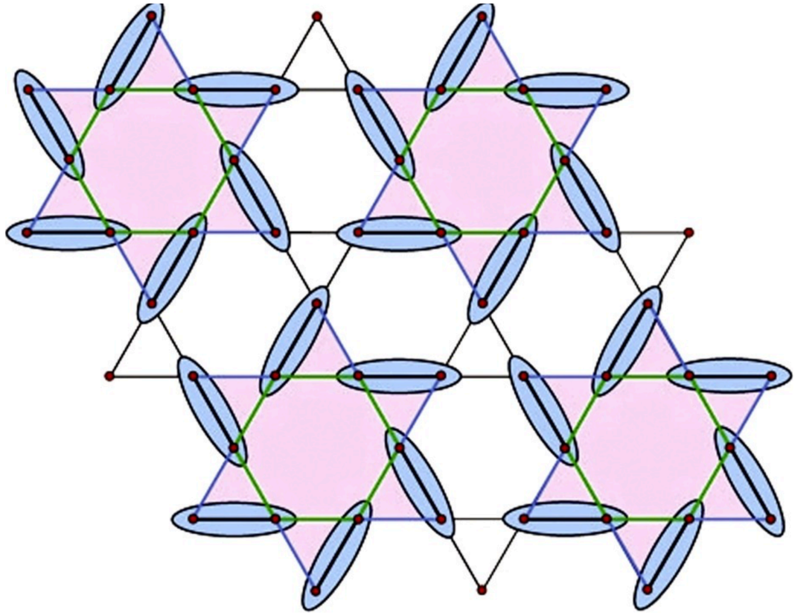
$i = 1, ..., N_f$ is indexing the species. Another discrete parameter is κ which counts the solenoid strength of the fermions (anyons).

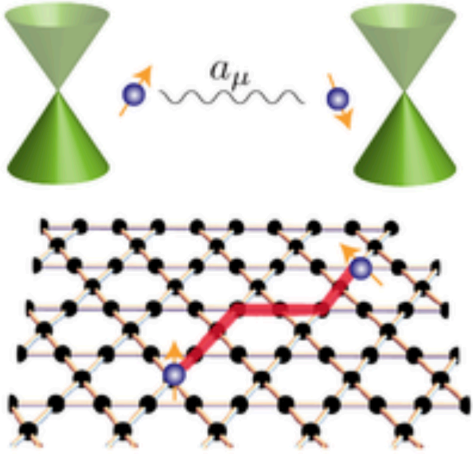
This theory has $U(N_f)$ symmetry rotating the species.

It has found many applications:

- FQH states
- Herbertsmiththtite $ZnCu_3(OH)_6Cl_2$.
- Kagome Lattice
- etc.





