Formally Verified Timing Computation for Non-deterministic Horizontal Turns During Aircraft Collision Avoidance Maneuvers

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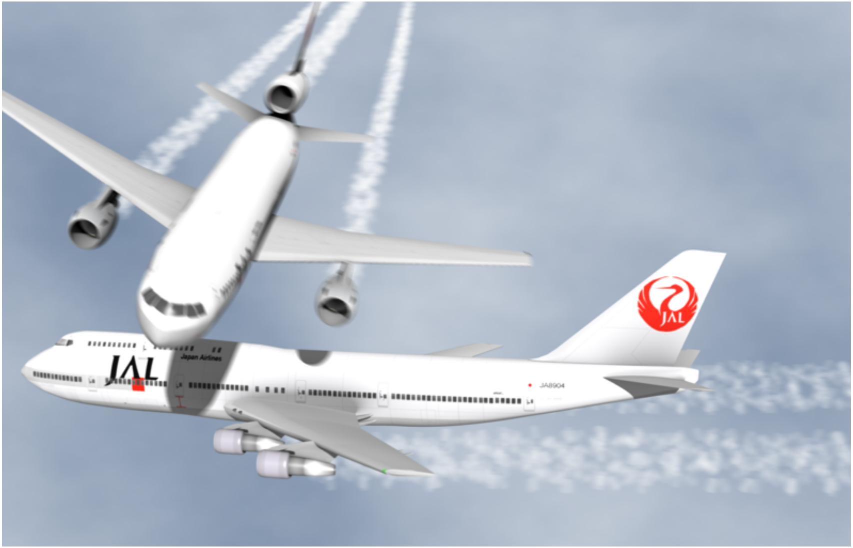
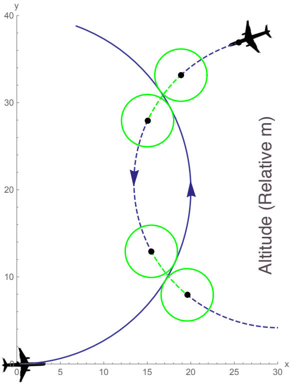
OUTLINE

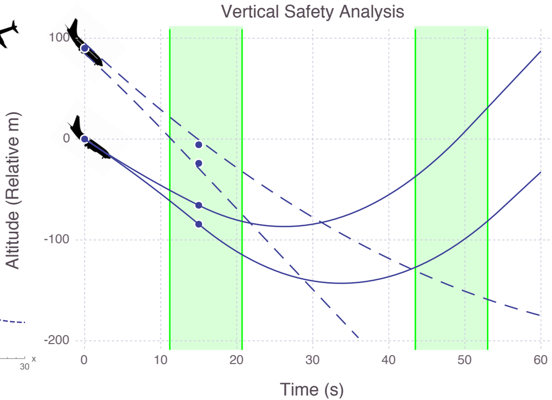
- Objectives
- Approach
- Vertical Safety Analysis
- Horizontal Safety Analysis