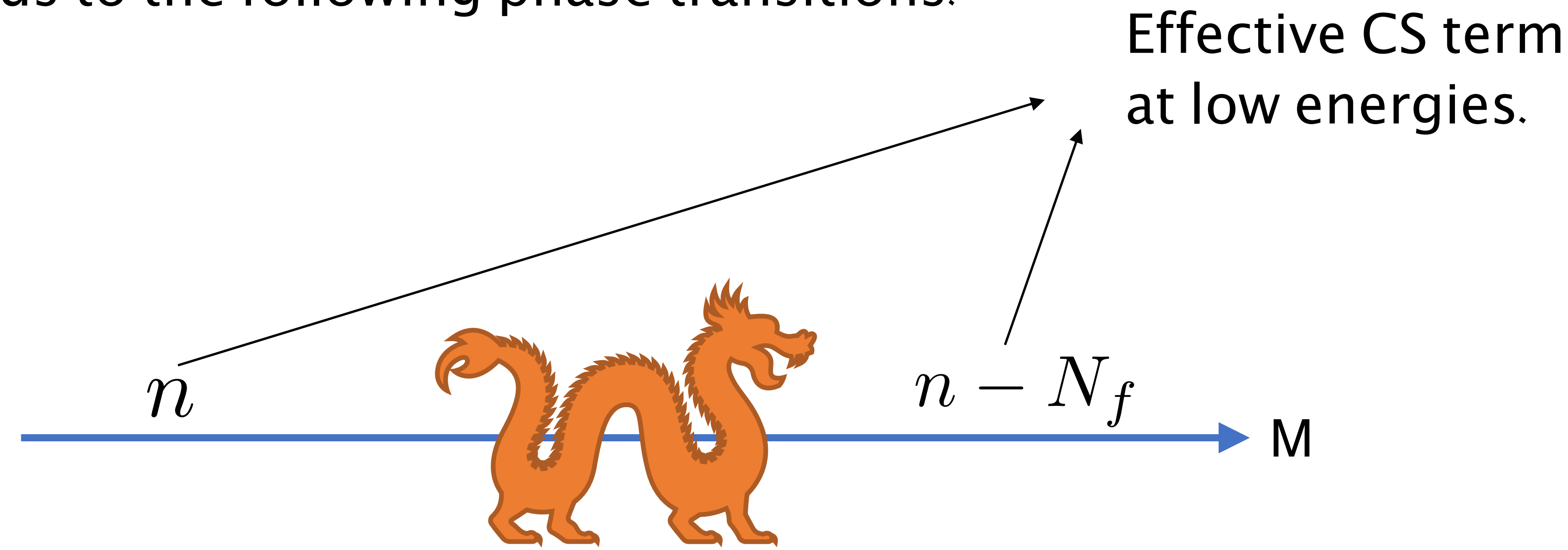


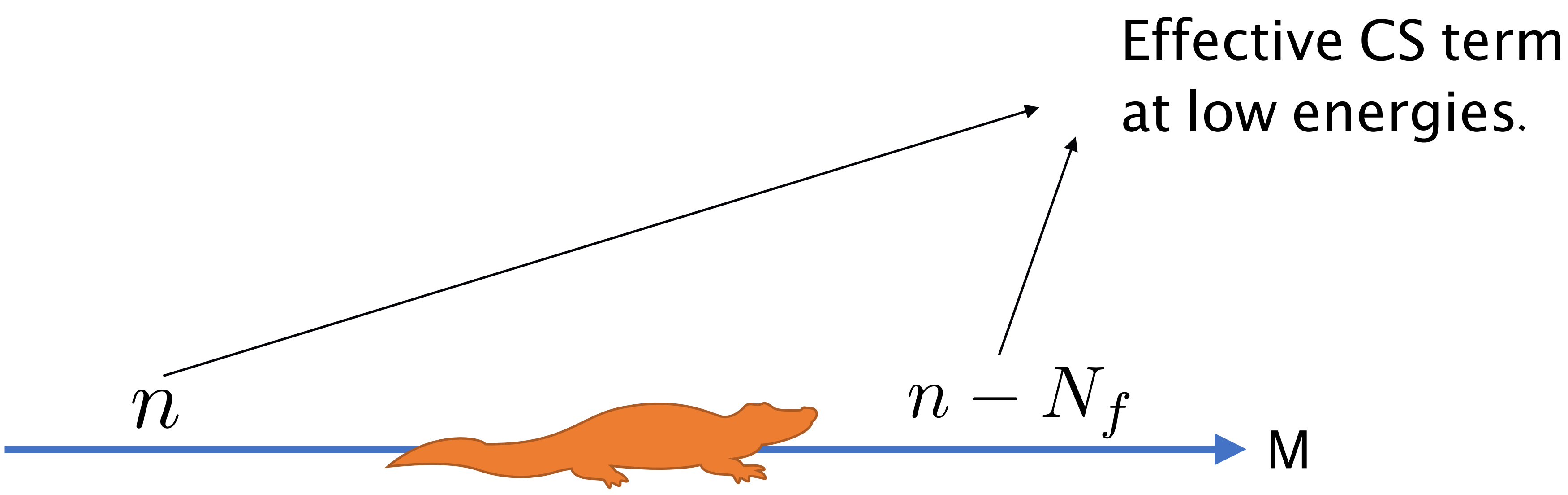
Y-C. He et al.

We must remember that in the phases with massive anyons the Chern–Simons term must be quantized

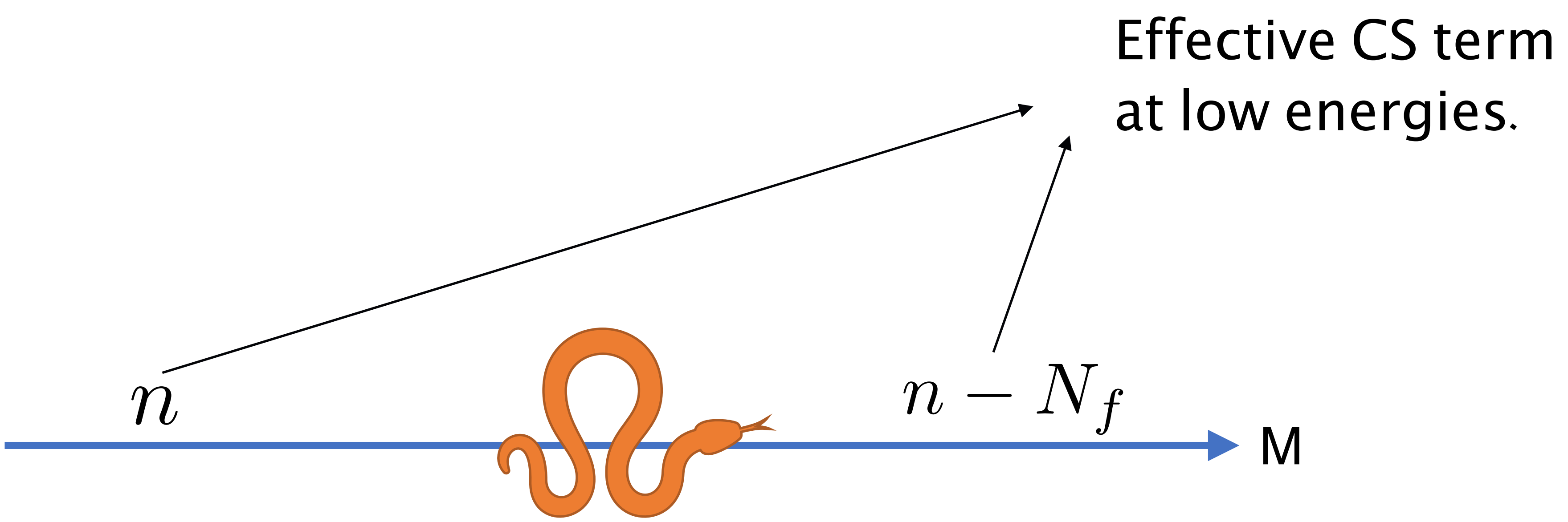
$$\kappa = \frac{e^2}{2\pi\hbar} n \,, \quad n \in \mathbb{Z}$$

QED3 leads to the following phase transitions:





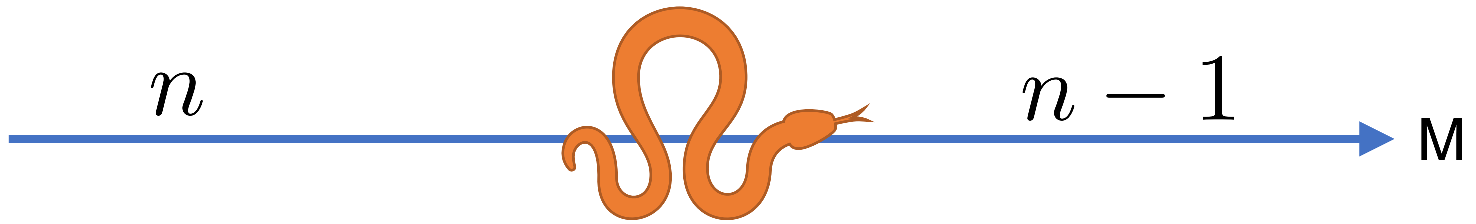
Clearly there is **some** phase transition since the type of anyons changes. It can be symmetry breaking or preserving and it can be first or second order. It is generically non-Landau-Ginzburg.



We know for sure that for large n and\or large N_f these transitions are all non-Landau-Ginzburg, second order with critical exponents very close to the free Dirac cone, $\Delta(\Psi^\dagger\Psi) \sim 2$.

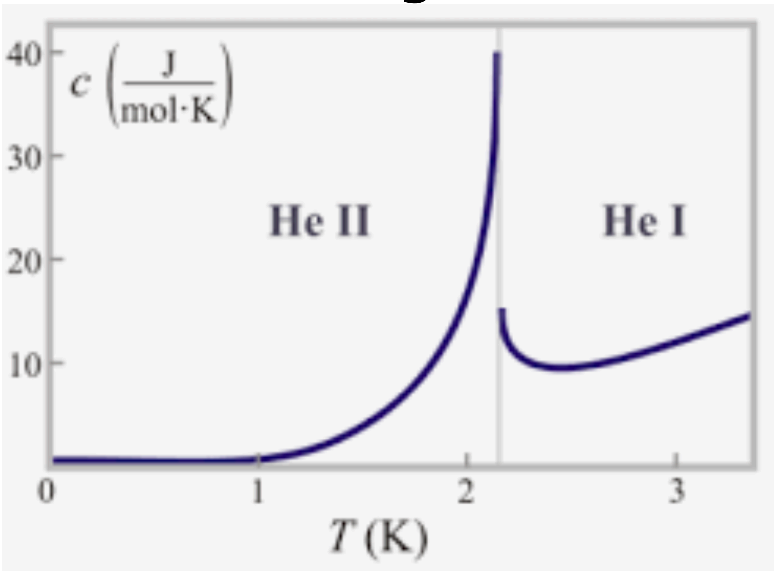
For small n and N_f one must make conjectures — the guiding principles are symmetries, anomalies, unitarity, RG monotonicity etc.