How can we guarantee safety? Image from airlive.net

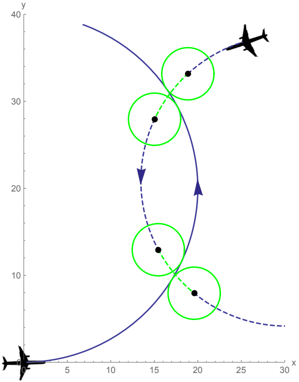


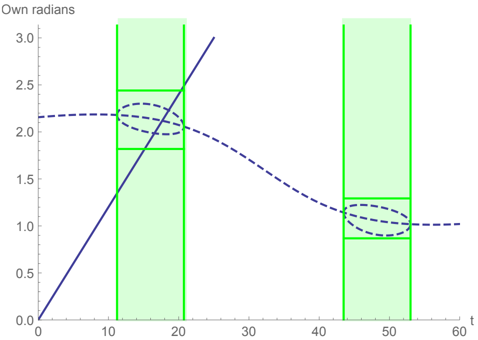
Safety Analysis for Example Scenario





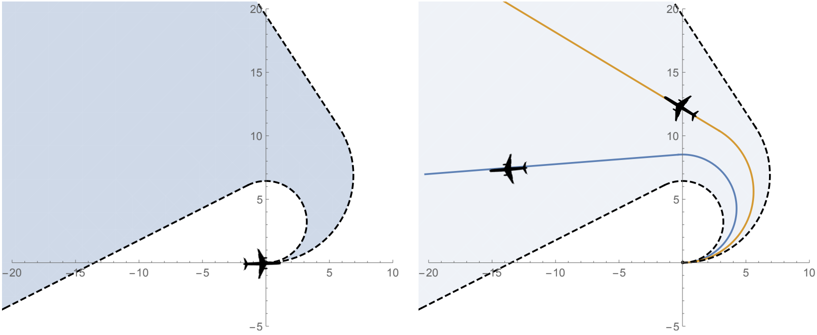
Horizontal Timing Calculation





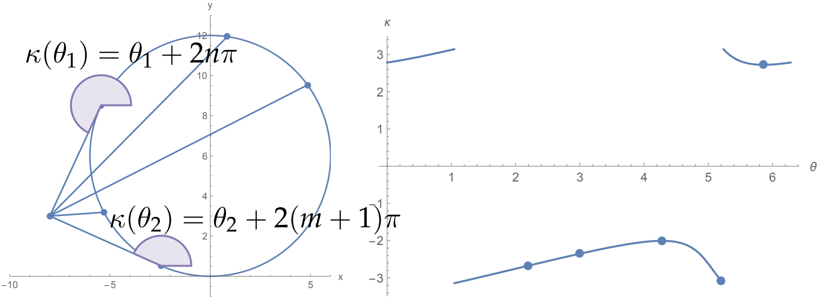
Non-deterministic one-turn-to-bearing motion

A set of trajectories representing a range of future motion possibilities, characterized by a tuple $(x_0, y_0, \theta_0, r_\alpha, r_\beta, \theta_\alpha, \theta_\beta, s_\alpha, s_\beta)$.



Trigonometric Properties

Geometric intuition which might seem simple does not always translate naturally to formal analysis in a proving environment. Encode the way circular turns may be combined with straight paths that exit the turns on a tangent.



We can define a function that computes the distance of the path for a deterministic, left-turning turn-to-bearing trajectory starting from the origin with orientation $\theta_0 = 0$, passing through (x, y) with orientation θ, using a turn of radius r:

$$L(x, y, \theta, r) = r\theta + \|(x, y) - r(\sin\theta, 1 - \cos\theta)\| \quad (1)$$

Turn-to-bearing kinematics constrain the parameters for L, i.e. its arguments cannot all be chosen independently.

Path Length Properties

- A central insight here is that for paths with the same starting and ending points, the path with a larger angle of approach will have a larger radius; and the path with a larger radius will be longer. More precisely:
- Approach angle orders turn-to-bearing path radii. Given two turn-to-bearing paths, (r_1, θ_1) and (r_2, θ_2) that pass through the same point (x, y), if $\theta_1 > \theta_2 > 0$, then the radius of the first path r_1 is longer than the radius of the second path r_2, i.e. $r_1 > r_2$:

$$\theta_1 > \theta_2 > 0 \rightarrow R(x, y, \theta_1) > R(x, y, \theta_2) \tag{2}$$

- Radius orders turn-to-bearing path lengths: Given two turn-to-bearing paths, (r_1, θ_1) and (r_2, θ_2) that pass through the same point (x, y), if $r_1 > r_2 > 0$, then the first path length L_1 is greater than the second path length L_2, i.e. $L_1 > L_2$:

$$r_1 > r_2 > 0 \rightarrow L(x, y, \Theta(x, y, r_1), r_1) > L(x, y, \Theta(x, y, r_2), r_2)$$