$N_f = 1$



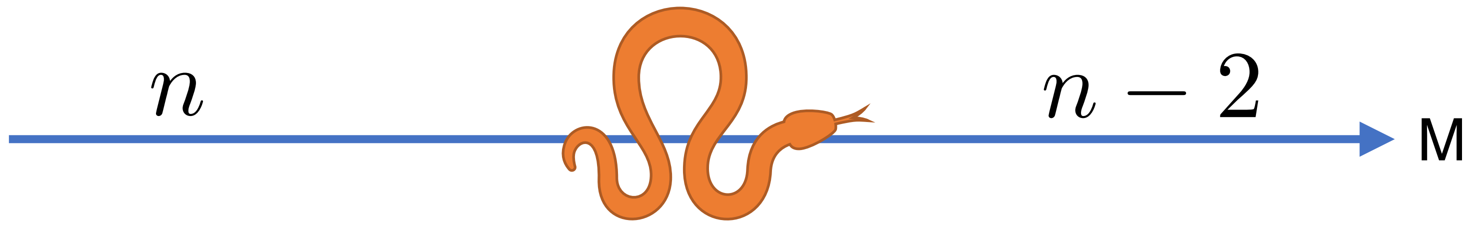
For n=1 the phase on the left does not have anyons and the phase on the right is just an ordinary massless particle (the single photon polarization). Therefore it is believed that this is nothing but the usual LG superfluid transition.

[…Son, Karch-Tong,
Seiberg, Senthil,
Wang, Witten…]



$N_f = 2$

This follows [Seiberg, ZK]
[Gomis, ZK, Seiberg]
[Armoni, Dumitrescu, Festuccia, ZK]



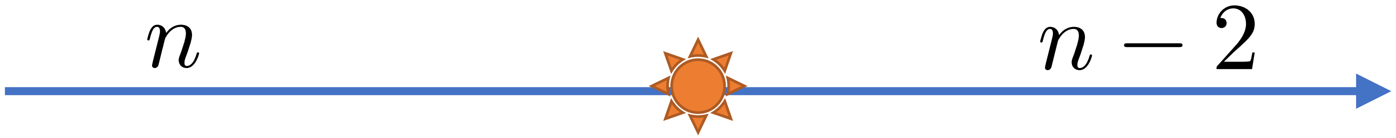
It has a chance for being LG for n=1. Here is a proposal:

$$\frac{U(2)}{U(1)\times U(1)}$$

The Wilson-Fisher transitions are some usual order/disorder transitions with scalar order parameters.