Max and min pathlength functions

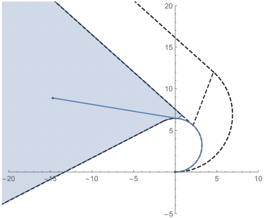
For turn-to-bearing kinematics, given interval constraints $[\theta_\alpha, \theta_\beta]$ and $[r_\alpha, r_\beta]$, and a reachable point (x, y),

$$d_{\min}(x,y) = \begin{cases} L(x,y,\Theta(x,y,r_\alpha),r_\alpha) & \theta_\alpha \leq \Theta(x,y,r_\alpha) \leq \theta_\beta \\ L(x,y,\theta_\alpha,R(x,y,\theta_\alpha)) & \theta_\alpha < \theta_m \wedge \Theta(x,y,r_\alpha) < \theta_\alpha \\ L(x,y,\theta_m,r_m) & r_\alpha \leq r_m \leq r_\beta \wedge \theta_m \leq \theta_\alpha \end{cases} \tag{4}$$

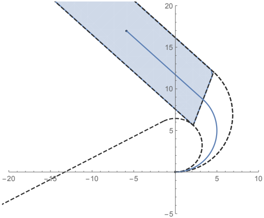
and

$$d_{\max}(x,y) = \begin{cases} L(x,y,\Theta(x,y,r_\beta),r_\beta) & \theta_\alpha \leq \Theta(x,y,r_\beta) \leq \theta_\beta \\ L(x,y,\theta_\beta,R(x,y,\theta_\beta)) & \theta_\beta < \theta_m \wedge \theta_\beta < \Theta(x,y,r_\beta) \\ L(x,y,\theta_m,r_m) & r_\alpha \leq r_m \leq r_\beta \wedge \theta_m \leq \theta_\beta \end{cases} \tag{5}$$

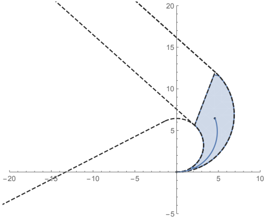
Max and min pathlength function pieces



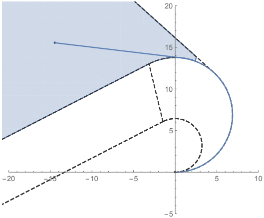
(a) $r = r_\alpha$



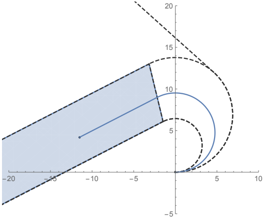
(b) $\theta_c = \theta_\alpha$



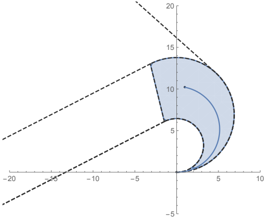
(c) $r = r_m \wedge \theta_m \leq \theta_c$



(d) $r = r_\beta$

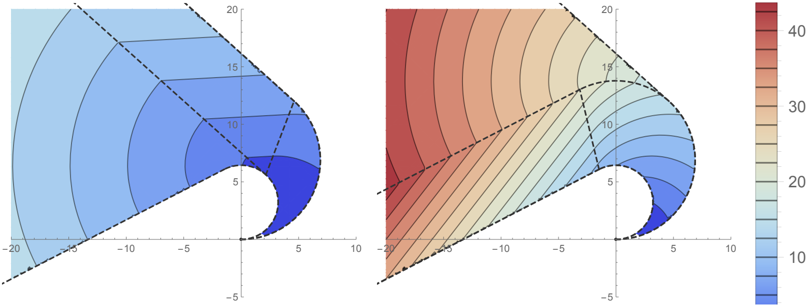


(e) $\theta_c = \theta_\beta$



(f) $r = r_m \wedge \theta_m \leq \theta_c$

Horizontal Timing Analysis



Horizontal Timing Analysis for Safety