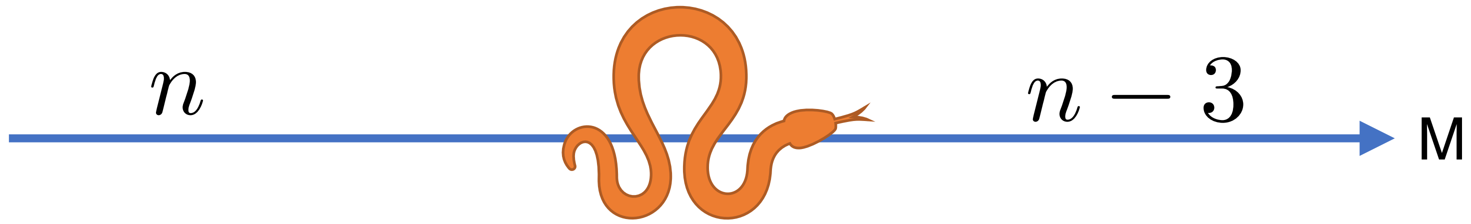
$\underline{N_f = 2}$

$n \neq 1 :$

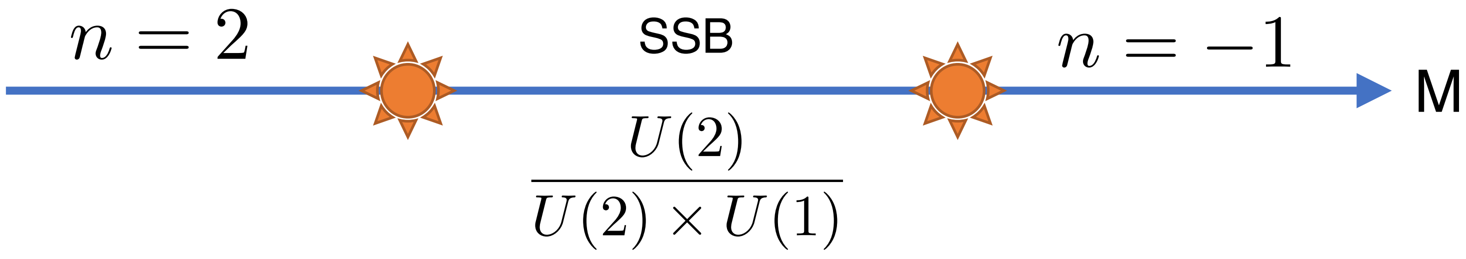


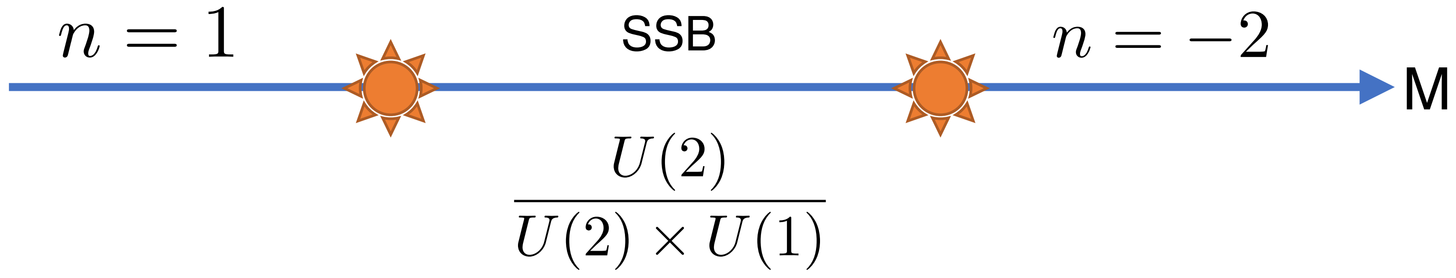
A single second-order non-LG transition. No fermion condensate.

$N_f = 3$

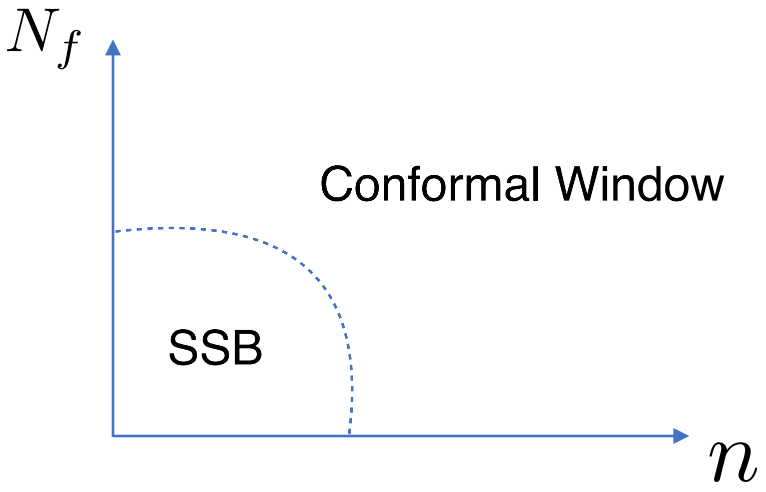


This never has a chance of being LG.





For other values of n there is no fermion condensate and a single transition. This pattern continues to higher N_f, until fermion condensates stop developing altogether. Perhaps already for $N_f < 4$. [Zhijin Li] At higher N_f the conformal window starts.



One can similarly develop conjectural phase diagrams for other gauge groups and other matter contents, e.g. with bosons. I will not go into it here.

We see that interacting anyons give rise to many new phenomena, including

- Non-standard anyonic condensates
- Non-LG transitions
- Dualities
- Fractional quantum Hall phases
- Many tractable models with supersymmetry [Aharony,…Giveon, Kutasov…]
- etc.

Some of it is already confirmed in simulation and even experiments, but a lot remains to be seen.

THANK YOU!